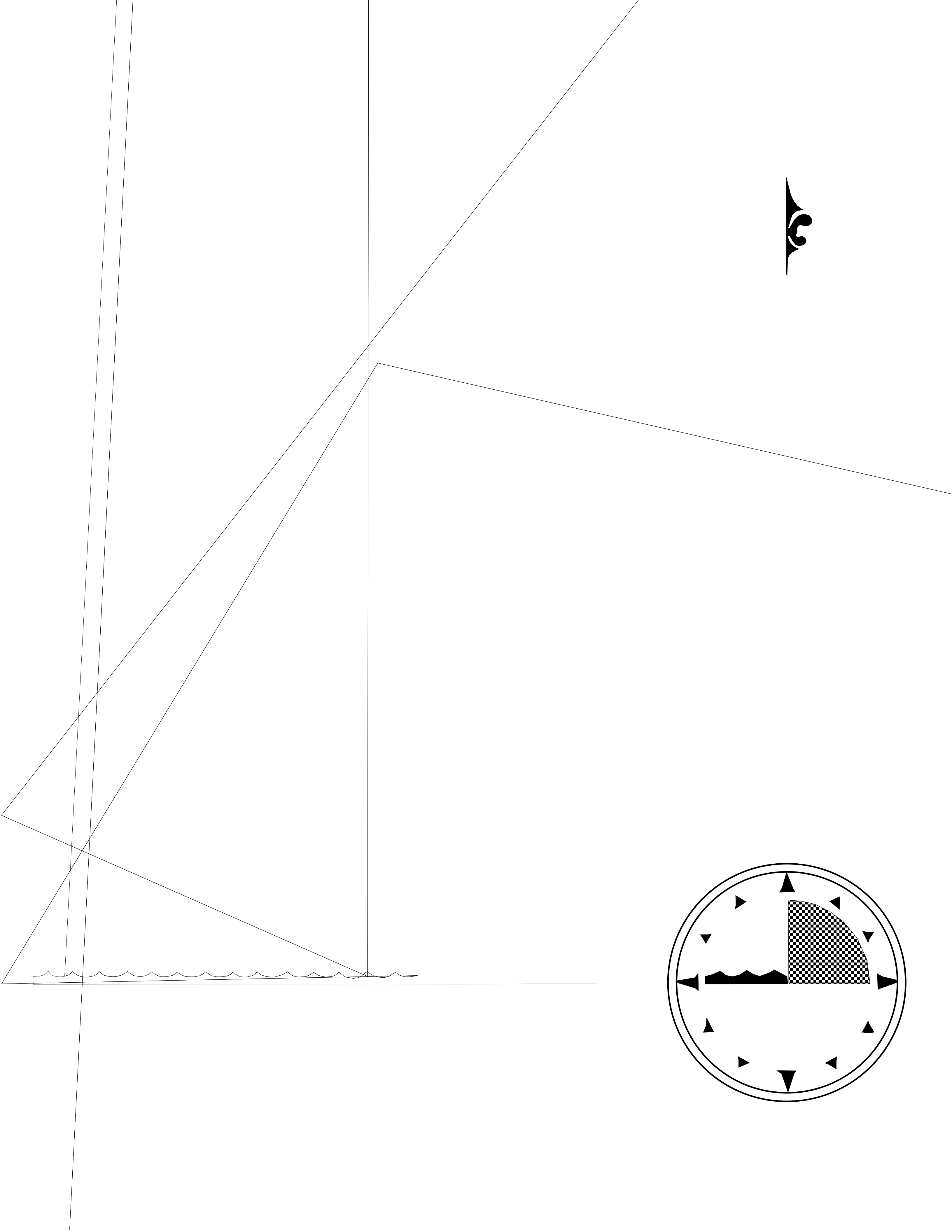

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Pacific Marine Science Report 77-10

MANUAL FOR
TIDAL HEIGHTS ANALYSIS AND PREDICTION

by

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1977
Revised September 1979
Reprinted May 1984
Revised November 1993
Revised July 1996
Revised October 2004

CONTENTS

PREFACE

This report is intended to serve as a user's manual to G. Godin's tidal heights analysis and predictions programs, revised along lines suggested by Godin. In addition to describing input and output of these programs, the report gives an outline of the methods used; a full presentation of which can be found in Godin (1972) and Godin and Taylor (1973).

Users who wish to receive updates of these programs and manual should send their names, addresses, and type of computer used, to the author.

ACKNOWLEDGEMENTS

The writer wishes to thank G. Godin for his guidance during the computer program revisions, J. Taylor and R.F. Henry for their helpful suggestions, and R. Rutka for transferring the manuscript to TEX.

1 USE OF THE TIDAL HEIGHTS ANALYSIS COMPUTER PROGRAM

1.1 General Description

This program analyses the hourly height tide gauge data for a given period of time. Amplitudes and Greenwich phase lags are calculated via a least squares fit method coupled with nodal modulation for only those constituents that can be resolved over the length of the record. Unless specified otherwise, a standard data package of 69 constituents will be considered for inclusion in the analysis. However, up to 77 additional shallow water constituents can be requested. If the record length is such that certain important constituents are not included directly in the analysis, provision is made for the inference of the amplitude and phase of these constituents from others. Gaps within the tidal record are permitted.

1.2 Routines Required

(1) **MAIN** ...... reads in some of data, controls most of the output and calls other routines.

(2) **INPUT** ...... reads in the hourly height data for the desired time period and checks for errors.

(3) **UCON** ...... chooses the constituents to be included in the analysis via the Rayleigh criterion

(4) **SCFIT2** ...... finds the least squares fit to an equally spaced time series using sines and cosines of specified frequencies as fitting functions.

(5) **VUF** ...... reads required information and calculates the nodal corrections for all constituents.

(6) **INFER** ...... reads required information and calculates the amplitude and phase of inferred constituents, as well as adjusting the amplitude and phase of the constituent used for the inference.

(7) **CHLSKY** ...... solves the symmetric positive definite matrix equation resulting from a linear least squares fit.

(8) **GDAY** ...... returns the consecutive day number from a specific origin for any given date and vice versa.

(9) **ASTR** ...... calculates ephermides for the sun and moon.

(10) **OUTPUT** ...... writes predicted hourly heights to the output file.

(11) **SCULP** ...... scales up amplitudes to compensate for moving average filters.

1.3 Data Input

For a computer run of the tidal heights analysis program, two logical units are used for data input. Logical unit number 8 contains the tidal constituent information while logical unit 4 contains the hourly heights and information relating to the type of analysis and output required. A listing of the standard constituent information for logical unit 8 and a sample set of input for logical unit 4 are given in Appendices 7.1 and 7.2 respectively.

Logical unit 8 expects four types of data:

(i) One card each for all possible constituents, `KONTAB`, to be included in the analysis along with their frequencies, `FREQ`, in cycles/h and the constituent with which they should be compared under the Rayleigh criterion, `KMPR`. The format used is `(4X,A5,3X,F13.10,4X,A5)`. Unless `KONTAB` is specifically designated on logical unit 4 for inclusion, a blank data field for `KMPR` results in the constituent not being included in the analysis.

A blank card terminates this data type.

(ii) Two cards specifying values for the astronomical arguments `SO,HO,PO,ENPO,PPO,DS,DH,DP,DNP,DPP` in the format `(5F13.10)`.

`SO` = mean longitude of the moon (cycles) at the reference time origin;
`HO` = mean longitude of the sun (cycles) at the reference time origin;
`PO` = mean longitude of the lunar perigee (cycles) at the reference time origin;
`ENPO` = negative of the mean longitude of the ascending node (cycles) at the reference time origin;
`PPO` = mean longitude of the solar perigee (perihelion) at the reference time origin.

`DS,DH,DP,DNP,DPP` are their respective rates of change over a 365-day period at the reference time origin.

Although these argument values are not used by the program that was revised in October 1992, in order to maintain consistency with earlier programs, they are still required as input. Polynomial approximations are now employed to more accurately evaluate the astronomical arguments and their rates of change.

(iii) At least one card for all the main tidal constituents specifying their Doodson numbers and phase shifts along with as many cards as are necessary for the satellite constituents. The first card for each such constituent is in the format `(6X,A5,1X,6I3,F5.2,I4)` and contains the following information:

`KON` = constituent name;
`II,JJ,KK,LL,MM,NN` = the six Doodson numbers for `KON`;
`SEMI` = the phase correction for `KON`;
`NJ` = the number of satellite constituents.

A blank card terminates this data type.

If `NJ>0`, information on the satellite constituents follows, three satellites per card, in the format `(11X,3(3I3,F4.2,F7.4,1X,I1,1X))`. For each satellite the values read are:

`LDEL,MDEL,NDEL` = the last three Doodson numbers of the main constituent subtracted from the last three Doodson numbers of the satellite constituent;

`PH` = phase correction of the satellite constituent relative to the phase of the main constituent;

`EE` = amplitude ratio of the satellite tidal potential to that of the main constituent;

`IR` = 1 if the amplitude ratio has to be multiplied by the latitude correction factor for diurnal constituents,

= 2 if the amplitude ratio has to be multiplied by the latitude correction factor for semidiurnal constituents,

= otherwise if no correction is required to the amplitude ratio.

(iv) One card specifying each of the shallow water constituents and the main constituents from which they are derived. The format is `(6X,A5,I1,2X,4(F5.2,A5,5X))` and the respective values are:

`KON` = name of the shallow water constituent;

`NJ` = number of main constituents from which it is derived;

`COEF,KONCO` = combination number and name of these main constituents.

The end of these shallow water constituents is denoted by a blank card.

Logical unit 4 contains six types of data:

(i) One card for the variables `IOUT1,RAYOPT,ZOFF,ICHK,OBSFAC,INDPR,NSTRP` in the format `(I2,2X,F4.2,2X,F10.0,I2,3X,F10.7,2I5)`.

`IOUT1` = 6 if the only output desired is a line printer listing of results,

= 2 if both analysis output and listing are desired;

`RAYOPT` = Rayleigh criterion constant value if different from 1.0;

`ZOFF` = constant to be subtracted from all the hourly heights;

`ICHK` = 0 if the hourly height input data is to be checked for format errors,

= otherwise if this checking to be waived;

`OBSFAC` = scaling factor, if different from 0.01, which will multiply the hourly observations, in order to produce the desired units for the final constituent amplitudes. (e.g. if the hourly observations are in mm/s and the final units are to be ft/sec, then this variable would be set to 0.0032808.);

`INDPR` = 1 if hourly height predictions based on the analysis results are to be calculated and written onto device number 10. If there is inference, this parameter value will also give the rms residual error after inference adjustments have been made,

= 0 if no such predictions are desired;

`NSTRP` = number of successive moving average filters that have been applied to the original data.

If `NSTRP>0`, then `TIMINT` and `(LSTRP(I),I=1,NSTRP)` will be read on a following card, in the format `(F10.0,10I5)`, and suitable amplitude corrections will be applied to compensate for the smoothing effect of these filters.

`TIMINT` = sampling interval, in minutes, of the original unfiltered record;

(LSTRP(J),J=1,NSTRP) = number of consecutive observations used in computing each of the NSTRP moving average filters.

(ii) One card for each possible inference pair. The format is (2(4X,A5,E16.10),2F10.3) and the respective values read are:

KONAN & SIGAN = name and frequency of the analysed constituent to be used for the inference;
KONIN & SIGIN = name and frequency of the inferred constituent;
R = amplitude ratio of KONIN to KONAN;
ZETA = Greenwich phase lag of the inferred constituent subtracted from the Greenwich phase lag of the analysed constituent.

These are terminated by one blank card.

(iii) One card for each shallow water constituent, other than those in the standard 69 constituent data package, to be considered for inclusion in the analysis. The Rayleigh comparison constituent is also required and the additional shallow water constituent must be found in data type (i) of logical unit 8, but have a blank data field where the Rayleigh comparison constituent is expected. The format is (6X,A5,4X,A5) and a blank card is required at the end.

(iv) One card in the format (I1,1X,10I2) specifying the following information on the period of the analysis:

INDY = 8 indicates an analysis is desired for the upcoming period;
= 0 indicates no further analyses are required;
IHH1,IDD1,IMM1,IYY1,ICC1 = hour, day, month, year and century of the beginning of the analysis (measured in time ITZONE of input data (v));
IHHL,IDDL,IMML,IYYL,ICCL = hour, day, month, year and century of the end of the analysis.

If ICC1 or ICCL are zero, their value is reset to 19.

(v) One card in the format (I1,4X,A5,3A6,A4,A3,1X,2I2,I3,I2,5X,A5) containing the following information on the tidal station:

INDIC = 1 if J card output is desired (no longer used),
= otherwise if not;
KSTN = tidal station number;
(NA(J),J=1,4) = tidal station name (22 characters maximum length);
ITZONE = time zone of the hourly observations;
LAD,LAM = station latitude in degrees and minutes;
LOD,LOM = station longitude in degrees and minutes;
IREF = reference station number.

(vi) The hourly height data cards contain the following information in the format (I1,1X,I5,4X,I2,1X,3I2,12A4).

KOLI = 1 or 2 indicates whether this specific card is the first or second

	one for that day,
=	otherwise indicates a non-data card which is ignored;
JSTN =	tidal station number;
IC,ID,IM,IY =	century, day, month and year of the heights on this card. If IC=0, it is reset to 19;
(KARD(J),J=1,12) =	hourly heights in integer form. The final constituent amplitudes unless a are in units 1/100 of those for the hourly height nonzero for OBSFAC is read (see (i)). Missing values should be specified as a blank field or 9999.

When KOLI=1, the first hourly height on the data card is assumed to be at 0100 h and when KOLI=2, it is assumed to be at 1300 h. The time zone of these observations determines the nature of the Greenwich phase lag (see Section 2.3.1).

After the initial analysis of a computer run is completed, control returns to input (iv). Successive cards are read then until either a 0 or 8 value is found for INDY.

The hourly height data cards need not begin and end so as to include exactly the analysis period. The program ignores data outside this range. However if more than one analysis is desired from a single job submission and hourly height data cards do extend beyond the first analysis period, care should be taken to ensure that one of these cards does not have KOLI=0 or blank, otherwise the job will be terminated. This is because all successive cards after the one containing the last hour of the desired analysis period are read in input (iv) format.

1.4 Output

Three logical units are used for the output of results from the tidal heights analysis program. Device number 6 is the line printer, 2 is used for analysis results and 10 contains hourly synthesized values based on the analysis results; 6 is required for all program runs whereas the use of 2 and 10 is controlled by the input variables IOUT1 and INDPR which are read from device 4.

Recommendations for the use of moving average filters on the elevation data prior to submission for analysis, and the scaling compensation method used in the improved analysis program are found in Foreman (1978) or Godin (1972).

When IOUT1 is 6, INDPR is other than 1, and there are no inferred constituents, the only output is two pages on the line printer. The first of these lists the constituents included in the least squares fit, their frequencies in cycles/h (although eight decimal places are given, depending on computer accuracy, less than this number may be significant), the C and S coefficient values (see Section 2.2.1) measured in units OBSFAC times those for the hourly heights, and their respective standard deviation estimates. It also specifies the number of hourly height observations (excluding gaps) within the analysis period, the average and standard deviation of the original observations, the root mean square residual error, and the matrix condition number. In the columns titled AL, GL, A, and G, the second page respectively lists the amplitudes and phases (degrees) obtained for each constituent from the C and S coefficient values, and the same amplitudes and phases after nodal modulation and astronomical argument adjustments. The initial and final hour of the analysis are also specified along with the Rayleigh criterion constant ('separation'), the midpoint of the analysis period, the total number of possible hourly observations in the analysis period, and the total number of possible observations used in the analysis. This last value includes gaps in the record and is the largest odd number less than or equal to the total number of possible hourly observations (if the total number of possible hourly observations is an even number, the last hour is ignored). If there is at least one inferred constituent, page 2 results are repeated with the inclusion of inferred constituents and appropriate adjustments to the constituents from which the inferences were made. Appendix 7.3 lists the final page of results obtained from the input value of Appendix 7.2.

The only effect of changing the value of IOUT1 to 2 (regardless of INDPR's value) is to store on file 2, the same information as the second (and third) page(s) of the line printer. The list of constituent names, amplitudes and Greenwich phase lags begins on line 5 of this file and is in the correct format for input to the tidal heights prediction program, namely (5X,A5,28X,F8.4,F7.2).

When INDPR equals 4, device 10 will contain hourly predictions calculated from the analysis results. Values are specified only for the analysis period, including those intervals where there were gaps in the original record, and are in the same measurement units and scaling as the original data. The format used is the same as for input type (vi) of logical unit 4.

1.5 Program Conversion, Modifications, Storage and Dimension Guidelines

The source program and constituent data package described in this manual have been tested on various mainframe, PC and workstation computers at the Institute of Ocean Sciences, Patricia Bay. Although as much of the program as possible was written in basic FORTRAN, some changes may be required before the program and data package can be used on other installations. Please write or call the author if any problems are encountered.

The program in its present form requires approximately 68,000 bytes for the storage of its instructions and arrays.

Changing the number or type of constituents in the standard data package may require some alterations to the analysis program. If constituents are added to the standard data package, the dimensions of several arrays may have to be altered. Restrictions on the minimum dimension of such arrays are now given.

Let

MTOT be the total number of possible constituents contained in the data package (presently 146),

M be the number of constituents considered for inclusion in the analysis (presently 69 plus the number of shallow water constituents specifically designated for inclusion),

MCON be the number of main constituents in the standard data package (presently 45),

MSAT be the sum of the total number of satellites for these main constituents and the number of main constituents with no satellites (presently 162 plus 8 for the version of the constituent data package, listed in Appendix 7.1, that contains no third-order satellites for both N_2 and L_2),

MSHAL be the sum for all shallow water constituents, of the number of main constituents from which each is derived (presently 251).

Then in the main program, arrays KONTAB,FREQ and KMPR should have minimum dimension MTOT+1; arrays KON,C,S,SIG,ERC,ERS,A,EPS,KO,AA and GD should have minimum dimension M; array NKON should have dimension at least as large as the number of extra shallow water constituents specifically designated for analysis inclusion (its present maximum is 15); and arrays

Z and XP should be large enough to contain the hourly heights (and gaps) in the analysis period (its present maximum is 375 days).

In subroutine **INPUT** array Z should be dimensioned the same as in the main program, while KARD and IHT should be dimensioned 12.

In the other subroutine **OUTPUT**, Z is in a common block and should be dimensioned as in the main program, XP is in the argument list and need only have dimension 2, and arrays MONTH and IHT should have dimension 12 and 24 respectively.

In subroutine **VUF**, arrays VU and F should have minimum dimension MTOT; arrays KON and NJ should have minimum dimension MTOT+1; arrays II,JJ,KK,LL,MM,NN and SEMI should have minimum dimension MCON+1; arrays EE,LDEL,MDEL,NDEL,IR and PH should have minimum dimension MSAT; and KONCO, COEF should have minimum dimension MSHAL+4.

In subroutine **INFER**, arrays KONAN,KONIN,SIGAN,SIGIN,R and ZETA can presently accommodate a maximum of nine inferred constituents.

In subroutine **SCFIT2**, arrays X,XP,C,S,ERC,ERS and F should have the same dimension as Z,XP,C,S,ERC,ERS and SIG in the main program and arrays RHS and A should have minimum dimension 2M-1 and M(2M-1) respectively. AC and AS should have the size of A and care should be taken that through their equivalence relationships, neither AC and AS, nor RHSC and RHSS overlap.

Finally, in subroutine **CHLSKY**, arrays A and F should have minimum dimensions M(2M-1) and 2M-1 respectively.

2 TIDAL HEIGHTS ANALYSIS PROGRAM DETAILS

2.1 Constituent Data Package

2.1.1 Astronomical variables

The astronomical variables required by the tidal analysis program were used by Doodson (1921) in his development of the tidal potential. From them one can calculate the position of the sun or moon, and hence the tide generating forces, at any time. These varaibles are:

$$
\begin{aligned}
S(t) &= \text{ mean longitude of the moon;} \\
H(t) &= \text{ mean longitude of the sun;} \\
P(t) &= \text{ mean longitude of the lunar perigee;} \\
N'(t) &= \text{ negative of the longitude of the mean ascending node;} \\
P'(t) &= \text{ mean longitude of the solar perigee (perihelion).}
\end{aligned}
$$

For H, N' and P' these longitudes are measured along the ecliptic eastward from the mean vernal equinox position at time t; while for S and P they are measured in the ecliptic eastward from the mean vernal equinox position at time t to the mean ascending mode of the lunar orbit, and then along this orbit. Together with the rates of change of these variables, τ the local mean lunar time, and the Doodson numbers for each tidal constituent, one can calculate the constituent frequencies, their astronomical argument phase angles, V, and their nodal modulation phase, u, and amplitude, f, corrections.

The values of the astronomical variables and constituent frequencies in the program are calculated using the power series expansion formulae given on pages 98 and 107 of the *Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac* (1961). These formulae were derived from Newcomb's *Tables of the Sun* and a revision of Brown's lunar theory (used in the development of his *Tables of Motion of the Moon*) so that it is in accord with Newcomb's.

(For those interested, even higher ordered approximations can be found in *Astronomical Formulae for Calculators* by Jean Meeus.) In particular, the astronomical variables and frequencies are calculated at the central hour of the analysis period and in order to gain precision t_0, the reference time origin, is taken to be 0000 ET.[1] This latter date, it was felt, would be closer to the analysis period of most records than the previous choice of 0000 ET January 1, 1901, and hence would yield more accurate results via the linear approximation.

In keeping with the choice of reference time origin and astronomical variable specifications, t should be measured in Ephemeris time. However, the correction from Universal time is irregular and in most cases small, so it has been assumed for computational purposes that all observations are recorded in ET.

2.1.2 Choice of constituents and Rayleigh comparison pairs

There is a maximum of 146 possible tidal constituents that can be included in the tidal analysis, 45 of these are astronomical in origin (main constituents) while the remaining 101 are

[1] Ephemeris Time (ET) is the uniform measure of time defined by the laws of dynamics and determined in principle from the orbital motion of the Earth as represented by Newcomb's *Tables of the Sun*. Universal or Greenwich Mean Time is defined by the rotational motion of the Earth and is not rigorously uniform.

shallow water constituents.[2] Because computation time (and cost) of the computer program increases approximately as the square of the number of constituents included in the analysis, and because for many tidal stations, most of the shallow water constituents are insignificant, a smaller standard package was seen as adequate for general use. Based on the suggestions of G. Godin, it was decided that this package contain all the main constituents and 24 of the shallow water. However, provision was made so that other shallow water constituents among the 77 remaining could be included if desired.

The Rayleigh comparison constituent is used for the purpose of deciding whether or not a specific constituent should be included in the analysis. If F_0 is the frequency of such a constituent, F_1 is the frequency of its Rayleigh comparison constituent and T is the time span of the proposed record to be analysed, then the constituent will be included in the analysis only if $|F_0 - F_1|T \geq RAY$. RAY is commonly given the value 1 although it can be specified differently in the program.

In order to determine the set of Rayleigh comparison pairs, it is important to consider, within a given constituent group (e.g. diurnal or semidiurnal), the order of constituent inclusion in the analysis as T (the time span of the record to be analysed) increases. Assuming this point of view, the specific objectives used when constructing the set listed in Appendix 7.1 were:

(i) within each constituent group, when possible, have the order of constituent selection correspond with decreasing magnitude of tidal potential amplitude (as calculated by Cartwright and Edden (1973)),

(ii) when possible, compare a candidate constituent with whichever of the neighbouring, already selected constituents, that is nearest in frequency,

(iii) when there are two neighbouring constituents of relatively equal tidal potential amplitude, rather than waiting until the record length is sufficient to permit the selection of both at the same time (i.e. by comparing them to each other), choose a representative of the pair whose inclusion will be as early as possible. This will give information sooner about that frequency range, and via inference, still enable some information to be obtained on both constituents.

The Rayleigh comparison pairs chosen for the low frequency, diurnal, semidiurnal and terdiurnal constituent groups are given in Tables 1, 2, 3 and 4 respectively. Figures given for the length of record required for constituent inclusion assume a Rayleigh criterion constant value (input variable `RAYOPT`) of 1.0.

$2Q_1$ and SIG_1 provide an example of objective (iii). Because $2Q_1$ has a greater frequency separation for Q_1 and hence would appear in an analysis of shorter record length than SIG_1, it was chosen as the representative.

However, it can be seen in several cases, that it was not possible or feasible to adhere to all the objectives just outlined. Choosing a Rayleigh comparison constituent from the list of those constituents already included in the analysis proved to be difficult near the frequency edges of constituent groups. Upward arrows indicate failure to uphold this objective. OO_1 is such a case. For it, the potential comparison pairs were SO_1, K_1 and J_1. The first of these would result in both SO_1 and OO_1 appearing at the same later time than had J_1 or K_1 been

[2] The criterion for selecting these main constituents was to include all the diurnal and semidiurnal constituents with Cartwright and Edden (1973) tidal potential amplitudes greater than 0.00250, along with M_3 and the most important low frequency constituents. Section 2.1.3 gives the analogous shallow water constituent criterion.

Table 1 Order of Slower-than-Diurnal Constituent Selection in Accordance with the Rayleigh Criterion.
Tidal Potential Amplitude for Main Constituents Shown within Brackets.
Lines with Arrows Denote Links with Rayleigh Comparison Pairs.

Length of Record (h) Required for Constituent Inclusion	Frequency Differences (cycles/h) $\times 10^3$ between Neighbouring Constituents						
		0.11407	0.11408	1.08162	0.20237	1.30978	0.22816
	ZO	SA	SSA	MSM	MM	MSF	MF
13	ZO						
355						(1369) MSF	
764					(8254) MM		
4383			(7281) SSA				(15647) MF
4942				(1579) MSM			
8766		(1156) SA					

Table 2 Order of Constituent Selection in Accordance with the Rayleigh Criterion. Tidal Potential Amplitude for Main Constituents is Shown within Brackets. Lines with Arrows Denote Links with Rayleigh Comparison Pairs.

Frequency Differences (cycles/h) $\times 10^3$ between Neighbouring Constituents

ALP_1–$2Q_1$	$2Q_1$–SIG_1	SIG_1–Q_1	Q_1–RHO_1	RHO_1–O_1	O_1–TAU_1	TAU_1–BET_1	BET_1–NO_1	NO_1–CHI_1	CHI_1–PI_1	PI_1–P_1	P_1–S_1	S_1–K_1	K_1–PSI_1	PSI_1–PHI_1	PHI_1–THE_1	THE_1–J_1	J_1–SO_1	SO_1–OO_1	OO_1–UPS_1
1.30978	0.20237	1.30978	0.20237	1.30978	0.22816	1.08162	0.22816	0.20237	0.96758	0.11407	0.11408	0.11407	0.11407	0.11408	1.08162	0.20237	1.30978	0.22816	151215

Length of Record (h) Required for Constituent Inclusion	ALP_1	$2Q_1$	SIG_1	Q_1	RHO_1	O_1	TAU_1	BET_1	NO_1	CHI_1	PI_1	P_1	S_1	K_1	PSI_1	PHI_1	THE_1	J_1	SO_1	OO_1	UPS_1
24														(53011) K_1							
328						(37694) O_1															
651																				(1624) OO_1	
662		(955) $2Q_1$		(7217) Q_1					(2964) NO_1									(2964) J_1			(311) UPS_1
764	(278) ALP_1																				
4383							(493) TAU_1	(278) BET_1				(17543) P_1				(755) PHI_1			SO_1		
4942			(1152) SIG_1		(1371) RHO_1					(567) CHI_1							(567) THE_1				
8767											(1028) PI_1		(416) S_1		(422) PSI_1						

Table 3 Order of Semidiurnal Constituent Selection in Accordance with the Rayleigh Criterion. Tidal Potential Amplitude for Main Constituents is Shown within Brackets. Lines with Arrows Denote Links with Rayleigh Comparison Pairs.

Length of Record (h) Required for Constituent Inclusion	Frequency Differences (cycles/h) $\times 10^3$ between Neighbouring Constituents																		
	OQ_2	EPS_2	$2N_2$	MU_2	N_2	NU_2	GAM_2	H_1	M_2	H_2	MKS_2	LDA_2	L_2	T_2	S_2	R_2	K_2	MSN_2	ETA_2
13									(90809) M_2										
355															(42248) S_2				
662					(17386) N_2														(643) ETA_2
764		(671) EPS_2		(2776)									(2597)						

chosen. Hence, information about OO_1 would be unnecessarily delayed. Although, due to the tidal potential amplitude of J_1, objective (i) is violated with both the second and third choices, it was felt that the third was a better compromise. With it, OO_1 only appears 11 h sooner than J_1.

K_2 is an example of an unavoidable violation of objective (i). Because it is so close in frequency to S_2, its importance as a major semidiurnal constituent does not insure it an early inclusion in the analysis package.

Because shallow water constituents do not have a tidal potential amplitude, objective (i) does not apply to them. However, based on his experience, Godin was able to suggest a hierarchy of their relative importance. A further criteria used when selecting comparison pairs for them was that no shallow water constituent should appear in an analysis before all the main constituents, from which it is derived, have also been selected. Table 5 shows that this has

Table 5 Shallow Water Constituents in the Standard Data Package.

Shallow Water Constituent	Record Length (h) Required for Constituent Inclusion	Component Main Constituents and Record Lengths (h) Required for Their Inclusion in the Analysis					
SO_1	4383	S_2	355	O_1	328		
MKS_2	4383	M_2	13	K_2	4383	S_2	356
MSN_2	4383	M_2	13	S_2	355	N_2	662
MO_3	656	M_2	13	O_1	328		
SO_3	4383	S_2	355	O_1	328		
MK_3	656	M_2	13	K_1	24		
SK_3	355	S_2	355	K_1	24		
MN_4	662	M_2	13	N_2	662		
M_4	25	M_2	13				
SN_4	764	S_2	355	N_2	662		
MS_4	355	M_2	13	S_2	355		
MK_4	4383	M_2	13	K_2	4383		
S_4	355	S_2	355				
SK_4	4383	S_2	355	K_2	4383		
$2MK_5$	24	M_2	13	K_1	24		
$2SK_5$	178	S_2	355	K_1	24		
$2MN_6$	662	M_2	13	N_2	662		
M_6	26	M_2	13				
$2MS_6$	355	M_2	13	S_2	355		
$2MK_6$	4383	M_2	13	K_2	4383		
$2SM_6$	355	S_2	355	M_2	13		
MSK_6	4383	M_2	13	S_2	355	K_2	4383
$3MK_7$	24	M_2	13	K_1	24		
M_8	26	M_2	13				

been upheld for all shallow water constituents in the standard 69 constituent data package.

We recommend that the objectives outlined here be employed when choosing the Rayleigh comparison constituent for any additions to the list of possible constituents to be included in the analysis.

2.1.3 Satellite constituents and nodal modulation

Doodson's (1921) development of the tidal potential contains a very large number of constituents. Due to the great length of record required for their separation, several of these can be considered, for all intents and purposes, unanalysable. The standard approach to this problem is to form clusters consisting of all constituents with the same first three Doodson numbers. The major contributor in terms of tidal potential amplitude lends its name to the cluster and the lesser constituents are called satellites.

The method of analysis uses this main and satellite constituent approach in the following manner. The Rayleigh criteria is applied to the main constituent frequencies to determine whether or not they are to be included in the analysis. For each of those so chosen, we analyse at its frequency and obtain an apparent amplitude and phase. However, because these results are actually due to the cumulative effect of all the constituents in that cluster, an adjustment is made so that only the contribution due to the main constituent is found. This adjustment is called the nodal modulation.

In order to make the nodal modulation correction to the amplitude and phase of a main constituent, it is necessary to know the relative amplitudes and phases of the satellites. As is commonly done, it is assumed in this program that the same relationship as is found with the equilibrium tide (tidal potential), holds with the actual tide. That is, the tidal potential amplitude ratio of a satellite to its main constituent is assumed to be equal to the corresponding tidal heights amplitude ratio, and the difference in tidal potential phase equals the difference in tidal height phase.

The source of the tidal potential amplitude ratio, as found in the constituent data package of Appendix 7.1, is Cartwright and Tayler (1971) and Cartwright and Edden (1973). Using new computation methods and the latest values for the astronomical constants, they obtained more accurate results than those from the previously used Doodson computations. It should be noted that in several cases (whenever the satellite arises via the third-order term), this version of the constituent data package requires that the amplitude ratio be multiplied by a latitude correction factor.

Phase differences between satellites and main constituents arise when the tidal potential development yields different trigonometric terms for these constituents. The common convention is to express all terms in cosine form and so an extra $-\frac{1}{4}$ cycle phase shift is introduced if the term was originally a sine. Satellites requiring such a shift are called third order. A further $\frac{1}{2}$ cycle change is also introduced when all negative amplitudes are made positive.

Because several test analyses indicate less consistent results when third-order satellites are included in the N_2 and L_2 nodal modulation, Godin has decided to delete these from the present standard constituent data package. Instead he suggests that the results of analyses with this package should be compared with those of previous analyses in order to find the most suitable adjustment for these constituents.

The only other main constituents that do not have all their satellites included for nodal modulation are the slow frequency constituents. For them, no satellites are specified. Because low frequency noise may be as much as an order of magnitude greater than the satellite contributions, and M_m, M_{sf} and M_f when they are detectable are often of shallow water origin, the effect of making corrections for the expected satellites would be to obscure further, rather than clarify the actual low frequency periodic signal.

Section 2.3.2 gives further details on the nodal modulation correction.

2.1.4 Shallow water constituents

Shallow water tidal constituents arise from the distortion of main constituent tidal oscillations in shallow water. Because the speed of propagation of a progressive wave is approximately proportional to the square root of the depth of water in which it is travelling, shallow water has the effect of retarding the trough of a wave more than the crest. This distorts the original sinusoidal wave shape and introduces harmonic signals that are not predicted in tidal potential development. The frequencies of these derived harmonics can be found by calculating the effect of non-linear terms in the hydrodynamic equations of motion on a signal due to one or more main constituents (see Godin (1972), pp. 154–164 for further details).

The shallow water constituents chosen for inclusion in the standard 69 constituent data package were suggested by G. Godin. They are listed in Table 5 and are derived only from the largest main constituents, namely M_2, S_2, N_2, K_2, K_1 and O_1, using the lowest types of possible interaction. The 77 additional shallow water constituents that can be included in the analysis if so desired are derived from lesser main constituents and higher types of interaction. In the constituent data package listing of Appendix 7.1, they can be spotted by their lack of a Rayleigh comparison constituent.

When shallow water effects are noticeable, main constituents, if they are close in frequency, may coexist or be masked by constituents of non-linear origin. The resultant nodal modulation will be due to the pair and thus will not coincide to the calculated modulation of the main constituent. In suspected cases, the effectiveness of nodal corrections in a series of successive analyses will indicate the presence of pairs or emphasize the predominance of one constituent over the other. Table 6 (taken from unpublished notes of Godin) lists compound constituents which may coexist with or mask constituents of direct astronomical origin. In all cases except SO_1 and MO_3, the main rather than the compound constituent is included in the standard constituent data package.

2.2 The Least Squares Method of Analysis

2.2.1 Formulation of the problem

The first stage in the actual analysis of tidal records is the least squares fit for constituent amplitude and phase. If the tidal record is of minimum length 13 h, the present program and data package insure that the constant constituents Z_0 and M_2 are always included in the analysis. If σ_j for $j = 1, M$ are the frequencies (cycles/h) of the other tidal constituents chosen for inclusion in the analysis by the Rayleigh criterion, then the problem is to find the amplitudes, A_j, and phases, ϕ_j, of the function $C_0 + \sum_{j=1}^{M} A_j \cos[2\pi(\sigma_j t_i - \phi_j)]$ that best fit the series of observations $y(t_i)$, $i = 1, N$.[3] Assuming $N > 2M + 1$ we see that it is impossible to

[3] In order to minimize the loss of accuracy due to round off, the average of the hourly heights observations is subtracted from all original values. The $y(t_i)$ values mentioned in all computations henceforth are actually the resultant deviations. At the end of all calculations, C_0 is adjusted by this mean value.

Table 6 Shallow Water Constituents that May Mask Main Constituents.

Main Constituent	Component Constituent which May Coexist at or Near its Frequency
Q_1	NK_1
O_1	NK_1**
TAU_1	MP_1**
NO_1*	NO_1**
P_1	SK_1**
K_1	MO_1
J_1	MQ_1
SO_1	SO_1
OQ_2	OQ_2**
EPS_2	MNS_2
$2N_2$	O_2**
MU_2	$2MS_2$
N_2	KQ_2**
GAM_2	OP_2**
M_2	KO_2**
L_2	$2MN_2$**
S_2	KP_2
K_2	K_2
MO_3	MO_3**
M_3	NK_3**

* With M_1 as a satellite.

** The modulation or frequency of the compound constituent is sufficiently different that the pair could be separated if a long enough record of high precision were available.

solve the system $y(t_i) = C_0 + \sum_{j=1}^{M} A_j \cos[2\pi(\sigma_j t_i - \phi_j)]$ exactly because it is overdetermined. Hence, it is necessary to adopt a criterion which will enable unique optimum values for the parameters A_j and ϕ_j to be found. The most common optimization criterion used, and the one chosen here, is the least squares technique.

Re-expressing $\sum_{j=1}^{M} A_j \cos[2\pi(\sigma_j t_i - \phi_j)]$ as

$$\sum_{j=1}^{M} [C_j \cos(2\pi\sigma_j t_i) + S_j \sin(2\pi\sigma_j t_i)] \, ,$$

where $A_j = (C_j^2 + S_j^2)^{1/2}$ and $2\pi\phi j = \arctan S_j/C_j$, so that the fitting function is linear in the parameters S_j and C_j and hence more easily solved, and rewriting $y(t_i)$ as y_i, the objective of the least squares technique is to minimize

$$T = \sum_{i=1}^{M} \left[y_i - C_0 - \sum_{j=1}^{M} (C_j \cos 2\pi\sigma_j t_i + S_j \sin 2\pi\sigma_j t_i) \right]^2 ,$$

Figure 1 The matrix equation $B\mathbf{x} = \mathbf{y}$ resulting from the least squares fit for constituent amplitudes and phases.

for C_0 and all $C_j, S_j \quad j = 1, M$. This is done by solving the following $2M + 1$ simultaneous equations for $j = 1, M$:

$$0 = \frac{\partial T}{\partial C_0} = 2 \sum_{i=1}^{N} \left(y_1 - C_0 - \sum_{j=1}^{M} C_j \cos 2\pi\sigma_j t_i - \sum_{j=1}^{M} S_j \sin 2\pi\sigma_j t_i \right) (-1);$$

$$0 = \frac{\partial T}{\partial C}$$

Gaps in the data record (i.e. missing hourly observations) are easily handled by the least squares method because it is not necessary that the observation times, t_i, for $i = 1, N$ be evenly spaced. For example, if the analysis covers the total time period of 100 h but hours 50 to 74 inclusive are missing, then t_{50} will correspond to the seventy-fifth hour. However, because the following identities which simplify the summations require that the observation times be evenly spaced, it is necessary that each of the matrix terms be calculated as the sum of contributions over the data periods that contain no gaps. Assuming that $[n_0, n_1]$ is the hour range of a section of record containing no gaps, we can substitute $t_k = k$ in the matrix coefficients expressions since the times are at successive hours.

Using the relationships

$$\begin{aligned}
\cos a \cos b &= \tfrac{1}{2}[\cos(a+b) + \cos(a-b)] \\
\sin a \sin b &= \tfrac{1}{2}[\cos(a-b) - \cos(a+b)] \\
\sin a \cos b &= \tfrac{1}{2}[\sin(a+b) + \sin(a-b)],
\end{aligned}$$

the formula for the sum of a geometric series, namely

$$\frac{a + ar +, \ldots, + ar^n = a(r^{n+1} - 1)}{(r-1)},$$

and expressing $\cos x$ and $\sin x$ as the real and imaginary parts of e^{ix}, we obtain the identities:

$$\sum_{k=n_0}^{n_1} \cos kx = \frac{\sin\{[(n_1 - n_0 + 1)x]/2\} \cos\{[(n_1 + n_0)x]/2\}}{\sin(x/2)},$$

and

$$\sum_{k=n_0}^{n_1} \sin kx = \frac{\sin\{[(n_1 - n_0 + 1)x]/2\} \sin\{[(n_1 + n_0)x]/2\}}{\sin(x/2)}.$$

Hence the summation expressions in the least squares matrix can be simplified (with regard to computer execution time) as follows.

$$\begin{aligned}
\sum_{k=n_0}^{n_1} \cos(2\pi\sigma_1 k)\cos(2\pi\sigma_2 k) &= \frac{1}{2}\sum_{k=n_0}^{n_1} \{\cos[2\pi k(\sigma_1+\sigma_2)] + \cos[2\pi k(\sigma_1-\sigma_2)]\} \\
&= \frac{1}{2}\Bigg(\frac{\sin[(n_1-n_0+1)\pi(\sigma_1+\sigma_2)]\cos[(n_1+n_0)\pi(\sigma_1+\sigma_2)]}{\sin\pi(\sigma_1+\sigma_2)} \\
&\qquad + \frac{\sin[(n_1-n_0+1)\pi(\sigma_1-\sigma_2)]\cos[(n_1+n_0)\pi(\sigma_1-\sigma_2)]}{\sin\pi(\sigma_1-\sigma_2)}\Bigg),
\end{aligned}$$

$$\begin{aligned}
\sum_{k=n_0}^{n_1} \sin(2\pi\sigma_1 k)\sin(2\pi\sigma_2 k) &= \frac{1}{2}\sum_{k=n_0}^{n_1} \{\cos[2\pi k(\sigma_1-\sigma_2)] - \cos[2\pi k(\sigma_1+\sigma_2)]\} \\
&= \frac{1}{2}\Bigg(\frac{\sin[(n_1-n_0+1)\pi(\sigma_1-\sigma_2)]\cos[(n_1+n_0)\pi(\sigma_1-\sigma_2)]}{\sin\pi(\sigma_1-\sigma_2)} \\
&\qquad - \frac{\sin[(n_1-n_0+1)\pi(\sigma_1+\sigma_2)]\cos[(n_1+n_0)\pi(\sigma_1+\sigma_2)]}{\sin\pi(\sigma_1+\sigma_2)}\Bigg),
\end{aligned}$$

$$\begin{aligned}\sum_{k=n_0}^{n_1} \sin(2\pi\sigma_1 k)\cos(2\pi\sigma_2 k) &= \frac{1}{2}\sum_{k=n_0}^{n_1}\{\sin[2\pi k(\sigma_1+\sigma_2)] + \sin[2\pi k(\sigma_1-\sigma_2)]\} \\ &= \frac{1}{2}\left(\frac{\sin[(n_1-n_0+1)\pi(\sigma_1+\sigma_2)]\sin[(n_1+n_0)\pi(\sigma_1+\sigma_2)]}{\sin\pi(\sigma_1+\sigma_2)}\right. \\ &\quad \left. + \frac{\sin[(n_1-n_0+1)\pi(\sigma_1-\sigma_2)]\sin[(n_1+n_0)\pi(\sigma_1-\sigma_2)]}{\sin\pi(\sigma_1-\sigma_2)}\right).\end{aligned}$$

With these substitutions made in Figure 1, we have the least squares matrix equation $B\mathbf{x} = \mathbf{y}$ generated in subroutine **SCFIT2**. Because B is symmetric it is sufficient to store only its upper triangle consisting of $2M^2+3M+1$ elements instead of the entire matrix of $(2M+1)^2$ elements.

Partitioning the matrix equation $B\mathbf{x} = \mathbf{y}$ into the form

$$\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}\begin{pmatrix} \mathbf{c} \\ \mathbf{s} \end{pmatrix} = \begin{pmatrix} \mathbf{y}_c \\ \mathbf{y}_s \end{pmatrix},$$

where B_{11}, B_{12}, B_{21}, B_{22}, $\mathbf{c}$, $\mathbf{s}$, $\mathbf{y}_c$, $\mathbf{y}_s$ have dimensions $(M+1)\times(M+1)$, $(M+1)\times M$, $M\times(M+1)$, $M\times M$, $(M+1)\times 1$, $M\times 1$, $(M+1)\times 1$, $M\times 1$ respectively, it is easily seen when $n_0 = -n_1$ that B_{12} and B_{21} become zero matrices and two smaller matrix equations, $B_{11}\mathbf{c} = \mathbf{y}_c$ and $B_{22}\mathbf{s} = \mathbf{y}_s$, result. The combined computation time to solve these equations is less than that of the original (see Section 2.2.2) so it is desirable to

It is worth mentioning that the overdetermined system $\mathbf{y} = A\mathbf{x} + \mathbf{e}$ can be solved in many ways, depending on the criterion chosen for minimizing $\mathbf{e}$. For our purposes, those methods which solve the system without changing the form of the matrix are impractical from a storage, processing time and rounding error point of view because the first dimension of A (= the number of hourly observations) is commonly 9000. However, minimizing $\mathbf{e}^T\mathbf{e}$ is equivalent to the least squares criterion adopted here.

An important result for any positive definite symmetric matrix B is that it can be uniquely decomposed in the form $B = GG^T$, where G is a lower triangular matrix with positive diagonal elements.[4] Expanding this relationship leads to the matrix element equalities:

$$
\begin{aligned}
b_{jj} &= \sum_{k=1}^{j} g_{jk}^2, \\
b_{ij} &= \sum_{k=1}^{j} g_{ik} g_{jk} \qquad \text{for all } i > j.
\end{aligned}
$$

The algorithm resulting from using these equations in the proper order to find the elements of G is known as Cholesky's square root method for factoring a positive definite matrix (also attributed to Banachiewicz; see Faddeev and Faddeeva (1963)). Unlike other matrix decomposition methods such as Gaussian elimination, it does not have to search for, and divide by pivots. Such techniques must insure that the reduced matrix elements are not too large so that rounding errors and loss of accuracy do not occur. In Cholesky's method however, we can see that $|g_{ij}| \leq \sqrt{b_{ii}}$ for all i, j and so upper bounds for the elements of G always exist.

Once B has been decomposed into the upper and lower triangular matrices, it is a relatively easy matter to solve the matrix solution. This is done by breaking down the equation $GG^T\mathbf{x} = \mathbf{y}$ into $G\mathbf{b} = \mathbf{y}$ and $G^T\mathbf{x} = \mathbf{b}$. Because of the triangular nature of G, these equations can be solved by forward and backward substitution for $\mathbf{b}$ and $\mathbf{x}$ respectively.

The amount of arithmetic in a matrix algorithm is usually measured by the number of multiplicative operations (i.e. multiplications and divisions) used, since there are normally approximately the same number of additive operations. For a matrix of dimension $n \times n$, the Cholesky factorization algorithm requires n square roots and approximately $\frac{1}{6}n^3$ multiplications. This compares favourably with the $\frac{1}{3}n^3$ multiplications required by Gaussian elimination (Wilkinson, 1967) to produce a triangular matrix.

Wilkinson (1967) suggests a factorization of B into LDL^T, where L is a lower triangular matrix and D is a positive diagonal matrix, that involves no more multiplications than Cholesky and avoids the square roots. However, assuming that the time ratio of a square root operation to a multiplication is 15:1 (approximate ratio for the IBM 370-168) and that all 69 constituents in the data package are included in the analysis (i.e. $n = 137$) the time saved by eliminating the square roots in only 0.5%. Furthermore, some of this gain would be replaced by time required for storing and retrieving information from the additional matrix D, and for the n additional division operations each time a solution is calculated by forward and backward substitution. Hence the factorization was not adopted in the present program.

Because the time required for the factorization of B varies as the cube of the number of unknowns, an approximate four-fold time reduction should result when the tidal record has no

[4] If B is symmetric but not positive definite a similar decomposition exists. However, some elements of G may be complex or, in the degenerate case, zero along the diagonal.

gaps and the partitioned rather than the original matrix equations are solved. However, as the following table of execution times for sections of subroutine **SCFIT2** demonstrates, significant improvements can also be expected in the time required for matrix generation, and error calculation. The values shown in Table 7 were obtained on an IBM 370-168 computer with a 34-constituent analysis of a 38-day tidal record.

A rough indication of the round-off difficulties associated with solving the equation $Bx = y$ is given by the matrix condition number. Although several different definitions for a condition number exist, an appropriate one for our purposes, in the sense that it pertains to least squares matrices and is easily calculated, is specified by Davis and Rabinowitz (1961). Its development is as follows.

Table 7 Comparison of Processing Times between the Partitioned and Non-Partitioned Matrix Equation Solutions.

Components of Matrix Solution	Partitioned Matrix System Times (s)	Non-Partitioned Matrix System (s)
Parameter initializations and right-hand generation	0.347	0.346
Matrix generation	0.059	0.178
Matrix factorization	0.049	0.146
Solution	0.010	0.018
Error calculation	0.128	0.403

If $\{\mathbf{b}_1, \dots \mathbf{b}_n\}$ are n-dimensional vectors such that the matrix

$$B = \begin{pmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_n \end{pmatrix} (\mathbf{b}_1 \quad \dots \quad \mathbf{b}_n) = \begin{pmatrix} \mathbf{b}_1 \cdot \mathbf{b}_1 & \dots & \mathbf{b}_1 \cdot \mathbf{b}_n \\ \vdots & & \vdots \\ \mathbf{b}_1 \cdot \mathbf{b}_1 & \dots & \mathbf{b}_1 \cdot \mathbf{b}_n \end{pmatrix},$$

then it can be shown that $0 \leq \det(B) \leq \|\mathbf{b}_1\| \, \|\mathbf{b}_2\|, \dots, \|\mathbf{b}_n\|$ where if $\mathbf{b}_j = (b_{j1}, \dots, b_{jn})$, the norm $\|\mathbf{b}_j\| = (\sum_{i=1}^{n} b_{ji}^2)^{1/2}$. Furthermore, $\det(B) = 0$ if and only if the vectors are linearly dependent, and $\det(B) = \|\mathbf{b}_1\|, \dots, \|\mathbf{b}_n\|$ if and only if they are orthogonal (i.e. $\mathbf{b}_i \cdot \mathbf{b}_j = 0$ for $i \neq j$). This determinant is known as the Gram determinant of the system $\{\mathbf{b}_1, \dots, \mathbf{b}_n\}$ and is the square of the n-dimensional volume of the parallelepiped whose edges are these vectors.

Since it can be shown that all least squares matrices can be expressed in this manner, this result can be applied to our situation. In particular when the vectors are normalized so that $\|\mathbf{b}_i\| = 1$, the actual value of $\det(B)$ will always be bounded and provide a measure of the linear independence of the system, and hence round-off difficulties encountered in solving the equation. A value close to 1 will mean near orthogonality, a virtually diagonal matrix for B, and thus an easy solution. On the other hand, a value close to 0 will mean that at least two rows are near scalar multiples of one another, and thus greater accuracy problems will occur when their difference is calculated during the equation solution.

For our particular case observe that $\det(B) = \det(GG^T) = (\det G)^2 = \prod_{i=1}^{n} g_{ii}^2$, and that B can be written as

$$GG^T = \begin{pmatrix} \mathbf{g}_1 \cdot \mathbf{g}_1 & \dots & \mathbf{g}_1 \cdot \mathbf{g}_n \\ \vdots & & \vdots \\ \mathbf{g}_n \cdot \mathbf{g}_n & \dots & \mathbf{g}_n \cdot \mathbf{g}_n \end{pmatrix},$$

where

$$G^T = \begin{pmatrix} g_{11} & g_{21} & \cdots & g_{n1} \\ 0 & \ddots g_{22} & \cdots & g_{n2} \\ \vdots & \ddots & \ddots & \\ 0 & \cdots & \ddots 0 & \ddots g_{nn} \end{pmatrix} = (\mathbf{g}_1, \mathbf{g}_2 \ldots \mathbf{g}_n).$$

Since $b_{jj} = \sum_{k=1}^{j} g_{jk}^2$, $\|\mathbf{g}_j\| = \sqrt{b_{jj}}$ and the determinant of the matrix resulting from normalizing the $\mathbf{g}_j$ vectors is $\prod_{i=1}^{n}(g_{ii/b_{ii}}^2)$. The square root of this value is the volume of the n-dimensional parallelepiped whose edges are these normalized vectors and is the quantity calculated as the condition number of the matrix B.

The statistical properties of the least squares fit solution can be found in any analysis of variance or regression model text. They are outlined briefly as follows.

Reverting to the overdetermined problem statement, the least squares objective can be stated as finding the vector $\mathbf{x}$ in $\mathbf{y} = A\mathbf{x} + \mathbf{e}$ such that $\mathbf{e}^T\mathbf{e}$ is minimized. This yields the solution $\hat{\mathbf{x}} = (A^TA)^{-1}A^T\mathbf{y}$.

The total sum of squares is $\mathbf{y}^T\mathbf{y}$ and the sum of squares due to regression is $\hat{\mathbf{x}}^TA^Ty$. Their difference is the residual error sum of squares and this difference divided by the degrees of freedom in the fit is the residual mean square error (MSE). "Degrees of freedom" is the difference between the number of hourly observations (excluding gaps) and A the number of parameters fit in the analysis. If there were M constituents including Z_0 chosen for the analysis, the degrees of freedom would be $N - 2M + 1$.

If it is assumed, as is commonly done, that the vector $\mathbf{e}$ is distributed normally with 0 standard deviation and $\sigma^2 I$ variance, where I is the unit diagonal matrix, then the variance of $\hat{\mathbf{x}}$ is $(A^TA)^{-1}\sigma^2$. Since the mean square residual error is an unbiased estimator for σ^2, an estimate of the standard deviation of $\hat{x}_i$, the ith element of $\hat{\mathbf{x}}$, is

$$\sqrt{(\boldsymbol{\mu}_i^T(A^TA)^{-1}\boldsymbol{\mu}_i)\mathrm{MSE}}\,,$$

where $\boldsymbol{\mu}_i$ is the vector with one in the ith position of zeros elsewhere.

2.3 Modifications to the Least Squares Analysis Results

2.3.1 Astronomical argument and Greenwich phase lag

Instead of regarding each tidal constituent as the result of some particular component of the tidal potential, an artificial causal agent can be attributed to each constituent in the form of a fictitious star which travels around the equator with an angular speed equal to that of its corresponding constituent. Making use of this conceptual aid, the astronomical argument, $V(L,t)$, of a tidal constituent can then be viewed as the angular position of this fictitious star relative to longitude, L, and at time, t. Although the longitudinal dependence is easily calculated, for historical reasons L is generally assumed to be the Greenwich meridian, and V is reduced to a function of one variable.

The Greenwich phase lag, g, is the difference between this astronomical argument for Greenwich and the phase of the observed constituent signal. Its value is dependent upon the time zone in which the hourly heights of the record were taken. This means that when phases at various stations, not necessarily in the same time zone, are compared, they must be reduced to a common zone in order to avoid spurious differences due to difference relative times. Specifically, if σ is the constituent frequency and $g(j+\Delta_j)$ and $g(j)$ are the Greenwich phase lags evaluated for time zones $j+\Delta_j$ and j respectively (e.g. Pacific Standard Time is +8), then

$$g(j+\Delta_j) = g(j) - (\Delta_j)\sigma.$$

Although these adjustments are easily calculated, they can be tedious because each constituent must be handled individually. Therefore, to avoid possible misinterpretation of phases from nearby stations of subsequent phase alterations, it is suggested that all observations be recorded in, or converted to, GMT.

The calculation of g (see Section 2.3.3) requires that the astronomical argument need only be evaluated at one time, the central hour of the analysis period. For a particular main constituent, it is calculated as

$$V = i_0\tau + j_0 S + k_0 H + l_0 P + m_0 N' + n_0 P',$$

where $i_0, j_0, k_0, l_0, m_0, n_0$ are the Doodson numbers of the constituent and S, H, P, N', P' are the astronomical variables defined in Section 2.1.1. The variable, τ, the number of mean lunar days from an absolute time origin is calculated as sum of the local mean solar time from this origin and $(H-S)$, and so need not be read from the data cards.

For shallow water constituents, the astronomical argument is calculated as the linear combination of the coefficient number and the astronomical argument of the main constituents from which it is derived. For example,

$$V_{\mathrm{MSN_2}} = V_{\mathrm{M_2}} + V_{\mathrm{S_2}} - V_{\mathrm{N_2}} \quad \text{and} \quad V_{\mathrm{2MK_5}} = 2V_{\mathrm{M_2}} + V_{\mathrm{K_1}}.$$

2.3.2 Nodal corrections

Most of this section has been taken from the unpublished notes of G. Godin which were written subsequent to the Cartwright and Tayler (1971) and Cartwright and Edden's (1973) recalculation of the tide-generating potential. The material presented here is intended to give greater detail than that of Section 2.1.3.

Due to the presence of satellites in a given cluster, it is known from tidal potential theory that the analysed signal found at the frequency, σ_j, of the main constituent is actually the result of

$$a_j \sin(V_j - g_j) + \sum_k A_{jk} a_{jk} \sin(V_{jk} - g_{jk}) + \sum_l A_{jl} a_{jl} \cos(V_{jl} - g_{jl})$$

for the diurnal and terdiurnal constituents of direct gravitational origin, and

$$a_j \cos(V_j - g_j) + \sum_k A_{jk} a_{jk} \cos(V_{jk} - g_{jk}) + \sum_l A_{jl} a_{jl} \sin(V_{jl} - g_{jl})$$

for the slow and semidiurnal constituents. The variables, a, g and V, are the true amplitude, Greenwich phase and astronomical argument, respectively, at the central time of the record for all the constituents. Single j subscripts refer to the major contributor while jk and jl subscripts refer to satellites originating from tidal potential terms of the second and third order respectively. A is the element of the interaction matrix resulting from the interference of a satellite with the main constituent.

It is the convention in tides and an assumption for our least squares fit that all constituents arise through a cosine term and positive amplitude, i.e. the contribution for a constituent whose astronomical argument is V_j and whose Greenwich phase is g_j, is expected to be in the form $a_j \cos(V_j - g_j)$ for $a_j > 0$. However, the diurnal and terdiurnal constituents, assuming that they are due to second order terms in the tidal potential, actually arise through a $b_j \sin(V_j - g_j)$ term where b_j may be negative. Hence a phase correction (variable `SEMI` read in data input (iii) from logical unit 8) of either $-\frac{1}{4}$ or $-\frac{3}{4}$ cycles is necessary, i.e.

$$\begin{aligned} b_j \sin(V_j - g_j) &= |b_j| \cos\left(V_j - g_j - \tfrac{1}{4}\right) & \qquad b_j \geq 0, \\ &= |b_j| \cos\left(V_j - g_j - \tfrac{3}{4}\right) & \qquad b_j < 0. \end{aligned}$$

Similarly, an adjustment of $\frac{1}{2}$ cycle will only be necessary for slow and semidiurnal main constituents if the tidal potential amplitude is negative.

Making these changes, the combined result of a constituent cluster in the diurnal and terdiurnal cases is

$$|a_j| \cos(V_j' - g_j) + \sum_k A_{jk} a_{jk} \cos(V_{jk}' + \alpha_{jk} - g_k) + \sum_l A_{jl} a_{jk} \cos(V_{jl}' + \alpha_{jl} - g_{jl})$$

where if

$$a_j < 0, \qquad V' = V - \tfrac{3}{4}, \quad \alpha_{jk} = \tfrac{1}{2}, \quad \alpha_{jl} = \tfrac{3}{4},$$

and if

$$a_j > 0, \qquad V' = V - \tfrac{1}{4}, \quad \alpha_{jk} = 0, \quad \alpha_{jl} = \tfrac{1}{4}.$$

A further phase adjustment to satellite constituents can be made if we wish to ensure that their amplitudes are positive. This convention was adopted for the data package of Appendix 7.1 (variable `PH` read in data input (iv) from logical unit 8). Replacing a_{jk} and a_{jl} by their absolute values we now see that

$$\begin{aligned} \alpha_{jk} &= 0 & \quad &\text{if both } a_{jk} \text{ and } a_j \text{ have the same sign,} \\ &= \tfrac{1}{2} & &\text{otherwise;} \\ \alpha_{jkl} &= \tfrac{1}{4} & &\text{if both } a_{jl} \text{ and } a_j \text{ have the same sign,} \\ &= \tfrac{3}{4} & &\text{otherwise.} \end{aligned}$$

Similarly, for the slow and semidiurnal constituents, the cluster contribution can be written as

$$|a_j| \cos(V_j' - g_j) + \sum_k A_{jk} |a_{jk}| \cos(V_{jk}' + \alpha_{jk} - g_{jk}) + \sum_l A_{jl} |a_{jl}| \cos(V_{jl}' + \alpha_{jl} - g_{jl}),$$

where

$$\begin{aligned} V' &= V + \tfrac{1}{2} & \quad &\text{if } a_j < 0, \\ &V & &\text{otherwise;} \\ \alpha_{jk} &= 0 & &\text{if } a_{jk} \text{ and } a_j \text{ have the same sign,} \\ &\tfrac{1}{2} & &\text{otherwise;} \\ \alpha_{jl} &= -\tfrac{1}{4} & &\text{if } a_{jl} \text{ and } a_j \text{ have the same sign,} \\ &\tfrac{1}{4} & &\text{otherwise.} \end{aligned}$$

Special note should be made of the terdiurnal M_3 because both it and its only satellite are due to third-order terms in the tidal potential. Hence both contribute directly through a cosine term and so behave as if they were second order semidiurnals.

In order to determine the amplitude and phase of the major contributor, we assume that the result actually found in the analysis was $f_j a_j \cos(V_j' - g_j + u_j)$, where f_j and u_j are called the nodal modulation corrections in amplitude and phase respectively. To avoid a possible misunderstanding, it is worth mentioning here that the term nodal modulation is actually a misnomer. It and the symbols f and u were first used before the advent of modern computers to designate corrections for the moon's nodal progression that were not incorporated into the calculations of the astronomical argument for the main constituent. However, now the term satellite modulation is more appropriate because our correction is due to the presence of satellite constituents differing not only in the contribution of the lunar node to their astronomical argument, but also in the lunar and solar perigee effect.

For the purpose of calculating f_j and u_j it is assumed that the admittance is very nearly a constant over the frequency range within a constituent cluster, and so $g_j = g_{jk} = g_{jl}$; and $r_{jk} = |a_{jk}|/|a_j|$, $r_{jl} = |a_{jl}|/|a_j|$ are equal to the ratio of the tidal equilibrium amplitudes of the satellite to the major contributor. These ratios are latitude dependent when satellites of the third order are involved, necessitating the correction factors mentioned in Section 2.1.3. However, the ratios are usually small and the correction is slight.

Dropping the 'prime' notation and grouping the second- and third-order terms in one summation, the relationship between the analysed results for a main constituent and the actual cluster contribution is

$$f_j|A_j|\cos(V_j + u_j - g_j) = |a_j|\Big[\cos(V_j - g_j) + \sum_k A_{jk} r_{jk} \cos(V_j - g_j + \Delta_{jk} + \alpha_{jk})\Big],$$

where $\Delta_{jk} = V_{jk} - V_j$.

Expanding this result and observing that it must be true for all $V_j(t)$, the following explicit formulae are found for f and u:

$$f_j = \left[\Big(1 + \sum_k A_{jk} r_{jk} \cos(\Delta_{jk} + \alpha_{jk})\Big)^2 + \Big(\sum_k A_{jk} r_{jk} \sin(\Delta_{jk} + \alpha_{jk})\Big)^2\right]^{1/2},$$

$$u_j = \arctan\left[\frac{\sum_k A_{jk} r_{jk} \sin(\Delta_{jk} + \alpha_{jk})}{1 + \sum_k A_{jk} r_{jk} \cos(\Delta_{jk} + \alpha_{jk})}\right].$$

For an analysis carried out over $2N + 1$ consecutive observations, Δt time units apart, A_{jk} is given by

$$A_{jk} = \frac{\sin[(2N+1)\Delta t(\sigma_{jk} - \sigma_j)/2]}{(2N+1)\sin[\Delta t(\sigma_{jk} - \sigma_j)/2]},$$

where σ_j is the frequency of the main contributor and σ_{jk} is that of its satellite. However, A_{jk} is very nearly one, even for a one-year analysis, and in the program it is approximated by this value.

For a shallow water constituent whose frequency is calculated as $\sum_{j=1}^{N_0} c_j \sigma_j$, where σ_j is the frequency of the jth main constituent from which it is derived and c_j is the linear coefficient, the nodal modulation corrections for amplitude and phase are computed as

$$f = \prod_{j=1}^{N_0} f_j^{|c_j|} \quad \text{and} \quad u = \sum_{j=1}^{N_0} c_j u_j.$$

2.3.3 Final amplitude and phase results

The result of the least squares analysis was to find for a constituent with frequency σ_j, the optimal amplitude A_j and phase ϕ_j value for the tidal signal $A_j \cos 2\pi(\sigma_j t - \phi_j)$. However, due to nodal corrections, when the astronomical argument is calculated at the central time origin $t = 0$ of the record, we know that the actual contribution of the constituent cluster is $f_j a_j \cos 2\pi(V_j + u_j - g_j)$. Hence the amplitude and Greenwich phase lag of the constituent corresponding to frequency σ_j can be calculated as $a_j = A_j/f_j$ and $g_j = V_j + u_j + \phi_j$.

2.3.4 Inferred constituents

In accordance with previous notation, tidal signals in this section are assumed to be real in nature. However, an alternative presentation using complex numbers and the basis for the following development is given by Godin (1972).

If the length of a specific tidal record is such that certain important constituents will not be included directly in the analysis, provision is made via the data input on logical unit 4 to include these constituents indirectly by inferring their amplitudes and phases from neighbouring constituents that are included. If accurate amplitude ratios and phase differences are specified, inference has the effect of significantly reducing any periodic behaviour in the amplitudes and phases of the constituent used for the inference. This is due to the removal of interaction from the neighbouring inferred constituent. If it so happens that a constituent specified for inference is included directly in the analysis, the program will ignore the inference calculations.

The actual adjustments are as follows. Assume that the constituent with frequency, σ_2, is to be inferred from the constituent with frequency, σ_1, and that the least squares fit analysis found the latter's contribution to be $A_1^0 \cos 2\pi(\sigma_1 t - \phi_1^0)$, where A_1^0 and ϕ_1^0 are the amplitude and phase respectively (σ_1 and ϕ_1^0 are measured in cycles/h and cycles respectively). Letting

VU_1 be the astronomical argument + nodal modulation phase correction,
g_1 be the Greenwich phase lag,
f_1 be the nodal modulation amplitude correction factor,
and a_1 be the corrected amplitude.

then from Section 2.3.3 we know that

$$-\phi_1 = VU_1 - g_1$$

and

$$a_1 = A_1/f_1.$$

Assuming that A_1 and ϕ_1 are the post-inference amplitude and phase respectively for the constituent with frequency, σ_1,

$$r_{12} = \frac{a_2}{a_1} = \frac{(A_2/f_2)}{(A_1/f_1)}$$

and

$$\zeta = g_1 - g_2 = VU_1 + \phi_1 - VU_2 - \phi_2$$

(the latter two being data input variables `R` and `ZETA` respectively), then the presence of the inferred constituent in the analysed signal yields the relationship:

$$A_1^0 \cos 2\pi(\sigma_1 t - \phi_1^0) = A_1 \cos 2\pi(\sigma_1 t - \phi_1) + A_2 \cos 2\pi(\sigma_2 t - \phi_2)$$

$$
\begin{aligned}
&= A_1 \cos 2\pi(\sigma_1 t - \phi_1) \\
&\qquad \left\{ 1 + r_{12}\left(\frac{f_2}{f_1}\right) \cos 2\pi[(\sigma_2 - \sigma_1)t + VU_2 - VU_1 + \zeta] \right\} \\
&\quad - A_1 \sin 2\pi(\sigma_1 t - \phi_1) \\
&\qquad \left\{ r_{12}\left(\frac{f_2}{f_1}\right) \sin 2\pi[(\sigma_2 - \sigma_1)t + VU_2 - VU_1 + \zeta] \right\}.
\end{aligned}
$$

Since the constituent with frequency σ_2 was not chosen for inclusion in the least squares analysis, $|\sigma_2 - \sigma_1|N < RAY$, where N is the record length in hours and RAY is the Rayeigh criterion constant (usually 1.0). Assuming in general that $|\sigma_2 - \sigma_1|N$ is small, good approximations to $\cos 2\pi[(\sigma_2 - \sigma_1)t + VU_2 - VU_1 + \zeta]$ and $\sin 2\pi[(\sigma_2 - \sigma_1)t + VU_2 - VU_1 + \zeta]$ are their average values over the interval $[-N/2, N/2]$, namely $\sin[\pi N(\sigma_2 - \sigma_1)] \cos[2\pi(VU_2 - VU_1 + \zeta)]/[\pi N(\sigma_2 - \sigma_1)]$ and $\sin[\pi N(\sigma_2 - \sigma_1)] \sin[2\pi(VU_2 - VU_1 + \zeta)]/[\pi N(\sigma_2 - \sigma_1)]$ respectively. Making these substitutions and setting

$$S = r_{12}\left(\frac{f_2}{f_1}\right) \sin[\pi N(\sigma_2 - \sigma_1)] \sin[2\pi(VU_2 - VU_1 + \zeta)]/[\pi N(\sigma_2 - \sigma_1)]$$

and

$$C = 1 + r_{12}\left(\frac{f_2}{f_1}\right) \sin[\pi N(\sigma_2 - \sigma_1)] \cos[2\pi(VU_2 - VU_1 + \zeta)]/[\pi N(\sigma_2 - \sigma_1)],$$

we obtain

$$\frac{A_1^0}{A_1} \cos[2\pi(\sigma_1 t - \phi_1^0)] = C \cos[2\pi(\sigma_1 t - \phi_1)] - S \sin[2\pi(\sigma_1 t - \phi_1)].$$

Expanding and regrouping this result yields

$$
\begin{aligned}
&\cos 2\pi\sigma_1 t \left(\frac{A_1^0}{A_1} \cos 2\pi\phi_1^0 - C \cos 2\pi\phi_1 - S \sin 2\pi\phi_1 \right) \\
&\qquad\qquad = \sin 2\pi\sigma_1 t \left(-\frac{A_1^0}{A_1} \sin 2\pi\phi_1^0 + C \sin 2\pi\phi_1 - S \cos 2\pi\phi_1 \right).
\end{aligned}
$$

Now since this relationship must hold for all t, both terms in brackets are equal to zero. Hence

$$
\begin{aligned}
\frac{A_1^0}{A_1} \cos 2\pi\phi_1^0 &= C \cos 2\pi\phi_1 + S \sin 2\pi\phi_1, \\
\frac{A_1^0}{A_1} \sin 2\pi\phi_1^0 &= C \sin 2\pi\phi_1 - S \cos 2\pi\phi_1
\end{aligned}
$$

and so

$$
\begin{aligned}
A_1 &= \frac{A_1^0}{\sqrt{C^2 + S^2}}, \\
\phi_1 &= \phi_1^0 + \frac{\arctan(S/C)}{2\pi}.
\end{aligned}
$$

The relative phase and amplitude of the inferred constituent are then calculated as

$$\phi_2 = VU_1 - VU_2 + \phi_1 - \zeta$$

and

$$A_2 = r_{12} A_1 \left(\frac{f_2}{f_1}\right).$$

3 USE OF THE TIDAL HEIGHTS PREDICTION COMPUTER PROGRAM

3.1 General Description

This program produces tidal height values at a given location for a specified period of time. Amplitudes and Greenwich phase lags of the tidal constituents to be used in the prediction are required as input and either equally spaced heights or all the high and low values can be produced.

3.2 Routines Required

(1) **MAIN** ...... reads in tidal station and time period information, amplitudes and Greenwich phases of constituents to be used in the prediction, and calculates the desired tidal heights.

(2) **ASTRO** ...... reads the standard constituent data package and calculates the frequencies, astronomical arguments, and nodal corrections for all constituents.

(3) **PUT** ...... controls the output for high–low predictions.

(4) **HPUT** ...... controls the output for equally spaced predictions.

(5) **GDAY** ...... returns the consecutive day number from a specific origin for any given date and vice versa.

(6) **ASTR** ...... calculates ephermides for the sun and moon.

3.3 Data Input

All input data required by the tidal heights prediction program is from logical unit 8. A sample set is given in Appendix 7.4. Although data types (i), (ii) and (iii) are identical to types (ii), (iii) and (iv) expected in logical unit 8 by the analysis program, for completeness they are repeated here.

(i) Two cards specifying values for the astronomical arguments `SO,HO,PO,ENPO,PPO,DS,DH,DP,DNP,DPP` in the format `(5F13.10)`.

`SO` = mean longitude of the moon (cycles) at the reference time origin;
`HO` = mean longitude of the sun (cycles) at the reference time origin;
`PO` = mean longitude of the lunar perigee (cycles) at the reference time origin;
`ENPO` = negative of the mean longitude of the ascending node (cycles) at the reference time origin;
`PPO` = mean longitude of the solar perigee (perihelion) at the reference time origin.

DS,DH,DP,DNP,DP are their respective rates of change over a 365-day period at the reference time origin.

Although these argument values are not used by the program that was revised in October 1992, in order to maintain consistency with earlier programs, they are still required as input. Polynomial approximations are now employed to more accurately evaluate the astronomical arguments and their rates of change.

(ii) At least one card for all the main tidal constituents specifying their Doodson numbers and phase shift, along with as many cards as are necessary for the satellite constituents. The first card for each such constituent is in the format (6X,A5,1X,6I3,F5.2,I4) and contains the following information:

KON = constituent name;
II,JJ,KK,LL,MM,NN = the six Doodson numbers for KON;
SEMI = phase correction for KON;
NJ = number of satellite constituents.

A blank card terminates this data type.

If NJ>0, information on the satellite constituents follows, three satellites per card, in the format (11X,3(3I3,F4.2,F7.4,IX,I1,1X)). For each satellite the values read are:

LDEL,MDEL,NDEL = the last three Doodson numbers of the main constituent subtracted from the last three Doodson numbers of the satellite constituent;
PH = phase correction of the satellite constituent relative to the phase of the main constituent;
EE = amplitude ratio of the satellite tidal potential to that of the main constituent;
IR = 1 if the amplitude ratio has to be multiplied by the latitude correction factor for diurnal constituents,
= 2 if the amplitude ratio has to be multiplied by the latitude correction factor for semidiurnal constituents,
= otherwise if no correction is required to the amplitude ratio.

(iii) One card specifying each of the shallow water constituents and the main constituents from which they are derived. The format is (6X,A5,I1,2X,4(F5.2,A5,5X)) and the respective values read are:

KON = name of the shallow water constituent;
NJ = number of main constituents from which it is derived;
COEF,KONCO = combination number and name of these main constituents.

The end of these shallow water constituents is denoted by a blank card.

(iv) One card with the tidal station information ISTN,(NA(J),J=1,4),ITZONE,LAD,LAM,LOD, LOM in the format (5X,I4,1X,3A6,A4,A3,1X,I2,1X,I2,2X,I3,1X,I2).

ISTN = station number;
(NA(J),J=1,4) = station name;
ITZONE = time zone reference for the "Greenwich" phases;

`LAD,LAM` = station latitude in degrees and minutes;
`LOD,LOM` = station longitude in degrees and minutes.

(v) One card for each constituent to be included in the prediction with the constituent name (`KON`), amplitude (`AMP`) and phase lag (`G`) in the format (`5X,A5,28X,F8.4,F7.2`). (This format is compatible with the analysis program results produced on output device 2). The phase lag units should be degrees (measured in time zone `ITZONE` while the units of the predicted tidal heights will be the same as those of the input amplitudes. The last constituent is followed by a blank card.

(vi) One card containing the following information on the period and type of prediction desired. The format is (`3I3,1X,3I3,1X,A4,F9.5,2X,2I3`).

`IDYO,IMOO,IYRO` = first day, month and year of the prediction period;
`IDYE,IMOE,IYRE` = first day, month and year of the prediction period;
`ITYPE` = `'EQUI'` if equally spaced predictions are desired,
= `'EXTR'` if all the high and low tide times and heights are desired;
`DT` = time spacing of the predicted values if `ITYPE='EQUI'`,
= time step increment used to initially bracket a high or low value if `ITYPE='EXTR'`;
`ICEO,ICEE` = century number for the beginning and end of the prediction. (Blank values for `ICEO` or `ICEE` will be reset to 19.)

Equally spaced predictions begin at `DT` hours on the first day and extend to 2400 h (assuming 24 is a multiple of `DT`) of the last day. When `ITYPE='EXTR'`, Godin and Taylor (1973) recommend using the following values for `DT`: 3 h for a semidiurnal tide, 6 h for a diurnal tide and 0.5 h for a mixed tide.

Type (vi) data may be repeated any number of times. One blank card following a type (vi) record will return the program to type (iv) input, while two blank cards will end the program execution.

3.4 Output

Two logical units are used for the output of results in the tidal heights prediction program. Device number 6 is the line printer and 10 is a data file. Both equally spaced and high–low predictions are put onto both devices with the same format. However the line printer also records the station name and location along with the amplitudes and phase lags of the constituents used in the prediction. Appendix 7.5 lists device 10 output resulting from the input of Appendix 7.4.

When daily high–low values are desired, the date, station number and a series of up to six heights and occurrence times are listed per record. Each record begins with the variable HL whose value is zero if the first height for that day is a high (i.e. larger than the second height) and one if the first height is a low. If there are less than six high–low values for a day, they are padded up to six with the values 9999 and 99.9 for the times and heights respectively. On device 10, the format used for the variables HL, the station number, the day, month, year, and the six pairs of times and heights is (`1X,I1,I5,2I3,I2,6(I5,F5.1)`).

When equally spaced heights are requested, 8 values are listed on each record preceded by the station number, the time, day, month and year of the first value, and followed by the time increment between heights. On device number 10, the format for these variables is `(1X,I4,F8.4,I3,2I2,8F6.3,F12.4)`

3.5 Program Conversion, Modifications, Storage and Dimension Guidelines

The source program and constituent data package described in this manual have been tested on various mainframe, PC and workstation computers at the Institute of Ocean Sciences, Patricia Bay. Although as much of the program as possible was written in basic FORTRAN, some changes may be required before the program and data package can be used on other installations. Please write or call the author if any problems are encountered.

The program in its present form requires approximately 33,000 bytes for the storage of its instructions and arrays respectively. As with the analysis program, changing the number or type of constituents in the standard data package may require alteration to the dimensions of some arrays. Restrictions on the minimum dimension of all arrays are now given.

Let

- `MTAB` be the total number of possible constituents contained in the data package (presently 146),
- `M` be the number of constituents to be included in the prediction,
- `MCON` be the number of main constituents in the standard data package (presently 45),
- `MSAT` be the sum of the number of satellites for these main constituents and the number of main constituents with no satellites (presently 162 plus 8 for the version of the constituent data package, listed in Appendix 7.4, that contains no third-order satellites for both N_2 and L_2),
- `MSHAL` be the sum for all shallow water constituents of the number of main constituents from which each is derived (presently 251),
- `NITER` be the iterations required to reduce the time interval within which it is known that a high or low tide exists, to a desired length (with the largest initial interval size of 6 h and a 6-min final interval, `NITER` is 6).

Then in the main program, arrays `SIGTAB,V,U` and `F` should have minimum dimension `MTAB`; array `KONTAB` should have minimum dimension `MTAB+1`; arrays `SIG,INDX,TWOC,CH,CHP,CHA,CHB,CHM,ANGO` and `AMPNC` should have minimum dimension `M`; arrays `KON,AMP` and `G` should have minimum dimension `M+1`; and the two-dimensional array `BTWDC` should have a minimum dimension of `M` by `NITER`. Array `COSINE` which stores pre-calculated cosine function values over the range of 0° to 360° and is used as a look-up table, presently has 2002 elements.

In subroutine **ASTRO**, the arrays `FREQ,V,U` and `F` should have minimum dimension `MTAB`; arrays `KON` and `NJ` should have minimum dimension `MTAB+1`; arrays `II,JJ,KK,LL,MM,NN` and `SEMI` should have minimum dimension `MCON+1`; arrays `EE,LDEL,MDEL,NDEL,IR` and `PH` should have minimum dimension `MSAT`; and arrays `KONCO` and `COEF` should have minimum dimension `MSHAL+4`.

In subroutine **PUT**, the dimensions of arrays `HGTK` and `ITIME` should be at least as large as the maximum number of high and low values per day (this is presently assumed to be 9).

In subroutine **HPUT**, the dimension of array `H` should be at least equal to the number of equally spaced tidal height values per output record of logical unit 10 or 6 (presently, this is 8).

In subroutine **CDAY**, both arrays `NDM` and `NDP` should have dimension 12.

4 TIDAL HEIGHTS PREDICTION PROGRAM DETAILS

4.1 Problem Formulation and the Equally Spaced Predictions Method

The tidal height, $h(t)$, at a particular station may be represented by the harmonic summation (see Section 2.3.3)

$$h(t) = \sum_{j=1}^{m} f_j(t) A_j \cos\left[2\pi(V_j(t) + u_j(t) - g_j)\right], \tag{1}$$

where

$$\begin{aligned} A_j,\ g_j &= \text{amplitude and phase lag of constituent, } j, \\ f_j(t),\ u_j(t) &= \text{nodal modulation amplitude and phase correction factors for constituent, } j, \\ V_j(t) &= \text{astronomical argument for constituent, } j. \end{aligned}$$

Expanding $V(t)$ as in Section 2.3.1 and using the first-order Taylor approximations for the astronomical arguments as in Section 2.1.1, $V(t)$ can be re-expressed as

$$\begin{aligned} V(t) &= i\tau(t) + jS(t) + kH(t) + lP(t) + mN'(t) + nP'(t) \\ &= i\tau(t_0) + jS(t_0) + kH(t_0) + lP(t_0) + mN'(t_0) + nP'(t_0) \\ &\qquad + (t - t_0)\frac{\partial}{\partial t}\left[i\tau(t) + jS(t) + kH(t) + lP(t) + mN'(t) + nP'(t)\right]_{t=t_0} \\ &= V(t_0) + (t - t_0)\sigma, \end{aligned}$$

where t_0 is the reference time origin and σ is the constituent frequency at this time origin. It follows from this result that $V(t_2) = V(t_1) + (t_2 - t_1)\sigma$ for arbitrary times, t_1, t_2, and so $V_j(t)$ can be replaced in (1) by $V_j(t_1) + (t - t_1)\sigma_j$ for some convenient time, t_1.

From Section 2.3.2 it is seen that $f(t)$ and $u(t)$ are time dependent only through the $\Delta_{jk}(t)$ variable. Since satellites differ from main constituents in only the last three Doodson numbers (see Section 2.1.3),

$$\begin{aligned} \Delta_{jk}(t) &= V_{jk}(t) - V_j(t) \\ &= \Delta l P(t) + \Delta m N'(t) + \Delta n P'(t). \end{aligned}$$

Using the first order Taylor approximations for P, N' and P', it follows that over a time period $[t_1, t_2]$ the change in $\Delta_{jk}(t)$ is

$$\begin{aligned} \Delta_{jk}(t_2) - \Delta_{jk}(t_1) &= \Delta l[P(t_2) - P(t_1)] + \Delta m[N'(t_2) - N'(t_1)] + \Delta n[P'(t_2) - P'(t_1)] \\ &= (t_2 - t_1)\frac{d}{dt}[\Delta l P(t) + \Delta m N'(t) + \Delta n P'(t)]_{t=t_0} \\ &= (t_2 - t_1)(\sigma_{jk} - \sigma_j). \end{aligned}$$

Since $d/dt[P(t) + N'(t) + P'(t)]_{t=t_0}$ is 0.16668884 cycles/356 days and $|\Delta l|$, $|\Delta m|$, $|\Delta n|$ are always less than or equal to 4, if $|t_2 - t_1| \leq 16$ days, $|\Delta_{jk}(t_2) - \Delta_{jk}(t_1)| \leq 0.03$ cycles. This small variation in $\Delta_j k(t)$ leads to a similar behaviour in $\cos[\Delta_{jk}(t)]$ and $\sin[\Delta_{jk}(t)]$, and hence $f(t)$ and $u(t)$. Thus only a small loss in accuracy but a considerable calculation time saving will result if $f(t)$ and $u(t)$ are approximated by a constant value throughout the period of a month. Consequently $f(t)$ and $u(t)$ are assumed to equal their value at 0000 h of the sixteenth day of the month for the entire monthly period; for convenience, $V(t)$ is set to $V(t_{16}) + (t - t_{16})\sigma$, where t_{16} is this same time.

The procedure for calculating a series of tidal heights is then as follows. Since the tidal prediction data package does not contain constituent frequencies, they must be calculated via the astronomical variable derivatives and the constituent Doodson numbers. The values f, u and V are then calculated for the sixteenth day of the first month of the desired prediction period and, as required, for subsequent months. Tidal heights for the desired values of t can then be calculated as

$$h(t) = \sum_{j=1}^{m} f_j(t_{16}) A_j \cos[2\pi(V_j(t_{16}) + (t - t_{16})\sigma_j + u_j(t_{16}) - g_j)]. \tag{2}$$

In order to avoid calling a trigonometric library function for each new value of t, when a sequence of equally spaced heights are required, the following Chebyshev iteration formula is used for each constituent contribution,

$$f(n+1) = 2\cos(\sigma\Delta t) f(n) - f(n-1), \tag{3}$$

where $f(n) = \cos(n\sigma\Delta t)$ or $\sin(n\sigma\Delta t)$.

4.2 The High and Low Tide Prediction Method

The material presented here is taken from Godin and Taylor (1973).

In Section 4.1 we saw that the tidal height at a given location can be represented by the harmonic sum

$$h(t) = \sum_{j=1}^{m} f_j(t_0) A_j \cos[2\pi(V_j(t_0) + (t - t_0)\sigma_j + u(t_0) - g_j)] \tag{1}$$

where

$$\begin{aligned} A_j,\, g_j,\, \sigma_j &= \text{amplitude, phase lag and frequency of constituent, } j, \\ f_j(t_0),\, u_j(t_0) &= \text{nodal modulation amplitude and phase correction factors for constituent,} \\ &\quad\ j, \text{ at the time origin } t_0, \\ V_j(t_0) &= \text{astronomical argument for constituent } j \text{ at the time origin } t_0. \end{aligned}$$

Letting $D(t)$ be the derivative of $h(t)$, i.e.

$$D(t) = -\sum_{j=1}^{m} f_j(t_0) A_j 2\pi\sigma_j \sin[2\pi(V_j(t_0) + (t - t_0)\sigma_j + u(t_0) - g_j)], \tag{2}$$

the high–low tide prediction method uses the following calculus results. If $D(t)$ is a continuous function on the interval $[t_1, t_2]$ and t_k is a point in this interval, then:

(i) $D(t_k) = 0$ if and only if t_k is an extreme point or saddle point,[5] or $h(t)$ is constant in the neighbourhood of t_k;

(ii) if $D(t_1)$ and $D(t_2)$ have opposite signs, then there exists a t_k in (t_1, t_2) with $D(t_k) = 0$.

[5] An example of a saddle point is $x = 0$ for the function $f(x) = x^3$.

Now for computational purposes we can assume that saddle points do not exist. That is to say, due to accuracy limitations of the computer, a zero derivative will be approximated by a number with a very small absolute value and thus perturb a saddle point so that it becomes either a maximum or minimum, or a near saddle point (in the neighbourhood of a "near saddle point", the derivative is of constant sign and almost assumes the value zero). And since, from its definition, we can reasonably assume that $h(t)$ is not constant over any arbitrarily small interval, the continuity of $D(t)$ everywhere implies that an interval $[t_1, t_2]$ with $D(t_1)$ and $D(t_2)$, having opposite signs, contains an extremum.

However, this result alone is not sufficient to guarantee the location of all extrema because it does not eliminate the possibility of having more than one extremum in an interval whose endpoints have different signs, nor does it imply that if the endpoints have the same derivative sign there is no extremum in the interval. In order to ensure these conditions and thus be assured of bracketing all extreme values, it is necessary that a minimum interval size be specified in which we can assume that there exists, at most, one high or low tide.

Clearly, the interval size, Δt, will be dependent upon the nature of the tide at a particular station. The time between successive high and low waters for predominantly semidiurnal and diurnal tides is approximately 6 and 12 h respectively. However, if the tide is mixed, the pattern of extremes is more complicated. Figure 2 shows the water level at Victoria, British Columbia between July 24 and 31, 1976. It is a mixed tide where the shorter period fluctuations override the major diurnal oscillations with a continuous shift in their position and amplitude.

One characterization of the tide may be obtained by calculating the ratio of the amplitudes of the major harmonic constituents, M_2, S_2 K_1 and O_1. This value is called the form number (Dietrich, 1963) and is defined precisely as

$$F = \frac{\mathrm{K}_1 + \mathrm{O}_1}{\mathrm{M}_2 + \mathrm{S}_2}.$$

The tide is then said to be

(i) semidiurnal if $0 \leq F \leq 0.25$,
(ii) mixed if $0.25 < F \leq 3.00$,
(iii) diurnal if $F > 3.00$.

For Victoria, $F = 2.1$.

In accordance with this determination, Godin suggests the following maximum time interval values in which it can be assumed that there exists at most one extremum:

(i) $\Delta t = 3$ h for semidiurnal tide,
(ii) $\Delta t = 0.5$ h for mixed tide,
(iii) $\Delta t = 6$ h for diurnal tide.

Although in fact, a mixed tide may have extrema closer than 0.5 h, he feels that for practical purposes it is sufficient to note just one of them.

With these values of Δt we can then bracket all extrema by moving forward in time with steps of size, Δt, and comparing signs of the interval endpoints. Once such upper and lower bounds have been found, the extreme point can be located exactly by any one of a number of search techniques. Because it requires a minimal amount of time, the one chosen is Bolzano's method of bisection coupled with linear interpolation. Although the bisection method does not take the minimal number of iterations when compared to more sophisticated search techniques,

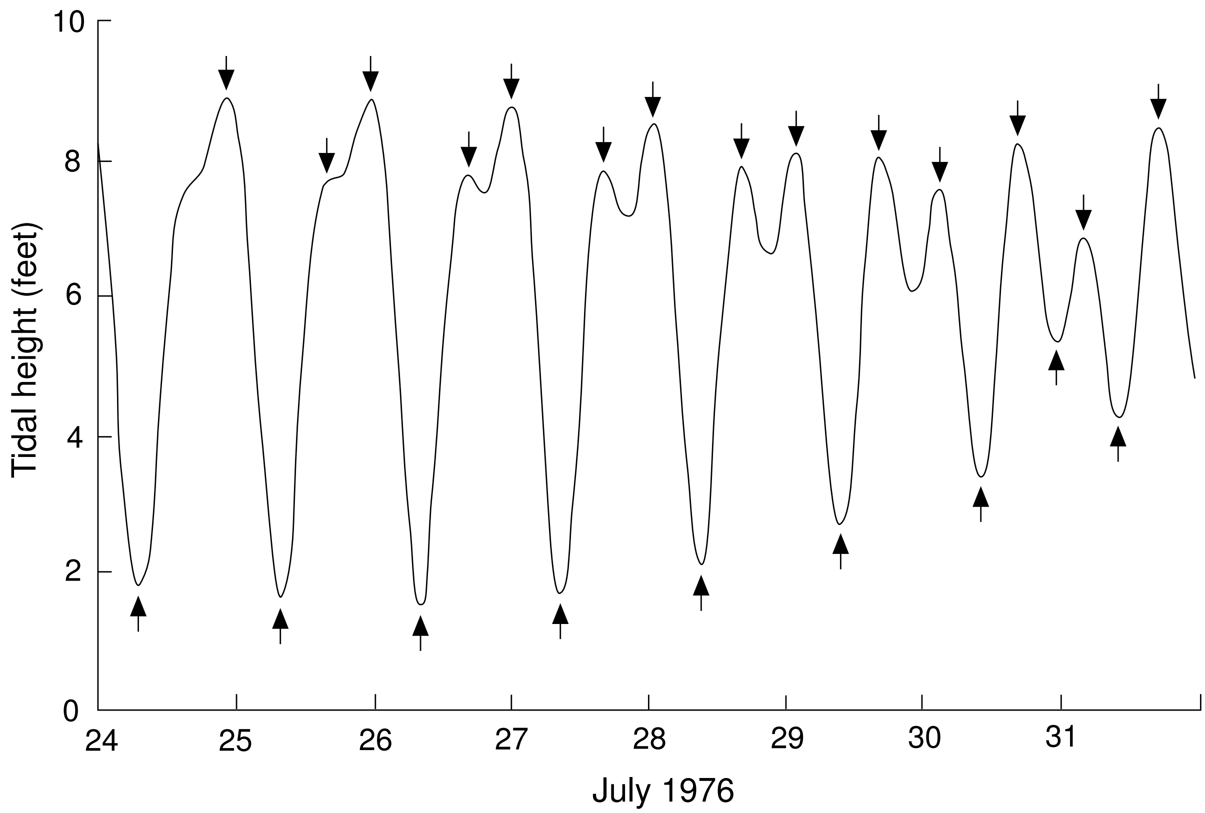

Figure 2 Synthesized water level at Victoria, British Columbia over the period July 24 to 31, 1976. The tide is of a mixed character with $F = 2.1$. The arrows indicate the time and height of the extrema predicted using the method described in Section 4.2. (Redrawn from C. Wallace)

it is able to make significant time savings by computing new sine function values as a linear combination of old ones and thus, unlike the other methods, avoid calls to the FORTRAN library function `SIN`.

In more detail, the search algorithm for an extremum is then as follows:

(i) Move forward in time from the origin, or the last extremum, in steps of Δt until either a change in sign exists between the derivative values at the endpoints of the interval (t_a, t_b), or t_b extends beyond the desired prediction period. Each constituent contribution in the summation $D(t)$ is evaluated by the Chebyshev iteration formula (3) of Section 4.1. When an interval containing an extremum is located, set $k = 1$ and proceed to (ii).

(ii) Calculate $t_k = t_a + \frac{1}{2^k}\Delta t$ and for each constituent in the sum evaluate $D(t_k)$ by using the formula

$$\sin(t_k) = \frac{\sin(t_a) + \sin(t_b)}{2\cos(1/2^k \Delta t)}.$$

If $|D(t_k)| \leq 10^{-16}$, set $D(t_k) = 10^{-16}$.

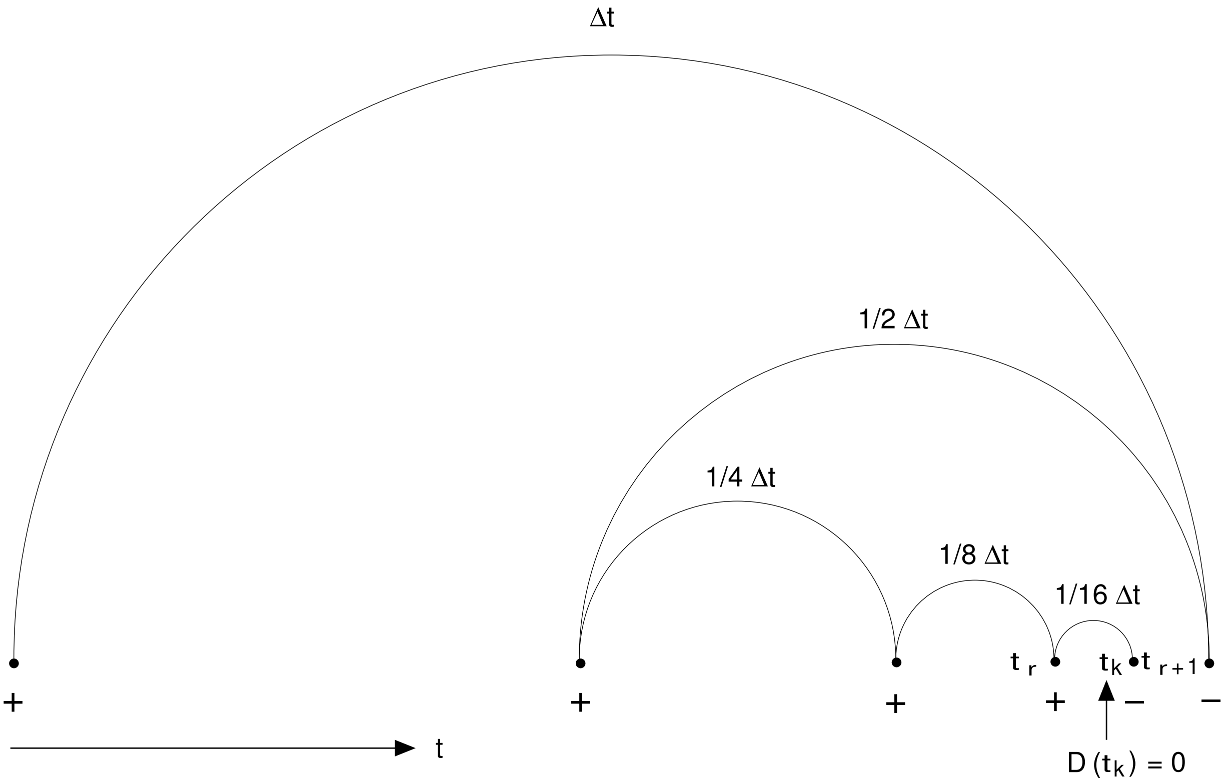

Figure 3 An example of the sequence of steps involved in locating a zero t_k of the derivative, $D(t)$. The sign of $D(t)$ at the various points tested is denoted by a plus or minus. After a step, Δt, the sign has changed; by a retrogression of $\frac{1}{2}\Delta t$, the sign has reverted to plus, forcing a forward step of $\frac{1}{4}\Delta t$ where the sign is still unchanged. Two further forward steps of $\frac{1}{8}\Delta t$ and $\frac{1}{16}\Delta t$ locate the minimum width interval (t_r, t_{r+1}) over which the position of t_k is determined by linear interpolation from the values of $D(t)$ at t_r and t_{r+1}. (Redrawn from C. Wallace)

(iii) Re-assign whichever of t_a or t_b has the same derivative sign as $D(t_k)$, by t_k. If the new interval length $t_b - t_a$ is less than 0.1 h, proceed to (iv). Otherwise set $k = k+1$ and return to (ii).

(iv) Use the following linear interpolation formula to find the extremum t_E,

$$t_E = t_a + [D(t_a)(t_b - t_a)]/[D(t_a) - D(t_b)],$$

and evaluate $h(t_E)$ via (1). For each constituent term in this sum, obtain the function value by using a pre-calculated stored table of 2002 cosine values with arguments in the range of 0° to 360°. Return to (i).

Figure 3 illustrates an example of the sequence of steps involved in the search for an extreme value. It is easily calculated that the number of iterations required to reduce the bracketing interval from Δt to 0.1 h is six for diurnal tides, three for mixed tides, and five for semidiurnal tides.

Arrows in Figure 2 indicate the extrema predicted for Victoria using the technique just described; the shaft of the arrow locates the time abscissa while the tip ends at the predicted height. The predicted hourly heights and the times and heights of all extrema are listed in Appendix 7.5.

5 CONSISTENCY OF THE ANALYSIS AND PREDICTION PROGRAMS

Although consistency between the tidal heights analysis program and the tidal heights prediction program was a major objective in their revision, they do have one difference. In

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Appendix 7.1 Standard Constituent Input Data for the Tidal Heights Analysis Computer Program. This Data is Read by the Program from Logical Unit 8.

```
Z0       0.0              M➚
SA       0.00011 0_ 1     SSA
SSA      0.000➚➚ 1ↄ91     Z0
MSM      0.001 09_ 0      MM
MM       0.001ↄ1➚1ↄ1      MSF
MSF      0.00➚ ➚19 ➚_     Z0
MF       0.00 0ↄ0091      MSF
ALP1     0.0   96ↄ699     ➚Q1
➚Q1      0.0 ↄ_06 ↄ0_     Q1
SIG1     0.0 ↄ90 _➚1      ➚Q1
Q1       0.0 _➚1 ↄ0➚6     O1
RHO1     0.0 _ ➚0 _ 6     Q1
O1       0.0  _ 06ↄ       K1
TAU1     0.0  9ↄ  1 6     O1
BET1     0.0 00 0  ↄ      NO1
NO1      0.0 0➚6 ↄ9       K1
CHI1     0.0 0 _096ↄ      NO1
PI1      0.0 1   ↄ1 0     P1
P1       0.0 1ↄↄ➚ↄ _1     K1
S1       0.0 16666_➚1     K1
K1       0.0 1_ 0_ 6➚     Z0
PSI1     0.0 1 9  ➚0      K1
PHI1     0.0 ➚00 90ↄ      K1
THE1     0.0  090ↄ➚_0     J1
J1       0.0  ➚9➚ 9 1     K1
➚PO1     0.0    _ ↄ19
SO1      0.0  60➚6_ 9     OO1
OO1      0.0    0   0     J1
UPS1     0.0 6  ➚9 9      OO1
ST 6     0.0_  ↄↄ   ↄ
➚NS➚     0.0_ 66ↄ16
ST _     0.0_  6_ↄ ↄ
ST1      0.0_  9   ➚
OQ➚      0.0_ↄ9_ 9 ↄ1     EPS➚
EPS➚     0.0_61__ 161     ➚N➚
ST➚      0.0_6 0ↄ _ↄ
ST       0.0__➚  1 9
O➚       0.0__ 61 0 9
➚N➚      0.0__  _09_0     MU➚
MU➚      0.0__6 9 6 0     N➚
SNK➚     0.0_ __10 9_
N➚       0.0_ 999➚        M➚
NU➚      0.0_9➚01619      N➚
ST       0.0_9 ↄↄ6_0
OP➚      0.0 0➚  ➚ 16
GAM➚     0.0 0 090➚96     H1
H1       0.0 0 9_ ➚66     M➚
M➚       0.0 0ↄ11 00_     Z0
H➚       0.0 06➚ↄ _       M➚
MKS➚     0.0 0_ 9ↄↄ9      M➚
STↄ      0.0 096__1 9
ST6      0.0 1ↄ9 0➚➚
LDA➚     0.0 1 ➚11 1ↄ     L➚
L➚       0.0 ➚0➚ ↄↄ➚ↄ     S➚
```

⚑SK⚑	0.0 10ↄ1‗ ⚑	
T⚑	0.0 ⚑19⚑ↄ9⚑	S⚑
S⚑	0.0	M⚑
R⚑	0.0 ‗ 0‗	S⚑
K⚑	0.0 ↄ61 9⚑	S⚑
MSN⚑	0.0 ↄ ↄ⚑	ETA⚑
ETA⚑	0.0 ↄ0‗ 6	K⚑
ST‗	0.0 ↄ 01 0	
⚑SM⚑	0.0 61ↄↄ⚑660	
ST	0.0 6 ↄ‗6 ‗0	
SKM⚑	0.0 6 ⚑ↄ1	
⚑SN⚑	0.0 ‗66‗ 1‗9	
NO	0.11‗‗⚑990	
MO	0.119⚑ ⚑0ↄↄ1	M
M	0.1⚑0‗6‗1010	M⚑
NK	0.1⚑0‗‗999ↄ0	
SO	0.1⚑⚑06 9 ‗	MK
MK	0.1⚑⚑⚑9⚑1 69	M
SP	0.1⚑ ↄ9⚑0	
SK	0.1⚑ↄ11 0‗96	MK
ST	0.1ↄ66 ‗16	
N	0.1ↄ‗99 9‗6	
MS	0.1ↄ ⚑00 6 ‗	
ST 9	0.1ↄ9⚑ ⚑ 90	
MN	0.1ↄ9ↄ106 9ↄ	M
ST9	0.1ↄ9‗ 0 6	
ST 0	0.160‗9 6 ⚑⚑	
M	0.1610⚑⚑ 01	M
ST10	0.161⚑ↄ0960	
SN	0.16⚑ ⚑ↄ ⚑1	M
KN	0.16⚑ↄ60‗ 1	
MS	0.16 ‗ 0	M
MK	0.16 0‗⚑ 9 1	MS
SL	0.16ↄ ↄ6 ↄ	
S	0.166666666‗	MS
SK	0.166 9 ⚑ↄ	S
MNOↄ	0.19 ⚑ 1 0 9	
⚑MOↄ	0.199‗ↄ ↄↄ	
MPↄ	0.1999 161 9	
MNKↄ	0.⚑01⚑91 9ↄ‗	
⚑MPↄ	0.⚑0⚑ↄ‗ↄ	
⚑MKↄ	0.⚑0⚑ 0 ↄ ‗ↄ	M
MSKↄ	0.⚑0ↄ6⚑ↄ 0⚑	
KMↄ	0.⚑0ↄ ↄ 6 9	
⚑SKↄ	0.⚑0 ‗ 1⚑9	⚑MKↄ
ST11	0.⚑ ‗⚑⚑ↄ90ↄ6	
⚑NM6	0.⚑ ↄ09 9	
ST1⚑	0.⚑ ‗ 0ↄ‗	
⚑MN6	0.⚑ 00⚑⚑0ↄ01	M6
ST1	0.⚑ 0⚑ↄ0⚑09	
ST 1	0.⚑ 1 060 ⚑9	
M6	0.⚑ 1ↄ ⚑0⚑0	⚑MKↄ
MSN6	0.⚑ ⚑ 9 ⚑	
MKN6	0.⚑ 0‗⚑1 19	
ST ⚑	0.⚑ 1⚑‗9‗ↄ6	
⚑MS6	0.⚑ ↄ61 ‗	M6

```
➚MK6      0.➚  ↄ  ➚9      ➚MS6
NSK6      0.➚ ↄ 9 0‗ 6
➚SM6      0.➚ ‗1‗ 06‗     ➚MS6
MSK6      0.➚ ‗ 06➚➚6     ➚SM6
S6        0.➚ↄ00000000
ST1       0.➚‗ ‗ↄ➚‗0 6
ST1ↄ      0.➚ 0➚906  ↄ
M‗        0.➚ 1‗ 990➚
ST16      0.➚  0 6‗ 91
 MK‗      0.➚   1 9  ➚    M6
ST1‗      0.➚ 61 6  09
ST1       0. 190➚1➚990
 MN       0. ➚0ↄ   ↄ0
ST19      0. ➚0‗616099
M         0. ➚➚0 ↄ60➚‗     MK‗
ST➚0      0. ➚  ↄↄ   ↄ
ST➚1      0. ➚ ↄ  ↄ ➚6
 MS       0. ➚  6‗ↄ ↄ
 MK       0. ➚ↄ09ↄ69
ST➚➚      0. ➚6 0ↄ ‗ↄ
ST➚       0. ➚‗6 9 6 0
ST➚       0. ➚‗91‗6➚‗1
ST➚ↄ      0. 60 0➚0 ↄ➚
ST➚6      0. 6➚ 1 19‗0
 MK9      0. 6  ➚6   9
ST➚‗      0. 666  ➚ 1ↄ
ST➚       0. 010   ↄ1ↄ
M10       0. 0➚ↄↄ‗00
ST➚9      0. 0  66‗  1
ST 0      0. 0ↄ ‗ 9 60
ST 1      0. 06916 ‗ↄ9
ST ➚      0. 0 ➚00 6 ‗
ST        0.   ‗1ↄ96 ➚➚
M1➚       0.   06  0 0
ST        0.  ↄ 90  6‗
ST ↄ      0.   ‗ ➚ ➚‗66

  .‗ ➚ ‗9‗0ↄↄ  .‗‗‗1900 ➚9  .ↄ1 ‗0ↄ1 0   . 6 1ↄ ➚ↄ9➚  .‗  ‗990160  000GMT 1/1/‗6
1 . ↄ9 019 6   .999  6 9 ↄ  .11➚9ↄ1‗9 ➚  .0ↄ 6 9 0ↄ6  .0000 ‗‗ 1   INCR./ 6ↄDAYS
    Z0      0  0  0  0  0  0 0.0     0
    SA      0  0  1  0  0 ,1 0.0     0
    SSA     0  0  ➚  0  0  0 0.0     0
    MSM     0  1 ,➚  1  0  0  .00    0
    MM      0  1  0 ,1  0  0 0.0     0
    MSF     0  ➚ ,➚  0  0  0 0.0     0
    MF      0  ➚  0  0  0  0 0.0     0
    ALP1    1 ,     ➚  1  0  0 ,.➚ↄ    ➚
    ALP1  ,1  0  0 .‗ↄ 0.0 60R1   0 ,1  0 .00 0.1906
    ➚Q1     1 ,     0  ➚  0  0,0.➚ↄ    ↄ
    ➚Q1   ,➚ ,➚  0 .ↄ0 0.006     ,1 ,1  0 .‗ↄ 0.0➚ 1R1  ,1  0  0 .‗ↄ 0.060‗R1
    ➚Q1    0 ,➚  0 .ↄ0 0.006      0 ,1  0 .0  0.1  ↄ
    SIG1    1 ,     ➚  0  0  0,0.➚ↄ
    SIG1  ,1  0  0 .‗ↄ 0.009ↄR1   0 ,➚  0 .ↄ0 0.0061     0 ,1  0 .0  0.1
    SIG1   ➚  0  0 .ↄ0 0.00 ‗
    Q1      1 ,➚  0  1  0  0,0.➚ↄ  10
    Q1    ,➚ ,    0 .ↄ0 0.000‗    ,➚ ,➚  0 .ↄ0 0.00 9    ,1 ,➚  0 .‗ↄ 0.0010R1
```

```
Q1     1  1  0 .   0.011 R1    1  0  0 .   0.0 9 R1    0     0 . 0 0.00
Q1     1  0  1 .0  0.000       0  1  0 .0  0.1         1  0  0 .   0.001 R1
Q1        0  0 . 0 0.00
RHO1    1       1  0  0 0.
RHO1   0     0 . 0 0.00        0  1  0 .0  0.1         1  0  0 .   0.01 1R1
RHO1      0  0 . 0 0.0  6         1  0 .0  0.01
O1      1  1  0  0  0  0 0.
O1     1  0  0 .   0.000 R1    0     0 . 0 0.00        0  1  0 .0  0.1
O1     1  1  0 .   0.000 R1    1  0  0 .   0.00 9R1    1  1  0 .   0.000 R1
O1        0  0 . 0 0.006          1  0 . 0 0.0010
TAU1    1  1     0  0  0 0.
TAU1      0  0 .0  0.0  6      1  0  0 .   0.0  6R1    0  1  0 . 0 0.0
TAU1   0  1  0 . 0 0. 1 0      0     0 . 0 0.01
BET1    1  0     1  0  0  . .     1
BET1   0  1  0 .00 0.  66
NO1     1  0  0  1  0  0 0.     9
NO1          0 . 0 0.00           1  0 .0  0.066          0  0 .0  0.  96
NO1    1  1  0 .   0.0  1R1    1  0  0 .   0.   R1     1  1  0 .   0.0 90R1
NO1    0  1  0 . 0 0.0 90      0  1  0 .0  0. 00       0     0 . 0 0.00
CHI1    1  0     1  0  0 0.
CHI1   0  1  0 . 0 0.0         0  1  0 .0  0. 1
PI1     1  1        0  0  1 0.     1
PI1    0  1  0 . 0 0.00
P1      1  1     0  0  0 0.     6
P1     0     0 .0  0.000       0  1  0 . 0 0.011       0  0    . 0 0.000
P1     1  0  0 .   0.000 R1       0  0 . 0 0.001          1  0 . 0 0.000
S1      1  1  1  0  0  1 0.
S1     0  0    .0  0.          0  1  0 . 0 0.0 6
K1      1  1  0  0  0  0 0.   10
K1        1  0 .0  0.000       1  1  0 .   0.0001R1    1  0  0 .   0.000 R1
K1     1  1  0 .   0.0001R1    0     0 .0  0.0001      0  1  0 . 0 0.019
K1     0  1  0 .0  0.1  6      0     0 . 0 0.00 9      1  0  0 .   0.000 R1
K1     1  1  0 .   0.0001R1
PSI1    1  1  1  0  0  1 0.     1
PSI1   0  1  0 .0  0.0190
PHI1    1  1     0  0  0 0.
PHI1      0  0 .0  0.0            1  0 .0  0.0106      0  0    .0  0.01
PHI1   0  1  0 . 0 0.0         0     0 . 0 0.01
THE1    1        1  0  0  . .
THE1      1  0 .00  .0 00      1  0  0 .   0.01 1R1    0  1  0 . 0  .0 1
THE1   0  1  0 .00  .199
J1      1     0  1  0  0 0.   10
J1     0  1  0 . 0 0.0 9       0  1  0 .0  0.19 0      0     0 . 0 0.00
J1     1  1  0 .   0.00  R1    1  0  0 .   0.0 16R1    1  1  0 .   0.0  1R1
J1     1     0 .   0.00  R1       0  0 . 0 0.01           1  0 . 0 0.009
J1           0 . 0 0.00
OO1     1     0  0  0  0 0.
OO1       1  0 . 0 0.00           0  0 .0  0.1 96         1  0 .0  0.0 96
OO1    1  0  0 .   0.0  0R1    1  1  0 .   0.0099R1    0  1  0 .0  0.6 9
OO1    0     0 .0  0.1         0     0 .0  0.00 6
UPS1    1     0  1  0  0  . .
UPS1      0  0 .00 0.0611      0  1  0 .00 0.6 99      0     0 .00 0.1 1
UPS1   1  0  0 .   0.0  9R1    1  1  0 .   0.0  R1
OQ                0     0  0 0.0
OQ     1  0  0 .   0.10  R     0  1  0 . 0 0.0  6
EPS                   1  0  0 0.0
EPS    1  1  0 .   0.00  R     1  0  0 .   0.0 0 R     0  1  0 . 0 0.0
```

```
?N?      ? ?  0  ?  0  0 0.0
?N?     ? ?  0 .?0 0.0061     1 1  0 .??0 0.011?R?  1  0  0 .??0 0.06? R?
?N?     0 1  0 .?0 0.0 ?
MU?      ? ?  ?  0  0  0 0.0
MU?     1 1  0 .??0 0.001 R?  1  0  0 .??0 0.010 R?   0 1  0 .?0 0.0 ??
N?       ? 1  0  1  0  0 0.0
N?      ? ?  0 .?0 0.00 9     1  0  1 .00 0.000       0 ?  0 .00 0.000?
N?      0 1  0 .?0 0.0 ?
NU?      ? 1  ? 1  0  0 0.0
NU?     0 1  0 .?0 0.0 ?      1  0  0 .??0 0.00 ?R?   ?  0  0 .0  0.00 ?
NU?     ? 1  0 .?0 0.00 6
GAM?     ? 0  ?  ?  0  0 ..?0
GAM?    ? ?  0 .00 0.1 ?9     1  0  0 .??0 0.0?9 R?   0 1  0 .?0 0.0  0
H1       ? 0  1  0  0  1 0.?0   ?
H1      0 1  0 .?0 0.0??      1  0  1 .?0 0.0  ?
M?       ? 0  0  0  0  0 0.0    9
M?      1 1  0 .??0 0.0001R?  1  0  0 .??0 0.000 R?   0 ?  0 .0  0.000?
M?      0 1  0 .?0 0.0 ?      1 1  0 .??0 0.0001R?   1  0  0 .??0 0.0009R?
M?      1 1  0 .??0 0.000?R?   ?  0  0 .0  0.0006      ?  1  0 .0  0.000?
H?       ? 0  1  0  0  1 0.0    1
H?      0 1  0 .?0 0.0?1?
LDA?     ? 1  ?  1  0  0 0.?0   1
LDA?    0 1  0 .?0 0.0
L?       ? 1  0  1  0  0 0.?0   ?
L?      0 1  0 .?0 0.0 66      ? 1  0 .00 0.00 ?      ?  0  0 .?0 0.??0?
L?      ? 1  0 .?0 0.110?      ?  ?  0 .?0 0.01?6
T?       ? ?     0  0  1 0.0    0
S?       ? ?  ?  0  0  0 0.0
S?      0 1  0 .0  0.00??     1  0  0 .??0 0.0001R?   ?  0  0 .0  0.0001
R?       ? ?  1  0  0  1 0.?0   ?
R?      0 0  ? .?0 0.?? ?     0  1  ? .0  0.01 1
K?       ? ?  0  0  0  0 0.0    ?
K?      1 0  0 .??0 0.00? R?  1  1  0 .??0 0.000 R?   0 1  0 .?0 0.01?
K?      0 1  0 .0  0.?9 0     0  ?  0 .0  0.0 ?
ETA?     ?    0  1  0  0 0.0    ?
ETA?    0 1  0 .?0 0.01 ?     0  1  0 .0  0. ??       0  ?  0 .0  0.0 6?
ETA?    1 0  0 .??0 0.0? ?R?  1  1  0 .??0 0.0  ?R?   1  ?  0 .??0 0.009 R?
ETA?    ? 0  0 .?0 0.00?
M          0  0  0  0  0 ..?0   1
M       0 1  0 .?0  .0?6

?PO1 ?   ?.0 P1        1.0 O1
SO1  ?   1.0 S?        1.0 O1
ST 6     ?.0 M?        1.0 N?        ?.0 S?
?NS? ?   ?.0 N?        1.0 S?
ST ? ?    .0 M?        ?.0 S?
ST1      ?.0 N?        1.0 K?        ?.0 S?
ST?      1.0 M?        1.0 N?        1.0 K?        ?.0 S?
ST       ?.0 M?        1.0 S?        ?.0 K?
O?   1   ?.0 O1
ST       ?.0 K?        1.0 N?        ?.0 S?
SNK?     1.0 S?        1.0 N?        1.0 K?
OP?  ?   1.0 O1        1.0 P1
MKS?     1.0 M?        1.0 K?        1.0 S?
ST?      1.0 M?        ?.0 K?        ?.0 S?
ST6      ?.0 S?        1.0 N?        1.0 M?        1.0 K?
?SK? ?   ?.0 S?        1.0 K?
```

MSN2		1.0 M2	1.0 S2	-1.0 N2	
ST7		2.0 K2	1.0 M2	-1.0 S2	-1.0 N2
2SM2	2	2.0 S2	-1.0 M2		
ST		2.0 M2	1.0 S2	-2.0 N2	
SKM2		1.0 S2	1.0 K2	-1.0 M2	
2SN2	2	2.0 S2	-1.0 N2		
NO	2	1.0 N2	1.0 O1		
MO	2	1.0 M2	1.0 O1		
NK	2	1.0 N2	1.0 K1		
SO	2	1.0 S2	1.0 O1		
MK	2	1.0 M2	1.0 K1		
SP	2	1.0 S2	1.0 P1		
SK	2	1.0 S2	1.0 K1		
ST		2.0 M2	1.0 N2	-1.0 S2	
N	1	2.0 N2			
MS	2	.0 M2	-1.0 S2		
ST 9		1.0 M2	1.0 S2	1.0 N2	-1.0 K2
MN	2	1.0 M2	1.0 N2		
ST 0		2.0 M2	1.0 S2	-1.0 K2	
ST9		1.0 M2	1.0 N2	1.0 K2	-1.0 S2
M	1	2.0 M2			
ST10		2.0 M2	1.0 K2	-1.0 S2	
SN	2	1.0 S2	1.0 N2		
KN	2	1.0 K2	1.0 N2		
MS	2	1.0 M2	1.0 S2		
MK	2	1.0 M2	1.0 K2		
SL	2	1.0 S2	1.0 L2		
S	1	2.0 S2			
SK	2	1.0 S2	1.0 K2		
MNO5		1.0 M2	1.0 N2	1.0 O1	
2MO5	2	2.0 M2	1.0 O1		
MP5	2	.0 M2	-1.0 P1		
MNK5		1.0 M2	1.0 N2	1.0 K1	
2MP5	2	2.0 M2	1.0 P1		
2MK5	2	2.0 M2	1.0 K1		
MSK5		1.0 M2	1.0 S2	1.0 K1	
KM5		1.0 K2	1.0 K1	1.0 M2	
2SK5	2	2.0 S2	1.0 K1		
ST11		.0 N2	1.0 K2	-1.0 S2	
2NM6	2	2.0 N2	1.0 M2		
ST12		2.0 N2	1.0 M2	1.0 K2	-1.0 S2
ST 1		.0 M2	1.0 S2	-1.0 K2	
2MN6	2	2.0 M2	1.0 N2		
ST1		2.0 M2	1.0 N2	1.0 K2	-1.0 S2
M6	1	.0 M2			
MSN6		1.0 M2	1.0 S2	1.0 N2	
MKN6		1.0 M2	1.0 K2	1.0 N2	
2MS6	2	2.0 M2	1.0 S2		
2MK6	2	2.0 M2	1.0 K2		
NSK6		1.0 N2	1.0 S2	1.0 K2	
2SM6	2	2.0 S2	1.0 M2		
MSK6		1.0 M2	1.0 S2	1.0 K2	
ST 2		2.0 M2	2.0 S2	-1.0 K2	
S6	1	.0 S2			
ST1		2.0 M2	1.0 N2	1.0 O1	
ST15		2.0 N2	1.0 M2	1.0 K1	
M7	1	.5 M2			

ST16	2.0 M2	1.0 S2	1.0 O1	
MK7 2	.0 M2	1.0 K1		
ST17	1.0 M2	1.0 S2	1.0 K2	1.0 O1
ST1 2	2.0 M2	2.0 N2		
MN 2	.0 M2	1.0 N2		
ST19	.0 M2	1.0 N2	1.0 K2	1.0 S2
M 1	.0 M2			
ST20	2.0 M2	1.0 S2	1.0 N2	
ST21	2.0 M2	1.0 N2	1.0 K2	
MS 2	.0 M2	1.0 S2		
MK 2	.0 M2	1.0 K2		
ST22	1.0 M2	1.0 S2	1.0 N2	1.0 K2
ST2 2	2.0 M2	2.0 S2		
ST2	2.0 M2	1.0 S2	1.0 K2	
ST25	2.0 M2	2.0 N2	1.0 K1	
ST26	.0 M2	1.0 N2	1.0 K1	
MK9 2	.0 M2	1.0 K1		
ST27	.0 M2	1.0 S2	1.0 K1	
ST2 2	.0 M2	1.0 N2		
M10 1	5.0 M2			
ST29	.0 M2	1.0 N2	1.0 S2	
ST 0 2	.0 M2	1.0 S2		
ST 1	2.0 M2	1.0 N2	1.0 S2	1.0 K2
ST 2 2	.0 M2	2.0 S2		
ST	.0 M2	1.0 S2	1.0 K1	
M12 1	6.0 M2			
ST 2	5.0 M2	1.0 S2		
ST 5	.0 M2	1.0 N2	1.0 K2	1.0 S2

Appendix 7.2 Sample Tidal Station Input Data for the Analysis Program.

The following sample input for logical unit 4 will produce an analysis of Tuktoyaktuk, Northwest Territories data for the period 1600 MST July 6, 1975 to 1400 MST September 9, 1975 inclusive, with constituents P_1 and K_2 inferred, shallow water constituent M_{10} specifically designated for analysis inclusion and only line printer output of the results. The final analysis results are listed in Appendix 7.3.

```
6   1.0        0.0
   K1        0.0 1_ 0_ 6➚    P1        0.0 1ↄↄ➚ↄ _1    0.  09     . _.0_
   S➚        0.0                K➚        0.0  ↄ61 9➚    0.➚_➚1ↄ   .➚➚. 0

     M10       M

  16060__ↄ  1 0909_ↄ
     6  ↄ  TUKTOYUKTUK NWT      MST 69➚_   0➚
1  6  ↄ        6 __ↄ
➚  6  ↄ        6 __ↄ             ➚1ↄ ➚➚  ➚1ↄ ➚0➚ ➚1ↄ ➚➚_ ➚   ➚ ➚ ➚
1  6  ↄ        _ __ↄ ➚➚9 ➚1  ➚06 ➚00 19  1 _ 1_9 1_6 1   199 ➚1ↄ ➚ 1
➚  6  ↄ        _ __ↄ ➚ↄ➚ ➚6  ➚60 ➚   ➚10 1_6 1ↄ  1 ↄ 1ↄ  16➚ 1 ➚ ➚0
1  6  ↄ          __ↄ ➚➚1 ➚ ➚ ➚ 0 19ↄ 1ↄ  119 10ↄ 11ↄ 1 ➚ 1ↄ9 19➚ ➚1
➚  6  ↄ          __ↄ ➚ 6 ➚6➚ ➚6  ➚ↄ➚ ➚➚  19_ 166 1ↄ  1ↄ9 1_  ➚01 ➚➚9
1  6  ↄ        9 __ↄ ➚ↄ1 ➚6_ ➚91 ➚ↄ_ ➚➚ↄ ➚0  1   1_6 1   1   ➚0  ➚ ➚
➚  6  ↄ        9 __ↄ ➚ↄↄ ➚_➚ ➚ ↄ ➚9  ➚96 ➚ↄ  199 1ↄ➚ 11➚ 111 1 ➚ 16_
1  6  ↄ       10 __ↄ ➚01 ➚➚1 ➚➚_ ➚➚  ➚01 166 1 1  99  _0    ➚ 1➚1 161
➚  6  ↄ       10 __ↄ ➚09 ➚6   0➚  ➚1  ➚9  0   ➚ↄ  ➚0ↄ 16  1   16  1 0
1  6  ↄ       11 __ↄ ➚1➚ ➚   ➚_1 ➚ ➚ ➚_  ➚ↄ  ➚➚1 169 1 ↄ 1➚6 1   1ↄ6
➚  6  ↄ       11 __ↄ 1 ➚ ➚19 ➚ 9 ➚ↄ_ ➚6➚ ➚   ➚0ↄ 16  1 ↄ 110 10ↄ 11
1  6  ↄ       1➚ __ↄ 1 ➚ 1_  ➚1   ➚ ➚ ➚ _ ➚   ➚0  1ↄ9 119   ↄ  _➚   9
➚  6  ↄ       1➚ __ↄ 116 1   1 0 ➚0ↄ ➚➚  ➚➚➚ 1 6 1 _ 10ↄ  66      ↄↄ
1  6  ↄ       1  __ↄ  _  10  1 6 16_ 19  199 1 ➚ 1   10_  _➚  ↄ   61
➚  6  ↄ       1  __ↄ   _ 10  1 9 16ↄ 1 ➚ 190 1 ↄ 1ↄ  1➚ↄ   9  ↄ9   9
1  6  ↄ       1  __ↄ  ↄↄ      11  1   16  1 ➚ 19  1   1ↄ  11        66
➚  6  ↄ       1  __ↄ  66    _ 11_ 1 6 16ↄ 1 0 1 1 16  1   100  6➚    0
1  6  ↄ       1ↄ __ↄ   6      _➚ 10  1➚9 1ↄ_ 16_ 16_ 1ↄ6 1 1 10ↄ    _
➚  6  ↄ       1ↄ __ↄ  _1   _➚   0  9ↄ 11  1➚  1 _ 1   1 ➚ 110       6
1  6  ↄ       16 __ↄ   ➚   ➚   ➚9  ↄ_  9  1➚ↄ 1 _ 1ↄ6 1ↄↄ 1   10ↄ    ➚
➚  6  ↄ       16 __ↄ  _0  6   6   _    9ↄ 116 1 1 1 ↄ 1 0 11ↄ  9    _ↄ
1  6  ↄ       1_ __ↄ  60  ↄ0  ↄ0  61      111 1 ↄ 1 9 1ↄ  1ↄ  1    1➚➚
➚  6  ↄ       1_ __ↄ  99   1   _➚   _     ↄ  99 11ↄ 1➚_ 1➚_ 11ↄ 10   9
1  6  ↄ       1  __ↄ   _   ↄ9  ↄ0  ↄ0  6     1 100 1 ➚ 16  1   1   1_9
➚  6  ↄ       1  __ↄ 1ↄ9 1 _ 1    1 ↄ 1 _ 1    1 _ 1   1ↄ  161 1_0 161
1  6  ↄ       19 __ↄ 1 _ 1 1 1 ➚ 1➚   1 0 1    160 1__ 19   ➚11 ➚ 0 ➚ 6
➚  6  ↄ       19 __ↄ ➚➚6 ➚0   1 1 16ↄ 1ↄ_ 161 1_  1_  1_   1 6 196 19
1  6  ↄ       ➚0 __ↄ ➚01 190 1_➚ 1ↄↄ 1     1 0 1 6 1ↄ6 1_   199 ➚➚_ ➚ ↄ
➚  6  ↄ       ➚0 __ↄ ➚ↄ   ➚ↄ6 ➚ ↄ 199 16➚ 1 1 1➚9 1    1ↄ_ 1    ➚06 ➚➚0
1  6  ↄ       ➚1 __ↄ ➚19 ➚➚➚ ➚10 19   1 ➚ 169 1_1 1    ➚06 ➚ 0 ➚ↄↄ ➚6ↄ
➚  6  ↄ       ➚1 __ↄ ➚    ➚96 ➚9➚ ➚ ➚ ➚6➚ ➚    ➚1➚ 190 1 6 19   ➚➚0 ➚ 0
1  6  ↄ       ➚➚ __ↄ ➚ↄ9 ➚6   ➚_1 ➚6   ➚ 9 ➚➚   ➚0  1    1 _ ➚06 ➚ 0 ➚ↄ_
➚  6  ↄ       ➚➚ __ↄ ➚    ➚9   ➚9ↄ ➚ ➚ ➚61 ➚ ➚ ➚0   1 ➚ 16ↄ 1_1 19➚ ➚1
1  6  ↄ       ➚  __ↄ ➚ ➚ ➚ _ ➚ↄↄ ➚ 9 ➚ 0 ➚0ↄ 1 1 1ↄ  1    1ↄ➚ 1 0 ➚09
➚  6  ↄ       ➚  __ↄ ➚   ➚60 ➚_➚ ➚61 ➚ 1 196 160 1 0 111 109 1➚ↄ 1ↄ_
1  6  ↄ       ➚  __ↄ 1 _ ➚09 ➚➚  ➚ 1 ➚09 1 1 1ↄↄ 1➚ↄ 110 111 1 0 1ↄ9
➚  6  ↄ       ➚  __ↄ 19ↄ ➚➚_ ➚ 9 ➚ↄ0 ➚   ➚00 161 1➚   9     _  9_ 1➚
```

```
1 6 ?   ?? ??? 1? 1 196 ?0? 19? 1? 1 101 ?1 ? 60 ?
? 6 ?   ?? ??? 1?? 1?9 1 ? ?0? 199 1?9 1 10 66 0 ?
1 6 ?   ?6 ??? ?? 10 1 ? 1?1 160 1?? 1?9 9 66 9 ?
? 6 ?   ?6 ??? ?9 11 1 16 1?? 16? 1?1 11? ? ?0 ?0 19
1 6 ?   ?? ??? 9 ? 10? 1 6 1 1? 1 1 11 9 ? ?9 16
? 6 ?   ?? ??? 1 ?6 10? 1 1 9 ?0? 196 1 ? 1 ? 16? 160 16
1 6 ?   ? ??? 16 1 ? ??? ?? ?60 ??? ? 1 ?6 ??6 ? 1 ??1 19
? 6 ?   ? ??? ?0 ? 0 ?? ?6 ? ? 01 ?91 ??0 ? ? ?1? 1 1?6
1 6 ?   ?9 ??? 1 ?00 ?? ? ? ??6 ?69 ? 0 ??0 ? ?16 19 16
? 6 ?   ?9 ??? 16 1?? ?01 ? ? ?6 ? ? ? 1 ?90 ??9 ? ?0? 1?9
1 6 ?   0 ??? 1?9 1 ?0? ??6 ? ? ??? ? 1 ??9 ?6 ? ?0? ??9
? 6 ?   0 ??? 16 1 ?10 ? ? ? ? ?? ?6 ??9 ? ??1 19 1
1 6 ?   1 ??? 1 0 1?6 19 ?0 ?1? ?? ? ? ? ? 1 ??? ?0? 1
? 6 ?   1 ??? 1?6
1 6 ?   1 ??
? 6 ?   1 ??
1 6 ?   ? ??
? 6 ?   ? ?? 10 9? 9 9? 10 11? 11 11 116 10
1 6 ?   ?? 9? 6 ?6 ?1 ? ?? 9? 11? 1 0 1 1 9
? 6 ?   ?? 1 1?0 10 ? ?1 ?6 ?? 66 1 9 109 10?
1 6 ?   ?? 9 ?? 9 ? 1 ? 1? ?0 9 110
? 6 ?   ?? 11? 116 10? ?1 ?? 6 60 10 1??
1 6 ?   ? ?? 1 1 6 11 6 ?0 6? 6? ?9 11 1 1?? ?0
? 6 ?   ? ?? ? ??6 ?66 ? 0 ?0 1?9 1 11? 11 1 6 16? 1 6
1 6 ?   6 ?? ?? ? ??? ?0 1 0 1? 1? 1?0 ?01 ??? ? ?
? 6 ?   6 ?? ?? ?60 ? ? ? 1 ?11 1 160 1 1 ? 1 ? 16? 19?
1 6 ?   ? ?? ??1 ? 9 ? 9 ? 9 ??? 1 1 111 10? 1?9 1?0 ?01
? 6 ?   ? ?? ? ??? ?60 ??? ??? 19? 1?6 1? 10? 11 1 9 1 0
1 6 ?   ?? ?11 ? ? ? ? ??? ??9 ?00 1?1 1 10? 9? 1?? 16
? 6 ?   ?? ?0? ?? ?9? 69 ? 1 ? ? ??1 1 16? 1??
1 6 ?   9 ?? ?1? ? 0 ?60 ? ? ? ??9 ??9 196 1? 1?6 1 ? ?0
? 6 ?   9 ?? ? ? ?9 ?6 69 ?0 ? ? 1 ? 9 ?9 ?9 ? ?
1 6 ?   10 ?? ?9 0 ?6 1 ? ? 1 ? ? ? ?? 1
? 6 ?   10 ?? ? 6? 0 ?0 ? ? ?6 1 ?
1 6 ?   11 ?? 6 ? 91 ?0? ? ?? 9 ?? ?? 9 90
? 6 ?   11 ?? 9 0 ?1 1? ?9 ? 00 ?6? ? ?
1 6 ?   1? ?? ? ? ??? ??? 0 ?? 9 9 0 ??? ?0 1 ? 1 ?
? 6 ?   1? ?? ?0 ? ? ?60 ? 1 19 1? ?9? ?? ? ? 19 16 1?
1 6 ?   1 ?? 1?? 1?1 19? ?1? ? 9 ??? ?? ?? ? ? ??? ?0? 1?9
? 6 ?   1 ?? 16? 1?? 190 ?1? ? ? ??? ?66 ?6 ? ?1? 1 ? 1??
1 6 ?   1 ?? 1 ? 1 16 19? ?? ? 6 ??9 ??6 ? 6 ?09 1 0 1?0
? 6 ?   1 ?? 1?9 1?? 1 6 161 1 ?00 ?0? ?0? 19? 1?? 1? 1 6
1 6 ?   1? ?? 116 10? 10 11? 1 0 16 19 ?0 ?16 ?0 196 1 ?
? 6 ?   1? ?? 1?9 1 ? 1 ? 16 1?? 1 19? ?0? ?0? ?0? 19? 1?6
1 6 ?   16 ?? 160 1 ? 1 ? 1 6 1?? 1?? 19? ?11 ?? ?? ??? ?10
? 6 ?   16 ?? 199 1 6 1?1 16? 16 169 1 0 190 ?01 ?0 ?00 19
1 6 ?   1? ?? 1 ? 1?? 16? 1?? 1?6 169 ?01 ??? ? 9 ??? ? ? ?
? 6 ?   1? ?? ?9? ? 0 ??9 ? 1 ??? ?11 ?11 ??6 ? ? ?6 ? 6 ?9?
1 6 ?   1 ?? ?96 ??? ? ? ?1 196 19 ?09 ??6 ? 9 ? ? ? ?
? 6 ?   1 ?? ? 6 ? 9 ??9 ?1 ?01 1 16? 1? 160 1 ?0? ??1
1 6 ?   19 ?? ??? ?? ?09 1 ? 1?9 1 1 1 1 9 16? 1 ? ?09 ??
? 6 ?   19 ?? ? 9 ? ? ? ?1? 1 1?? 1?9 119 1 ? 16? 19 ?1
1 6 ?   ?0 ?? ? ? ? 1 ? 0 ?0? 1? 1?1 1 0 11 1?? 1 ? 1?? ?0
? 6 ?   ?0 ?? ??6 ? ? ? ? ?? 19? 16? 1 1 10 10 11 1 1?
1 6 ?   ?1 ?? ?0 ??? ??9 ?? ?00 1?? 1?0 1?9 1 1 1 6 1? ?0?
? 6 ?   ?1 ?? ? 6 ?? ?6 ??6 ? 19 16? 1 ? 1?? 1 1?9 190
1 6 ?   ?? ?? ??1 ? 1 ??? ??? ? 1 ?00 16? 1 ? 119 11 1 166
? 6 ?   ?? ?? ?01 ? ??6 ?6 ? 9 ?1? 1?6 1 0 111 10 11? 1 0
```

```
1 6 ↄ ↗ _ↄ 1_1 ↗0 ↗ ↗ ↗ ↗ ↗ ↗1 1 0 1 6 1↗1 11ↄ 1↗6 1ↄ_
↗ 6 ↄ ↗ _ↄ 1 _ ↗11 ↗ ↄ ↗ 9 ↗ _ ↗↗9 19 1ↄ1 11 _ 110
1 6 ↄ ↗ _ↄ 1 1__ ↗06 ↗ 0 ↗ _ ↗↗ 1 6 1 0 9 6 66 9
↗ 6 ↄ ↗ _ↄ 1↗9 16ↄ 19_ ↗1ↄ ↗↗0 ↗0ↄ 1__ 1 11↗ 0 100
1 6 ↄ ↗ↄ _ↄ 1 6 1_ ↗0 ↗ ↗ↄ↗ ↗ ↗1_ 1 1 1 9 10ↄ 9 9
↗ 6 ↄ ↗ↄ _ↄ 1↗1 1ↄ9 1 199 ↗00 1 ↄ 1ↄↄ 1↗1 6 ↄ ↗
1 6 ↄ ↗6 _ↄ ↗ _↗ 1↗1 1_ ↗1ↄ ↗ _ ↗11 19_ ↗ ↗ 1_6 196
↗ 6 ↄ ↗6 _ↄ ↗ↄ0 ↗19 ↗_↗ 61 91 _6 ↄↄ 9 _0 ↗1 00 ↗ ↄ
1 6 ↄ ↗_ _ↄ ↗ ↗ ↄ0 0 ↗↗ 1ↄ 0ↄ 9 1↗ 0 ↄ ↄ09
↗ 6 ↄ ↗_ _ↄ ↄↄ_ ↄↄ9 ↄ ↄ60 ↄ_6 ↄↄ_ ↄ1 9 6↗ ↗↗
1 6 ↄ ↗ _ↄ _1 9 1 19 _↗ ↗ 0 0 ↗
↗ 6 ↄ ↗ _ↄ ↗ 0 ↗ 6 00 09 1↗ ↗ 19 ↗99 ↗_0 ↗↗_ ↗0 1 1
1 6 ↄ ↗9 _ↄ 19 ↗ 0 ↗ 1 1_ ↄ↗ ↄ1 61 ↄ0 ↄ ↄ ↄ0
↗ 6 ↄ ↗9 _ↄ ↗6 1_ 16 ↗ ↗_ 1 ↗9 ↗ ↗6 ↗ ↗ 0 ↗1ↄ
1 6 ↄ 0 _ↄ 19 19 ↗1_ ↗ 1 ↗ↄ6 ↗6↗ ↗61 ↗ↄ9 ↗ _ ↗↗9 ↗1 19ↄ
↗ 6 ↄ 0 _ↄ 1_ↄ 16 16 1__ 1 _ 19 ↗0 ↗0 191 1_1 1 119
1 6 ↄ 1 _ↄ 9_ 9↗ 10↗ 1↗ↄ 1ↄ0 16 1_6 1 19_ 19_ ↗06 ↗0↗
↗ 6 ↄ 1 _ↄ 191 1 6 19↗ ↗00 19_ 199 ↗06 ↗0ↄ ↗0_ ↗0 ↗0ↄ 19
1 6 ↄ 1 9_ↄ 1 ↄ 1 _ 19 ↗09 ↗ ↗ↄↄ ↗_ↄ ↗ ↄ 0ↄ ↗_ ↗ ↗0
↗ 6 ↄ 1 9_ↄ 01 ↗9ↄ ↗91 ↗_ↄ ↗__ ↗9 1↗ ↗ ↄ ↗1 ↗
1 6 ↄ ↗ 9_ↄ ↗ 1ↄ ↗ 16 1 ↗9 ↗1 1 1_ ↗9 6 6
↗ 6 ↄ ↗ 9_ↄ ↗_ 16 01 ↗ ↗6 ↗ ↄ ↗ 6 ↗ 1 ↗ ↗ 0 ↗ↄ0 ↗6↗
1 6 ↄ 9_ↄ ↗61 ↗ↄ0 ↗↗_ ↗0↗ 1_↗ 1ↄ 1ↄ 16↗ 1_1 1_↗ 190 ↗1
↗ 6 ↄ 9_ↄ ↗↗6 ↗↗ ↗1 1 6 160 1 1 ↗ 1ↄↄ 1_ ↗01 ↗ 6 ↗ↄↄ
1 6 ↄ 9_ↄ ↗_ ↗ ↗ ↗ ↗ↄↄ ↗16 1 16ↄ 1_9 ↗0 ↗ 1 ↗ↄ ↗9
↗ 6 ↄ 9_ↄ ↗_ 6 ↄ ↗ ↗99 ↗6↗ ↗↗_ ↗0_ ↗19 ↗ 0 ↗ↄ6 ↗_ↄ
1 6 ↄ ↄ 9_ↄ 0↗ 09 ↗9 ↗_ ↗ 0 196 1ↄ9 1 ↗ 1ↄ 19↗ ↗↗↗ ↗ 9
↗ 6 ↄ ↄ 9_ↄ ↗_0 ↗ 0 ↗ ↗ ↗69 ↗ 9 19_ 1ↄ 110 99 1↗ↄ 1ↄ9 1 _
1 6 ↄ 6 9_ↄ ↗1 ↗ ↄ ↗ 6 ↗↗1 1 9 1ↄ 11 6 ↄ↗ 6ↄ 106
↗ 6 ↄ 6 9_ↄ 1 ↗ 1ↄ1 16ↄ 1_ↄ 169 1 11ↄ _ ↗ 1 ↄ 0
1 6 ↄ _ 9_ↄ 6 1↗ 1 9 ↗1 19 16_ 1↗6 1 ↄ↗ 0 ↄ6 1
↗ 6 ↄ _ 9_ↄ 1↗1 1_ ↗0 ↗11 199 1_ 1 _ 100 _↗ ↄ ↄ_ 9↗
1 6 ↄ 9_ↄ 1 ↄ 1_9 ↗1 ↗ ↗ 0 ↗↗ 190 1 6 91 ↄ 6ↄ
↗ 6 ↄ 9_ↄ 99 1 0 1_ 19 19ↄ 1_ↄ 1 1 96 ↄ_ 0 ↄ0
1 6 ↄ 9 9_ↄ 6 1↗9 1_ ↗0 ↗1_ ↗0↗ 1_1 1↗ↄ _9 _ 0 ↄ9
↗ 6 ↄ 9 9_ↄ 1↗1
```

Appendix 7.3 Final Analysis Results Arising from the Input Data of Appendix 7.2 and the Standard Constituent Data Package of Appendix 7.1.

ANALYSIS OF HOURLY TIDAL HEIGHTS STN 6 ↄ 16H 6/ ¬/¬ↄ TO 1 H 9/ 9/¬ↄ
NO.OBS.= 1ↄↄ9 NO.PTS.ANAL.= 1ↄↄ9 MIDPT= H / /¬ↄ SEPARATION =1.00

NO	NAME	FREQUENCY	STN	M¸Y/ M¸Y	A	G		AL	GL
1	Z0	0.00000000	6 ↄ	¬¬ↄ/ 9¬ↄ	1.9 06	0.00		1.9 06	0.00
↗	MM	0.001ↄ1↗1ↄ	6 ↄ	¬¬ↄ/ 9¬ↄ	0.↗1↗1	↗6 .		0.↗1↗1	↗ .ↄ0
	MSF	0.00↗ ↗19	6 ↄ	¬¬ↄ/ 9¬ↄ	0.1ↄ61	1 . 0		0.1ↄ61	11ↄ.1ↄ
	ALP1	0.0 96ↄ¬	6 ↄ	¬¬ↄ/ 9¬ↄ	0.01ↄ↗	.9ↄ		0.01 1	1 0.96
ↄ	↗Q1	0.0 ↄ¬06 ↄ	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0↗ 6	↗.69		0.0↗↗6	↗ 6. ↗
6	Q1	0.0 ¬↗1 ↄ0	6 ↄ	¬¬ↄ/ 9¬ↄ	0.01ↄ	6ↄ.¬		0.01	↗ↄ↗.¬ↄ
¬	O1	0.0 ¬ 06ↄ	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0¬6	¬ .↗		0.069	↗ .
	NO1	0.0 0↗6 ↄ9	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0↗90	↗ .1		0.0 0	↗¬ↄ. ↄ
9	P1	0.0 1ↄↄ↗ↄ9	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0 6ↄ	¬1.¬6	INF FR K1	0.0 6	↗ↄ↗.↗0
10	K1	0.0 1¬ 0¬ↄ	6 ↄ	¬¬ↄ/ 9¬ↄ	0.1 06	6 .69		0.1 ↗	1 ↄ.ↄ
11	J1	0.0 ↗9↗90	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0↗ↄ	¬. ↗		0.0↗	10 .6
1↗	OO1	0.0 0	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0ↄ 1	↗ ↄ.¬		0.0 6	ↄ . ¬
1	UPS1	0.0 6 ↗99	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0↗9	91.¬		0.0↗	↗ 9.1↗
1	EPS↗	0.0¬61¬¬ 1	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0↗11	1 .ↄ9		0.0↗16	109.9
1ↄ	MU↗	0.0¬¬6 9 ¬	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0 19	.↗		0.0 ↗	0.06
16	N↗	0.0¬ 999↗ↄ	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0	.ↄ↗		0.0 ↄ¬	06. ↄ
1¬	M↗	0.0 0ↄ11 0	6 ↄ	¬¬ↄ/ 9¬ↄ	0. 90	¬¬.¬0		0.ↄ00¬	. 0
1	L↗	0.0 ↗0↗ ↄↄ	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0↗1	ↄ.↗1		0.01¬	16 .0
19	S↗	0.0	6 ↄ	¬¬ↄ/ 9¬ↄ	0.↗19ↄ	1↗6.6ↄ		0.↗19	6.¬
↗0	K↗	0.0 ↄ61 9	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0ↄ9¬	1 9.0ↄ	INF FR S↗	0.0ↄ1ↄ	1 1.1ↄ
↗1	ETA↗	0.0 ↄ0¬ 6	6 ↄ	¬¬ↄ/ 9¬ↄ	0.00¬1	↗ 6.0ↄ		0.00ↄ9	↗ ↄ.
↗↗	MO	0.119↗ ↗06	6 ↄ	¬¬ↄ/ 9¬ↄ	0.01	↗ .9¬		0.01	11. 6
↗	M	0.1↗0¬6¬10	6 ↄ	¬¬ↄ/ 9¬ↄ	0.01↗	↗61.ↄ¬		0.01↗6	1.91
↗	MK	0.1↗↗↗9↗1ↄ	6 ↄ	¬¬ↄ/ 9¬ↄ	0.00 9	1.60		0.00	9.1ↄ
↗ↄ	SK	0.1↗ↄ11 0	6 ↄ	¬¬ↄ/ 9¬ↄ	0.00↗	↗ ¬.69		0.00↗↗	↗↗ .6
↗6	MN	0.1ↄ9ↄ1066	6 ↄ	¬¬ↄ/ 9¬ↄ	0.009↗	↗ↄ6. ¬		0.0096	ↄ.00
↗¬	M	0.1610↗↗ 0	6 ↄ	¬¬ↄ/ 9¬ↄ	0.01↗6	↗91.¬		0.01 1	1 ↄ.1¬
↗	SN	0.16↗ ↗ↄ9	6 ↄ	¬¬ↄ/ 9¬ↄ	0.00	↗¬0. ↄ		0.00 ↄ	↗.¬
↗9	MS	0.16 ¬	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0010	9. ↄ		0.0011	1¬6.1
0	S	0.1666666¬	6 ↄ	¬¬ↄ/ 9¬ↄ	0.00 ¬	↗99.ↄ6		0.00 ¬	119.¬ↄ
1	↗MKↄ	0.↗0↗ 0 ↄↄ	6 ↄ	¬¬ↄ/ 9¬ↄ	0.001	10.10		0.001	↗ .
↗	↗SKↄ	0.↗0 ¬	6 ↄ	¬¬ↄ/ 9¬ↄ	0.00 ↄ	10 .00		0.00	ↄ.0
	↗MN6	0.↗ 00↗↗0ↄ	6 ↄ	¬¬ↄ/ 9¬ↄ	0.00 ↄ	↗¬1.↗		0.00	↗6. 6
	M6	0.↗ 1ↄ ↗0	6 ↄ	¬¬ↄ/ 9¬ↄ	0.001¬	1ↄ . 9		0.001	↗9 .9¬
ↄ	↗MS6	0.↗ ↄ61	6 ↄ	¬¬ↄ/ 9¬ↄ	0.00ↄ6	06.10		0.00ↄ9	69.ↄ9
6	↗SM6	0.↗ ¬1¬ 0	6 ↄ	¬¬ↄ/ 9¬ↄ	0.00↗	↗9 .9↗		0.00↗	ↄ. 0
¬	MK¬	0.↗ 1 9	6 ↄ	¬¬ↄ/ 9¬ↄ	0.00 6	↗1↗.↗ↄ		0.00 6	¬ .↗0
	M	0. ↗↗0 ↄↄ9	6 ↄ	¬¬ↄ/ 9¬ↄ	0.00 0	↗.		0.00	109.↗↗
9	M10	0. 0↗ↄↄ699	6 ↄ	¬¬ↄ/ 9¬ↄ	0.0009	19 .↗		0.0010	191.¬1

Appendix 7.4 Sample Input for the Tidal Heights Prediction Program.

The following sample input for logical unit 8 will synthesize hourly heights and the times and heights of all extrema at Victoria, British Columbia for the period 0100 PST July 1, 1976 to 2400 PST July 31, 1976 inclusive. The output results are listed in Appendix 7.5.

```
 .¬ ➶ ¬9¬0ↄↄ  .¬¬¬1900 ➶9  .ↄ1 ¬0ↄ1 0   . 6 1ↄ ➶ↄ9➶  .¬  ¬990160  000GMT 1/1/¬6
1 . ↄ9 019 6   .999  6 9 ↄ  .11➶9ↄ1¬9 ➶  .0ↄ 6 9 0ↄ6  .0000 ¬¬ 1   INCR./ 6ↄDAYS
     Z0       0  0  0  0  0  0 0.0     0
     SA       0  0  1  0  0 ,1 0.0     0
     SSA      0  0  ➶  0  0  0 0.0     0
     MSM      0  1 ,➶  1  0  0  .00    0
     MM       0  1  0 ,1  0  0 0.0     0
     MSF      0  ➶ ,➶  0  0  0 0.0     0
     MF       0  ➶  0  0  0  0 0.0     0
     ALP1     1 ,   ➶  1  0  0 ,.➶ↄ    ➶
     ALP1    ,1  0  0 .¬ↄ 0.0 60R1   0 ,1  0 .00 0.1906
     ➶Q1      1 ,   0  ➶  0  0,0.➶ↄ    ↄ
     ➶Q1     ,➶ ,➶  0 .ↄ0 0.006      ,1 ,1  0 .¬ↄ 0.0➶ 1R1  ,1  0  0 .¬ↄ 0.060¬R1
     ➶Q1     0 ,➶  0 .ↄ0 0.006       0 ,1  0 .0  0.1  ↄ
     SIG1     1 ,   ➶  0  0  0,0.➶ↄ
     SIG1    ,1  0  0 .¬ↄ 0.009ↄR1   0 ,➶  0 .ↄ0 0.0061      0 ,1  0 .0  0.1
     SIG1    ➶  0  0 .ↄ0 0.00 ¬
     Q1       1 ,➶  0  1  0  0,0.➶ↄ  10
     Q1      ,➶ ,   0 .ↄ0 0.000¬     ,➶ ,➶  0 .ↄ0 0.00 9     ,1 ,➶  0 .¬ↄ 0.0010R1
     Q1      ,1 ,1  0 .¬ↄ 0.011ↄR1  ,1  0  0 .¬ↄ 0.0➶9➶R1   0 ,➶  0 .ↄ0 0.00ↄ¬
     Q1      ,1  0  1 .0  0.000       0 ,1  0 .0  0.1        1  0  0 .¬ↄ 0.001 R1
     Q1      ➶  0  0 .ↄ0 0.00➶
     RHO1     1 ,➶  ➶ ,1  0  0,0.➶ↄ    ↄ
     RHO1    0 ,➶  0 .ↄ0 0.00ↄ       0 ,1  0 .0  0.1  ➶      1  0  0 .¬ↄ 0.01 1R1
     RHO1    ➶  0  0 .ↄ0 0.0ↄ¬6      ➶  1  0 .0  0.01¬ↄ
     O1       1 ,1  0  0  0  0,0.➶ↄ
     O1      ,1  0  0 .➶ↄ 0.000 R1   0 ,➶  0 .ↄ0 0.00ↄ       0 ,1  0 .0  0.1  ↄ
     O1      1 ,1  0 .➶ↄ 0.000 R1   1  0  0 .¬ↄ 0.00➶9R1   1  1  0 .➶ↄ 0.000 R1
     O1      ➶  0  0 .ↄ0 0.006       ➶  1  0 .ↄ0 0.0010
     TAU1     1 ,1  ➶  0  0  0,0.¬ↄ    ↄ
     TAU1    ,➶  0  0 .0  0.0  6     ,1  0  0 .➶ↄ 0.0 ➶6R1   0 ,1  0 .ↄ0 0.0➶
     TAU1    0  1  0 .ↄ0 0.➶1¬0      0  ➶  0 .ↄ0 0.01 ➶
     BET1     1  0 ,➶  1  0  0 ,.¬ↄ    1
     BET1    0 ,1  0 .00 0.➶➶66
     NO1      1  0  0  1  0  0,0.¬ↄ    9
     NO1     ,➶ ,➶  0 .ↄ0 0.00ↄ¬     ,➶ ,1  0 .0  0.066ↄ     ,➶  0  0 .0  0. ↄ96
     NO1     ,1 ,1  0 .¬ↄ 0.0  1R1  ,1  0  0 .➶ↄ 0.➶➶➶¬R1  ,1  1  0 .¬ↄ 0.0➶90R1
     NO1     0 ,1  0 .ↄ0 0.0➶90      0  1  0 .0  0.➶00       0  ➶  0 .ↄ0 0.00ↄ
     CHI1     1  0  ➶ ,1  0  0,0.¬ↄ    ➶
     CHI1    0 ,1  0 .ↄ0 0.0➶ ➶      0  1  0 .0  0.➶1 ¬
     PI1      1  1 ,    0  0  1,0.➶ↄ    1
     PI1     0 ,1  0 .ↄ0 0.00¬
     P1       1  1 ,➶  0  0  0,0.➶ↄ    6
     P1      0 ,➶  0 .0  0.000       0 ,1  0 .ↄ0 0.011➶      0  0  ➶ .ↄ0 0.000
     P1      1  0  0 .¬ↄ 0.000 R1   ➶  0  0 .ↄ0 0.001ↄ      ➶  1  0 .ↄ0 0.000
     S1       1  1 ,1  0  0  1,0.¬ↄ    ➶
     S1      0  0 ,➶ .0  0. ↄ        0  1  0 .ↄ0 0.0➶6
```

```
K1      1  1  0  0  0  0 0.  10
K1         1  0 .0  0.000      1  1  0 .  0.0001R1    1  0  0 .  0.000 R1
K1       1  1  0 .  0.0001R1   0     0 .0  0.0001     0  1  0 . 0 0.019
K1       0  1  0 .0  0.1 6     0     0 . 0 0.00 9     1  0  0 .  0.000 R1
K1       1  1  0 .  0.0001R1
PSI1     1  1  1  0  0  1 0.   1
PSI1     0  1  0 .0  0.0190
PHI1     1  1     0  0  0 0.
PHI1        0  0 .0  0.0          1  0 .0  0.0106     0  0    .0  0.01
PHI1     0  1  0 . 0 0.0       0     0 . 0 0.01
THE1     1        1  0  0  .
THE1        1  0 .00  .0 00    1  0  0 .  0.01 1R1    0  1  0 . 0  .0 1
THE1     0  1  0 .00  .199
J1       1     0  1  0  0 0.  10
J1       0  1  0 . 0 0.0 9     0  1  0 .0  0.19 0     0     0 . 0 0.00
J1       1  1  0 .  0.00 R1    1  0  0 .  0.0 16R1    1  1  0 .  0.0  1R1
J1       1     0 .  0.00 R1       0  0 . 0 0.01       1  0 . 0 0.009
J1             0 . 0 0.00
OO1      1     0  0  0  0 0.
OO1         1  0 . 0 0.00         0  0 .0  0.1 96        1  0 .0  0.0 96
OO1      1  0  0 .  0.0  0R1   1  1  0 .  0.0099R1    0  1  0 .0  0.6 9
OO1      0     0 .0  0.1       0     0 .0  0.00 6
UPS1     1     0  1  0  0  .
UPS1        0  0 .00 0.0611    0  1  0 .00 0.6 99     0     0 .00 0.1 1
UPS1     1  0  0 .  0.0  9R1   1  1  0 .  0.0  R1
OQ             0     0  0 0.0
OQ       1  0  0 .  0.10  R    0  1  0 . 0 0.0  6
EPS               1  0  0 0.0
EPS      1  1  0 .  0.00 R     1  0  0 .  0.0 0 R     0  1  0 . 0 0.0
N              0     0  0 0.0
N              0 . 0 0.0061    1  1  0 .  0.011 R     1  0  0 .  0.06  R
N        0  1  0 . 0 0.0
MU                0  0  0 0.0
MU       1  1  0 .  0.001 R    1  0  0 .  0.010 R     0  1  0 . 0 0.0
N           1  0  1  0  0 0.0
N              0 . 0 0.00 9    1  0  1 .00 0.000      0     0 .00 0.000
N        0  1  0 . 0 0.0
NU          1     1  0  0 0.0
NU       0  1  0 . 0 0.0       1  0  0 .  0.00  R        0  0 .0  0.00
NU          1  0 . 0 0.00 6
GAM         0        0  0  . 0
GAM            0 .00 0.1  9    1  0  0 .  0.0 9 R     0  1  0 . 0 0.0  0
H1          0  1  0  0  1 0. 0
H1       0  1  0 . 0 0.0       1  0  1 . 0 0.0
M           0  0  0  0  0 0.0    9
M        1  1  0 .  0.0001R    1  0  0 .  0.000 R     0     0 .0  0.000
M        0  1  0 . 0 0.0       1  1  0 .  0.0001R     1  0  0 .  0.0009R
M        1  1  0 .  0.000 R       0  0 .0  0.0006        1  0 .0  0.000
H           0  1  0  0  1 0.0    1
H        0  1  0 . 0 0.0 1
LDA         1     1  0  0 0. 0   1
LDA      0  1  0 . 0 0.0
L           1  0  1  0  0 0. 0
L        0  1  0 . 0 0.0 66       1  0 .00 0.00          0  0 . 0 0. 0
L           1  0 . 0 0.110              0 . 0 0.01 6
T                  0  0  1 0.0   0
S                  0  0  0 0.0
S        0  1  0 .0  0.00      1  0  0 .  0.0001R        0  0 .0  0.0001
```

```
R          1  0  0  1 0. 0
R       0  0   . 0 0.         0  1    .0  0.01 1
K             0  0  0  0 0.0
K        1  0  0 .  0.00  R    1  1  0 .   0.000 R    0  1  0 . 0 0.01
K       0  1  0 .0  0. 9 0     0    0 .0  0.0
ETA            0  1  0  0 0.0
ETA     0  1  0 . 0 0.01       0  1  0 .0  0.          0     0 .0  0.0 6
ETA     1  0  0 .   0.0   R    1  1  0 .   0.0   R     1     0 .   0.009 R
ETA        0  0 . 0 0.00
M          0  0  0  0  0  . 0   1
M       0  1  0 . 0  .0 6

PO1      .0 P1         1.0 O1
SO1     1.0 S          1.0 O1
ST 6     .0 M          1.0 N          .0 S
NS       .0 N          1.0 S
ST       .0 M           .0 S
ST1      .0 N          1.0 K          .0 S
ST      1.0 M          1.0 N         1.0 K          .0 S
ST       .0 M          1.0 S          .0 K
O    1   .0 O1
ST       .0 K          1.0 N          .0 S
SNK     1.0 S          1.0 N         1.0 K
OP      1.0 O1         1.0 P1
MKS     1.0 M          1.0 K         1.0 S
ST      1.0 M           .0 K          .0 S
ST6      .0 S          1.0 N         1.0 M         1.0 K
SK       .0 S          1.0 K
MSN     1.0 M          1.0 S         1.0 N
ST       .0 K          1.0 M         1.0 S         1.0 N
SM       .0 S          1.0 M
ST       .0 M          1.0 S          .0 N
SKM     1.0 S          1.0 K         1.0 M
SN       .0 S          1.0 N
NO      1.0 N          1.0 O1
MO      1.0 M          1.0 O1
NK      1.0 N          1.0 K1
SO      1.0 S          1.0 O1
MK      1.0 M          1.0 K1
SP      1.0 S          1.0 P1
SK      1.0 S          1.0 K1
ST       .0 M          1.0 N         1.0 S
N    1   .0 N
 MS      .0 M          1.0 S
ST 9    1.0 M          1.0 S         1.0 N         1.0 K
MN      1.0 M          1.0 N
ST 0     .0 M          1.0 S         1.0 K
ST9     1.0 M          1.0 N         1.0 K         1.0 S
M    1   .0 M
ST10     .0 M          1.0 K         1.0 S
SN      1.0 S          1.0 N
KN      1.0 K          1.0 N
MS      1.0 M          1.0 S
MK      1.0 M          1.0 K
SL      1.0 S          1.0 L
S    1   .0 S
SK      1.0 S          1.0 K
```

```
MNO5      1.0 M2      1.0 N2      1.0 O1
2MO5 2    2.0 M2      1.0 O1
 MP5 2     .0 M2      1.0 P1
MNK5      1.0 M2      1.0 N2      1.0 K1
2MP5 2    2.0 M2      1.0 P1
2MK5 2    2.0 M2      1.0 K1
MSK5      1.0 M2      1.0 S2      1.0 K1
 KM5      1.0 K2      1.0 K1      1.0 M2
2SK5 2    2.0 S2      1.0 K1
ST11       .0 N2      1.0 K2      1.0 S2
2NM6 2    2.0 N2      1.0 M2
ST12      2.0 N2      1.0 M2      1.0 K2      1.0 S2
ST 1       .0 M2      1.0 S2      1.0 K2
2MN6 2    2.0 M2      1.0 N2
ST1       2.0 M2      1.0 N2      1.0 K2      1.0 S2
M6   1     .0 M2
MSN6      1.0 M2      1.0 S2      1.0 N2
MKN6      1.0 M2      1.0 K2      1.0 N2
2MS6 2    2.0 M2      1.0 S2
2MK6 2    2.0 M2      1.0 K2
NSK6      1.0 N2      1.0 S2      1.0 K2
2SM6 2    2.0 S2      1.0 M2
MSK6      1.0 M2      1.0 S2      1.0 K2
ST 2      2.0 M2      2.0 S2      1.0 K2
S6   1     .0 S2
ST1       2.0 M2      1.0 N2      1.0 O1
ST15      2.0 N2      1.0 M2      1.0 K1
M    1     .5 M2
ST16      2.0 M2      1.0 S2      1.0 O1
 MK  2     .0 M2      1.0 K1
ST1       1.0 M2      1.0 S2      1.0 K2      1.0 O1
ST1  2    2.0 M2      2.0 N2
 MN  2     .0 M2      1.0 N2
ST19       .0 M2      1.0 N2      1.0 K2      1.0 S2
M    1     .0 M2
ST20      2.0 M2      1.0 S2      1.0 N2
ST21      2.0 M2      1.0 N2      1.0 K2
 MS  2     .0 M2      1.0 S2
 MK  2     .0 M2      1.0 K2
ST22      1.0 M2      1.0 S2      1.0 N2      1.0 K2
ST2  2    2.0 M2      2.0 S2
ST2       2.0 M2      1.0 S2      1.0 K2
ST25      2.0 M2      2.0 N2      1.0 K1
ST26       .0 M2      1.0 N2      1.0 K1
 MK9 2     .0 M2      1.0 K1
ST2        .0 M2      1.0 S2      1.0 K1
ST2  2     .0 M2      1.0 N2
M10  1    5.0 M2
ST29       .0 M2      1.0 N2      1.0 S2
ST 0 2     .0 M2      1.0 S2
ST 1      2.0 M2      1.0 N2      1.0 S2      1.0 K2
ST 2 2     .0 M2      2.0 S2
ST         .0 M2      1.0 S2      1.0 K1
M12  1    6.0 M2
ST   2    5.0 M2      1.0 S2
ST 5       .0 M2      1.0 N2      1.0 K2      1.0 S2
```

```
          _1 0 VICTORIA HARBOUR BC   PST             1
          Z0                                 6.06_0      .00
          Q1                                  .19_0 1 0. 0
          O1                                 1. 110 1 _.00
          NO1                                0.11 0 1 0. 0
          P1                                  .6_ 0 1   . 0
          S1                                  .09 0 1  .10
          K1                                  .0_00 1 9. 0
          J1                                  .11_0 166. 0
          N                                   . 9 0  6 . 0
          M                                  1. 1 0    _.00
          S                                   .   0  9 .90
00100_0_6 0 100_0_6 EQUI  1.0
00100_0_6 0 100_0_6 EXTR  0.
```

Appendix 7.5 Tidal Heights Prediction Results Arising from the Input Data of Appendix 7.4. Figure 2 is the Plot of These Hourly Heights over the Period 0100 PST July 24, 1976 to 2400 PST July 31, 1976.

```
 STN   1ST HR  DATE    1     ➚                       ↄ     6     _                 DT HRS

_1➚0   1.0000  1 __6 _. ↄ9  _._ 6  _.9➚6 _.   6  _.ↄ1  6._99 ↄ._9_  .6ↄ          1.0000
_1➚0   9.0000  1 __6  .ↄ_   ➚._ↄ9 ➚. 61 ➚.  _0  .06    .0 _ ↄ.➚➚6 6. 9          1.0000
_1➚0 1_.0000  1 __6 _. ↄ6  _.9_9   .➚11   .09➚ _._ ➚ _.➚    6. 96 6.6_6          1.0000
_1➚0   1.0000  ➚ __6 6.66   6. ➚   _.0ↄ1 _.➚16 _.1 9 6.   _ 6.➚96 ↄ.   ➚          1.0000
_1➚0   9.0000  ➚ __6  .ↄ 1  ._66   .➚10   .0      . ➚    .01➚  .99ↄ 6.09_          1.0000
_1➚0 1_.0000  ➚ __6 _.1➚➚ _. 99   . 1    .       .0➚➚ _.      6. _  6. ↄ1          1.0000
_1➚0   1.0000    __6 6.011 ↄ.900 ↄ.9    6.1 6 6. _ↄ 6.  6 6.➚ 9 ↄ.91          1.0000
_1➚0   9.0000    __6 ↄ. 6   ._ _   .➚11   . 96   .911  .➚9   ↄ.00   ↄ.9          1.0000
_1➚0 1_.0000    __6 6.919 _._ ➚   . _0   .ↄ 6   . 0_ _. 9   _.16   6. _ↄ          1.0000
_1➚0   1.0000    __6 ↄ.6 0 ↄ.19ↄ  .9_6 ↄ.009 ↄ.➚➚0 ↄ. 96 ↄ._➚➚ ↄ. 0_          1.0000
_1➚0   9.0000    __6 ↄ._1➚ ↄ. 6➚ ↄ.1 6   . ↄ1   ._➚_   . ↄ_ ↄ.➚__ ↄ.9ↄ_          1.0000
_1➚0 1_.0000    __6 6._9_ _.6ↄ➚   . ↄ_   .__0   ._9     . ➚    _._06 6.__0          1.0000
_1➚0   1.0000  ↄ __6 ↄ.__    . 99   .➚6_   .9ↄ    .9_ↄ   .➚ↄↄ   .6     ↄ.1          1.0000
_1➚0   9.0000  ↄ __6 ↄ.ↄ09 ↄ._1   ↄ._61 ↄ.690 ↄ.ↄ9   ↄ.ↄ96 ↄ.__ↄ 6.1 ➚          1.0000
_1➚0 1_.0000  ↄ __6 6._9   _.ↄ 9   .➚_1   .   6 9.09    .9      .  1  _. ↄ          1.0000
_1➚0   1.0000  6 __6 6. 01 ↄ.10➚  .0 6   .➚9➚ ➚.9 6 ➚.996   . 09   .0ↄ          1.0000
_1➚0   9.0000  6 __6  .__ↄ ↄ.  ➚ ↄ.9➚  6.➚11 6. ➚ↄ 6. ↄ0 6. 99 6.ↄ_          1.0000
_1➚0 1_.0000  6 __6 6.9 ↄ  _. _9   .1 1   ._6➚ 9.➚10 9. ➚9 9.0➚0   .➚6          1.0000
_1➚0   1.0000  _ __6 _.1➚_ ↄ._6     . _6   .1 0 ➚. ↄ_ ➚.019 ➚.1 _ ➚._ _          1.0000
_1➚0   9.0000  _ __6  .6_➚   .661 ↄ.ↄ_6 6.➚ _ 6._ _ 6.9ↄ0 _.01   _.0ↄ          1.0000
_1➚0 1_.0000  _ __6 _.1     _. 9   _.9_6   .ↄ61 9.11  9. 6ↄ 9. ↄ    .991          1.0000
_1➚0   1.0000    __6  .0ↄ0 6._➚0 ↄ.1_6   .6     ➚.  __ 1.ↄ60 1. 1   1.6          1.0000
_1➚0   9.0000    __6 ➚. 6ↄ   .ↄ 9   . 0   ↄ.90   6._     _.➚ↄ9 _.    ➚ _.ↄ16          1.0000
_1➚0 1_.0000    __6 _.ↄ0_ _.ↄ     _.   6   .➚ 6   . ➚_ 9. ➚0 9.ↄ    9. ↄ6          1.0000
_1➚0   1.0000  9 __6  .      _._ 0 6.➚ 1   .ↄ_➚ ➚.9_9 1._19 0.99   0.906          1.0000
_1➚0   9.0000  9 __6 1.     ➚. _    ._99 ↄ.16   6. ↄ➚  _.➚1ↄ _._01 _. 60          1.0000
_1➚0 1_.0000  9 __6 _. ➚1  _._     _.___   .00ↄ   . ➚➚   .9 ➚ 9. _1 9.ↄ          1.0000
_1➚0   1.0000 10 __6 9. 0➚   .ↄ _ _. 06 ↄ._1ↄ  .00➚ ➚.   _ 1.➚ ➚ 0._➚ↄ          1.0000
_1➚0   9.0000 10 __6 0.  6 1.ↄ96 ➚. 1➚   .➚ↄ  ↄ.6_ↄ 6.      _.6 ➚   .010          1.0000
_1➚0 1_.0000 10 __6  .0ↄ_ _.9➚   _._ ➚ _._ ➚ _.996   . 01    .  1 9.➚60          1.0000
_1➚0   1.0000 11 __6 9.  ↄ   .9 6   .1➚1 6. 00 ↄ.1 6   .ↄ➚➚ ➚.0 6 1.1➚1          1.0000
_1➚0   9.0000 11 __6 0._91 1.1     ➚.0_    . 0     .   6 6.➚ↄ   _. 06 _.9 1          1.0000
_1➚0 1_.0000 11 __6  .16    .0__ _. ↄ0 _.660 _.6     _. ↄ1   .➚ 0   .6_9          1.0000
_1➚0   1.0000 1➚ __6  .9     .9_    .ↄ19 _.ↄ 6 6.➚ↄↄ   ._0_   .191 1.9_1          1.0000
_1➚0   9.0000 1➚ __6 1.➚61 1.1    1._ 0 ➚. 0    .1_6 ↄ.ↄ91 6. 16 _.6 1          1.0000
_1➚0 1_.0000 1➚ __6  .116   .16   _.9    _.6 6 _. ➚    _. 9   _.ↄ9_ _.9ↄ6          1.0000
_1➚0   1.0000 1  __6  .      .ↄↄ    . ↄ➚ _.9    6.991 ↄ._➚9   .  _    .0ↄ          1.0000
_1➚0   9.0000 1  __6 ➚.11   1.6    1. ↄ   ➚.ↄ_     .699 ↄ.01ↄ 6.➚     _. 01          1.0000
_1➚0 1_.0000 1  __6 _.9 ↄ   .1ↄ    .0     _._1   _. ↄ6 _.11ↄ _.0_   _.➚ↄ          1.0000
_1➚0   1.0000 1  __6 _.ↄↄ9 _. ↄ   _.990 _. ➚0 _.➚     6. 00 ↄ.➚ _    .1➚          1.0000
_1➚0   9.0000 1  __6  .1 6 ➚.ↄ0➚ ➚. ↄ   ➚._➚➚   .ↄ _    .6 ➚ ↄ.  0 6. 9          1.0000
_1➚0 1_.0000 1  __6 _.66_   .06    .0 6 _. ➚    _. 1ↄ _.01   6._ↄ  6.69          1.0000
_1➚0   1.0000 1ↄ __6 6. ➚_ _.06   _.➚_9 _.       _.1    6.6 1 ↄ.  6    .9          1.0000
_1➚0   9.0000 1ↄ __6  .11➚   .  ➚   .096   .1      .691   .ↄ➚   ↄ.ↄ    6.ↄↄ0          1.0000
_1➚0 1_.0000 1ↄ __6 _. _9  _.906   .0 0 _.9 1 _.ↄↄ   _.0 1 6.6 9 6. 60          1.0000
_1➚0   1.0000 16 __6 6.➚60 6. ➚9 6. 90 6.6 6 6.6ↄ_ 6. _9 6.0    ↄ.ↄ1          1.0000
_1➚0   9.0000 16 __6  . _    .➚9ↄ   .91    .  ➚   .09➚   .66ↄ ↄ. ↄ9 6.          1.0000
_1➚0 1_.0000 16 __6 _.1 ↄ  _._ 0   .0➚9   .01➚ _._➚0 _.➚    6._1   6.➚ ↄ          1.0000
_1➚0   1.0000 1_ __6 ↄ.91ↄ ↄ._6➚ ↄ._69 ↄ.  _1 ↄ.9 1 6.016 ↄ.919 ↄ.6__          1.0000
_1➚0   9.0000 1_ __6 ↄ.  1   .9ↄ    .6ↄ    .ↄ➚     .6      .99➚ ↄ.ↄ66 6.➚69          1.0000
_1➚0 1_.0000 1_ __6 6.9 1  _.ↄ_   _.9ↄ9   .06➚ _.       _. 69 6.910 6. 16          1.0000
```

```
_1➚0  1.0000 1   __6 ↄ._90 ↄ. 0   ↄ.➚0ↄ ↄ.16  ↄ.➚ 9 ↄ. _  ↄ.  6 ↄ.ↄ➚➚      1.0000
_1➚0  9.0000 1   __6 ↄ. _1 ↄ. ↄↄ ↄ.➚➚6 ↄ.1ↄ  ↄ.➚0_ ↄ. ➚6 ↄ. 1_ 6.  ↄ      1.0000
_1➚0 1_.0000 1   __6 6.9    _.     _. 9   .09ↄ  .0 1  _._0   _.1_  6.ↄ➚ↄ      1.0000
_1➚0  1.0000 19  __6 ↄ. ↄↄ ↄ.➚66  .  ↄ   .60➚  .ↄ6   .6 1   . 9➚ ↄ.1➚       1.0000
_1➚0  9.0000 19  __6 ↄ.    ↄ.  1 ↄ.ↄ_1 ↄ.6 ↄ ↄ._➚1 ↄ.  0 6.1 6 6.ↄ➚➚      1.0000
_1➚0 1_.0000 19  __6 6.9 0 _. ↄ6 _. 6_  .1➚_  .16_ _.9ↄ➚ _.  9 6.          1.0000
_1➚0  1.0000 ➚0  __6 6.0_1 ↄ. 1   .66   .➚06  .9 _  .006  .➚➚ↄ  .ↄ_6      1.0000
_1➚0  9.0000 ➚0  __6  .9 0 ↄ. 66 ↄ.6 9 ↄ.9 ↄ 6.1➚➚ 6.➚91 6.  9 6._ↄ➚      1.0000
_1➚0 1_.0000 ➚0  __6 _.090 _. _9 _. 6   .16➚  .➚9ↄ  .19   _. ➚9 _.➚1ↄ      1.0000
_1➚0  1.0000 ➚1  __6 6. 1ↄ ↄ.ↄ 1  .6 6  .99_  .ↄↄ6  . 09  .ↄↄ   .9 ↄ      1.0000
_1➚0  9.0000 ➚1  __6  . 6  ↄ.0ↄ➚ ↄ.ↄ99 6.0 9 6.  ➚ 6.616 6._99 6.9 _      1.0000
_1➚0 1_.0000 ➚1  __6 _.➚➚   _.ↄ➚   _. 6ↄ  .1 ↄ  . 99  . ➚0  .1_9  _.6ↄ➚      1.0000
_1➚0  1.0000 ➚➚  __6 6. 6  ↄ.911  .90   .99➚  . 06 ➚.9 0 ➚.9 ↄ  .➚6ↄ      1.0000
_1➚0  9.0000 ➚➚  __6  . ↄ1  .ↄ_9 ↄ. ➚  ↄ.9   6. 91 6.  ↄ _.0 9 _.196      1.0000
_1➚0 1_.0000 ➚➚  __6 _.  9 _.ↄ60 _.  ➚   .16   .  _  .ↄ9➚  .ↄ0   .11➚      1.0000
_1➚0  1.0000 ➚   __6 _. 0_ 6.  0 ↄ. ➚6  .➚1_  .➚ 0 ➚.6ↄ6 ➚.     ➚.6➚      1.0000
_1➚0  9.0000 ➚   __6  .1 ↄ  .9    . 90 ↄ._ _ 6.    6.9 _ _.➚➚   _. 6      1.0000
_1➚0 1_.0000 ➚   __6 _. ↄ0 _.ↄ6_ _.__0  .06➚  . 90  .6ↄ    ._ ↄ  .ↄ➚      1.0000
_1➚0  1.0000 ➚   __6 _.9_  _.0 1 ↄ.9 6  .6 9  .ↄ➚ↄ ➚.6➚ↄ ➚.1 0 ➚.109      1.0000
_1➚0  9.0000 ➚   __6 ➚.ↄ ↄ  .  9  .  6 ↄ. 60 6.➚ↄↄ 6.91_ _. 1ↄ _.        1.0000
_1➚0 1_.0000 ➚   __6 _.ↄ➚   _.ↄ 9 _.6 6 _. 66  .19   .ↄ     ._9   . 0      1.0000
_1➚0  1.0000 ➚ↄ  __6  . 69  _._ 6 6.6__ ↄ.  6  .0ↄ_ ➚.90   ➚.111 1. 0      1.0000
_1➚0  9.0000 ➚ↄ  __6 ➚.0 0 ➚._16  ._➚➚   . ↄ6 ↄ.9➚ↄ 6.__ↄ _. ➚➚  _.ↄ_0      1.0000
_1➚0 1_.0000 ➚ↄ  __6 _.ↄ9_ _.ↄ➚9 _. 9ↄ _.ↄ9   _. ↄ   .➚➚9  .60_  .  ↄ      1.0000
_1➚0  1.0000 ➚6  __6  ._66  . 0   _. ➚9 6.➚➚0  .  0  .ↄ0ↄ ➚.  9 1. ➚      1.0000
_1➚0  9.0000 ➚6  __6 1._ↄ6 ➚.➚ ➚  .1 ➚   . 01 ↄ. 9  6.ↄ➚0 _.➚ 1 _.611      1.0000
_1➚0 1_.0000 ➚6  __6 _.6_1 _.ↄ 1 _. _1 _. 01 _. 1  _._➚   .1ↄ➚  .ↄ _      1.0000
_1➚0  1.0000 ➚_  __6  ._    .ↄ9_  .0➚   _.0   ↄ._ↄↄ  . 6   .11   ➚.➚11      1.0000
_1➚0  9.0000 ➚_  __6 1. ➚➚ ➚.00_ ➚._1   ._90 ↄ.0➚➚ 6.1 ➚ _.0 0 _.60      1.0000
_1➚0 1_.0000 ➚_  __6 _._ↄ  _.616 _.  ➚ _.0_  6.9 ➚ _.1➚   _. __ _.9      1.0000
_1➚0  1.0000 ➚   __6  .     . 9ↄ  .➚ ↄ _.6   6.61ↄ ↄ.      .0   ➚.9ↄ0      1.0000
_1➚0  9.0000 ➚   __6 ➚.➚_0 ➚.1 1 ➚.ↄↄ1  .  ↄ   .600 ↄ. 1ↄ 6. ↄ6 _.ↄↄ9      1.0000
_1➚0 1_.0000 ➚   __6 _. ↄ1 _._66 _.  ➚  6.99  6.66  6.ↄↄ1 6._0_ _.0 ↄ      1.0000
_1➚0  1.0000 ➚9  __6 _.ↄↄ6 _.9 6  .0    _.   _ _.➚01 6.➚16 ↄ.0ↄ➚  .9➚      1.0000
_1➚0  9.0000 ➚9  __6  .0ↄ  ➚.6 0 ➚._ 1  .     . ➚6 ↄ. 9➚ 6.60_ _. 6      1.0000
_1➚0 1_.0000 ➚9  __6 _.9 1 _.9_➚ _.6ↄ6 _.1➚  6.ↄ_  6.166 6.0➚➚ 6.169      1.0000
_1➚0  1.0000  0  __6 6.ↄ ↄ _.011 _. 9ↄ _.ↄ ↄ _. ➚  6._ↄ_ ↄ.90ↄ  .9         1.0000
_1➚0  9.0000  0  __6  .0 ↄ  .  9  .➚ↄ   .ↄↄ_  .➚    ↄ.➚9  6.     _.  6      1.0000
_1➚0 1_.0000  0  __6 _.9_   .19➚ _.991 _. 6➚ 6._66 6.091 ↄ.60➚ ↄ. 0ↄ      1.0000
_1➚0  1.0000  1  __6 ↄ.ↄ16 ↄ. 66 6. ➚1 6._16 6.90  6._9  6. _9 ↄ._         1.0000
_1➚0  9.0000  1  __6 ↄ.019   . 0➚  .0ↄ1  .0_9  .ↄ11 ↄ.➚ 1 6.➚ 1 _.19       1.0000
_1➚0 1_.0000  1  __6 _.9ↄ_   . 6   . ↄↄ _.9    _.➚➚  6. _ↄ ↄ.ↄ_   .99➚      1.0000
```

```
HL STN   DATE    TIME  HGT TIME  HGT TIME  HGT TIME  HGT TIME  HGT TIME  HGT

0 _1➚0   1    __6    ➚➚   _.9 111_  ➚.  190_   .➚ 9999 99.9 9999 99.9 9999 99.9
1 _1➚0   ➚    __6         6.6   ➚    _.➚ 11ↄ    .0 19      .  9999 99.9 9999 99.9
1 _1➚0        __6   ➚00   ↄ.9  ↄↄ0  6.   1➚➚    .9 ➚00➚    .6 9999 99.9 9999 99.9
1 _1➚0        __6    ➚1   ↄ.0  _ↄ9  ↄ.   1 0➚   ._ ➚0      .  9999 99.9 9999 99.9
1 _1➚0   ↄ    __6    ➚6    .9 10 _  ↄ.   1     ↄ.6 ➚110   9.1 9999 99.9 9999 99.9
1 _1➚0   6    __6   ↄ➚1   ➚.9 ➚1    9.   9999 99.9 9999 99.9 9999 99.9 9999 99.9
1 _1➚0   _    __6   609   ➚.0 ➚➚ 0  9.ↄ 9999 99.9 9999 99.9 9999 99.9 9999 99.9
1 _1➚0        __6   6ↄↄ   1.  1ↄ    _.ↄ 16    _.ↄ ➚ 1    9.6 9999 99.9 9999 99.9
1 _1➚0   9    __6   _     0.9 1611  _.9 1 19  _._ ➚ ↄ    9.ↄ 9999 99.9 9999 99.9
1 _1➚0  10    __6    19   0._ 16 0   .1 19 1  _.   9999 99.9 9999 99.9 9999 99.9
0 _1➚0  11    __6         9.    ↄ9  0.  1_09   .➚ ➚0 ↄ   _.6 9999 99.9 9999 99.9
0 _1➚0  1➚    __6   1➚9   9.0  9 _  1.1 1_ _   .➚ ➚1 _   _.   9999 99.9 9999 99.9
```

0	[illegible]	1	[illegible]	[illegible]	.6	101	[illegible]	1 06	[illegible]	[illegible]	[illegible]	9999	99.9	9999	99.9
0	[illegible]	1	[illegible]	00	.0	[illegible]	[illegible]	1	.1	[illegible]	[illegible]	9999	99.9	9999	99.9
0	[illegible]	[illegible]	[illegible]	6	[illegible]	111	.1	1900	.1	9999	99.9	9999	99.9	9999	99.9
1	[illegible]	16	[illegible]	[illegible]	6.		[illegible]	[illegible]	.	[illegible]	.1	9999	99.9	9999	99.9
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6.0	[illegible]	[illegible]	[illegible]	.1	9999	99.9	9999	99.9
1	[illegible]	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.1	9999	99.9	9999	99.9
1	[illegible]	19	[illegible]	[illegible]	.6	[illegible]	[illegible]	9999	99.9	9999	99.9	9999	99.9	9999	99.9
1	[illegible]	[illegible]	[illegible]	[illegible]	.0	[illegible]	.	9999	99.9	9999	99.9	9999	99.9	9999	99.9
1	[illegible]	[illegible]	[illegible]	[illegible]	.	[illegible]	.	9999	99.9	9999	99.9	9999	99.9	9999	99.9
1	[illegible]	[illegible]	[illegible]	6 1	[illegible]	[illegible]	.6	9999	99.9	9999	99.9	9999	99.9	9999	99.9
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9999	99.9	9999	99.9	9999	99.9	9999	99.9
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.	9999	99.9	9999	99.9	9999	99.9	9999	99.9
1	[illegible]	[illegible]	[illegible]	0	1.	16 9	[illegible]	[illegible]	[illegible]	9999	99.9	9999	99.9	9999	99.9
0	[illegible]	[illegible]	[illegible]	19	.9	[illegible]	[illegible]	16	[illegible]	[illegible]	[illegible]	9999	99.9	9999	99.9
0	[illegible]	[illegible]	[illegible]	10	.	911	1.	[illegible]	[illegible]	[illegible]	[illegible]	9999	99.9	9999	99.9
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6.6	9999	99.9	9999	99.9
0	[illegible]	[illegible]	[illegible]	[illegible]	.1	1019	[illegible]	[illegible]	.0	[illegible]	6.0	9999	99.9	9999	99.9
0	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	.	1 00	[illegible]	9999	99.9	9999	99.9	9999	99.9
1	[illegible]	1	[illegible]	[illegible]	[illegible]	[illegible]	6.9	[illegible]	.0	[illegible]	.	9999	99.9	9999	99.9