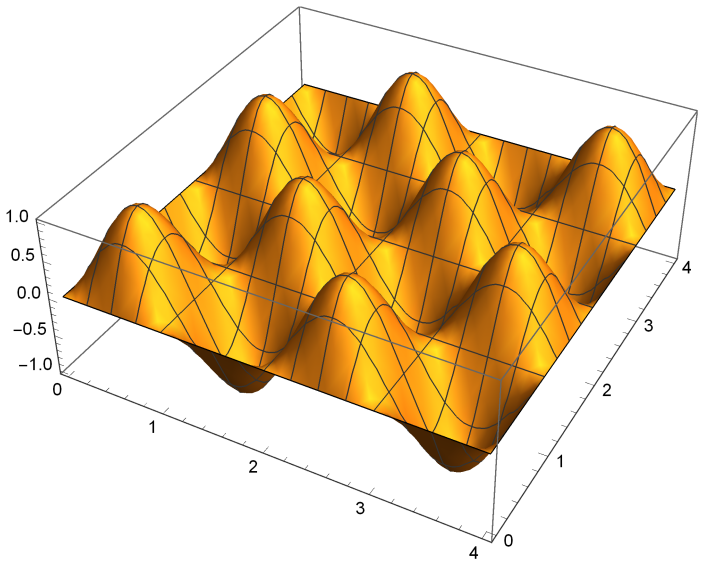
In[1]:=
```
Style["NDSolve solution", 20]
```
estilo resolve numéricamente equaçã

Out[1]= **NDSolve solution**

In[2]:=

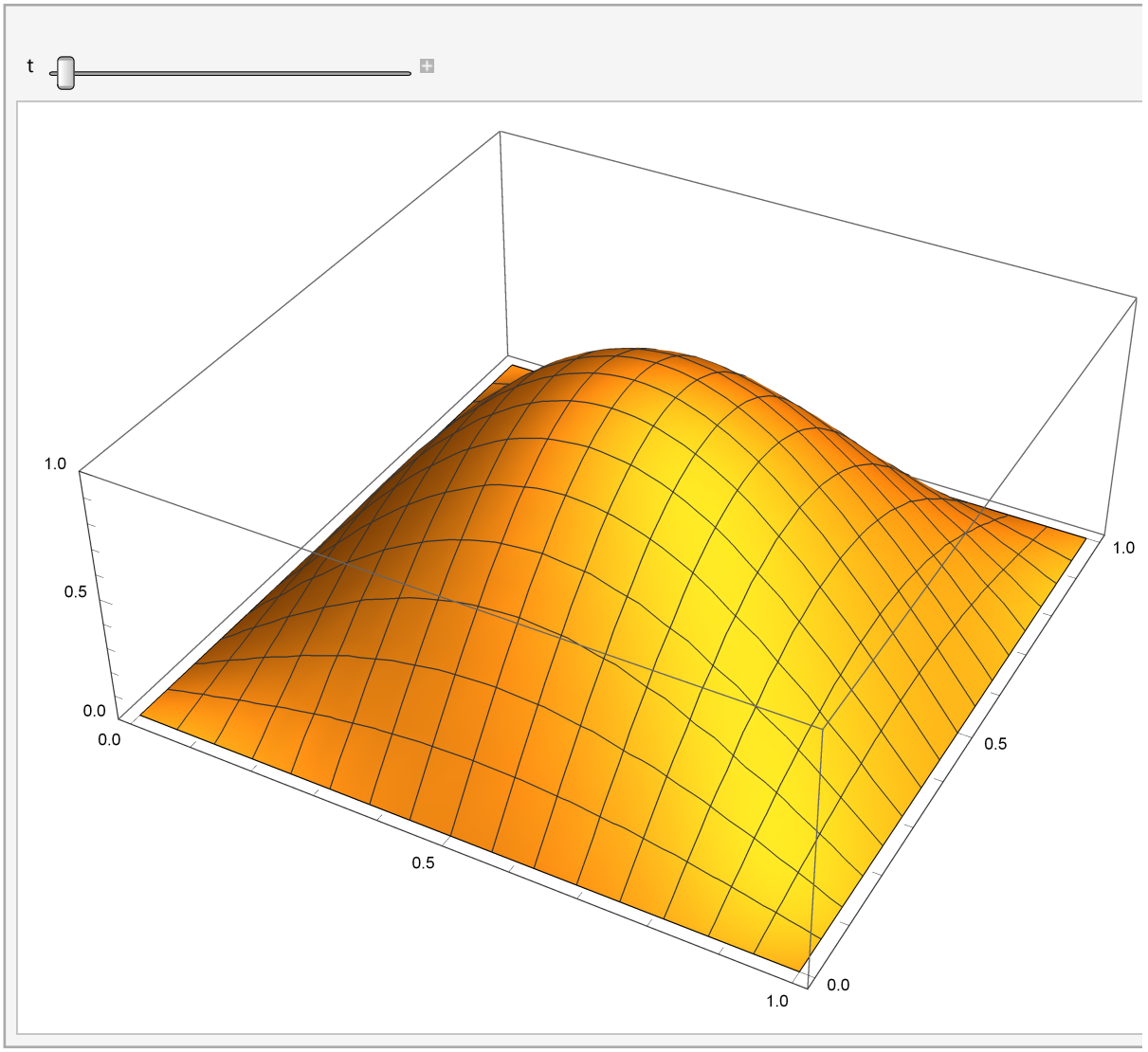
```
Clear[u, x, y, t, k, q, u0]
```
apaga
```
(*defining the heat equation and the contour conditions*)
eqn = D[u[x, y, t], {t, 1}] - k D[u[x, y, t], {x, 2}] - k D[u[x, y, t], {y, 2}] == q[x, y, t];
```
derivada derivada derivada
```
cc1 = u[x, y, 0] == u0[x, y];
cc2 = u[0, y, t] == 0;
cc3 = u[1, y, t] == 0;
cc4 = u[x, 0, t] == 0;
cc5 = u[x, 1, t] == 0;
k = 0.1;

u0[x_, y_] = (Sin[Pi x] Sin[Pi y]) (*(1-Cos[Pi (2x-1)]*(1-y)+y)/2*);
```
seno nú··· senc número pi co··· número pi
```
q[x_, y_, t_] = 0;
Plot3D[u0[x, y], {x, 0, 4}, {y, 0, 4}]
```
gráfico 3D
```
tf = 1;
uu = u /. NDSolve[{eqn, cc1, cc2, cc3, cc4, cc5}, u, {x, 0, 1}, {y, 0, 1}, {t, 0, tf}][[1]]
```
resolve numéricamente equação diferencial
```
Manipulate[
 Plot3D[uu[x, y, t], {x, 0, 1}, {y, 0, 1}, ImageSize → 600, PlotRange → {0, 1}], {t, 0, tf}]
```
manipula gráfico 3D tamanho da imagem intervalo do gráfico

Out[12]=



Out[14]= InterpolatingFunction[Domain: {{0., 1.}, {0., 1.}, {0., 1.}}
Output: scalar]

Out[15]=



In[16]:=

In[17]:=
```
Style["Third order error analysis", 20]
```
estilo

Out[17]=

Third order error analysis

In[18]:=
```
Clear[f, x, y, yy, t, Y, M, NN, w, Y0, xn, tn, X, T, RK, nx, a,
```
apaga
```
 tf, F, a1, a2, krk, krk1, krk2, MM, krk3, krk4, XX, aa1, aa2, W, v, MT]
X = {};
T = {};
MT = {};
f[x_, y_, t_] := k (D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]);
```
derivada derivada
```
(*defining variables and parameters*)
x2 = 1;
x1 = 0;
y2 = 1;
y1 = 0;
ti = 0;
tf = 1;
```

```
pk = 0.6;
qk = 1 - p;
rk = 3;
sk = 8;
d = 0.001;
c = 0.2;
pk = 0.6;
qk = 1 - p;
rk = 3;
sk = 8;
d = 0.001;
c = 0.2;


aa1 = 0.5;
aa2 = 1 - aa1;
a1 = 2.5 / 6;
a2 = 3 / 6;
a3 = 1 - a1 - a2;
M = {};
NN = 5;

Tempo = {};
Do[AppendTo[Tempo, Timing[
r··· adiciona a          cronometra

    JJ = 15; (*5n*)
    (*definitions*)
    xn := (2 x2 - x1) / (JJ - 1);
    yn := (2 y2 - y1) / (JJ - 1);
    tn := (tf - ti) / (NN - 1);
    X = Table[x, {x, x1, 2 x2 - xn, xn}];
        tabela
    Y = Table[y, {y, y1, 2 y2 - yn, yn}];
        tabela
    T = Table[t, {t, ti, tf, tn}];
        tabela
    W = {w};
    k = 0.1;
    p := k * tn / (xn^2);
    pk = 0.6;
    qk = 1 - p;
    rk = 3;
    sk = 8;
    d = 0.001;
    c = 0.2;
    Y0 = Table[u0[X, Y[[j]]], {j, 1, JJ - 1}];
         tabela
    Y0 = Flatten[Y0];
         achatar
    w = Y0;
    W = {Y0};
```

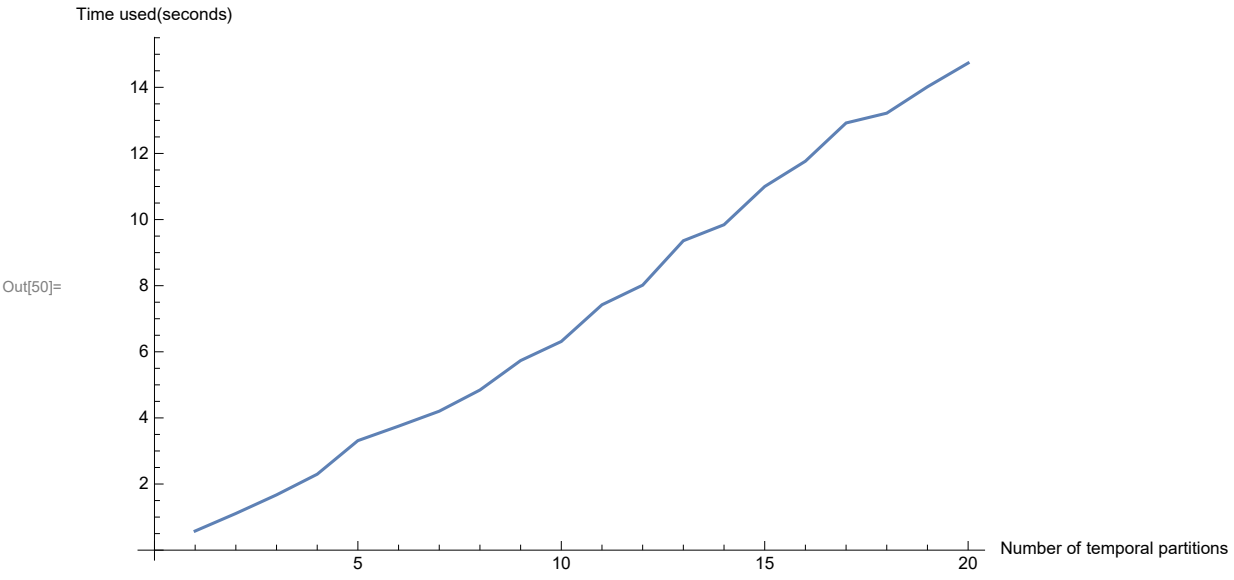
```
(*Temporal evolution matrix*)

A = {};
MT1 = Table[{}, (JJ - 1)];
      tabela

MZ = 0 IdentityMatrix[(JJ - 1)];
      matriz identidade
ML = p * IdentityMatrix[(JJ - 1)];
       matriz identidade
MC = -4 p * IdentityMatrix[(JJ - 1)];
            matriz identidade
MC[[1, (JJ - 1)]] = p;
MC[[(JJ - 1), 1]] = p;
Table[MC[[j, j + 1]] = p, {j, 1, (JJ - 2)}];
tabela
Table[MC[[j, j - 1]] = p, {j, 2, (JJ - 1)}];
tabela
Do[A = Table[MZ, (JJ - 1)];
      tabela
 A[[i]] = MC;
 If[i > 1, A[[i - 1]] = ML];
 se
 If[i == 1, A[[(JJ - 1)]] = ML];
 se
 If[i < (JJ - 1), A[[i + 1]] = ML];
 se
 If[i == (JJ - 1), A[[1]] = ML];
 se
 MT1[[i]] = Flatten[Join[Table[A[[j]], {j, 1, (JJ - 1)}]], 1], {i, 1, (JJ - 1)}];
            achatar  junta  tabela
MT = Join[
     junta
  Table[Flatten[Join[Table[MT1[[j, i]], {j, 1, (JJ - 1)}]], 1], {i, 1, (JJ - 1)^2}]];
       achatar  junta  tabela

F[t_, w_, tn_] := Module[{},
                  módulo de código
   MT.w];

RK[3][F_, t_, w_, tn_] := Module[{},
                          módulo de código

   krk1 = F[t, w, tn];
   krk2 = F[t + tn, w + krk1 / 2, tn];
   krk3 = F[t + rk, w + sk * krk2 / 2 + (rk - sk) * krk1 / 2, tn];
   w + (a1 * krk1 + a2 * krk2 + a3 * krk3)];


(*solutions*)
Clear[WW, n];
apaga
WW = {};
```

```
Do[
 AppendTo[W, RK[3][F, t, W[[n]], tn]],
 {n, 1, NN - 1}];

Do[
 AppendTo[WW,
   Table[Table[Partition[W[[n]], JJ - 1][[i, j]], {i, 1, JJ - 1}], {j, 1, JJ - 1}]]
 , {n, 1, NN}];
Clear[Uksn, uuRK];
Uksn = Flatten[Table[{{X[[i]], Y[[j]], T[[n]]}, WW[[n, i, j]]},
   {n, 1, NN}, {i, 1, (JJ - 1)}, {j, 1, (JJ - 1)}], 2];
uuRK = Interpolation[Uksn];
Animate[Plot3D[uuRK[x, y, t], {x, x1, x2}, {y, y1, y2}, PlotPoints → 15,
  ImageSize → 400, PlotRange → {0, 1}], {t, 0, tf}, AnimationRunning → False];
Manipulate[ListPlot3D[WW[[i]], PlotRange → {-1, 1}], {i, 1, NN, 1}];

(*error*)
Clear[WW3, n];
WW3 = {};
t = 0;
W = {Y0};
erro = {};
errot = {};
(*ordem3*)


xn := (2 x2 - x1) / (JJ - 1);
yn := (2 y2 - y1) / (JJ - 1);
tn := (tf - ti) / (NN - 1);
X = Table[x, {x, x1, 2 x2 - xn, xn}];
Y = Table[y, {y, y1, 2 y2 - yn, yn}];
T = Table[t, {t, ti, tf, tn}];
Y0 = Table[u0[X, Y[[j]]], {j, 1, JJ - 1}];
Y0 = Flatten[Y0];
w = Y0;
W = {Y0};
```

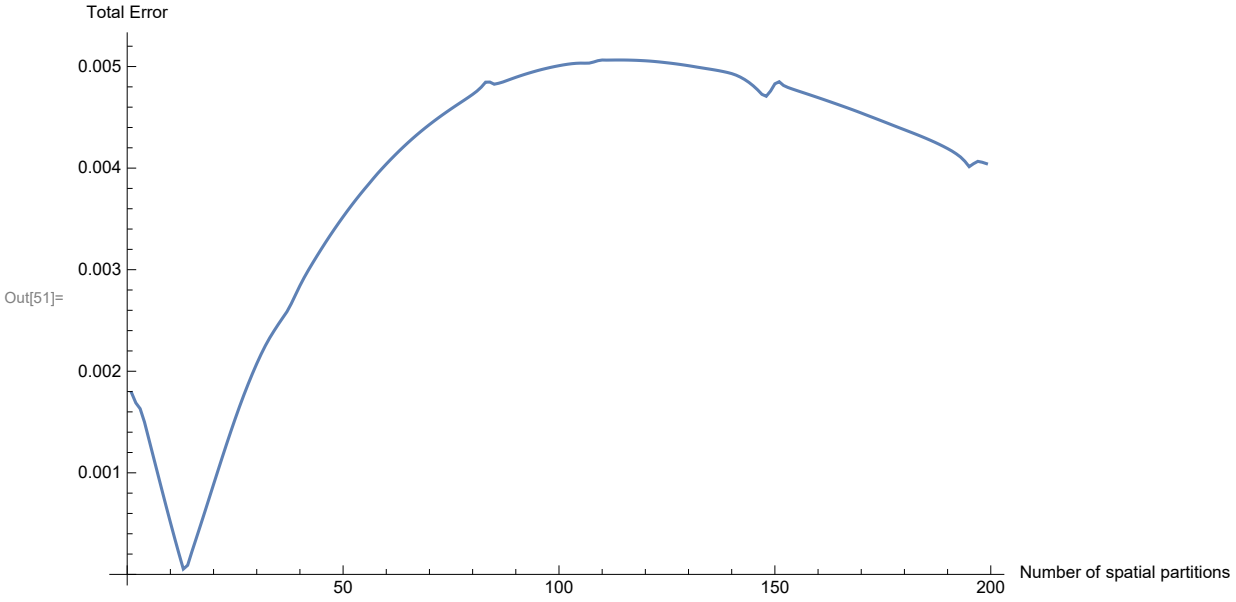
```
Do[AppendTo[W, RK[3][F, t, W[[n]], tn]], {n, 1, NN - 1}];
Do[AppendTo[WW3, Table[Table[Partition[W[[n]], JJ - 1][[i, j]], {i, 1, JJ - 1}],
    {j, 1, JJ - 1}]], {n, 1, NN}];
UU3 = Flatten[Table[{{X[[i]], Y[[j]], T[[n]]}, WW3[[n, i, j]]},
    {n, 1, NN}, {i, 1, JJ - 1}, {j, 1, JJ - 1} ], 2];
uu3 = Interpolation[UU3, InterpolationOrder → 2];
dif3 = Abs[uu3[x, y, T] - uu[x, y, T]];
Do[AppendTo[erro, Max[FindMaximum[dif3[[i]], {{x, 0.5}, {y, 0.5}}, AccuracyGoal →
        0.00001, PrecisionGoal → 0.00001]][[1]]], {i, 2, NN, 1}];][[1]]]
, {NN, 10, 200, 10}]
```

```
im1 = ListPlot[Tempo, ImageSize → 600, Joined → True,
  AxesLabel → {"Number of temporal partitions", "Time used(seconds)"}]
im2 = ListPlot[erro, ImageSize → 600, Joined → True,
  AxesLabel → {"Number of spatial partitions", "Total Error"}]
```

Out[50]=



Out[51]=


Total Error
0.005
0.004
0.003
0.002
0.001
50
100
150
200
Number of spatial partitions