1

$$\int x(x+1)(x-2)dx = \int x(x^2-x-2)dx = \int (x^3-x^2-2x)dx = \frac{x^4}{4}-\frac{x^3}{3}-x^2+C$$

2

$$\int \frac{x^2-x+1}{\sqrt{x}}dx = \int \frac{x^2}{\sqrt{x}}dx - \int \frac{x}{\sqrt{x}}dx + \int \frac{dx}{\sqrt{x}} = \int x^{\frac{3}{2}}dx - \int x^{\frac{1}{2}}dx + \int x^{-\frac{1}{2}}dx =$$

$$= \frac{2x^2\sqrt{x}}{5} - \frac{2x\sqrt{x}}{3} + 2\sqrt{x} + C$$

3

$$\int \sqrt{x\sqrt{x\sqrt{x}}}dx = \int x^{\frac{7}{8}}dx = \frac{8}{15}x^{\frac{15}{8}} + C$$

4

$$\int \frac{dx}{7+x^2} = \frac{1}{\sqrt{7}}\arctan\frac{x}{\sqrt{7}} + C$$

5

$$\int \frac{dx}{2x^2+3} = \frac{1}{2}\int \frac{dx}{x^2+\frac{3}{2}} = \frac{1}{2}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{2}{3}}x\right) + C$$

6

$$\int \frac{dx}{3x^2-7} = -\frac{1}{3}\int \frac{dx}{\frac{7}{3}-x^2} = -\frac{1}{3}\frac{1}{2\sqrt{\frac{7}{3}}}\ln\left|\frac{\sqrt{\frac{7}{3}}+x}{\sqrt{\frac{7}{3}}-x}\right| + C$$

7

$$\int \frac{x^2}{1-x^2}dx = \int \left(\frac{1}{1-x^2}-1\right)dx = \int \frac{dx}{1-x^2} - \int dx = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| - x + C$$

8

$$\int \frac{dx}{x^4-1} = \int \left(\frac{1}{2x^2-2} - \frac{1}{2x^2+2}\right) dx = -\frac{1}{2}\int \frac{dx}{1-x^2} - \frac{1}{2}\int \frac{dx}{x^2+1} =$$

$$= -\frac{1}{4}\ln\left|\frac{1+x}{1-x}\right| - \frac{1}{2}\arctan x + C$$

9

$$\int \frac{(1+x)^2}{x(1+x^2)}dx = \int \frac{x^2+2x+1}{x(1+x^2)}dx = \int \frac{x}{1+x^2}dx + 2\int \frac{dx}{1+x^2} + \int \frac{dx}{x(1+x^2)} =$$

$$= \left|\frac{1}{x(1+x^2)} = \frac{1}{x} - \frac{x}{1+x^2}\right| = \int \frac{x}{1+x^2}dx + 2\int \frac{dx}{1+x^2} + \int \frac{dx}{x} - \int \frac{x}{1+x^2}dx =$$

$$= \frac{1}{2}\ln|1+x^2| + 2\arctan x + \ln|x| - \frac{1}{2}\ln|1+x^2| + C = 2\arctan x + \ln|x| + C$$

10

$$\int \frac{dx}{\sqrt{2-x^2}} = \arcsin\frac{x}{\sqrt{2}} + C$$

11

$$\int (5^x - 2^x)^2 dx = \int (5^{2x} - 2*10^x + 4^x) dx = \int 25^x dx - 2\int 10^x dx + \int 4^x dx =$$

$$= \frac{25^x}{\ln 25} - \frac{2*10^x}{\ln 10} + \frac{4^x}{\ln 4} + C$$

12

$$\int \frac{2^x 3^{2x} 4^{3x}}{5^x 6^{2x}} dx = \int \left(\frac{32}{5}\right)^x dx = \left(\frac{32}{5}\right)^x \frac{1}{\ln\left(\frac{32}{5}\right)} + C$$

13

$$\int \frac{e^{3x}-1}{e^x-1}dx = \int \frac{(e^x-1)(e^{2x}+e^x+1)}{e^x-1}dx = \int e^{2x}dx + \int e^x dx + \int dx = \frac{1}{2}e^{2x} + e^x + x + C$$

14

$$\int \left(\sin\frac{x}{2}\right)^2 dx = \int \frac{1-\cos x}{2} dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos x dx = \frac{x-\sin x}{2} + C$$

15

$$\int \left(\cos\frac{x}{2}\right)^2 dx = \int \frac{1+\cos x}{2} dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos x dx = \frac{x}{2} + \frac{\sin x}{2} + C$$

16

$$\int \frac{dx}{(\sin x)^2(\cos x)^2} = 4\int \frac{dx}{(\sin 2x)^2} = |t = 2x dt = 2dx| = 2\int \frac{dt}{(\sin t)^2} =$$

$$-2\operatorname{ctg} t + C = -2\operatorname{ctg} 2x + C$$

17

$$\int (\operatorname{tg} x)^2 dx = \left|(\operatorname{tg} x)^2 = \frac{(\sin x)^2}{(\cos x)^2} = \frac{1-(\cos x)^2}{(\cos x)^2} = \frac{1}{(\cos x)^2} - 1\right| =$$

$$= \int \frac{dx}{(\cos x)^2} - \int dx = \operatorname{tg} x - x + C$$

18

$$\int (\operatorname{ctg} x)^2 dx = \left|(\operatorname{ctg} x)^2 = \frac{(\cos x)^2}{(\sin x)^2} = \frac{1-(\sin x)^2}{(\sin x)^2} = \frac{1}{(\sin x)^2} - 1\right| =$$

$$= \int \frac{dx}{(\sin x)^2} - \int dx = -\operatorname{ctg} x - x + C$$

19

$$\int \frac{dx}{2x+3} = |2x+3 = t, \quad dt = 2dx| = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln|2x+3| + C$$

20

$$\int \frac{x+3}{(x+2)(x-1)}dx = |t = x-1, \quad dt = dx| = \int \frac{t+4}{t(t+3)}dt =$$

$$= \int \frac{dt}{t+3} + 4\int \frac{dt}{t(t+3)} = \int \frac{dt}{t+3} + \frac{4}{3}(\int \frac{dt}{t} - \int \frac{dt}{t+3}) =$$

$$= |t+3 = q, \quad dq = dt| = \int \frac{dq}{q} + \frac{4}{3}\int \frac{dt}{t} - \frac{4}{3}\int \frac{dq}{q} =$$

$$= -\frac{1}{3}\ln|q| + \frac{4}{3}\ln|t| + C = -\frac{1}{3}\ln|x+2| + \frac{4}{3}\ln|x-1| + C$$

21

$$\int \frac{2+x}{1+x}dx = \int (\frac{1}{1+x} + 1)dx = \int \frac{dx}{1+x} + \int dx = |1+x = t, \quad dt = dx| =$$

$$= \int \frac{dt}{t} + \int dx = \ln|t| + x + C = \ln|1+x| + x + C$$

22

$$\int (2x+5)^{15}dx = |t = 2x+5, \quad dt = 2dx| = \frac{1}{2}\int t^{15}dt = \frac{t^{16}}{32} + C =$$

$$= \frac{(2x+5)^{16}}{32} + C$$

23

$$\int x(x-2)^5dx = |t = x-2, \quad x = t+2, \quad dt = dx| = \int (t+2)t^5dt = \int (t^6+2t^5)dt =$$

$$= \int t^6dt + 2\int t^5dt = \frac{t^7}{7} + \frac{t^6}{3} + C = \frac{(x-2)^7}{7} + \frac{(x-2)^6}{3} + C$$

24

$$\int (x+2)\sqrt{x-2}dx\, |t = x-2, \quad x+2 = t+4, \quad dt = dx| = \int (t+4)\sqrt{t}dt =$$

$$= \int t^{\frac{3}{2}}dt + 4\int t^{\frac{1}{2}}dt = \frac{2}{5}t^{\frac{5}{2}} + \frac{8}{3}t^{\frac{3}{2}} + C = \frac{2(x-2)^{\frac{5}{2}}}{5} + \frac{8(x-2)^{\frac{3}{2}}}{2} + C$$

25

$$\int \frac{2x-7}{\sqrt{1+3x}}dx = \left| t=1+3x, \quad dt=3dx, \quad 2x-7=\frac{2t-23}{3}, \quad x=\frac{t-1}{3} \right| =$$

$$= \frac{1}{9}\int \frac{2t-23}{\sqrt{t}}dt = \frac{2}{9}\int t^{\frac{1}{2}}dt - \frac{23}{9}\int t^{-\frac{1}{2}}dt = \frac{4}{27}(1+3x)^{\frac{3}{2}} - \frac{46}{9}\sqrt{1+3x} + C$$

26

$$\int \frac{x^2+1}{2x-1}dx = \left| t=2x-1, \quad dt=2dx, \quad x=\frac{t+1}{2} \right| = \frac{1}{4}(\int tdt + 2\int dt + 5\int dt) =$$

$$= \frac{1}{4}(\frac{t^2}{2}+2t+5t)+C = \frac{t^2}{8}+\frac{t}{2}+\frac{5t}{4}+C = \frac{(2x-1)^2}{8}+\frac{2x-1}{2}+\frac{2(2x-1)}{4}+C$$

27

$$\int (2x+3)^2(1-x)^8dx = |t=1-x, \quad dt=-dx, \quad x=1-t, \quad 2x+3=-2t+5| =$$

$$-\int(-2t+5)^2t^8dt = -\int(4t^2-20t+25)t^8dt = -4\int t^{10}dt + 20\int t^9dt - 25\int t^8dt =$$

$$= -\frac{4t^{11}}{11}+2t^{10}-\frac{25t^9}{9}+C = -\frac{4(1-x)^{11}}{11}+2(1-x)^{10}-\frac{25(1-x)^9}{9}+C$$

28

$$\int \frac{x-4}{\sqrt{x^2-2}}dx = \int \frac{x}{\sqrt{x^2-2}}dx - 4\int \frac{dx}{\sqrt{x^2-2}} =$$

$$= \left| t=x^2-2, \quad dt=2xdx \right| = \frac{1}{2}\int \frac{dt}{\sqrt{t}} - 4\int \frac{dx}{\sqrt{x^2-2}} = \sqrt{t} - 4\ln|x+\sqrt{x^2-2}| + C =$$

$$= \sqrt{x^2-2} - 4\ln|x+\sqrt{x^2-2}| + C$$

29

$$\int \frac{x+2}{\sqrt{x^2-10}}dx = \left| t=x^2-10, \quad dt=2xdx \right| = \frac{1}{2}\int \frac{dt}{\sqrt{t}} + 2\int \frac{dx}{\sqrt{x^2-10}} =$$

$$= \sqrt{t} + 2\ln|x+\sqrt{x^2-10}| + C = \sqrt{x^2-10} + 2\ln|x+\sqrt{x^2-10}| + C$$

30

$$\int x\sqrt{1-x^2}dx = \left|t = 1-x^2, \quad dt = -2xdx\right| = -\frac{1}{2}\int \sqrt{t}dt = -\frac{1}{2}\frac{2}{3}t^{\frac{3}{2}} + C =$$

$$= -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

31

$$\int x(1-x^2)^6 dx = \left|t = 1-x^2, \quad xdx = -\frac{1}{2}dt\right| = -\frac{1}{2}\int t^5 dt = -\frac{1}{12}t^6 + C =$$

$$= -\frac{(1-x^2)^6}{12} + C$$

32

$$\int \frac{xdx}{1+x^4} = \left|t = x^2, \quad dt = 2xdx\right| = \frac{1}{2}\int \frac{dt}{1+t^2} = \frac{1}{2}\arctan t + C = \frac{\arctan x^2}{2} + C$$

33

$$\int \frac{x^2}{x^6-5}dx = \left|t = x^3, \quad dt = 3x^2dx\right| = \frac{1}{3}\int \frac{dt}{t^2-5} = -\frac{1}{3}\int \frac{dt}{5-t^2} =$$

$$-\frac{1}{3}\frac{1}{2\sqrt{5}}\ln\left|\frac{\sqrt{5}+t}{\sqrt{5}-t}\right| + C = \frac{\sqrt{5}\ln\left|\frac{x^3+\sqrt{5}}{x^3-\sqrt{5}}\right|}{30} + C$$

34

$$\int \frac{x^4}{\sqrt{3+x^5}}dx = \left|t = x^5+3, \quad dt = 5x^4dx\right| = \frac{1}{5}\int \frac{dt}{\sqrt{t}} = \frac{2\sqrt{t}}{5} + C = \frac{2\sqrt{x^5+3}}{5} + C$$

35

$$\int \frac{dx}{x(\ln x)^5} = \left|t = \ln x, \quad dt = \frac{dx}{x}\right| = \int \frac{dt}{t^5} = -\frac{1}{4t^4} + C = -\frac{1}{4(\ln x)^4} + C$$

36

$$\int \frac{dx}{x\sqrt{\ln x}} = \left|t = \ln x, \quad dt = \frac{dx}{x}\right| \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + C = 2\sqrt{\ln x} + C$$

37

$$\int \frac{dx}{x(2+(\ln x)^2)} = \left| t = \ln x, \quad dt = \frac{dx}{x} \right| = \int \frac{dt}{2+t^2} = \frac{\arctan \frac{t}{\sqrt{2}}}{\sqrt{2}} + C =$$

$$= \frac{\arctan \frac{\ln x}{\sqrt{2}}}{\sqrt{2}} + C$$

38

$$\int \frac{\sqrt[3]{(\ln x)^2}}{x} dx = \left| t = \ln x, \quad dt = \frac{dx}{x} \right| = \int t^{\frac{2}{3}} dt = \frac{3}{5} t^{\frac{5}{3}} + C = \frac{3(\ln x)^{\frac{5}{3}}}{5} + C$$

39

$$\int \frac{e^x}{1+e^x} dx = \left| t = 1+e^x, \quad dt = e^X dx \right| = \int \frac{dt}{t} = \ln|t| + C = \ln|1+e^x| + C$$

40

$$\int \sin 5x dx = |t = 5x, \quad dt = 5dx| = \frac{1}{5} \int \sin t dt = -\frac{\cos t}{5} + C = -\frac{\cos 5x}{5} + C$$

41

$$\int x \sin x^2 dx = \left| t = x^2, \quad dt = 2xdx \right| = \frac{1}{2} \int \sin t dt = -\frac{1}{2} \cos t + C = -\frac{1}{2} \cos x^2 + C$$

42

$$\int \frac{\cos x}{1+\sin x} dx = |t = \sin x, \quad dt = \cos x dx| \int \frac{dt}{1+t} = \arctan \sqrt{t} + C = \arctan \sqrt{\sin x} + C$$

43

$$\int \frac{cos\sqrt{x}}{\sqrt{x}} dx = \left| t = \sqrt{x}, \quad dt = \frac{dx}{2\sqrt{x}} \right| = 2 \int \cos t dt = 2 \sin t + C = 2 \sin \sqrt{x} + C$$

44

$$\int (\sin x)^2 dx = \int \frac{1-\cos 2x}{2} dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x dx = |t = 2x, \quad dt = 2dx| =$$

$$= \frac{1}{2}\int dx - \frac{1}{4}\int \cos t dt = \frac{x - \frac{\sin t}{2}}{2} + C = \frac{x - \frac{\sin 2x}{2}}{2} + C$$

45