

## 1

$$\int x(x+1)(x-2)dx = \int x(x^2-x-2)dx = \int (x^3-x^2-2x)dx = \frac{x^4}{4} - \frac{x^3}{3} - x^2 + C$$

## 2

$$\begin{aligned}\int \frac{x^2-x+1}{\sqrt{x}}dx &= \int \frac{x^2}{\sqrt{x}}dx - \int \frac{x}{\sqrt{x}}dx + \int \frac{dx}{\sqrt{x}} = \int x^{\frac{3}{2}}dx - \int x^{\frac{1}{2}}dx + \int x^{-\frac{1}{2}}dx = \\ &= \frac{2x^2\sqrt{x}}{5} - \frac{2x\sqrt{x}}{3} + 2\sqrt{x} + C\end{aligned}$$

## 3

$$\int \sqrt{x\sqrt{x\sqrt{x}}}dx = \int x^{\frac{7}{8}}dx = \frac{8}{15}x^{\frac{15}{8}} + C$$

## 4

$$\int \frac{dx}{7+x^2} = \frac{1}{\sqrt{7}} \arctan \frac{x}{\sqrt{7}} + C$$

## 5

$$\int \frac{dx}{2x^2+3} = \frac{1}{2} \int \frac{dx}{x^2+\frac{3}{2}} = \frac{1}{2}\sqrt{\frac{2}{3}} \arctan \left( \sqrt{\frac{2}{3}}x \right) + C$$

## 6

$$\int \frac{dx}{3x^2-7} = -\frac{1}{3} \int \frac{dx}{\frac{7}{3}-x^2} = -\frac{1}{3} \frac{1}{2\sqrt{\frac{7}{3}}} \ln \left| \frac{\sqrt{\frac{7}{3}}+x}{\sqrt{\frac{7}{3}}-x} \right| + C$$

## 7

$$\int \frac{x^2}{1-x^2}dx = \int \left( \frac{1}{1-x^2} - 1 \right) dx = \int \frac{dx}{1-x^2} - \int dx = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| - x + C$$

## 8

$$\begin{aligned}\int \frac{dx}{x^4 - 1} &= \int \left( \frac{1}{2x^2 - 2} - \frac{1}{2x^2 + 2} \right) dx = -\frac{1}{2} \int \frac{dx}{1 - x^2} - \frac{1}{2} \int \frac{dx}{x^2 + 1} = \\ &= -\frac{1}{4} \ln \left| \frac{1+x}{1-x} \right| - \frac{1}{2} \arctan x + C\end{aligned}$$

## 9

$$\begin{aligned}\int \frac{(1+x)^2}{x(1+x^2)} dx &= \int \frac{x^2 + 2x + 1}{x(1+x^2)} dx = \int \frac{x}{1+x^2} dx + 2 \int \frac{dx}{1+x^2} + \int \frac{dx}{x(1+x^2)} = \\ &= \left| \frac{1}{x(1+x^2)} = \frac{1}{x} - \frac{x}{1+x^2} \right| = \int \frac{x}{1+x^2} dx + 2 \int \frac{dx}{1+x^2} + \int \frac{dx}{x} - \int \frac{x}{1+x^2} dx = \\ &= \frac{1}{2} \ln |1+x^2| + 2 \arctan x + \ln |x| - \frac{1}{2} \ln |1+x^2| + C = 2 \arctan x + \ln |x| + C\end{aligned}$$

## 10

$$\int \frac{dx}{\sqrt{2-x^2}} = \arcsin \frac{x}{\sqrt{2}} + C$$

## 11

$$\begin{aligned}\int (5^x - 2^x)^2 dx &= \int (5^{2x} - 2 * 10^x + 4^x) dx = \int 25^x dx - 2 \int 10^x dx + \int 4^x dx = \\ &= \frac{25^x}{\ln 25} - \frac{2 * 10^x}{\ln 10} + \frac{4^x}{\ln 4} + C\end{aligned}$$

## 12

$$\int \frac{2^x 3^{2x} 4^{3x}}{5^x 6^{2x}} dx = \int \left( \frac{32}{5} \right)^x dx = \left( \frac{32}{5} \right)^x \frac{1}{\ln \left( \frac{32}{5} \right)} + C$$

## 13

$$\int \frac{e^{3x} - 1}{e^x - 1} dx = \int \frac{(e^x - 1)(e^{2x} + e^x + 1)}{e^x - 1} dx = \int e^{2x} dx + \int e^x dx + \int dx = \frac{1}{2} e^{2x} + e^x + x + C$$

**14**

$$\int \left(\sin \frac{x}{2}\right)^2 dx = \int \frac{1 - \cos x}{2} dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos x dx = \frac{x - \sin x}{2} + C$$

**15**

$$\int \left(\cos \frac{x}{2}\right)^2 dx = \int \frac{1 + \cos x}{2} dx = \frac{1}{2} \int dx + \frac{1}{2} \int \cos x dx = \frac{x}{2} + \frac{\sin x}{2} + C$$

**16**

$$\begin{aligned} \int \frac{dx}{(\sin x)^2 (\cos x)^2} &= 4 \int \frac{dx}{(\sin 2x)^2} = |t = 2x dt = 2dx| = 2 \int \frac{dt}{(\sin t)^2} = \\ &= -2 \operatorname{ctg} t + C = -2 \operatorname{ctg} 2x + C \end{aligned}$$

**17**

$$\begin{aligned} \int (\operatorname{tg} x)^2 dx &= \left| (\operatorname{tg} x)^2 = \frac{(\sin x)^2}{(\cos x)^2} = \frac{1 - (\cos x)^2}{(\cos x)^2} = \frac{1}{(\cos x)^2} - 1 \right| = \\ &= \int \frac{dx}{(\cos x)^2} - \int dx = \operatorname{tg} x - x + C \end{aligned}$$

**18**

$$\begin{aligned} \int (\operatorname{ctg} x)^2 dx &= \left| (\operatorname{ctg} x)^2 = \frac{(\cos x)^2}{(\sin x)^2} = \frac{1 - (\sin x)^2}{(\sin x)^2} = \frac{1}{(\sin x)^2} - 1 \right| = \\ &= \int \frac{dx}{(\sin x)^2} - \int dx = -\operatorname{ctg} x - x + C \end{aligned}$$

**19**

$$\int \frac{dx}{2x+3} = |2x+3=t, \quad dt=2dx| = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln |t| + C = \frac{1}{2} \ln |2x+3| + C$$

## 20

$$\begin{aligned}\int \frac{x+3}{(x+2)(x-1)} dx &= |t = x-1, \quad dt = dx| = \int \frac{t+4}{t(t+3)} dt = \\&= \int \frac{dt}{t+3} + 4 \int \frac{dt}{t(t+3)} = \int \frac{dt}{t+3} + \frac{4}{3} \left( \int \frac{dt}{t} - \int \frac{dt}{t+3} \right) = \\&= |t+3 = q, \quad dq = dt| = \int \frac{dq}{q} + \frac{4}{3} \int \frac{dt}{t} - \frac{4}{3} \int \frac{dq}{q} = \\&= -\frac{1}{3} \ln |q| + \frac{4}{3} \ln |t| + C = -\frac{1}{3} \ln |x+2| + \frac{4}{3} \ln |x-1| + C\end{aligned}$$

## 21

$$\begin{aligned}\int \frac{2+x}{1+x} dx &= \int \left( \frac{1}{1+x} + 1 \right) dx = \int \frac{dx}{1+x} + \int dx = |1+x = t, \quad dt = dx| = \\&= \int \frac{dt}{t} + \int dx = \ln |t| + x + C = \ln |1+x| + x + C\end{aligned}$$

## 22

$$\begin{aligned}\int (2x+5)^{15} dx &= |t = 2x+5, \quad dt = 2dx| = \frac{1}{2} \int t^{15} dt = \frac{t^{16}}{32} + C = \\&= \frac{(2x+5)^{16}}{32} + C\end{aligned}$$

## 23

$$\begin{aligned}\int x(x-2)^5 dx &= |t = x-2, \quad x = t+2, \quad dt = dx| = \int (t+2)t^5 dt = \int (t^6 + 2t^5) dt = \\&= \int t^6 dt + 2 \int t^5 dt = \frac{t^7}{7} + \frac{t^6}{3} + C = \frac{(x-2)^7}{7} + \frac{(x-2)^6}{3} + C\end{aligned}$$

## 24

$$\begin{aligned}\int (x+2)\sqrt{x-2} dx &= |t = x-2, \quad x+2 = t+4, \quad dt = dx| = \int (t+4)\sqrt{t} dt = \\&= \int t^{\frac{3}{2}} dt + 4 \int t^{\frac{1}{2}} dt = \frac{2}{5} t^{\frac{5}{2}} + \frac{8}{3} t^{\frac{3}{2}} + C = \frac{2(x-2)^{\frac{5}{2}}}{5} + \frac{8(x-2)^{\frac{3}{2}}}{2} + C\end{aligned}$$

## 25

$$\begin{aligned} \int \frac{2x-7}{\sqrt{1+3x}} dx &= \left| t = 1+3x, \quad dt = 3dx, \quad 2x-7 = \frac{2t-23}{3}, \quad x = \frac{t-1}{3} \right| = \\ &= \frac{1}{9} \int \frac{2t-23}{\sqrt{t}} dt = \frac{2}{9} \int t^{\frac{1}{2}} dt - \frac{23}{9} \int t^{-\frac{1}{2}} dt = \frac{4}{27} (1+3x)^{\frac{3}{2}} - \frac{46}{9} \sqrt{1+3x} + C \end{aligned}$$

## 26

$$\begin{aligned} \int \frac{x^2+1}{2x-1} dx &= \left| t = 2x-1, \quad dt = 2dx, \quad x = \frac{t+1}{2} \right| = \frac{1}{4} \left( \int t dt + 2 \int dt + 5 \int dt \right) = \\ &= \frac{1}{4} \left( \frac{t^2}{2} + 2t + 5t \right) + C = \frac{t^2}{8} + \frac{t}{2} + \frac{5t}{4} + C = \frac{(2x-1)^2}{8} + \frac{2x-1}{2} + \frac{2(2x-1)}{4} + C \end{aligned}$$

## 27

$$\begin{aligned} \int (2x+3)^2 (1-x)^8 dx &= |t = 1-x, \quad dt = -dx, \quad x = 1-t, \quad 2x+3 = -2t+5| = \\ &= - \int (-2t+5)^2 t^8 dt = - \int (4t^2-20t+25) t^8 dt = -4 \int t^{10} dt + 20 \int t^9 dt - 25 \int t^8 dt = \\ &= -\frac{4t^{11}}{11} + 2t^{10} - \frac{25t^9}{9} + C = -\frac{4(1-x)^{11}}{11} + 2(1-x)^{10} - \frac{25(1-x)^9}{9} + C \end{aligned}$$

## 28

$$\begin{aligned} \int \frac{x-4}{\sqrt{x^2-2}} dx &= \int \frac{x}{\sqrt{x^2-2}} dx - 4 \int \frac{dx}{\sqrt{x^2-2}} = \\ &= |t = x^2-2, \quad dt = 2x dx| = \frac{1}{2} \int \frac{dt}{\sqrt{t}} - 4 \int \frac{dx}{\sqrt{x^2-2}} = \sqrt{t} - 4 \ln |x + \sqrt{x^2-2}| + C = \\ &= \sqrt{x^2-2} - 4 \ln |x + \sqrt{x^2-2}| + C \end{aligned}$$

## 29

$$\begin{aligned} \int \frac{x+2}{\sqrt{x^2-10}} dx &= |t = x^2-10, \quad dt = 2x dx| = \frac{1}{2} \int \frac{dt}{\sqrt{t}} + 2 \int \frac{dx}{\sqrt{x^2-10}} = \\ &= \sqrt{t} + 2 \ln |x + \sqrt{x^2-10}| + C = \sqrt{x^2-10} + 2 \ln |x + \sqrt{x^2-10}| + C \end{aligned}$$

**30**

$$\begin{aligned}\int x\sqrt{1-x^2}dx &= \left| t = 1-x^2, \quad dt = -2xdx \right| = -\frac{1}{2} \int \sqrt{t}dt = -\frac{1}{2} \frac{2}{3} t^{\frac{3}{2}} + C = \\ &= -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C\end{aligned}$$

**31**

$$\begin{aligned}\int x(1-x^2)^6 dx &= \left| t = 1-x^2, \quad xdx = -\frac{1}{2}dt \right| = -\frac{1}{2} \int t^5 dt = -\frac{1}{12} t^6 + C = \\ &= -\frac{(1-x^2)^6}{12} + C\end{aligned}$$

**32**

$$\int \frac{xdx}{1+x^4} = \left| t = x^2, \quad dt = 2xdx \right| = \frac{1}{2} \int \frac{dt}{1+t^2} = \frac{1}{2} \arctan t + C = \frac{\arctan x^2}{2} + C$$

**33**

$$\begin{aligned}\int \frac{x^2}{x^6-5} dx &= \left| t = x^3, \quad dt = 3x^2 dx \right| = \frac{1}{3} \int \frac{dt}{t^2-5} = -\frac{1}{3} \int \frac{dt}{5-t^2} = \\ &= -\frac{1}{3} \frac{1}{2\sqrt{5}} \ln \left| \frac{\sqrt{5}+t}{\sqrt{5}-t} \right| + C = \frac{\sqrt{5} \ln \left| \frac{x^3+\sqrt{5}}{x^3-\sqrt{5}} \right|}{30} + C\end{aligned}$$

**34**

$$\int \frac{x^4}{\sqrt{3+x^5}} dx = \left| t = x^5 + 3, \quad dt = 5x^4 dx \right| = \frac{1}{5} \int \frac{dt}{\sqrt{t}} = \frac{2\sqrt{t}}{5} + C = \frac{2\sqrt{x^5+3}}{5} + C$$

**35**

$$\int \frac{dx}{x(\ln x)^5} = \left| t = \ln x, \quad dt = \frac{dx}{x} \right| = \int \frac{dt}{t^5} = -\frac{1}{4t^4} + C = -\frac{1}{4(\ln x)^4} + C$$

**36**

$$\int \frac{dx}{x\sqrt{\ln x}} = \left| t = \ln x, \quad dt = \frac{dx}{x} \right| = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + C = 2\sqrt{\ln x} + C$$

**37**

$$\int \frac{dx}{x(2+(\ln x)^2)} = \left| t = \ln x, \quad dt = \frac{dx}{x} \right| = \int \frac{dt}{2+t^2} = \frac{\arctan \frac{t}{\sqrt{2}}}{\sqrt{2}} + C =$$

$$= \frac{\arctan \frac{\ln x}{\sqrt{2}}}{\sqrt{2}} + C$$

**38**

$$\int \frac{\sqrt[3]{(\ln x)^2}}{x} dx = \left| t = \ln x, \quad dt = \frac{dx}{x} \right| = \int t^{\frac{2}{3}} dt = \frac{3}{5} t^{\frac{5}{3}} + C = \frac{3(\ln x)^{\frac{5}{3}}}{5} + C$$

**39**

$$\int \frac{e^x}{1+e^x} dx = \left| t = 1+e^x, \quad dt = e^x dx \right| = \int \frac{dt}{t} = \ln |t| + C = \ln |1+e^x| + C$$

**40**

$$\int \sin 5x dx = \left| t = 5x, \quad dt = 5dx \right| = \frac{1}{5} \int \sin t dt = -\frac{\cos t}{5} + C = -\frac{\cos 5x}{5} + C$$

**41**

$$\int x \sin x^2 dx = \left| t = x^2, \quad dt = 2x dx \right| = \frac{1}{2} \int \sin t dt = -\frac{1}{2} \cos t + C = -\frac{1}{2} \cos x^2 + C$$

**42**

$$\int \frac{\cos x}{1+\sin x} dx = \left| t = \sin x, \quad dt = \cos x dx \right| = \int \frac{dt}{1+t} = \arctan \sqrt{t} + C = \arctan \sqrt{\sin x} + C$$