Homework 1

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1 Finite Differencing in Curved Coordinates

2 Differentiation and Integration with Noise

$$f(x) = sin(x)e^{cos(x)} \tag{1}$$
$$f'(x) = (cos(x) - sin^2(x))e^{cos(x)} \tag{2}$$

2.1 Differentiation using Stencils

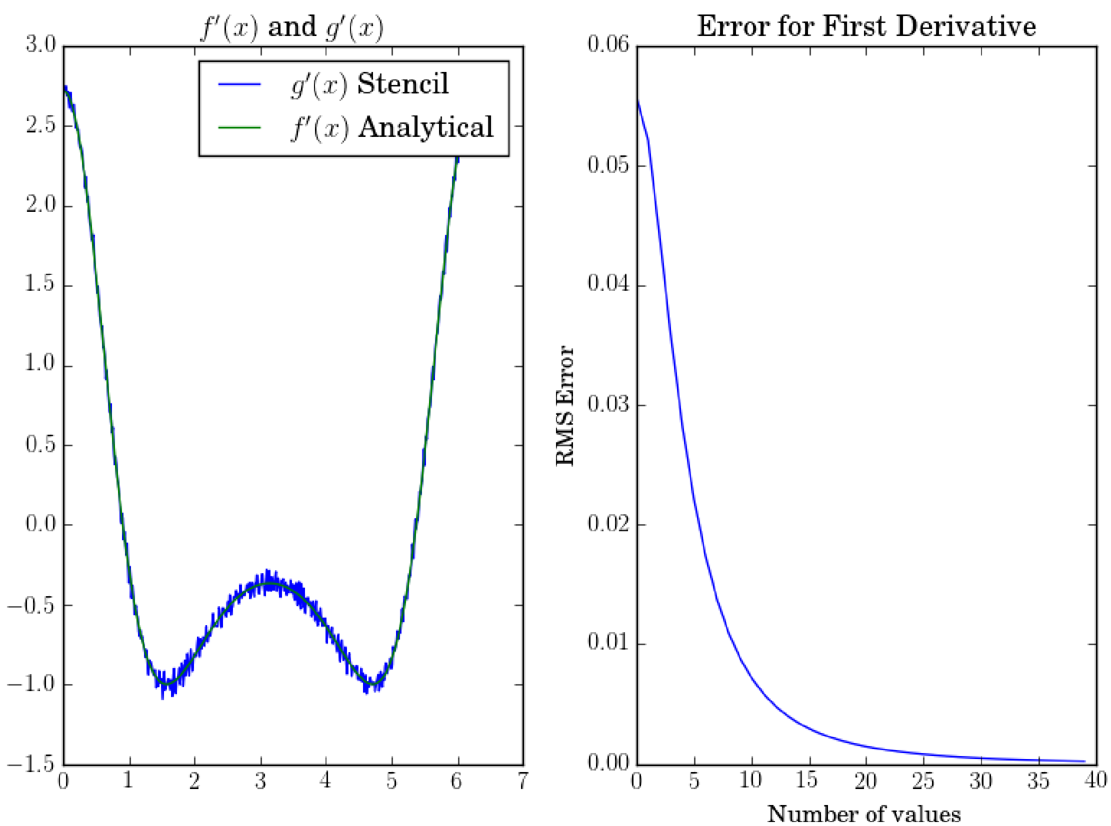

Figure 1: RMS Error in 5 point stencil for the first derivative

2.2 Integration with Simpson's Rule

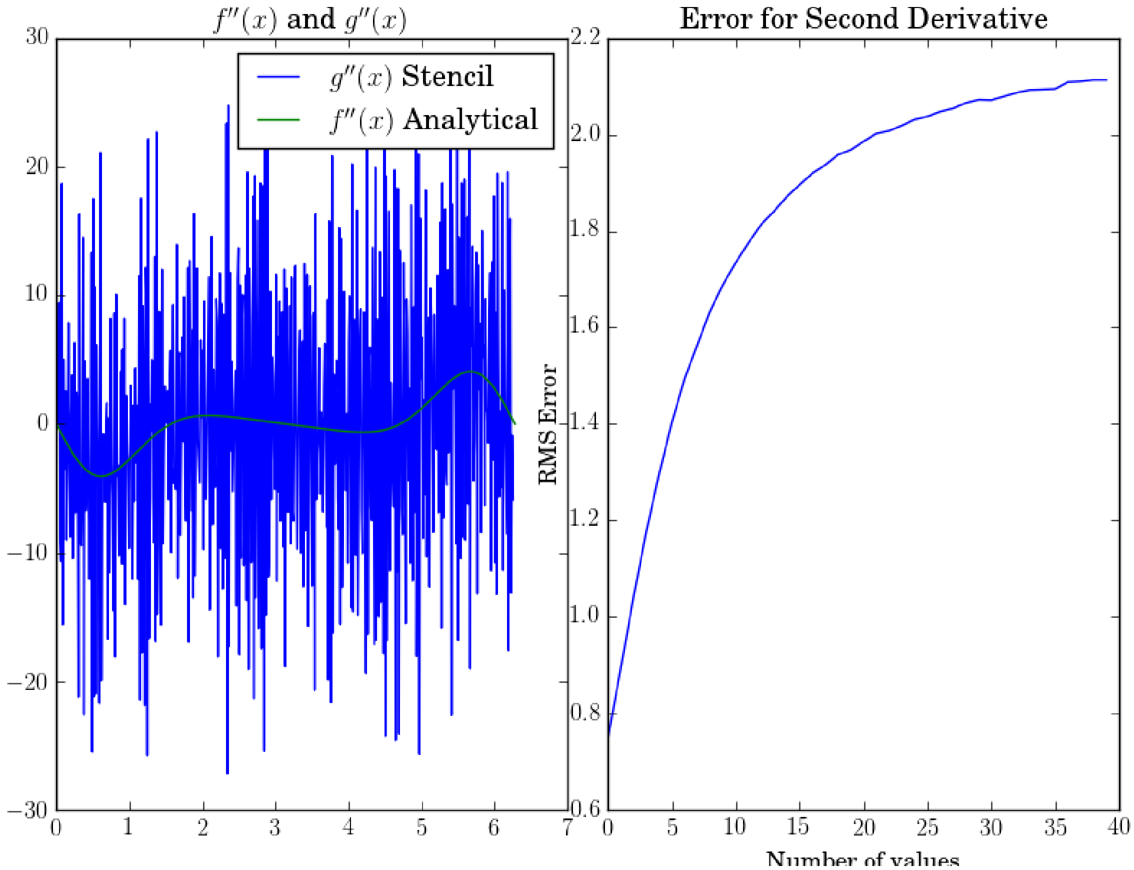

Figure 2: RMS Error in 5 point stencil for the second derivative

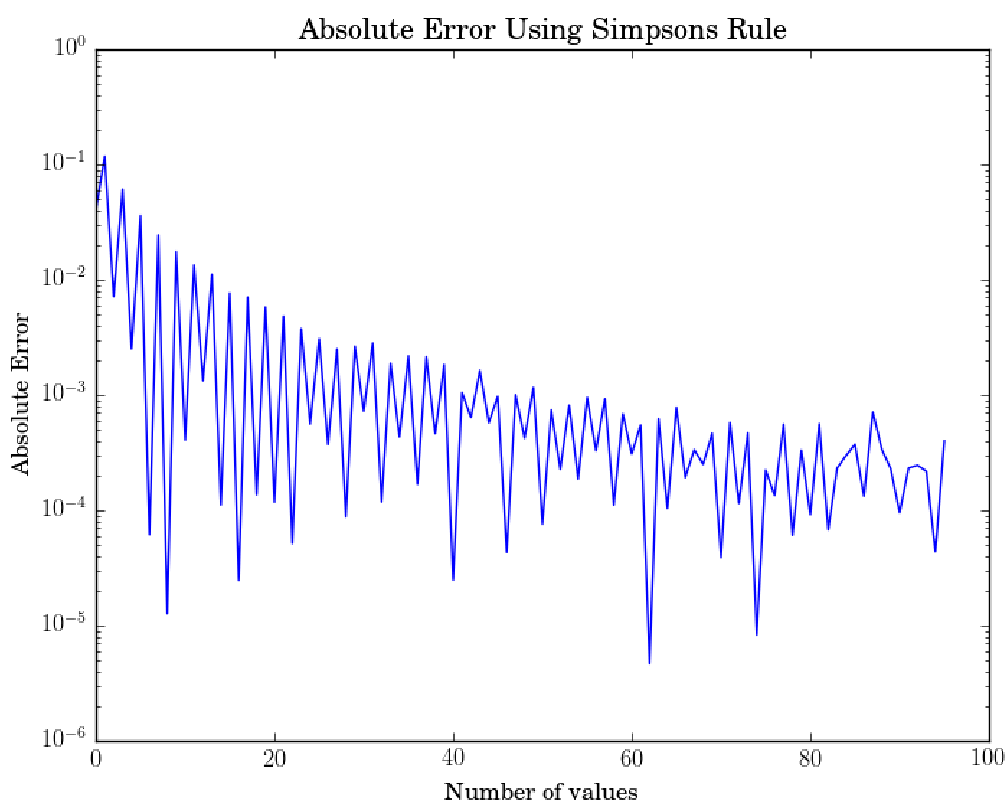

Figure 3: RMS Error using Simpson's Rule for varying numbers of points

3 Cepheid Lightcurve Integraiton

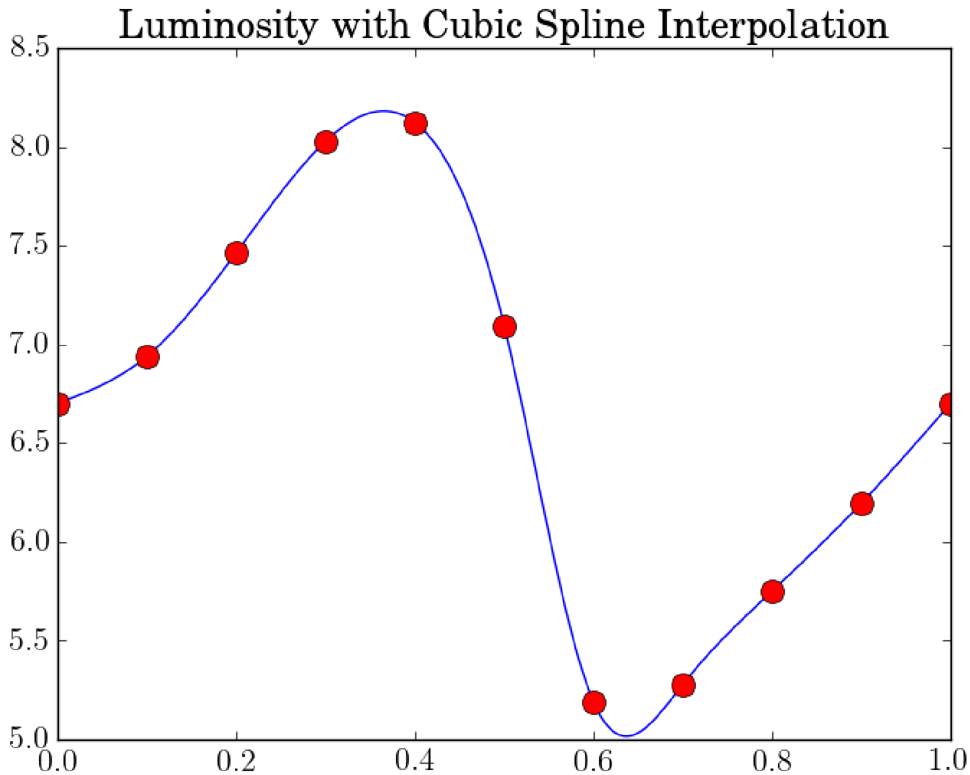

Figure 4: Magnitude converted to luminosity and interpolated with a cubic spline

Set Used	Simpson's Rule	Trapezoid Method
Data Only	6.0790	6.6789
Magnitude Spline	6.5974	6.5974
Luminosity Spline	6.6725	6.6725

Table 1: Luminosity-days evaluted for different methods of integration

4 Planck's Law

ν was redefined to be dimensionless:

$$\hat{\nu} \equiv \frac{h\nu}{k_B T}$$

$\hat{\nu}$ was transformed to $tan(\hat{\nu}$ over the range $(0, \pi/2)$ to evaluate the integral over infinity.

4.1 Total Number Density

$$
\begin{aligned}
n_{tot} &= \frac{8\pi}{c^3}\frac{k_BT}{h}\int_0^\infty \hat{\nu}^2\frac{1}{e^{\hat{\nu}}-1}d\hat{\nu} \\
&= \frac{8\pi}{c^3}\frac{k_BT}{h}\int_0^{\pi/2} \frac{tan(\hat{\nu})}{cos(\hat{\nu})}^2\frac{1}{e^{\hat{\nu}}-1}d\hat{\nu} \\
&= 2.40406\left(\frac{8\pi}{c^3}\frac{k_BT}{h}\right)
\end{aligned}
$$

4.2 Median Energy

$$
\begin{aligned}
median &= \frac{1}{2}\frac{8\pi}{c^3}\frac{k_BT}{h}\int_0^\infty n_\nu\hat{\nu}d\hat{\nu} \\
&= 1.31058\left(\frac{8\pi}{c^3}\frac{k_BT}{h}\right)^2
\end{aligned}
$$

4.3 Mean Energy

Using the frequency:

$$\bar{\nu} = 1.09030\left(\frac{k_BT}{h}\right)$$

Using the wavelength:

$$\bar{\lambda} = 0.64494\left(\frac{hc}{k_BT}\right)$$

The product should be c:

$$\left[0.64494\left(\frac{hc}{k_BT}\right)\right]\left[1.09030\left(\frac{k_BT}{h}\right)\right] = 0.70319c$$

Which it isn't (Error somewhere).

4.4 Standard Deviation

$$\sigma_\nu = 0.64756\left(\frac{k_BT}{h}\right)$$

5 Romberg Integration

Function	$R_{3,3}$	Actual
$cos^2(x)$	1.0118	1.4546
$xln(x+1)$	0.2454	0.3243
$sin^2(x) - 2xsin(x)$	-1.060	1.3667
$(xln(x))^{-1}$	0.4082	0.5266

Table 2: Integral approximations using Romberg integration

6 Gaussian Quadrature

$$a = \frac{7}{15}$$
$$b = \frac{16}{15}$$
$$c = \frac{7}{15}$$
$$d = \frac{1}{15}$$
$$e = -\frac{1}{15}$$