Project 2

Dan Kolbman

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1 Interpolation of Reflector Points

A cubic spline was used to interpolate given points on the reflector surface. The first derivatives of the y_1 and y_n points were used to apply clamped boundary conditions to the spline. The derivative of the last knot, $y_5\prime$, was doubled to show its effects on the complete spline. Only the interpolated line between the last 3 knots y_3 and y_5 were visibly affected by changing the boundary condition. Overall, using a cubic spline for interpolation yielded very good results.

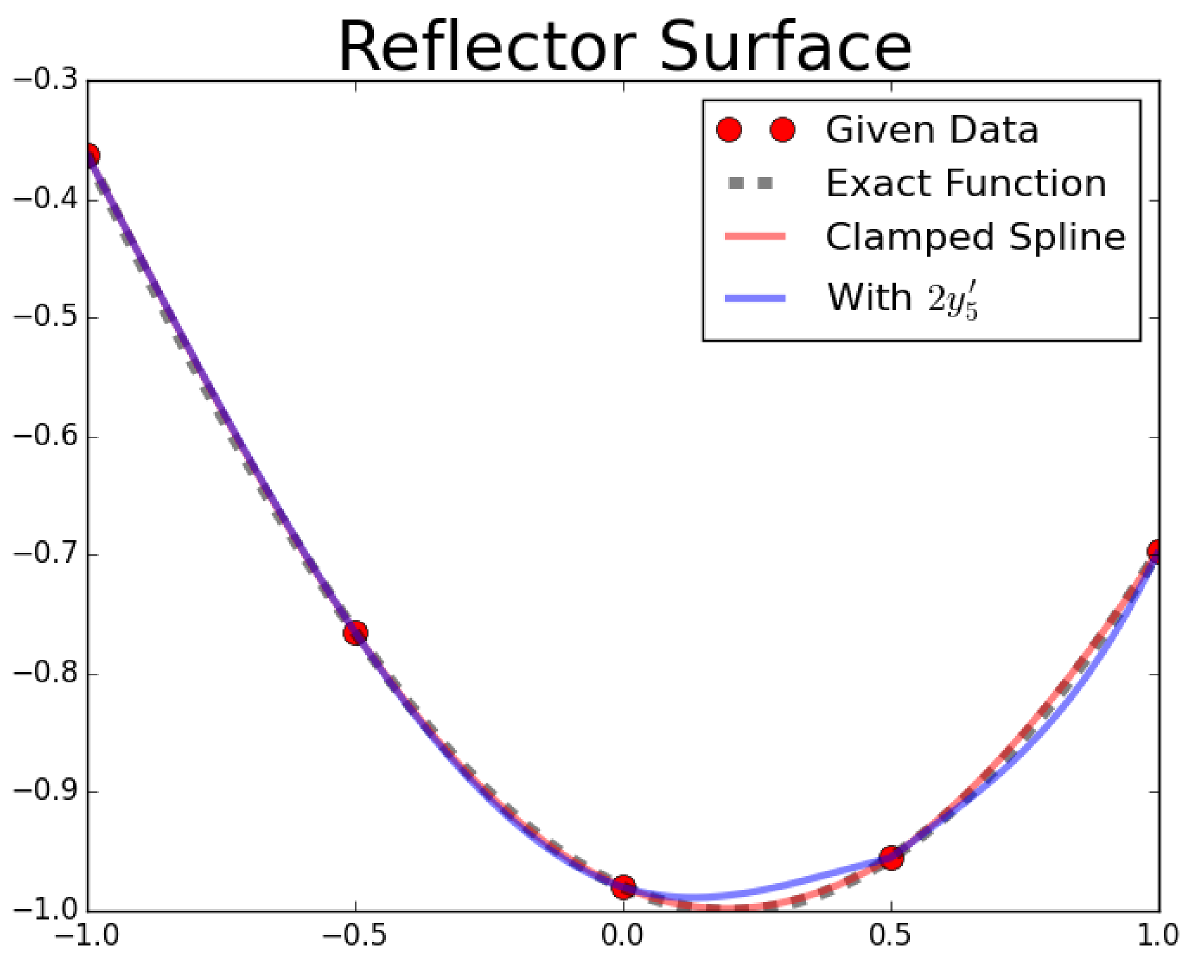

Figure 1: Fits for $f(x) = -cos(x - 0.2)$

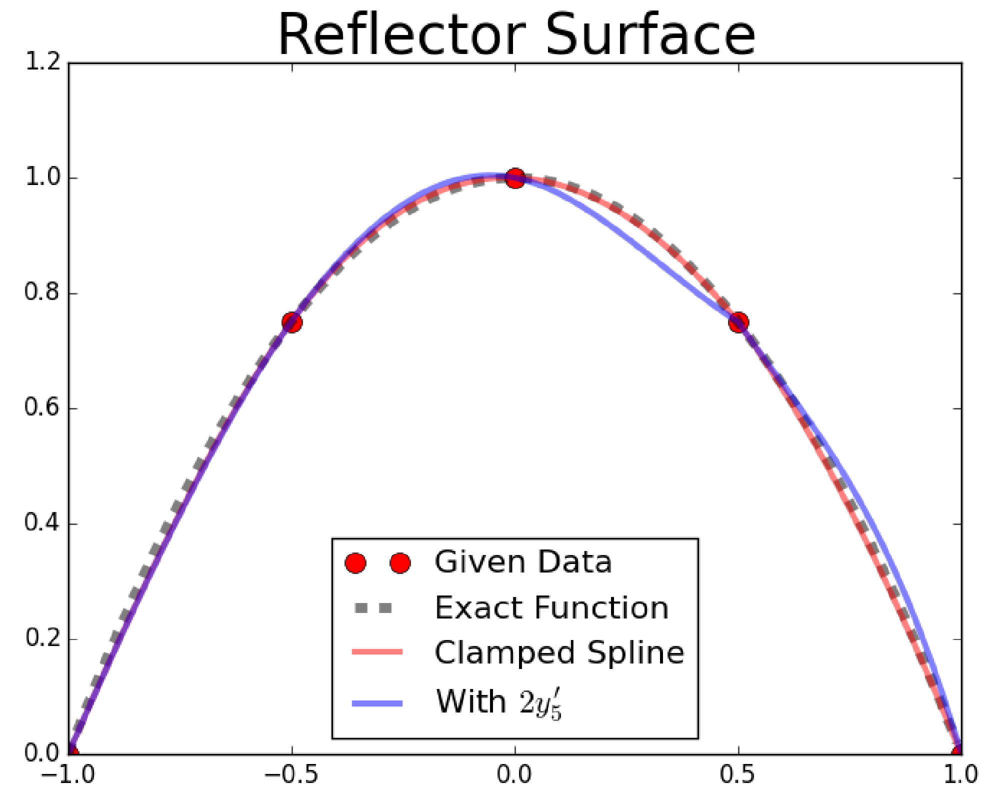

Figure 2: Fits for $f(x) = 1 - x^2$

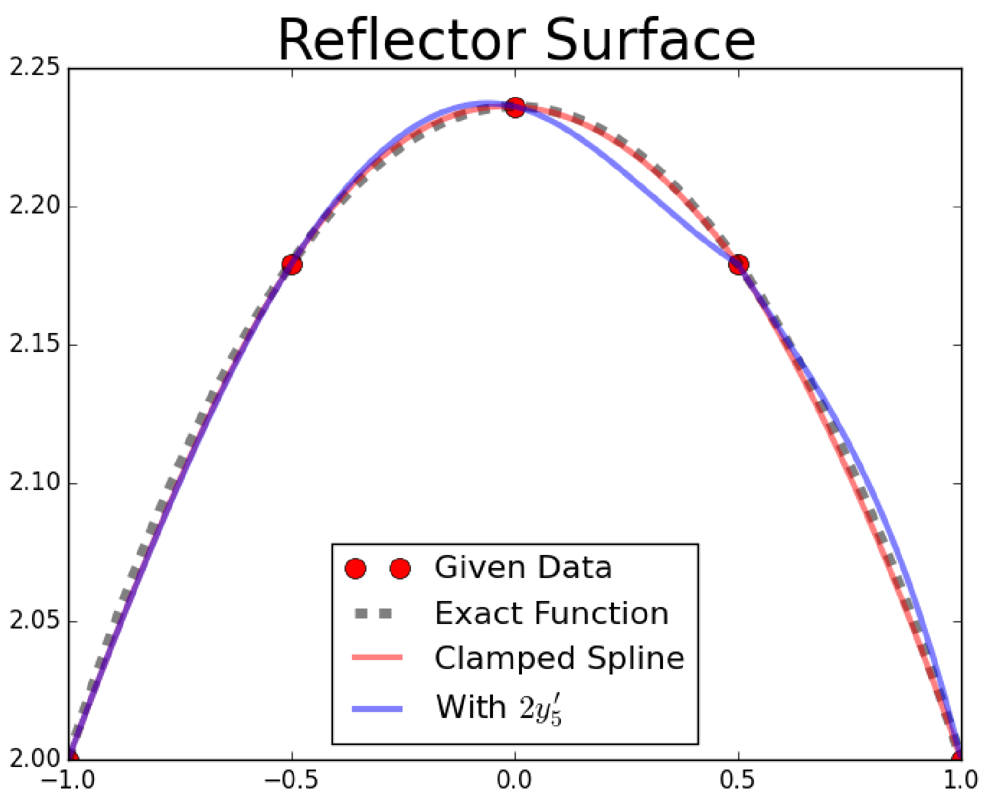

Figure 3: Fits for $f(x) = \sqrt{5 - x^2}$

2 Error Analysis

Error was calculated using the root mean square error (Eq 1) on all interpolated points.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{n}} \tag{1}$$

Doubling the derivative of the last knot resulted in a worse fit, as expected.

$f(x)$	RMSE (y_5')	RMSE ($2y_5'$)
$-cos(x-0.2)$	0.00357	0.00963
$1-x^2$	0.01047	0.02846
$\sqrt{5-x^2}$	0.00295	0.00697

3 Derivatives

The derivative of the interpolated spline was found using a central difference finite difference. Its accuracy largely depends on the resolution available from the spline. These results come from a spline with 200 points for each segment, or 800 total points. The error is expected te be atleast as bad as the function itself as new error will be introduced from the numerical derivative. It also must be worse as there were less conditions met for the derivative (2) than the function (5). Evaluating the error in the form of the RMSE reveals this is true.

$f(x)$	$f'(x)$	RMSE
$-cos(x-0.2)$	$-sin(0.2-x)$	0.02398
$1-x^2$	$-2x$	0.07096
$\sqrt{5-x^2}$	$\frac{-x}{\sqrt{5-x^2}}$	0.01971

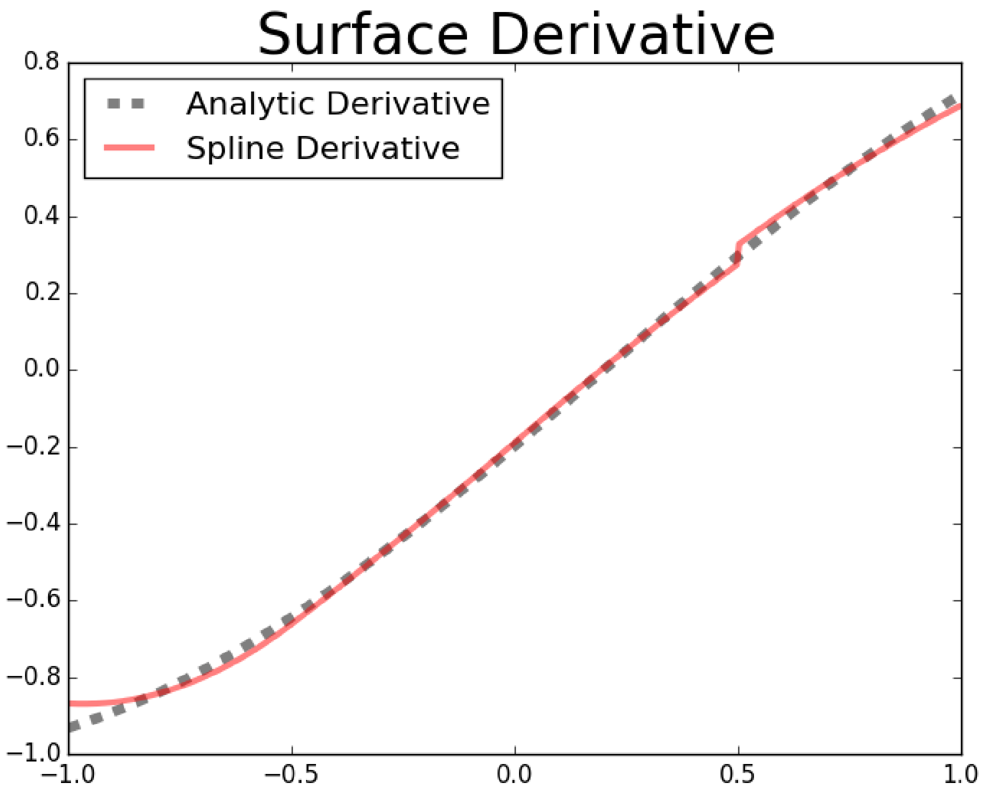

Figure 4: Fits for $f'(x) = -sin(0.2 - x)$

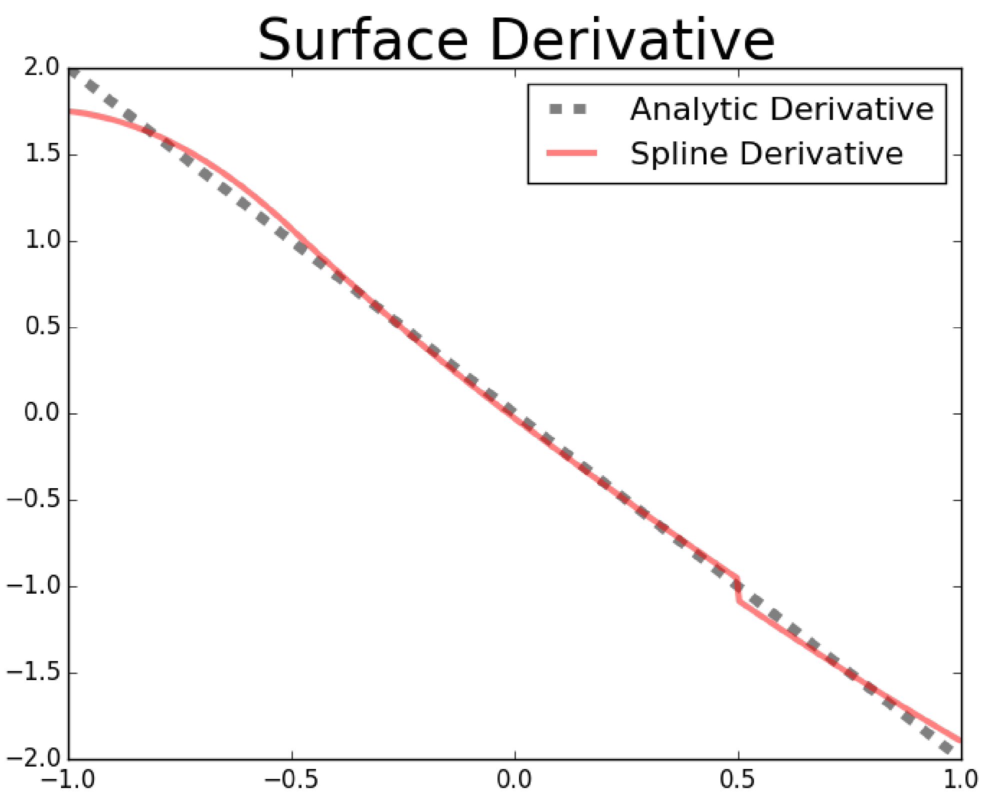

Figure 5: Fits for $f'(x) = -2x$

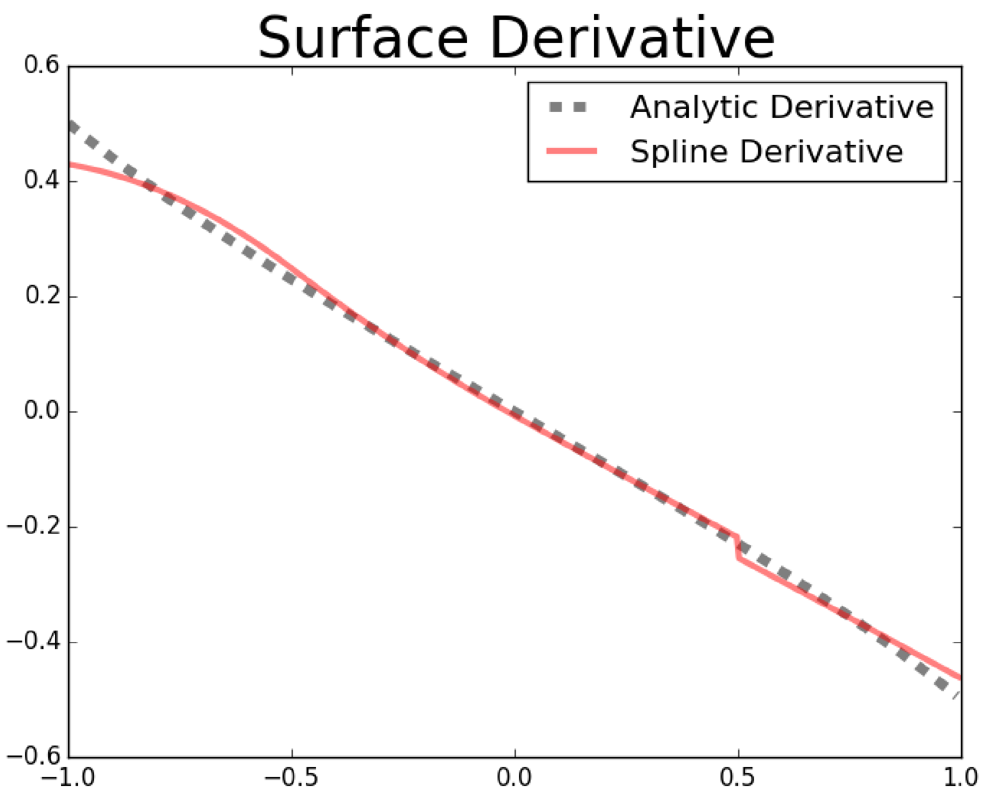

Figure 6: Fits for $f'(x) = \frac{-x}{\sqrt{5-x^2}}$