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Separating natural and epistemic uncertainty in flood frequency analysis

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Abstract

Although there are many sources of uncertainty it is important to recognise two basic kinds of uncertainty that are fundamentally different from each other: natural and epistemic uncertainty. Natural uncertainty stems from variability of the underlying stochastic process. Epistemic uncertainty results from incomplete knowledge about the process under study. The paper looks at the difference between these two kinds of uncertainty in flood frequency analysis. Natural uncertainty is incorporated in the distribution function of the annual maximum series from which the flood design criteria (e.g. annual failure probability, AFP) is derived. Sampling uncertainty and model uncertainty are two epistemic uncertainty sources. Sampling uncertainty is represented by probability distributions for AFP. The design criteria AFP is considered as random variable whereas the uncertainty of AFP depends on the knowledge of the analyst. It is shown how more data steepen the cumulative distribution function (cdf) of AFP, and, therefore, decrease the uncertainty about AFP. The uncertainty due to different distribution functions is incorporated by using probability bounds. They give a region within which the true but unknown distribution function is expected. The greater the uncertainty due to the distribution function type, the wider the bounds and the more difficult to make statements about frequencies of extreme events. By using a likelihood measure as indicator for the appropriateness of different distribution functions, distribution functions with low weights are eliminated. This considerably narrows the uncertainty bounds. This approach which separates between natural and epistemic uncertainty reveals the uncertainty which can be reduced by more knowledge (epistemic uncertainty) and the uncertainty which is not reducible (natural uncertainty).

Keywords: Flood estimation; Frequency analysis; Uncertainty; Variability

1. Introduction

1.1. Natural and epistemic uncertainty

Studies in risk analysis and decision analysis have shown that the consideration of uncertainty may be crucial for reliable results. The lack of quantitative

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characterisation of uncertainty may yield a qualitatively and quantitatively different answer than that derived from a reasoned treatment of uncertainty (Morgan and Henrion, 1990; Frank, 1999). There seems to be agreement that risk analyses should give an indication of the uncertainty of the risk quantifications: optimal decisions can only be expected when all relevant uncertainties are taken into consideration (USACE, 1992; Aven and Pörn, 1998).

There are comprehensive taxonomies of uncertainty in the literature which discuss in detail different sources and kinds of uncertainty (Morgan and Henrion, 1990; Rowe, 1994; Haimes, 1998; van Asselt and Rotmans, 2002). For the purpose of this paper, it is important to recognise two basic kinds of uncertainty that are fundamentally different from each other: natural and epistemic uncertainty. Natural uncertainty stems from variability of the underlying stochastic process. Epistemic uncertainty results from incomplete knowledge about the system under study. Natural uncertainty has also been termed (basic) variability, aleatory uncertainty, objective uncertainty, inherent variability, (basic) randomness, and type-A uncertainty. Terms for epistemic uncertainty are subjective uncertainty, lack-of-knowledge or limited-knowledge uncertainty, ignorance, specification error, prediction error, and type-B uncertainty (Tang and Yen, 1972; Morgan and Henrion, 1990; Plate, 1992, 1993a; Hoffman and Hammonds, 1994; NRC, 1995, 2000; Ferson and Ginzburg, 1996; Zio and Apostolakis, 1996; Haimes, 1998; Cullen and Frey, 1999; van Asselt and Rotmans, 2002).

Natural uncertainty refers to quantities that are inherently variable over time, space, or populations of individuals or objects. Variability exists, for example, in the amount of annual rainfall in consecutive years, in the clay content of a field, or in the body weight of adults. Such phenomena are characterised as random or stochastic, and probabilistic models are adopted for their description. Epistemic uncertainty is related to our ability to understand, measure, and describe the system under study. For example, if we use a mathematical model to describe a system, epistemic uncertainty may consist of model (or structural) uncertainty and parameter uncertainty. Model uncertainty can be thought of as addressing the uncertainty in the appropriateness of the structure of the model. Parameter uncertainty arises in the process of employing a specific value to the quantities of concern. Epistemic uncertainty or ignorance is partly due to the inability to measure variables: it is impossible to measure all necessary variables at all points in time and space. Furthermore, there are other constraints that result in noise in the measurements such as measurement costs, inadequate technology, or technological limitations (Moore and Brewer, 1972).

It is often stated that natural uncertainty is a property of the system, whereas epistemic uncertainty is a property of the analyst (Cullen and Frey, 1999). Different analysts, with different states of knowledge, different resources for obtaining data, etc. may have different levels of epistemic uncertainty regarding their predictions.

The central issue is that the differentiation in natural and epistemic uncertainty separates uncertainty which can be reduced (epistemic uncertainty) and uncertainty which is not reducible (natural uncertainty). For example, in predicting the impact of an accidental release of a volatile toxic substance from a chemical plant, the wind speed and direction at the release time are crucial. These quantities cannot be deterministically forecasted due to the inherent variability of the wind. The best that can be done is to quantify this variability by using an empirical frequency distribution of wind speed and direction. In addition, the parameters of this frequency distribution are uncertain. This epistemic uncertainty can be further reduced, e.g. by obtaining more data about the wind field.

From a practical viewpoint, it is rare to encounter only one type of uncertainty (Haimes, 1998; Cullen and Frey, 1999). Pure variability would mean that all relations and their parameters which describe the random process are exactly known. Pure epistemic uncertainty would mean that a deterministic process is considered but the relevant information cannot be obtained, e.g. due to the inability to measure the relevant parameters.

Many researchers recognise that epistemic and natural uncertainty should be treated separately (Hoffman and Hammonds, 1994; Ferson and Ginzburg, 1996; Helton, 1996; Hora, 1996; Parry, 1996; Winkler, 1996; Zio and Apostolakis, 1996; Haimes, 1998; Cullen and Frey, 1999; Frank, 1999; Hall, 2003). Both types of uncertainty are frequently described by probability distributions, however, with different interpretations: probability distributions for natural variables represent the relative frequency of values from a specified interval, whereas probability distributions for epistemic parameters represent the degree of belief or knowledge that a value is within a specified interval. Of course, epistemic uncertainty arises not only due to expert judgement about an uncertain parameter but also due to random sampling error and measurement errors.

The superposition of both types of uncertainty may lead to erroneous inferences (Cullen and Frey, 1999). As an example, Fig. 1 shows how the superposition of both uncertainties may mask important information. A population of river levee sections is located in the study area. Further, a levee section is assumed to fail if the river water level h_R exceeds the breaching height h_B. h_B may be smaller than the height of the levee due to failure mechanisms like water saturation and loss of soil stability, seepage through the levee foundation, and soil failure through rupturing. h_B is considered as a random variable. On the one hand, this is the result of the variability of geologic, soil and hydraulic properties from section to section. On the other hand, for a given levee section there is an epistemic uncertainty because it may not be possible to determine h_B exactly.

The variability of h_B in the levee section population may be described by the probability density function (pdf) $f_{h_B}(h_B)$ or the cumulative distribution function (cdf) $F_{h_B}(h_B)$. Because of epistemic uncertainty the distribution type and parameters of $f_{h_B}(h_B)$ may not be known with

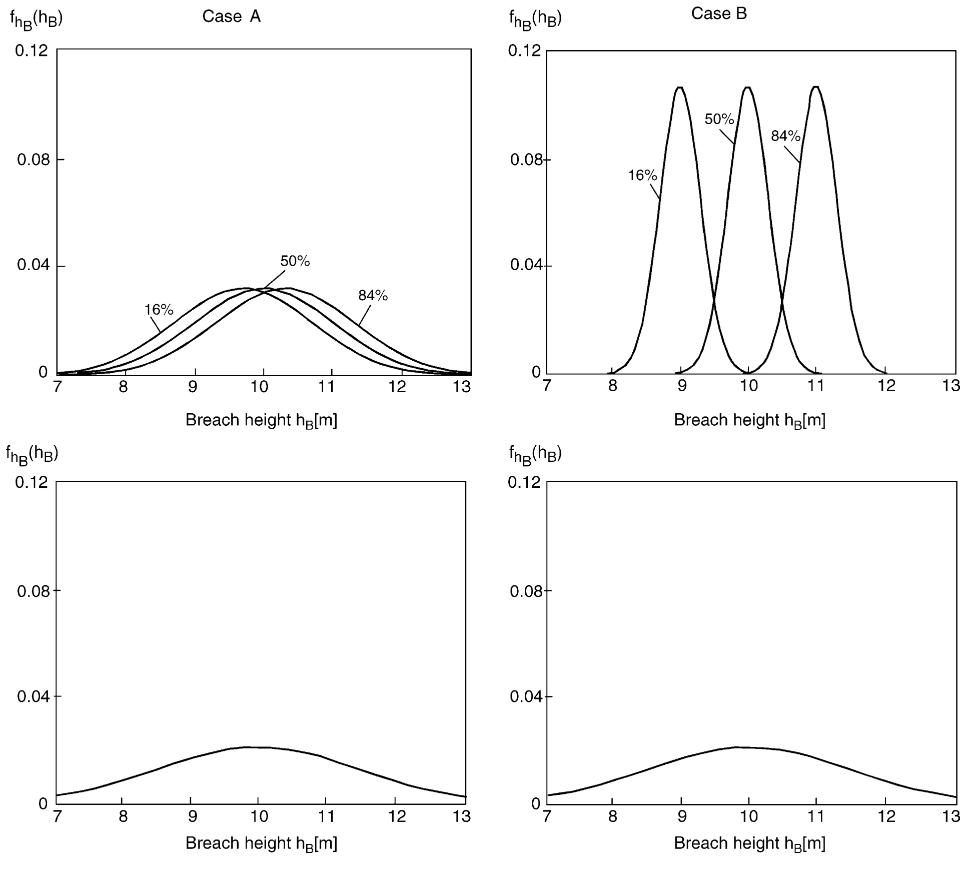

Fig. 1. Combining natural uncertainty, due to variability in the levee population, and epistemic uncertainty, due to incomplete knowledge about the levee breach process. Case A: large variability and small epistemic uncertainty; Case B: small variability and large epistemic uncertainty. Top: separation of natural (pdf for the breaching height h_B) and epistemic uncertainty (16, 50 and 84% percentiles of the pdf of h_B). Bottom: pdf of h_B for the combination of natural and epistemic uncertainty.

certainty. If it is assumed that, due to epistemic uncertainty, $f_{h_B}(h_B)$ is uncertain, a family of distributions may be constructed. Fig. 1 depicts this situation where the median is the pdf of h_B without taking into account the epistemic uncertainty. The percentile pdf illustrates the epistemic uncertainty, e.g. it is presumed that the true but unknown pdf is between the 16 and 84% percentile pdf with a probability of 68%. Fig. 1 A shows a situation where natural uncertainty dominates over epistemic uncertainty. h_B, the height when the levee sections fail, is relatively certain but h_B varies due to the large variation of the levee population. In case B a relatively homogeneous population is supposed, e.g. because the levee sections were built with the same soil material and they have experienced the same maintenance programme, but there is only little knowledge about the breaching process. Here, epistemic uncertainty dominates over natural uncertainty. Combining both types of uncertainty yields a hybrid distribution which does not inform about the two distinct sources of uncertainty.

Separating both types of uncertainty may help to make more informed management decisions. In case A, the available, but limited resources will be used for safety measurements for the most vulnerable levee sections. In that case the sections with the lowest values of h_B have to be identified. In case B, the available resources should be used in order to reduce process uncertainty, e.g. by performing experiments and collecting data about the breach process. Furthermore, safety measures will be uniformly distributed among the levee sections. This example shows that the superposition of the two types of uncertainty may yield an identical uncertainty whereas both cases are very different. The separation of uncertainty is more informative and gives a more complete representation of the process and the appropriate management response.

1.2. Uncertainty in flood frequency estimation for flood design

The discharge q_T corresponding to a return period T is a primary characteristic in the design of flood-control structures. Frequently, q_T is estimated by statistical flood frequency analysis which presumes that the observed flood discharges come from a parent population and can be described by a probability distribution. Table 1 shows the sources of uncertainty when q_T is estimated by flood frequency analysis. These sources are epistemic uncertainties because they depend on the analyst's knowledge about the stochastic process.

Table 1
Sources of uncertainty in flood-frequency estimation

Source	Examples
Measurement errors	Water level measurement errors, rating curve errors
Plotting position formula	Weibull, Hazen, Gringorten
Assumptions	Randomness, stationarity, homogeneity, independence
Selection of sample	Representative observation period, annual-flood series or partial duration series, consideration of historical events
Distribution function	Lognormal, Pearson type 3, Generalized Extreme Value
Parameter estimation method	Method of moments, method of L-moments, method of maximum likelihood
Sampling uncertainty	Length of time series

The first source of uncertainty results from measurement errors. Flow is commonly estimated indirectly by means of a rating curve which relates water level to discharge. In addition to the error in measuring the water level there is an error due to the transformation of the water levels to discharge values (Clarke, 1999). This error depends, among others, on the characteristic double-valued loop in the stage-discharge curve during the passage of a flood wave. Further measurement errors may arise due to changes in the rating curve, e.g. because of sediment deposition or erosion in the river bed. Unfortunately, the measurement errors are largest for extreme discharges which are the most valuable for flood frequency estimation (Clarke, 1999).

A second source of uncertainty are the assumptions on which flood frequency analysis is built. In particular, the assumption of stationarity has limited validity in geophysical time series (Booy and Lye, 1989) and, concerning flood estimation, this assumption has been questioned (Klemes, 1988, 1993; Arnell, 1989; Kaczmarek et al., 1996; Olsen et al., 1998; Milly et al., 2002). Changes in the watershed and the river system, such as urbanization, deforestation, or river training, affect the magnitude of annual floods and may result in a non-stationarity in the time series. There is also an indication of non-stationarity due to climate change (Milly et al., 2002). There are statistical tests, e.g. for testing independence or homogeneity, but the errors due to the underlying assumptions are difficult to evaluate (Bobee and Ashkar, 1988). Klemes (1986, 1988) gives a worthwhile evaluation of the assumptions of flood frequency estimation.

Further sources of uncertainty are the selection of the sample, the plotting position formula, the distribution function and the parameter estimation method. For example, for gauges with long observation records the assumption of stationarity is doubtful, and, therefore, a period representative for the conditions for which the frequency estimates will be made has to be selected. This selection of the sample can have large consequences (e.g. Lammersen et al., 2002). A large source of uncertainty arises due to the choice of the distribution functions (Tang, 1980; Cunnane, 1985; Resendiz-Carrillo and Lave, 1987; Afshar and Marino, 1990; Cluckie and Pessoa, 1990; Chbab, 1995; Malamud et al., 1996). Frequently, different distribution functions agree well with the observed data but give different extrapolation values. Tests of goodness of fit evaluate whether a certain pdf represents the sample. Unfortunately, due to the usually small flood samples, these tests are not powerful enough to discriminate between the different pdf's, and it is advisable not to count completely on statistical tests in choosing a pdf (Bobee and Ashkar, 1988).

Another source of uncertainty is the sampling uncertainty. Due to the limited information in the sample the estimation of q_T is combined with an error. This error is reduced with increasing sample size, but only if the assumptions of flood frequency analysis are valid for the studied site. Traditionally, sampling uncertainty is quantified by confidence intervals. The sampling uncertainty is expressed by means of intervals which contain the unknown but fixed population quantile, i.e. q_T, with a specified probability. Methods for deriving confidence intervals for q_T were discussed by Stedinger et al. (1992).

For each step in flood frequency analysis there are several possibilities and, in many cases, there is little guidance for these choices. One way to cope with this uncertain situation is to give recommendations, e.g. the Flood Estimation Handbook with generalized procedures for rainfall and flood frequency estimation in the UK (Institute of Hydrology, 1999). Some countries have agreed to adopt a certain distribution function (WRC, 1981). Thus, some degree of uniformity is reached although no attempt is made to explicitly describe the uncertainty. Frequently, such rules have built in some degree of conservatism.

Owing to the various sources of uncertainty, the design value q_T may be defined as a random variable that has a mean, variance and probability density (Haan, 1972). Due to the uncertainty of q_T the flood design involves an error which may result in an economic loss. In such cases the effects of uncertainty have been commonly compensated for by a margin of safety (e.g. freeboard). Bogárdi and Szidarovszky (1972) propose that a wide margin of safety is necessary where the discharge record available is short, the water regime is flashy in character and great economic interests are attached to the project.

Progress in quantifying the uncertainty in flood frequency analysis has been made by employing Bayesian statistics. Here, the uncertainty in unknown parameters can be described by a probability distribution that represents the analyst's degree of belief that the parameter has different values. The Bayesian point of view allows unknown model parameters to be treated as random variables. Further, the Bayesian approach allows incorporation of the uncertainty due to the choice of the distribution function in the frequency analysis. For example, Tang (1980) has proposed that different probability distributions could be correct. However, the model that has a larger discrepancy between observed flood data and the distribution function will have a larger standard deviation about the predicted value q_T. Consequently, this model is given less weight than models which better represent the sample.

The Bayesian view differentiates between descriptive distribution and predictive distribution (Booy and Lye, 1989). The descriptive distribution represents the variability of the process while the predictive distribution describes the probability that the random variable will be exceeded in a given time interval, taking into account that the parameters of the descriptive distribution are estimated from limited data. This approach allows the integration of natural variability and sampling uncertainty in a single risk estimation (Davis et al., 1976; Wood, 1977; Tang, 1980; Tung and Mays, 1981a; Booy and Lye, 1989). Other Bayesian approaches have incorporated the uncertainty due to the distribution function (Wood and Rodriguez-Iturbe, 1975; Bobo and Unny, 1976; Tang, 1980; Krzysztofowicz and Yakowitz, 1980; Tung and Mays, 1981a; Chbab et al., 2000). Thus, the Bayesian statistical theory allows to combine epistemic uncertainties from different sources (e.g. distribution function uncertainty, sampling uncertainty) with the natural uncertainty due to the inherent hydrological variability.

NRC (1995, 2000) has proposed that the distinction between natural and epistemic uncertainty is important in flood design. This distinction can be introduced in the design process by quantifying the epistemic uncertainty which is connected to the design criteria value. An example for this approach is the characterization of the design criteria EAD (expected annual damage). Epistemic uncertainty can be incorporated by considering EAD as random variable, due to imperfect knowledge. The calculation of EAD leads to different numerical results depending upon which uncertainties—natural uncertainty, epistemic uncertainty, or both—are included in the probabilistic averaging of the design procedure (NRC, 2000).

1.3. Purpose of the paper

This paper looks at some of the uncertainties which are connected to flood frequency estimation. In particular, the difference between the two fundamental kinds of uncertainty, epistemic and natural uncertainty, is highlighted. It is argued that natural and epistemic uncertainty should be distinguished in flood frequency estimation, especially when the results are used in the framework of flood design.

2. System outline and data

In this paper, the most simple flood design problem is considered, i.e. a flood defence wall which is designed to withstand the T-year flood discharge q_T. Given the return period T, the task is to estimate q_T. The probability of failure of the flood wall is defined as the probability that the system fails within a specified time interval. More specifically, it is defined as the annual probability that the maximal river discharge exceeds the critical discharge q_C above which flood damage occurs. q_C corresponds to the height of the flood wall. Thus the annual failure probability AFP reads

$$\mathrm{AFP} = P(q > q_C) = \int_{q_C}^{\infty} f_q(q)\mathrm{d}q \qquad (1)$$

where $f_q(q)$ is the pdf of the annual maximum flood peak. The only uncertainty considered in Eq. (1) is the natural uncertainty, i.e. the inherent randomness of flood occurrence represented by $f_q(q)$. The return period T (in years) is the average time separating two events with $a > q_T$, when the average is taken over a long period of time. For $q_T = q_C$, the return period reads $T = 1/\mathrm{AFP}$.

In what follows, the discharges of the Cologne gauge on at the river Rhine in Germany are considered. At this gauge a discharge series with daily data from 1880 to 1999 (120 years) is available. It was presumed that the assumptions on which flood frequency analysis is built, especially stationarity, are valid. For each hydrological year (from 1st November to 31st October), the maximum discharge was determined. This annual maximum series is further abbreviated AMS. Different (three-parameter) distribution functions were adapted to the AMS: Generalized Extreme-Value (GEV), Generalized Logistic (GL), Pearson type III (PE3), and Lognormal (LN3). PE3 refers to the Pearson type III, not the Log Pearson type III mandated for use in the US (WRC, 1981). These four distributions are widely used in flood frequency analysis (e.g. Stedinger et al., 1992; Institute of Hydrology, 1999). The parameters of the distributions were estimated by the method of L-moments given in Hosking and Wallis (1997). L-moments are frequently preferred for flood frequency analysis because of their robust properties in the presence of very small or large values (Institute of Hydrology, 1999). Fig. 2 shows the four distributions functions and the observed data by using six different plotting-position formulas (Weibull, Gringorten, Cunnane, Median, Blom, Hazen) given in Stedinger et al. (1992). Table 2 gives the 100-year floods for the different distribution functions.

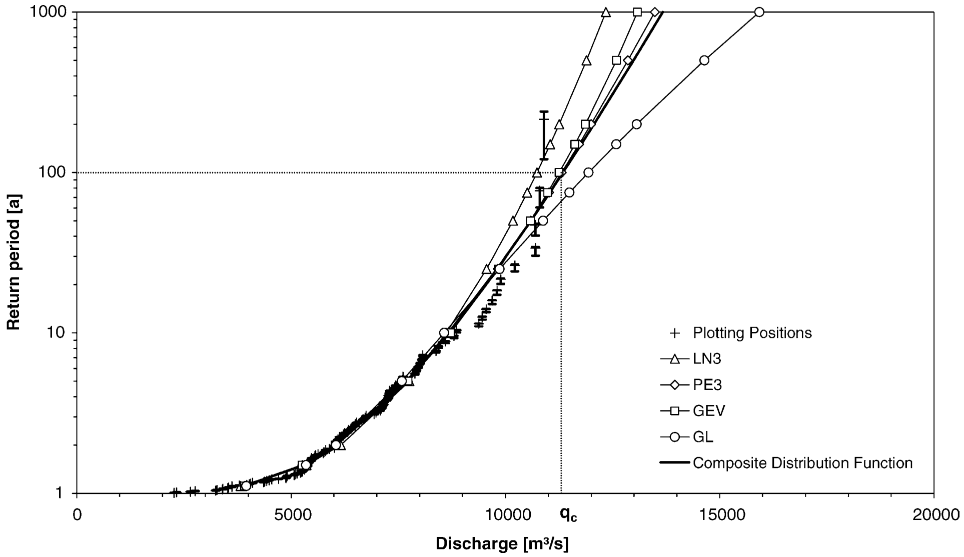

Fig. 2. Flood frequency estimation for AMS18801999 at the Cologne gauge on the river Rhine, using six plotting positions, four distributions functions and the composite distribution function derived in Section 3.2.

The critical discharge q_C equals the 100-year flood according to a Pearson type III distribution (q_{100}(PE3)). Within this design framework the annual failure probability AFP was determined considering natural uncertainty (variability), sampling uncertainty and distribution type uncertainty. Natural uncertainty or variability is inherent in the distribution function of the AMS. Distribution type uncertainty is addressed by considering the above mentioned distribution functions. Sampling uncertainty is considered by the variance Var(q_T) which equals the square of the standard error $s(q_T)$ for the mean estimated q_T. According to Stedinger et al. (1992) and Plate (1993b), it is assumed that q_T follows a normal distribution with mean q_T and variance $s^2(q_T)$

$$s^2(q_T) = \frac{s^2_{\mathrm{AMS}}}{n}\left[1 + K_T C_{SX} + \frac{K_T^2}{4}(K_X - 1)\right] \tag{2}$$

where n is sample size of the discharge series AMS, s_{AMS} and $\bar{q}_{\mathrm{AMS}}$ are the standard deviation and the mean of the discharge series AMS, respectively, K_X is the kurtosis, C_{SX} is the coefficient of skewness, and $K_T = q_T - \bar{q}_{\mathrm{AMS}}/s_{\mathrm{AMS}}$.

Table 2
Estimation of the 100-year flood for the Cologne gauge on the river Rhine, using different distribution functions and the composite distribution function (AMS 1880–1999)

Distribution function	100-year flood (m^3/s)
LN3	10,737
PE3	11,313
GEV	11,256
GL	11,940
Composite distribution	11,343

3. Combining natural and epistemic uncertainty in flood design

As shown in Table 1 there are different sources of epistemic uncertainty in flood-frequency analysis. In this paper two sources are considered: sampling uncertainty und distribution type uncertainty. Of course, it would be possible to consider other sources as well.

3.1. Combining natural uncertainty and sampling uncertainty

Following the Bayesian view, the predictive distribution (or Bayesian distribution) is obtained by application of the total probability theorem (Kuczera, 1999)

$$f_q^{\text{pred}}(q) = \int_\theta f_q^{\text{desc}}(q|\theta) f_\theta(\theta)\,d\theta \quad (3)$$

where $f_q^{\text{desc}}(q|\theta)$ is the pdf of the annual maximum discharge q conditional upon the uncertain parameter vector θ, and $f_\theta(\theta)$ is the pdf of θ.

Due to sampling uncertainty there is a probability that the true but unknown flood quantile $\hat{q}_T$ with return period T exceeds the estimated flood quantile q_T. If the system is designed such that $q_C = q_T$ this probability is

$$P(\hat{q}_T > q_C) = \int_{q_C}^{\infty} f_{q_T}(q_T)\,dq_T \quad (4)$$

where $f_{q_T}(q_T)$ is the sampling distribution corresponding to the T-year flood which is a function of the sample size n (Eq. (2)).

When both natural uncertainty and epistemic uncertainty due to sampling uncertainty are considered the Bayesian annual failure probability AFP yields

$$\text{AFP} = \int_{q_C}^{\infty} \left[\int_{q_C}^{\infty} f_{q_T}(q_T)\,dq_T \right] f_q(q)\,dq \quad (5)$$

where Eqs. (1) and (4) are combined by the total probability theorem given in (3). AFP is an expected exceedance probability for q_C with the expectation taken over all feasible values of θ (Wood, 1977; Kuczera, 1999).

For normally distributed variables Booy and Lye (1989) have derived the mean and variance of the predictive distribution. They showed that the mean of the predictive distribution is the same as the mean of the descriptive distribution. The variance of the predictive distribution Var^{pred} is

$$\text{Var}^{\text{pred}} = \left(1 + \frac{3}{2n}\right) \text{Var}^{\text{desc}} \quad (6)$$

where Var^{desc} is the variance of the descriptive distribution and n is the sample size. Fig. 3 compares the descriptive distribution with the predictive distribution for the lognormal distribution. Because the predictive distribution incorporates the sampling uncertainty it estimates a larger 100-year flood than the descriptive distribution. The influence of the sampling uncertainty decreases with increasing sample size and the predictive distribution approaches the descriptive distribution.

Fig. 4 shows the influence of the sampling uncertainty on the annual failure probability AFP. AFP decreases with increasing sample size and asymptotically approaches 0.01, which is the value if no sampling uncertainty is taken into account. This implies that a longer observation period leads to a safer system. Fig. 4 also illustrates the effects of

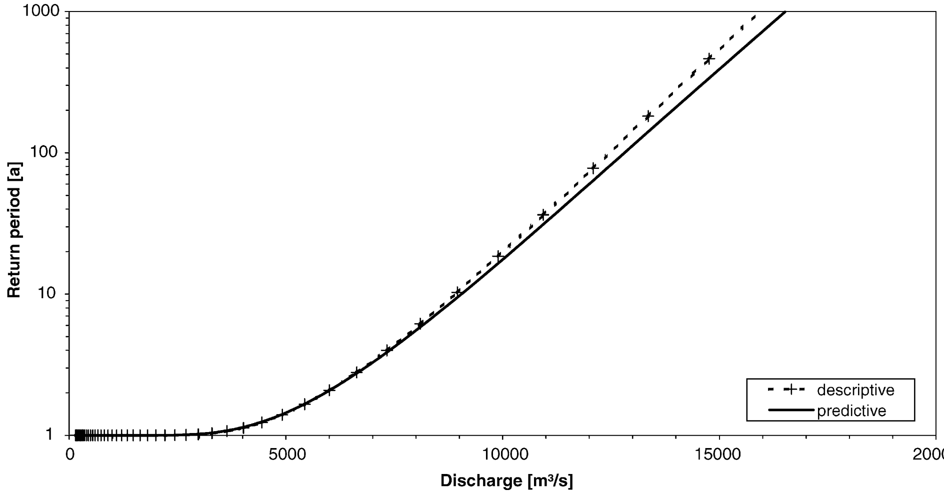

Fig. 3. Descriptive and predictive distribution for sample size $n = 20$ (lognormal distribution).

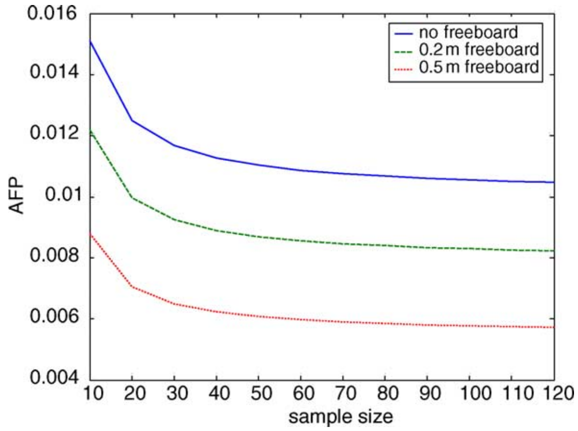

Fig. 4. Combining sampling uncertainty and natural uncertainty in the risk estimator AFP (lognormal distribution).

a freeboard on AFP. The provision of a freeboard increases the safety because it decreases the probability that the flood wall is overtopped. The combination of natural and epistemic uncertainty in one safety measure, e.g. AFP, may lead to wrong conclusions. For example, if an observation period of 10 years is available and no freeboard is used the AFP is 0.15. According to Fig. 4 the safety of the system could be improved to AFP = 0.12 by either heightening the flood wall by 20 cm or by obtaining an additional observation period of 15 years. This result is contrary to the intuitive knowledge that the system itself does not change if it is observed longer. More data do not increase the *safety* of the system but they increase the *knowledge* about the system and about its safety.

The combination of natural and epistemic uncertainty leads to a more conservative flood design because the incorporation of epistemic uncertainty increases the annual failure probability AFP and the expected annual flood damage, EAD, and it decreases the benefit-cost ratio BCR (Tung and Mays, 1981a,b; Bao et al., 1987; Booy and Lye, 1989; Yen and Tung, 1993; Tung, 1993; Kuczera, 1999; Chbab et al., 2000). In principle, a more conservative design makes sense if the epistemic uncertainty is large. However, the combination of both basic types of uncertainty masks important information about the safety of the system. It does not disclose the part of the uncertainty that can be reduced by gathering more information about the system. Combining both types of uncertainty implies that more information improves the safety of the system. But more information does not change safety. The probability that the area which is protected by the flood wall is inundated in a given year does not change by obtaining more data. What changes is the knowledge, and, therefore, the reliability of the assessment of the inundation probability.

3.2. Combining natural uncertainty and distribution function uncertainty

In flood frequency estimation probability plots and goodness-of-fit tests are frequently used to select appropriate distributions. Commonly, only some distributions can be rejected by these means and several distribution functions may be seen as potentially representing the investigated process. It has been proposed to use a composite distribution function which is a combination of the different selected distribution functions (Wood and Rodriguez-Iturbe, 1975; Bobo and Unny, 1976; Tung and Mays, 1981a; Russel, 1982; Chbab et al., 2000)

$$f_{\mathrm{com}}(q|\alpha,\theta) = \sum_{i=1}^{m} \alpha_i f_i(q|\theta_i) \tag{7}$$

where $f_{\mathrm{com}}(q|\alpha,\theta)$ is the composite distribution function, conditioned upon the composite model parameter set or weights α, and the parameter set θ, $f_i(q_i|\theta_i)$ is the pdf of the selected distribution function i and m is the number of distribution functions. In this way, the results from different distribution functions are averaged, weighted by the respective evidence which is attributed to each choice.

The weight α_i is seen as the relative probability that the distribution function $f_i(q_i|\theta_i)$ is the true function. It can be determined subjectively by expert judgement or by applying a measure of fit between model and observation. Following the Bayesian approach a prior probability is assigned to each selected distribution function. An uniform prior probability can be used which means that all models are assumed to be equally probable (Wood and Rodriguez-Iturbe, 1975; Bobo and Unny, 1976). Then the prior probabilities read: $\alpha'_i = 1/m$. In a second step, the data are considered in order to update the prior probabilities by the Bayes' theorem (Russel, 1982)

$$\alpha''_i = \frac{\alpha'_i L_i(\mathbf{q}|\theta_i)}{\sum_{i=1}^{m} \alpha'_i L_i(\mathbf{q}|\theta_i)} \tag{8}$$

where α''_i is the posterior probability. The data are incorporated by use of $L_i(\mathbf{q}|\theta_i)$ which is the likelihood of obtaining the data set $\mathbf{q}$ for the individual distribution i

$$L_i(\mathbf{q}|\theta_i) = \prod_{j=1}^{n} f_i(q_j)\Delta q \tag{9}$$

where n is the sample size. The posterior weights for the LN3, GEV, GL and PE3 distributions are calculated for the annual maximum series AMS 1880–1999 at the Cologne gauge on the river Rhine. The composite distribution function reads

$$f_{\mathrm{com}}(q|\alpha,\theta) = 0.34 f_{\mathrm{PE3}}(q) + 0.35 f_{\mathrm{GEV}}(q) + 0.12 f_{\mathrm{LN3}}(q) + 0.19 f_{\mathrm{GL}}(q) \tag{10}$$

where $f_{\mathrm{PE3}}(q)$ is the PE3 pdf, $f_{\mathrm{GEV}}(q)$ is the GEV pdf, $f_{\mathrm{GL}}(q)$ is the GL pdf, and $f_{\mathrm{IN3}}(q)$ is the LN3 pdf. The form of the composite Bayesian distribution is not fixed and changes as more data are available. Fig. 2 compares the composite distribution function and the individual functions.

In view of distribution type uncertainty, Wood and Rodriguez-Iturbe (1975) and Tung and Mays (1981a) suggest to use a composite distribution because it accounts for hydrological model uncertainty. The uncertainty due to the distribution function type can be incorporated in the estimation of the design value AFP by replacing the annual failure probability, which follows from a given distribution function, by the weighted annual failure probability, resulting from the composite distribution (Table 2).

Similar to Section 3.1 this approach combines epistemic uncertainty (distribution type uncertainty) with natural uncertainty. The design value AFP is a function of both types of uncertainty. Again, this approach does not show the part of the uncertainty that can be reduced by more knowledge.

4. Separating natural and epistemic uncertainty

Natural and epistemic uncertainty should be separated in flood frequency analysis. In this section, two sources of epistemic uncertainty, sampling uncertainty and distribution type uncertainty, are considered. Other sources may be treated accordingly.

4.1. Influence of sampling uncertainty

Due to sampling uncertainty there is a probability that the true but unknown T-year flood $\hat{q}_T$ is larger or smaller than the estimated flood quantile q_T. Hence, q_T is considered as a random variable (Section 3.1). q_T is transformed to the annual failure probability AFP by use of Eq. (1). Therefore, AFP is a random variable which depends on the sample size.

Figs. 5 and 6 show AFP as random variable for the GEV distribution for different sample sizes: the larger the sample, the smaller the sampling uncertainty, and the steeper the cdf of AFP. It is interesting to compare this representation of epistemic uncertainty with the one of Section 3.1, which combines epistemic uncertainty and natural uncertainty. In Fig. 4, AFP decreases with increasing sample size, that implies that the system becomes safer when more data are available. The separation of natural and epistemic uncertainty has another approach to represent the two types of uncertainty: natural uncertainty is represented by the mean value of AFP. This value is not influenced by the epistemic uncertainty (or sampling uncertainty). Epistemic uncertainty is represented by the variance of AFP (e.g. described by the interquartile range in Fig. 6) which decreases with increasing knowledge. This means that the probability that the flood defence system fails does not decrease if a longer discharge series is available, i.e. the flood defence system itself does not become safer. But the knowledge increases which is explicitly shown in the steeper cdf of AFP (Fig. 5).

4.2. Influence of sampling uncertainty and distribution type uncertainty

In a further step, another source of epistemic uncertainty, the uncertainty due to the type of distribution function, is considered. For a certain

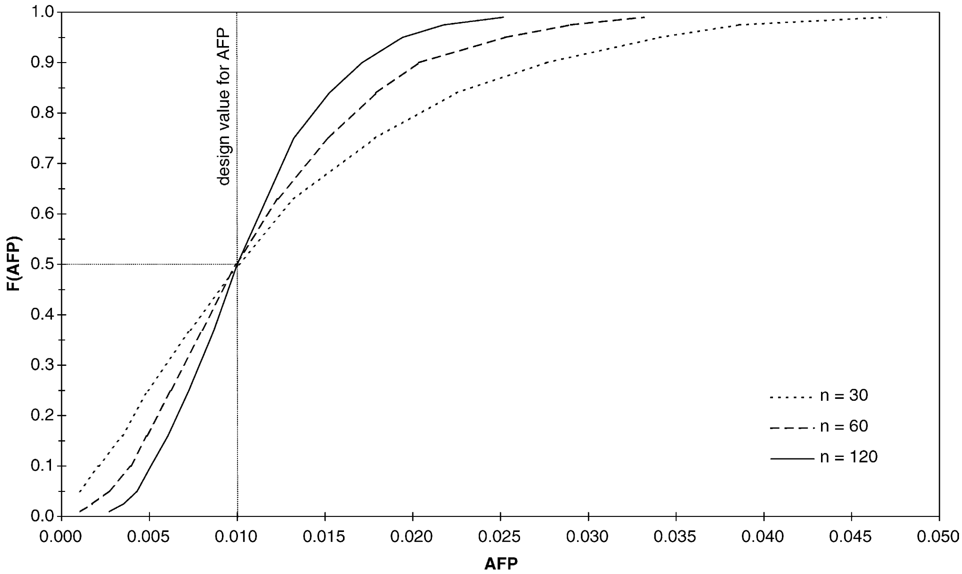

Fig. 5. Effect of the sample size on the cdf of AFP (GEV, L-Moments, $q_C = q_{100}(\mathrm{GEV}) = 11{,}256\ \mathrm{m^3/s}$).

design value the cdf of AFP can be derived for each distribution function which is seen as a plausible model. For the Cologne gauge on the river Rhine, Fig. 7 shows the cdf of AFP for four distribution functions (PE3, GEV, LN3, GL). The 100-year flood according to the GEV distribution has been chosen as design value. Now, the epistemic uncertainty due to the choice of the distribution function is given as probability bounds. These bounds limit the region within which the true but unknown cdf must lie (light grey area in Fig. 7). The greater the uncertainty about the 'correct' distribution function, the wider the bounds and the more difficult to derive precise statements about the frequency of extreme events.

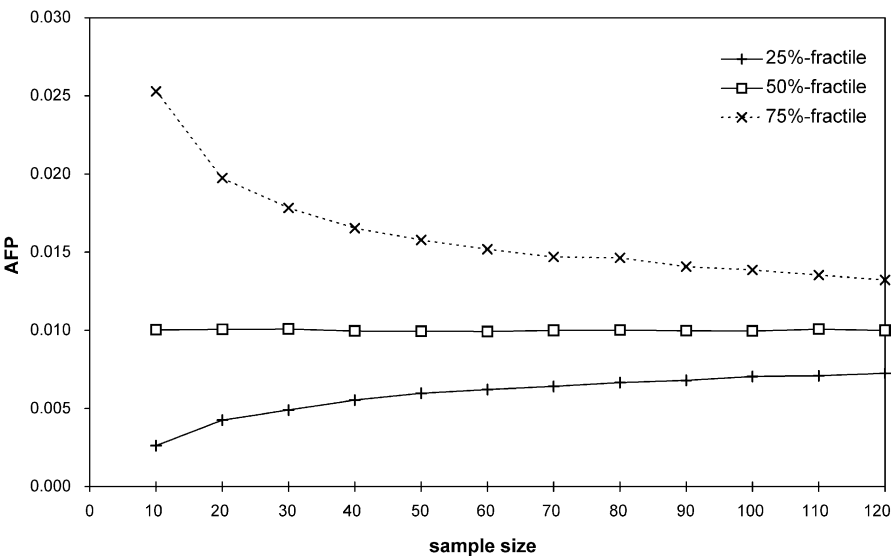

Fig. 6. Interquartile range (IQR) of AFP as representation of sampling uncertainty (GEV, L-Moments, $q_C = q_{100}(\mathrm{GEV}) = 11{,}256\ \mathrm{m^3/s}$).

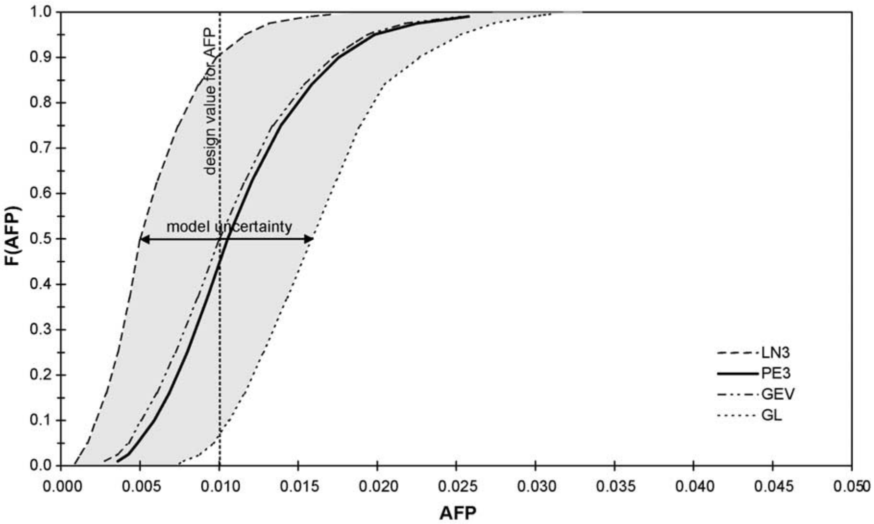

Fig. 7. Effect of different distribution functions on the cdf of AFP (L-Moments, $q_T = q_{100}$(GEV) = 11,256 m^3/s at Cologne gauge on the river Rhine, 120 years, 1880–1999).

Fig. 7 shows that the effects of model uncertainty on AFP may be considerable. The median of AFP due to the four distribution functions varies between 0.005 and 0.016. If the flood defence system is designed according to the 100-year flood and the PE3 distribution is used, the median of AFP may be as large as 0.016 if the GL distribution cannot be ruled out as suitable distribution function for this data set.

4.3. Examples

In a further step, the ideas which have been introduced in Sections 4.1 and 4.2 have been applied to runoff data of the Bratislava gauge on the river Danube, the Hermann MO gauge on the river Missouri, and the Dangar Bridge gauge on the Barwon River. Table 3 gives information about the data sets. These sites have been chosen in order to cover different characteristics, e.g. flood peak series with small and large skewness. Fig. 8 shows the probability bounds of the AFP for the three data sets. As in the case of the Cologne gauge, on the river Rhine, permitting different distribution functions may lead to large uncertainty bounds, especially for the data of Bratislava gauge and Hermann Mo. gauge (light grey areas of Fig. 8).

Now, we look at the decrease in uncertainty if the range of distribution functions is narrowed. Following

Table 3
Characteristics of the discharge gauges Bratislava/Danube, Hermann Mo./Missouri, Dangar Bridge/Barwon River, and Cologne/Rhine

River	Gauge	Catchment area (km^2)	Mean discharge (m^3/s)	Time series (AMS)	Number of years	Mean AMS (m^3/s)	Standard deviation AMS (m^3/s)	Coefficient of variation AMS	Skewness	Log skewness	q_{100} (m^3/s)
Danube	Bratislava	131,338	2042	1901–1992	92	5525	1410	0.26	0.82	0.09	9852 (GEV)
Missouri	Hermann Mo.	1,357,677	2351	1929–2000	72	8146	3705	0.45	0.96	0.46	19,773 (PE3)
Barwon River	Dangar Bridge	132,200	83	1913–1955	43	553	742	1.34	2.92	1.02	4050 (GEV)
Rhine	Cologne	144,232	2087	1880–1999	120	6200	1887	0.30	0.43	0.32	11,256 (GEV)

the approach of Section 3.2, the posterior weights (Eq. (8)) are used as a measure for the appropriateness of the different distribution functions. Table 4 shows the weights for the three AMS. These weights are seen as relative probability that the associated distribution function is the true distribution function. In all cases, this approach leads to the elimination of the LN3 distribution function, in the case of the Dangar Bridge gauge, the PE3 function is eliminated as well. This elimination considerably narrows the uncertainty bounds which is shown by the dark grey areas in Fig. 8.

Fig. 9 summarizes the ideas which have been developed in this paper. To illustrate the effect of the variability of the time series, the 120-year time series of Cologne gauge ($CV = 0.30$) has been scaled to

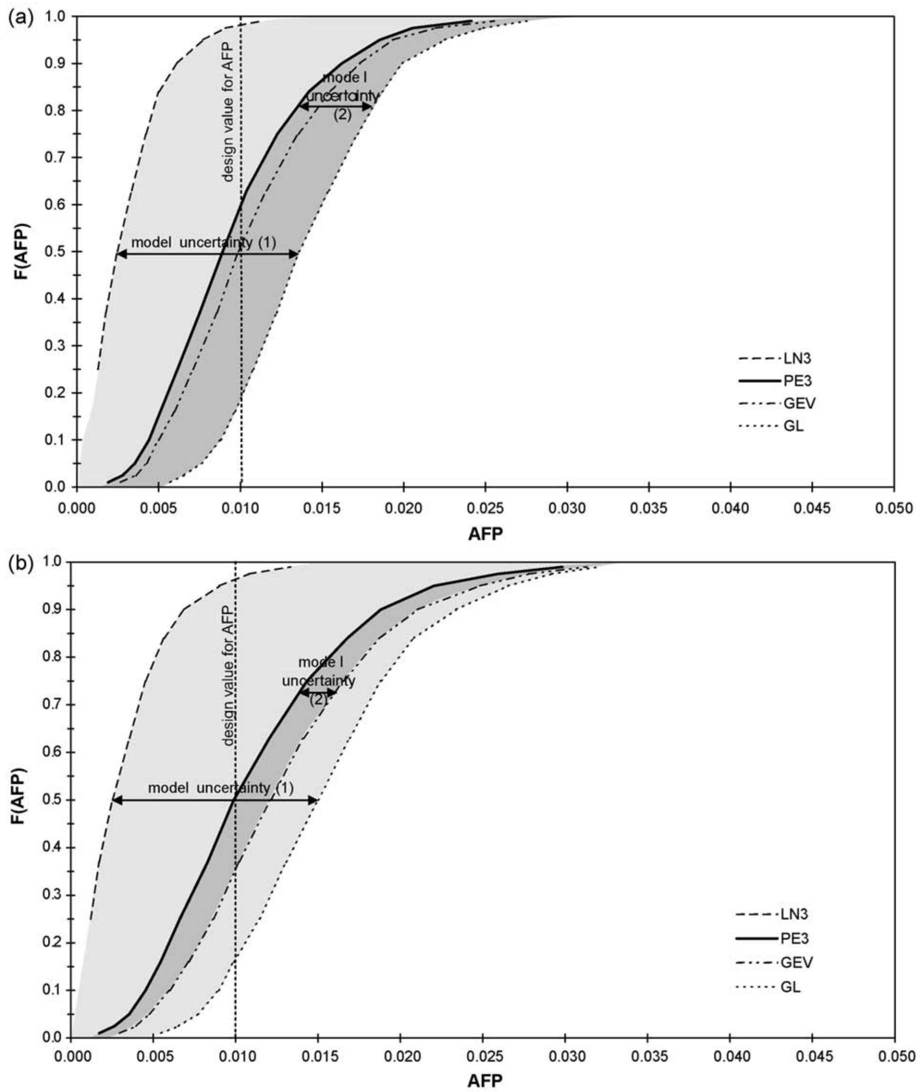

Fig. 8. Effect of distribution function uncertainty on the cdf of AFP. The epistemic uncertainty due to distribution function uncertainty is shown as distribution bounds (grey areas). The elimination of a distribution function may significantly reduce the uncertainty (the light grey area is reduced to the dark area). (a) Bratislava gauge on the river Danube (AMS 1901–1992); (b) Hermann Mo. gauge on the river Missouri (AMS 1929–2000). (c) Dangar Bridge gauge on the Barwon River (AMS 1913–1955).

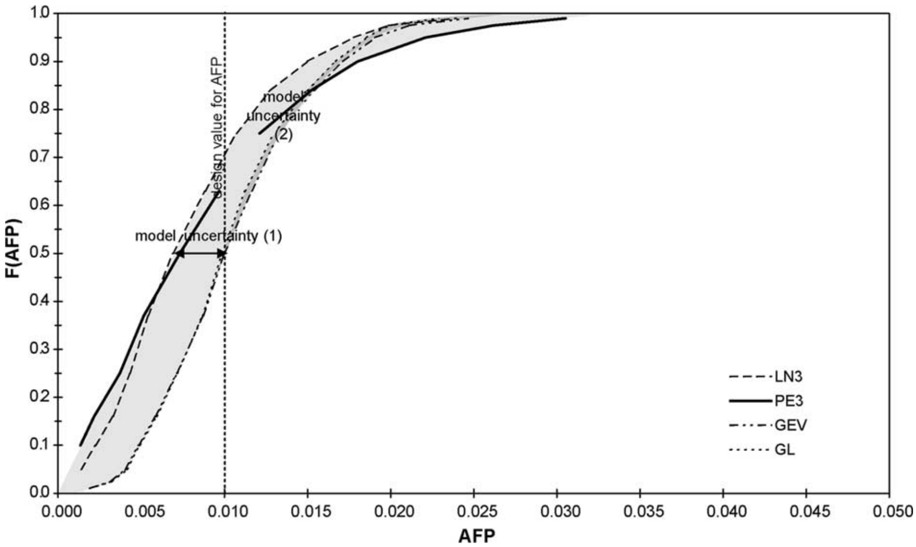

Fig. 8 (*continued*)

a time series with a rather small variability (CV = 0.26 like the Danube at Bratislava) and to a time series with a much larger variability (CV = 0.45, like the Missouri at Herrman MO). Plots A1 and A2 show the case of the smaller variability whereas plots B1 and B2 show the results for the larger variability. For both cases, the effects of increasing epistemic uncertainty and increasing safety are shown. Epistemic uncertainty is represented by the grey area which encompasses the range within which the true but unknown cdf of AFP must lie. For A1 and B1, these uncertainty bounds are very wide, due to the rather short observation period (30 years) and the use of four distribution functions. Extending the observation period to 120 years and eliminating probability functions with worse fit to the observed data (case A: elimination of LN3; case B: elimination of LN3 and GL), significantly steepens and narrows the uncertainty bounds (plots A2, B2). This sharpening of uncertainty bounds shows the reduction in uncertainty which can be obtained by more information or expert knowledge about the system under study.

Additionally, plots A2 and B2 show the effect of a freeboard on AFP. Adding a freeboard of 50 cm shifts the whole uncertainty range to the left. This shift is tantamount to an increase in safety. The probability that AFP is larger than the design value of AFP (0.01) is much smaller when a freeboard is added. Comparing cases A and B it becomes clear that the gain in safety, represented by the left shift of the probability bounds, depends on the variability of the time series. The smaller the variability of the time series, the larger is the gain in safety due to a given freeboard.

Table 4
Posterior weights of the different distribution functions for the Bratislava gauge on the river Danube, the Hermann Mo. gauge on the river Missouri, the Dangar Bridge gauge on the Barwon River, and the Cologne gauge on the river Rhine

Distribution function	Bratislava, Danube	Hermann Mo., Missouri	Dangar bridge, Barwon river	Cologne, Rhine
LN3	0.03	0.01	0.03	0.12
PE3	0.30	0.55	0.00	0.34
GEV	0.44	0.35	0.52	0.35
GL	0.23	0.09	0.45	0.19

5. Discussion and conclusions

Planners should know how much the estimates of flood design criteria (e.g. AFP, EAD) might be in error (NRC, 1995). An indication about the uncertainty may guide the flood defence strategy. For cases with large uncertainty it may be important to prepare for situations where the upper bound of the uncertainty range applies.

Flood risk analyses are based on assumptions and decisions about models, parameters, and data. In many cases it can be argued for different options. Transparency requires explicitly showing the uncertainty of each alternative. Even if the planning decision is based on the analyst's best estimate, an uncertainty analysis may significantly improve the quality of the decision because the analyst is forced to look at the whole range of unknowns. Nonlinear processes and limitations of human thinking may lead to 'best estimates' which are, in reality, not very good (Morgan and Henrion, 1990).

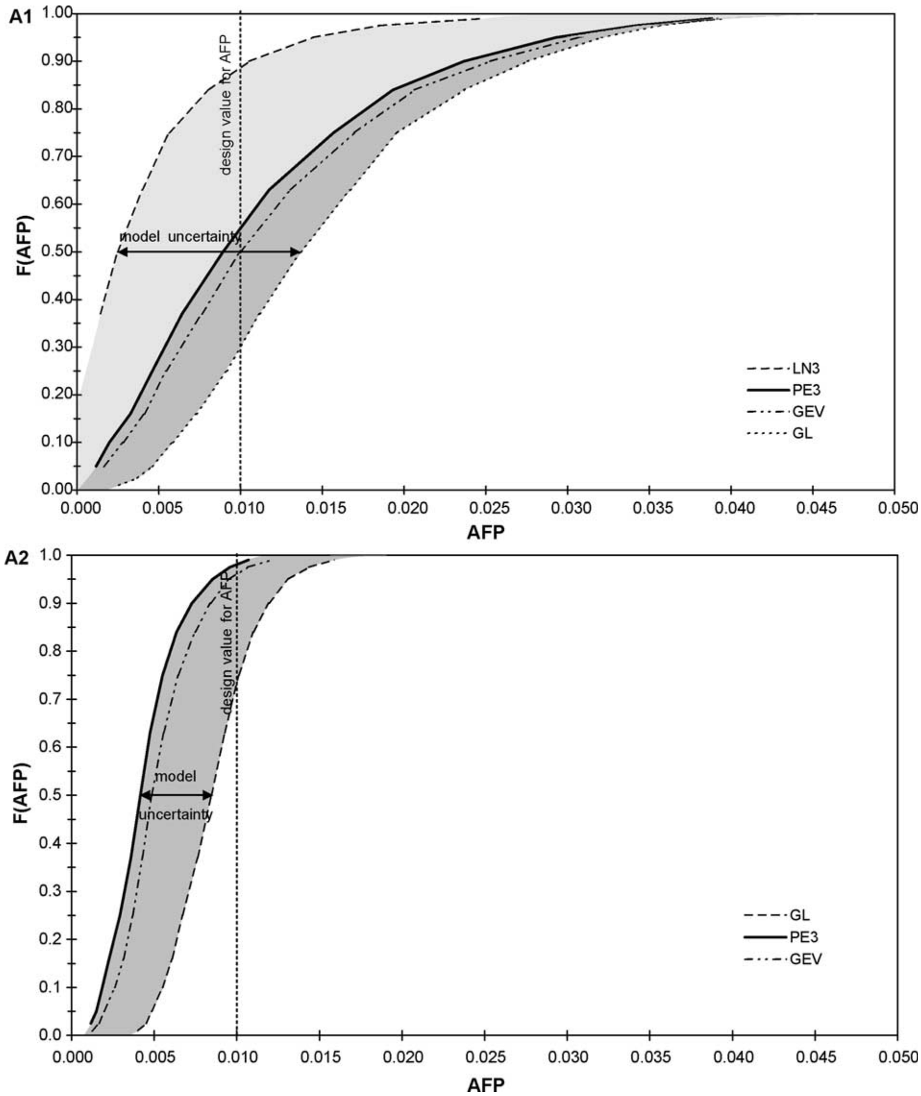

Fig. 9. Effect of decreasing epistemic uncertainty on AFP for time series with different variability. (A1) Large epistemic uncertainty (four distributions functions, 30 years of observation) and small variability (CV = 0.26); no freeboard. (A2) Small epistemic uncertainty (three distributions functions, 120 years of observation) and small variability (CV = 0.26); freeboard of 50 cm. (B1) Large epistemic uncertainty (4 distributions functions, 30 years of observation) and large variability (CV = 0.45); no freeboard. (B2) Small epistemic uncertainty (two distributions functions, 120 years of observation) and large variability (CV = 0.45); freeboard of 50 cm.

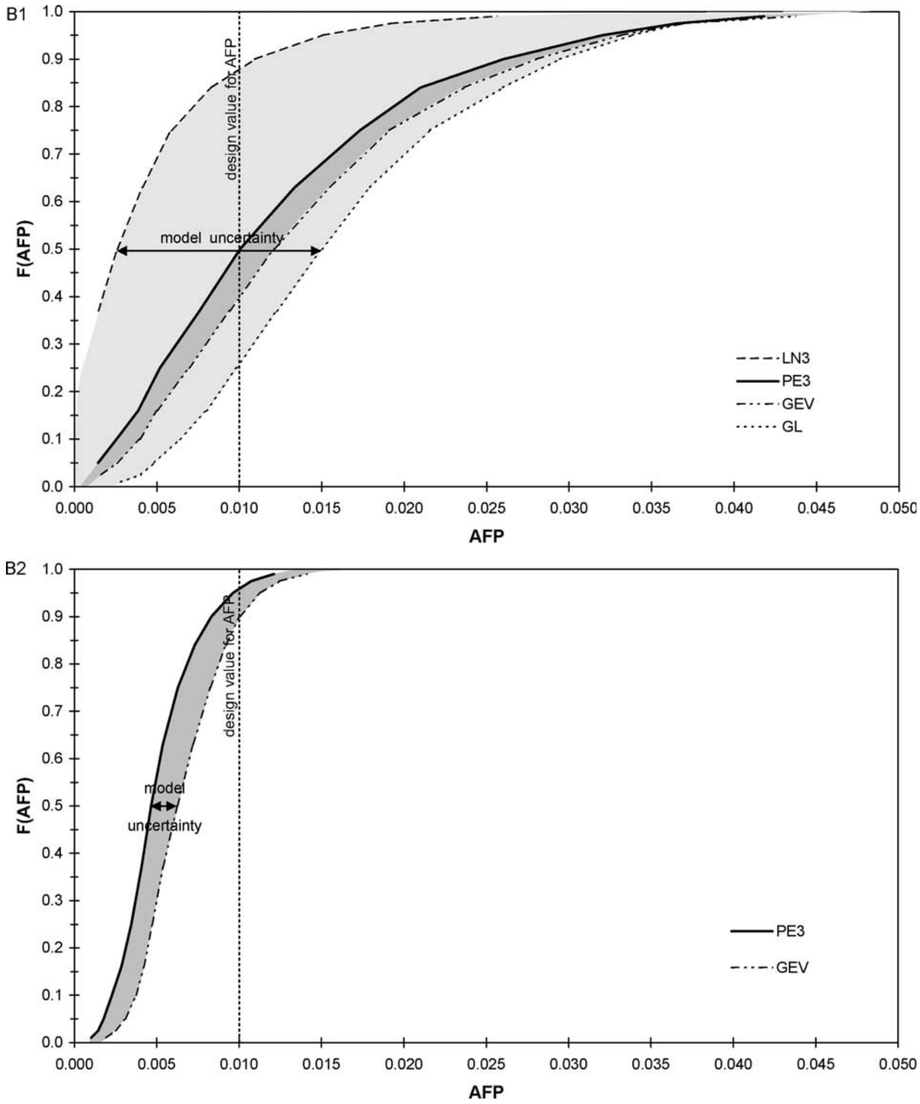

Fig. 9 (*continued*)

In this paper, it is argued that, in flood-frequency analysis, the two basic kinds of uncertainty, natural and epistemic uncertainty, should be separated. The central issue is that natural uncertainty is a property of the system and epistemic uncertainty is a property of the analyst. In principle, epistemic uncertainty can be reduced whereas natural uncertainty is not reducible. Therefore, the separation of the two uncertainty kinds gives a more differentiated picture of the complete uncertainty. This is illustrated by the examples in Sections 3 and 4. Combining natural and epistemic uncertainty in the design criteria AFP implies that the safety of a flood defence wall could be improved by obtaining additional observations. But in reality, the safety of the flood defence wall does not change. What changes is the knowledge about the system and about its safety.

If epistemic uncertainty can be described by a set of model parameters θ, the most simple and the most frequent procedure is to base the flood design on the analyst's best estimate of θ, denoted here as θ_{best}. Then, the design criteria are AFP(θ_{best}), EAD(θ_{best}), and BCR(θ_{best}). This procedure is satisfactory for situations when epistemic uncertainty is small. An alternative approach is to represent the epistemic uncertainty by a probability distribution for θ (NRC, 2000). In that case the planning criteria are random variables due to epistemic uncertainty and are described by their distribution functions. The design criteria, AFP, is considered as random variable whereas the uncertainty of AFP depends on the knowledge of the analyst. More knowledge, e.g. more data, decreases the uncertainty about AFP.

This argument can be extended to model uncertainty. In many cases in hydrology, available data are not sufficient to identify a 'correct' model. Beven (2001) argues that all possible models, which are consistent with the data and the theory, should be considered. Applying this view to flood frequency analysis we propose to quantify the uncertainty due to different distribution functions by using probability bounds. Probability bounds do not give a single cdf but rather a region within which the true but unknown cdf is expected. The greater the uncertainty due to the distribution type, the wider the bounds and the more difficult to make certain statements about frequencies of extreme events. Of course, the range of possibilities as shown by probability bounds is not particularly useful if some distribution functions have a low probability.

It has been proposed to average results from different model choices, e.g. different distribution functions, weighted by the respective evidence which is attributed to each choice. Others disagree and claim that is does not make sense to average the results of mutually exclusive models (Morgan and Henrion, 1990). If significant sources of disagreement exist it should be made explicit and should be communicated to the decision maker. Probability bounds clearly describe this source of epistemic uncertainty.

A frequent problem with uncertainty analyses is that the description of uncertainty is fairly subjective. Though it is often a challenge to determine a best estimate it is even more difficult to properly quantify the uncertainty of the best estimate. Combining natural and epistemic uncertainty as shown in Section 3 gives a best estimate which may be fairly subjective. Separating natural and epistemic uncertainty divides the description of the variability of the process from problems related to the knowledge about the process. Such an approach may guide efforts for obtaining more information. By separating the different sources of uncertainty the efforts for gaining more information can be prioritized.

The paper argues that natural and epistemic uncertainty should be separated in flood frequency analysis. This argument is based on calculations for both approaches, (1) combining natural and epistemic uncertainty (Section 3), and (2) separating the two kinds of uncertainty (Section 4). In flood design situations both approaches may lead to the same effect: the less knowledge, the more conservative the flood design. Even though both approaches may lead to comparable results, the separation of both kinds of uncertainty gives a much clearer picture about the process with its variability and the knowledge about the process.

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References

Afshar, A., Marino, M.A., 1990. Determining optimum spillway capacity based on estimated flood distribution. Water Power and Dam Construction January.

Arnell, N.W., 1989. Changing frequency of extreme hydrological events in northern and western Europe, in: Roald, L., Nordseth, K., Hassel, K.A. (Eds.), Friends in Hydrology, IAHS Publ. No. 187, pp. 237–249.

Aven, T., Pörn, K., 1998. Expressing and interpreting the results of quantitative risk analyses. Review and discussion. Reliability Engineering and System Safety 61, 3–10.

Bao, Y., Tung, Y.-K., Hasfurther, V.R., 1987. Evaluation of uncertainty in flood magnitude estimator on annual expected damage costs of hydraulic structures. Water Resources Research 23 (11), 2020–2029.

Beven, K., 2001. How far can we go in hydrological modelling?. Hydrology and Earth System Sciences 5 (1), 1–12.

Bobee, B., Ashkar, F., 1988. Review of statistical methods for estimating flood risk with special emphasis on the Log Pearson Type 3 distribution, in: El-Sabh, M.I., Murty, T.S. (Eds.), Natural and Man-made Hazards. D. Reidel Publishing Company, pp. 357–367.

Bobo, B., Unny, T.E., 1976. Model uncertainty in flood frequency analysis and frequency-based design. Water Resources Research 12 (6), 1109–1117.

Bogárdi, I., Szidarovszky, F., 1972. The margin of safety for compensating losses due to uncertainties in hydrological statistics. In: Proceedings of the International Symposium of Uncertainties in Hydrologic and Water Resources Systems, 11–14 December 1972, Tuscon, Arizona, vol. 2, pp. 835–852.

Booy, C., Lye, L.M., 1989. A new look at flood risk determination. Water Resources Bulletin 25 (5), 933–943.

Chbab, E.H., 1995. How extreme were the 1995 flood waves on the rivers Rhine and Meuse?. Physics and Chemistry of the Earth 20 (5–6), 455–458.

Chbab, E.H., van Noortwijk, J.M., Duits, M.T., 2000. Bayesian frequency analysis of extreme river discharges, in: Toensmann, F., Koch, M. (Eds.), River Flood Defence, Kassel Reports of Hydraulic Engineering. Herkules Verlag Kassel, , pp. F-51–F-60.

Clarke, R.T., 1999. Uncertainty in the estimation of mean annual flood due to rating-curve indefinition. Journal of Hydrology 222, 185–190.

Cluckie, I.D., Pessoa, M.L., 1990. Dam safety: an evaluation of some procedures for design flood estimation. Hydrological Sciences 35, 547–565.

Cullen, A.C., Frey, H.C., 1999. Probabilistic techniques in exposure assessment, A Handbook for Dealing with Variability and Uncertainty in Models and Inputs. Plenum Press, New York, London. 335 pp.

Cunnane, C., 1985. Factors affecting choice of distribution for flood series. Hydrological Sciences Journal 30, 25–36.

Davis, D.R., Duckstein, L., Fogel, M.M., 1976. Uncertainty in the return period of maximum hydrologic events: a Bayesian approach. Journal of Hydrology 31, 81–95.

Ferson, S., Ginzburg, L.R., 1996. Different methods are needed to propagate ignorance and variability. Reliability Engineering and System Safety 54, 133–144.

Frank, M.V., 1999. Treatment of uncertainties in space nuclear risk assessment with examples from Cassini mission applications. Reliability Engineering and System Safety 66, 203–221.

Haan, C.T., 1972. Evaluation of prediction error probabilities from stochastic models by simulation. In: Proceedings of the International Symposium of Uncertainties in Hydrologic and Water Resources Systems, 11–14 December 1972, Tuscon, Arizona, vol. 1, pp. 91–103.

Haimes, Y.Y., 1998. Risk modeling, assessment, and management, Wiley Series in Systems Engineering. Wiley, New York. 726 pp.

Hall, J.W., 2003. Handling uncertainty in the hydroinformatic process. Journal of Hydroinformatics 05.4, 215–232.

Helton, J.C., 1996. Probability, conditional probability and complementary cumulative distribution functions in performance assessment for radioactive waste disposal. Reliability Engineering and System Safety 54, 145–163.

Hoffman, F.O., Hammonds, J.S., 1994. Propagation of uncertainty in risk assessments: the need to distinguish between uncertainty due to lack of knowledge and uncertainty due to variability. Risk Analysis 14 (5), 707–712.

Hora, S.C., 1996. Aleatory and epistemic uncertainty in probability elicitation with an example from hazardous waste management. Reliability Engineering and System Safety 54, 217–223.

Hosking, J.R.M., Wallis, J.R., 1997. Regional Frequency Analysis, An Approach Based on L-Moments. Cambridge University Press, Cambridge.

Institute of Hydrology, 1999. Flood Estimation Handbook, Institute of Hydrology, Crowmarsh Gifford, Wallingford, UK, vol. 1–5.

Kaczmarek, Z., Arnell, N.W., Starkel, L., 1996. Climate, hydrology, and water resources, in: Kaczmarek, Z., Strzepek, K.M., Smolyody, L., Priazhinskaya, V. (Eds.), Water Resources Management in the Face of Climatic/Hydrologic Uncertainties. Kluwer Academic Publishers, Dordrecht, pp. 3–29.

Klemes, V., 1986. Dilettantism in hydrology: transition or destiny?. Water Resources Research 22 (9), 177S–188S.

Klemes, V., 1988. The improbable probabilities of extreme floods and droughts, in: Starosolszky, O., Melder, O.M. (Eds.), Proceedings of Technical Conference. WMO, Geneva, pp. 43–51.

Klemes, V., 1993. Probability of extreme hydrometeorological events—a different approach, in: Kundzewicz, Z.W., Rosbjerg, D., Simonovic, S.P., Takeuchi, K. (Eds.), Extreme Hydrological Events: Precipitation, Floods and Droughts, IAHS-Publ. No. 213, pp. 167–176.

Krzysztofowicz, R., Yakowitz, S., 1980. Large-sample methods for decision analysis of gamma variates. Water Resources Research 16 (3), 491–500.

Kuczera, G., 1999. Comprehensive at-site flood frequency analysis using Monte Carlo Baysian inference. Water Resources Research 35 (5), 1551–1557.

Lammersen, R., Engel, H., Van de Langemheen, W., Buiteveld, H., 2002. Impact of river training and retention measures on flood peaks along the Rhine. Journal of Hydrology 267 (1–2), 115–124.

Malamud, B.D., Turcotte, D.L., Barton, C.C., 1996. The 1993 Mississippi river flood: a one hundred or a one thousand year event?. Environmental and Engineering Geoscience II (4), 479–486.

Milly, P.C.D., Wetherald, R.T., Dunne, K.A., Delworth, T.L., 2002. Increasing risk of great floods in a changing climate. Nature 415, 514–517.

Moore, S.F., Brewer, J.W., 1972. Environmental control systems: treatment of uncertainty in models and data. In: Proceedings of the International Symposium of Uncertainties in Hydrologic and Water Resources Systems, 11–14 December 1972, Tuscon, Arizona, vol. 1, pp. 16–30.

Morgan, M.G., Henrion, M., 1990. Uncertainty: a guide to dealing with uncertainty in quantitative risk and policy analysis. Cambridge University Press, Cambridge, England. 332 pp..

NRC (National Research Council), 1995. Flood Risk Management and the American River Basin. An Evaluation, National Research Council. National Academy Press, Washington DC p. 235.

NRC (National Research Council), 2000. Risk Analysis and Uncertainty in Flood Damage Reduction Studies. National Academy Press, Washington DC. 202 pp.

Olsen, J.R., Lambert, J.H., Haimes, Y.Y., 1998. Risk of extreme events under nonstationary conditions. Risk Analysis 18 (4), 497–510.

Parry, G.W., 1996. The characterization of uncertainty in probabilistic risk assessment of complex systems. Reliability Engineering and Systems Safety 54, 119–126.

Plate, E.J., 1992, Stochastic design in hydraulics: concepts for a broader application, in Stochastic Hydraulics'92, Proceedings of the sixth IAHR International Symposium of stochastic Hydraulics, Taipei, Taiwan, May 18-20, 1992, J.-T. Kuo and G.-F. Lin, Eds., Department of Civil Engineering, National Taiwan University, Taipei, Taiwan, pp. 663–670.

Plate, E.J., 1993a. Some remarks on the use of reliability analysis in hydraulic engineering applications, in: Yen, B.C., Tung, Y.-K. (Eds.), Reliability and Uncertainty Analysis in Hydraulic Design. American Society of Civil Engineers, New York, pp. 5–15.

Plate, E.J., 1993b. Statistik und angewandte Wahrscheinlichkeitslehre für Bauingenieure. Ernst & Sohn, Verlag f. Architektur u. techn, Wissenschaften, Berlin.

Resendiz-Carrillo, D., Lave, L.B., 1987. Optimizing spillway capacity with an estimated distribution of floods. Water Resources Research 23 (11), 2043–2049.

Rowe, W.D., 1994. Understanding uncertainty. Risk Analysis 14, 743–750.

Russel, S.O., 1982. Flood probability estimation. Journal of the Hydraulics Division, ASCE 108 (HY1), 63–73.

Stedinger, J.R., Vogel, R.M., Foufoula-Georgiou, E., 1992. Frequency analysis of extreme events, in: Maidment, D.R. (Ed.), Handbook of Hydrology. McGraw-Hill, New York, pp. 18.1–18.66.

Tang, W.H., 1980. Bayesian frequency analysis. Journal of the Hydraulics Division, ASCE 106 (HY7), 1203–1218.

Tang, W.H., Yen, B.C., 1972. Hydrologic and hydraulic design under uncertainty. In: Proceedings of the International Symposium of Uncertainties in Hydrologic and Water Resources Systems, 11–14 December 1972, Tuscon, Arizona, vol. 2, pp. 868–882.

Tung, Y.-K., 1993. Confidence intervals of optimal risk-based hydraulic parameters, in: Yen, B.C., Tung, Y.K. (Eds.), Relieability and Uncertainty Analysis in Hydraulic Design: A Report Prepared by the Subcommittee on Uncertainty and Reliability Analysis in Design of Hydraulic Structures of the Technical Committee on Probabilistic Approaches to Hydraulics. ASCE, New York, pp. 81–96.

Tung, Y.-K., Mays, L.W., 1981a. Risk models for flood levee design. Water Resources Research 17 (4), 833–841.

Tung, Y.-K., Mays, L.W., 1981b. Optimal risk-based design of flood levee systems. Water Resources Research 17 (4), 843–852.

USACE (U.S. Army Corps of Engineers), 1992. Guidelines for risk and uncertainty analysis in water resources planning. Institute for Water Resources, IWR Report 92-R-1, Fort Belvoir, VA, 62 p.

van Asselt, M.B.A., Rotmans, J., 2002. Uncertainty in integrated assessment modelling. Climatic Change 54 (1), 75–105.

Winkler, R.L., 1996. Uncertainty in probabilistic risk assessment. Reliability Engineering and Systems Safety 54, 127–132.

Wood, E.F., 1977. An analysis of flood levee reliability. Water Resources Research 13 (3), 665–671.

Wood, E.F., Rodriguez-Iturbe, I., 1975. A Bayesian approach to analysing uncertainty among flood frequency models. Water Resources Research 11 (6), 839–843.

WRC (US Water Resources Council), 1981. Guidelines for determining flood-flow frequency. Bull. 17B. Hydrol. Comm., Washington, DC.

Yen, B.C., Tung, Y.K., 1993. Some recent progress in reliability analysis for hydraulic design, in: Yen, B.C., Tung, Y.K. (Eds.), Reliability and Uncertainty Analysis in Hydraulic Design: A Report Prepared by the Subcommittee on Uncertainty and Reliability Analysis in Design of Hydraulic Structures of the Technical Committee on Probabilistic Approaches to Hydraulics. ASCE, New York, pp. 35–79.

Zio, E., Apostolakis, G.E., 1996. Two methods for the structured assessment of model uncertainty by experts in performance assessments of radioactive waste repositories. Reliability Engineering and System Safety 54, 225–241.