CSI 445/660 – Part 6

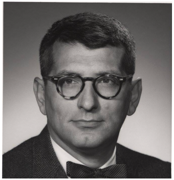
(Centrality Measures for Networks)

References

1. L. Freeman, “Centrality in Social Networks: Conceptual Clarification”, *Social Networks*, Vol. 1, 1978/1979, pp. 215–239.
2. S. Wasserman and K. Faust, *Social Network Analysis: Methods and Applications*, Cambridge University Press, New York, NY, 1994.
3. M. E. J. Newman, *Networks: An Introduction*, Oxford University Press, New York, NY, 2010.
4. Wikipedia entry on Centrality Measures:

   `https://en.wikipedia.org/wiki/Centrality`

A Pioneer on the Topic



- Alex Bavelas (1913–1993) **(??)**
- Received Ph.D. from MIT (1948) in Psychology.
- Dorwin Cartwright was a member of his Ph.D. thesis committee.
- Taught at MIT, Stanford and the University of Victoria (Canada).

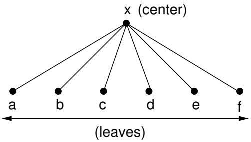
Centrality Measures for Networks

Centrality:

- Represents a “measure of importance”.
  - Usually for nodes.
  - Some measures can also be defined for edges (or subgraphs, in general).
- Idea proposed by Alex Bavelas during the late 1940’s.
- Further work by Harold Leavitt (Stanford) and Sidney Smith (MIT) led to **qualitative** measures.
- **Quantitative** measures came years later. (Many such measures have been proposed.)

Point Centrality – A Qualitative Measure

Example:



- The **center** node is "structurally more important" than the other nodes.

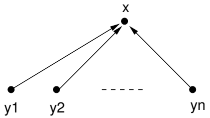
Reasons for the importance of the center node:

- The center node has the maximum possible degree.
- It lies on the shortest path ("geodesic") between any pair of other nodes (leaves).
- It is the closest node to each leaf.
- It is in the "thick of things" with respect to any communication in the network.

Degree Centrality – A Quantitative Measure

- For an **undirected** graph, the **degree** of a node is the number of edges incident on that node.
- For a **directed** graph, both **indegree** (i.e., the number of incoming edges) and **outdegree** (i.e., the number of outgoing edges) must be considered.

When does a large indegree imply higher importance?

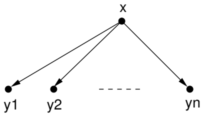


- Consider the Twitter network.
- Think of x as a **celebrity** and the other nodes as followers of x.

- For a different context, think of each node in the directed graph as a web page.
- Each of the nodes y_1, y_2, ..., y_n has a link to x.
- The larger the value of n, the higher is the "importance" of x (a crude definition of **page rank**).

Degree Centrality (continued)

When does a large outdegree imply higher importance?



- Consider the hierarchy in an organization.
- Think of x as the manager of y_1, y_2, $\ldots$, y_n.
- Large outdegree may mean more "power".

Undirected graphs:

- High degree nodes are called **hubs** (e.g. airlines).
- High degree may also also represent higher **risk**.

  Example: In disease propagation, a high degree node is more likely to get infected compared to a low degree node.

Normalized Degree

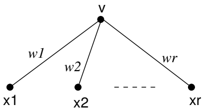
Definition: The **normalized degree** of a node x is given by

$$\text{Normalized Degree of } \ x \ = \ \frac{\text{Degree of } \ x}{\text{Maximum possible degree}}$$

- Useful in comparing degree centralities of nodes between two networks.

Example: A node with a degree of 5 in a network with 10 nodes may be relatively more important than a node with a degree of 5 in a network with a million nodes.

Weighted Degree Centrality (Strength):



- Weighted degree (or strength) of $v = w_1 + w_2 + \ldots + w_r$.

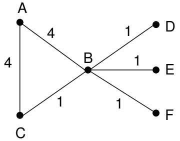
Degree Centrality (continued)

Assuming an **adjacency list** representation

- for an undirected graph $G(V, E)$, the degree (or weighted degree) of all nodes can be computed in **linear** time (i.e., in time $O(|V| + |E|)$) and
- for a directed graph $G(V, E)$, the indegree or outdegree (or their weighted versions) of all nodes can be computed in **linear** time.

Combining degree and strength: (**[Opsahl et al. 2009]**)

Motivating Example:



- A and B have the same strength.
- However, B seems more central than A.

Combining Degree and Strength (continued)

Proposed Measure by Opsahl et al.:

- Let d and s be the degree and strength of a node v respectively.
- Let α be a parameter satisfying the condition $0 \leq \alpha \leq 1$.
- The combined measure for node $v = d^{\alpha} \times s^{1-\alpha}$.
- When $\alpha = 1$, the combined measure is the **degree**.
- When $\alpha = 0$, the combined measure is the **strength**.
- A suitable value of α must be chosen for each context.