

CSI 445/660 – Part 2 (Strong and Weak Ties)

Ref: Chapter 3 of [Easley & Kleinberg].

Strong and Weak Ties

Importance: These notions help in understanding how “local” ties and processes in networks impact their “global” functioning.

Background:

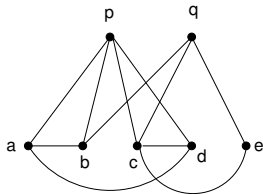


- Mark Granovetter (1943 –)
- Professor of Sociology, Stanford University
- During late 1960's, Granovetter interviewed many people who recently changed jobs.
- Main question: How did you find about the new job?
- Typical answer: Through personal contacts.

Background (continued)

- Many of these contacts were **acquaintances** rather than close friends.
- Granovetter wanted to understand/explain this social phenomenon (without being specific to the “job seeking” domain).
- Led to his work on the “strength of weak ties”.

Definition:

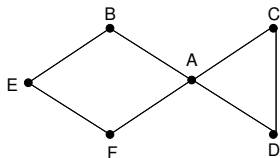


- Nodes a, b, c and d are the **neighbors** of p.
- Nodes b, c and e are the **neighbors** of q.

Triadic Closure

- Applicable to networks that evolve over time.
- Suggested by Georg Simmel (German Sociologist) in 1908 and developed further by Granovetter.

Example:



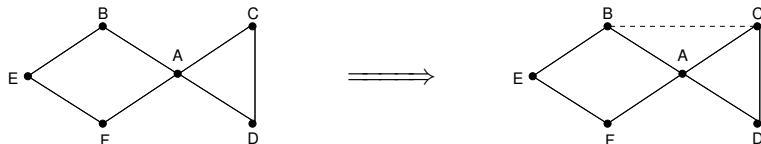
- A friendship network.
- **Question:** Why might this network grow over time?

Basic Principle: (Triadic Closure)

If two people have a common friend, then there is an increased likelihood that they will become friends at some point in the future.

Triadic Closure (continued)

Example:

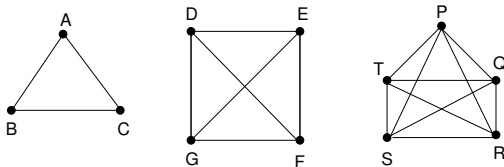


- Network on the left: B and C have a common friend (namely, A).
- By the triadic closure principle, nodes B and C are likely to become friends in the future.
- Nodes A, B and C would then form a **triangle**; edge {B, C} “closes” this triangle (network on the right).
- Examples of other future edges: {F, D} and {B, F}.

Quantifying Triadic Closure

- A common measure: **Clustering Coefficient**.
- Need some preliminaries before defining this measure.

Complete Graph (Clique):



- A clique contains all possible edges between its nodes.
- **Fact:** The number of edges in a clique with k nodes = $k(k-1)/2$.

Definition of Clustering Coefficient

Definition: Suppose the degree of node A is d and the number of edges among the neighbors of A is e . Then, the **clustering coefficient** of A , denoted by $CCF(A)$, is given by

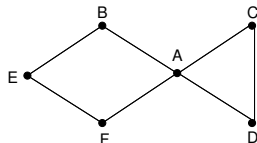
$$CCF(A) = \frac{e}{[d(d-1)/2]}$$

Notes:

- The expression $d(d-1)/2$ is the number of edges in a clique with d nodes.
- For any node A , $0 \leq CCF(A) \leq 1$.
- Also called **local clustering coefficient**.

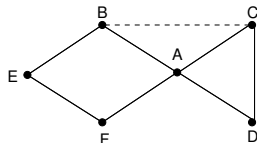
Examples: Clustering Coefficient Calculation

Example 1:



- Degree of A = 4.
- No. of edges among the neighbors of A = 1.
- $CCF(A) = 1/[4(4-1)/2] = 1/6$.

Example 2:

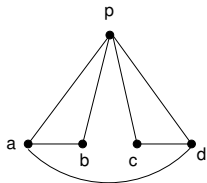


- Degree of A = 4.
- No. of edges among the neighbors of A = 2.
- $CCF(A) = 2/[4(4-1)/2] = 1/3$.
(Thus, triadic closure increases the clustering coefficient.)

Clustering Coefficient and Triadic Closure

Question: How is the definition of CCF related to triadic closure?

Example: Consider the value of $CCF(p)$ in the following graph.



- Degree of $p = 4$.
- No. of edges among the neighbors of $p = 3$.
- $CCF(p) = 3/[4(4 - 1)/2] = 1/2$.

- Each edge between a pair of neighbors of p forms a triangle that includes p .
- The maximum number of triangle that can include $p = 6$.
- So, we can also define $CCF(p)$ as the ratio

$$\frac{\text{No. of triangles that include } p}{\text{Maximum number of triangles that can include } p}$$

Some Sociological Reasons for Triadic Closure

Assumption: B and C are friends of A.

- 1 B and C have increased chances of meeting each other and becoming friends.
- 2 The friendship with A provides a basis for **mutual trust** between B and C.
- 3 A may have an **incentive** to make B and C friends. (If B and C are not friends, this may be a source of stress for A.)

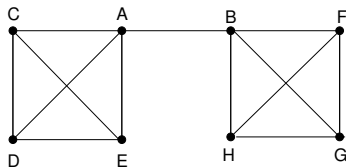
Empirical Evidence for Item 3:

- Bearman & Moody [2004] studied social networks of teenage girls in conjunction with public health records.
- Their finding: Girls whose CCFs are low are more likely to contemplate suicide than those whose CCFs are high.

Bridges and Local Bridges

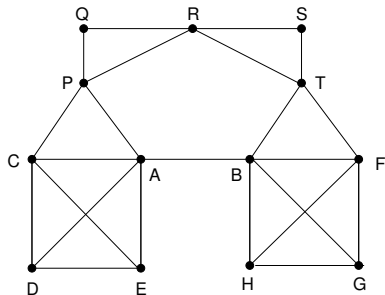
Definition: A **bridge** is an edge whose removal disconnects a network.

Example:



- Here, $\{A, B\}$ is a bridge.
- Note that A and B don't have any common neighbors.
- The set of nodes $\{A, C, D, E\}$ above form a “tightly knit” group.
- Edge $\{A, B\}$ allows A to “reach a different part” of the network (i.e., it may enable A to get other information that can't be obtained from C, D or E).
- Bridges are rare in social networks. Thus, A and B are likely to be joined through other (longer) paths.

Bridges and Local Bridges (continued)



- Several long paths between A and B.
- This structure is more common in practice.

Definition: An edge $\{x, y\}$ is a **local bridge** if x and y don't have any common neighbor.

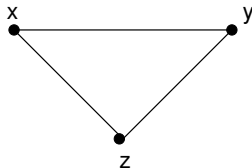
Example: In the above figure, $\{A, B\}$ is a local bridge.

Observation 1: Every bridge is a local bridge but a local bridge **need not** be a bridge.

Bridges and Local Bridges (continued)

Observation 2: If a local bridge $\{x, y\}$ is removed, then the shortest distance between x and y is **(strictly) larger than 2**.

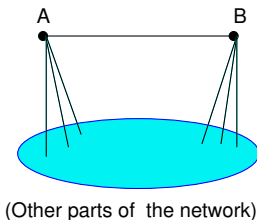
Example:



- Deleting $\{x, y\}$ shortest distance between x and y becomes 2.
- So, $\{x, y\}$ is **not** a local bridge.

Observation 3: An edge is a local bridge only when it **doesn't** form one edge of a triangle. (This is the connection to triadic closure.)

Role of Local Bridges



- Local bridge $\{A, B\}$ allows A to get information from B (or vice versa); without the local bridge, A and B will be **far away** from each other.
- All people in the “tightly knit” group that A belongs are likely to have the “same” information.
- So, A is more likely to get new information from a person such as B through a local bridge.

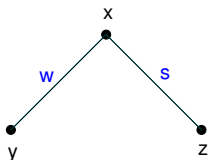
Note: So far, the discussion has not considered whether someone is an “acquaintance” or a “close friend”.

Strong and Weak Ties

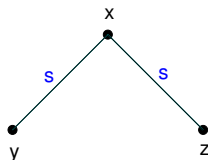
- Each edge of the network can be assigned a label “strong” (meaning “close friend”) or “weak” (meaning “acquaintance”).
- **Strong Triadic Closure (STC) Condition:** If a node x has **strong** ties to two other nodes y and z , then the graph contains the edge $\{y, z\}$.

Note: The STC condition does not specify the label of the edge $\{y, z\}$.

Examples:



(Satisfies STC)



(Violates STC)

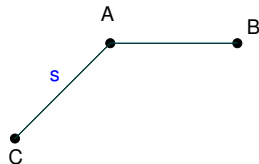
Granovetter's Assumption: Every node satisfies STC.

Strengths of Ties and Local Bridges

Theorem: If a node A satisfies STC and is involved in at least two strong ties, then every local bridge involving A must be a weak tie.

Proof: To be discussed in class.

An informal explanation:



- A local bridge $\{A, B\}$ is generally a weak tie.
- If not, STC would produce shortcuts that would eliminate its role as a local bridge (i.e., A and B would become part of the same “tight knit” community).

Summary: Strength of Weak Ties

- Local bridges help in getting information from other parts of the network.
- Under STC, local bridges represent weak ties.
- The formalism allows us to relate strengths of ties and the network structure.
- It also points out the **strength of weak ties**.
- Granovetter's study used small (manually constructed) social networks to support the conclusions.
- Other researchers have tested Granovetter's observations on large networks.