Epistemic : Deontic :: Additive : Intermediate

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1. The Puzzle. Kratzer (1978) makese a strong case that modal auxiliaries such as *must* are not semantically ambiguous. Even though *must* admits of (e.g.) epistemic and deontic interpretations, Kratzer argues that there is one lexical item and different readings are generated by changes in contextual parameters which capture background information relevant to the modal's interpretation. This analysis suggests a further question: are there *any* differences between epistemic and deontic modals that cannot be captured by shifting the contextual background? Kratzer (1991) influentially suggested that there are not: a single logic is operative for all modals, including both auxiliaries such as *must* and expressions of graded modality such as *likely* and *good*.

However, epistemic and deontic adjectives differ in at least one respect: (1) is valid, (2) is not.

(1) a. ϕ is as good as ψ.
b. ϕ is as good as χ.
c. **Valid**: ϕ is as good as $(\psi \vee \chi)$.

(2) a. ϕ is as likely as ψ.
b. ϕ is as likely as χ.
c. **Invalid**: ϕ is as likely as $(\psi \vee \chi)$.

For instance, let ϕ = *Bill wins the lottery*, ψ = *Mary wins*, and χ = *Sue wins*. (2c) could easily be false while the premises are true, for example, if Bill, Mary, and Sue all bought the same number of tickets. But it is hard to imagine how (1c) could fail to be true if (1a) and (1b) are. If Bill winning is at least as morally good as Mary winning (e.g., because of what each would do with the money), and likewise with Sue, then it is clearly at least as good if Bill wins as if either Mary or Sue does.

This might seem to be a problem for Kratzer's claim that modals can be given a unified semantics. I will argue that it is not (though it is a problem for Kratzer's specific proposal). We can explain the contrast between (1) and (2) in terms of a little-noticed parameter of variation which appears in adjectival scales quite generally, as evidenced by the contrast in (3)/(4).

(3) a. x is as hot as y.
b. x is as hot as z.
c. **Valid**: x is as hot as $(y \sqcup_i z)$.

(4) a. x is as heavy as y.
b. x is as heavy as z.
c. **Invalid**: x is as heavy as $(y \sqcup_i z)$.

$\sqcup_i$ is the join operation in the domain of individuals in Link's (1983) semantics for plurals, just as $\vee$ expresses join in the propositional domain. I'll start with an account of (3) and (4), arguing for the linguistic and inferential relevance of a distinction between 'additive' and 'intermediate' measurement. I'll argue that this distinction has important implications for modal semantics, providing a general theory of scale structure which gives theoretical grounding to recent proposals to place *likely* and *good* on scales of probability and expected utility, respectively.

2. Plurality and measurement. Measurement Theory (Krantz et al. 1971) has devoted much attention to the distinction between additive and non-additive measurement. Examples of additive scales include weight, length, and volume, which have the property that the measure assigned to the concatenation of two objects is the sum of the measures assigned to the objects individually. As pointed out by Krifka (1989), the measurement-theoretic concatenation operation can be thought of as Link's $\sqcup_i$ operation restricted to non-overlapping individuals. In other words, the weight of a plural individual such as the one denoted by *the boys* is the sum of the weights of its atoms (the individual boys). Similarly, if I pour the contents of one bowl of soup into another, the result (assuming no spillage) is a bowl whose volume of soup is the sum of the volumes of the two bowls that I started with. Additive measures obey the following constraint:

(5) μ is **additive** iff $\mu(x \sqcup y) = \mu(x) + \mu(y)$ when $x \sqcup y$ is defined and x and y do not overlap.

Note that I use $\sqcup$ as a cover symbol for the join operations $\sqcup_i$ and $\vee$, in preparation for the extension to modals and their propositional arguments below.

The fact that *heavy* is associated with an additive measure straightforwardly explains why (4) is not valid. For example, if Sam, Joe, and Al are all exactly the same weight, then Sam will be as heavy as each of Joe and Al but less heavy than the two put together; thus (4) must be invalid. Note the precise parallel with the lottery example used above as a counter-example to (2).

Measurement theorists have also analyzed non-additive scales such as beauty, happiness, and temperature. However, little attention has been paid to the empirical question of how measurement relates to concatenation/pluralization in such scales. In the case of beauty, there does not seem to be any systematic relationship between the beauty of two objects considered separately and their beauty when considered together. It may not even be sensible to talk about the happiness of a plural individual; if it is, certainly no systematic relation will obtain. With measurements of heat, however, there is a clear relationship. Suppose the mean temperature of the soup in bowl x is d, and the mean temperature of the soup in bowl y is $d' < d$. If I combine them, the result will surely not be a bowl whose temperature is $d + d'$; instead, its temperature will be *intermediate* between d and d'.

(6) μ is **intermediate** iff $\mu(x) > \mu(x \sqcup y) > \mu(y)$ whenever $x \sqcup y$ is defined and $\mu(x) > \mu(y)$.

In the case of temperature we can be more precise: ignoring heat loss and chemical reactions, the temperature of a compound object will be a *weighted average* (WA) of the temperatures of its component parts, where the weight $w(x)$ of each x is given by its volume.

(7) μ is a **WA-measure** iff there is a weight function w such that, whenever $x \sqcup_i y$ is defined and x and y do not overlap, $\mu(x \sqcup y) = [\mu(x) \times w(x) + \mu(y) \times w(y)] / [w(x) + w(y)]$.

Every WA-measure is also intermediate (for non-zero weights), but the converse does not hold.

With intermediate properties, inferences of the type illustrated in (4) are valid. The reason is that the degree to which a compound object has P is at most the same as the degree of its component part which has the greatest degree of P; in general, it will be even less. So if the soup in bowl x is as hot as the soup in bowl y, and it is also as hot as the soup in bowl z, it will be as hot as the result of combining the contents of y and z, since the latter cannot be hotter than the hotter of y and z. This is how the distinction between additive and intermediate properties explains the contrast in (3)/(4).

3. Additive and intermediate modals. One of Lewis' (1973) several equivalent versions of his semantics for counterfactuals starts with a reflexive, transitive, connected ordering on worlds $\succcurlyeq$; this is lifted to an ordering on propositions $\succcurlyeq^P$ by the rule: $\phi \succcurlyeq^P \psi$ iff $\forall u \in \psi\ \exists v \in \phi [v \succcurlyeq u]$. Lewis also discusses treating this as a theory of deontic comparison, reading $\succcurlyeq^P$ as "at least as good as". As Lewis notes, this rule entails that, if ϕ is better than ψ, then ϕ is exactly as good as $\phi \vee \psi$; this is because the best worlds in ϕ are also the best worlds in $\phi \vee \psi$, and so determine the position of both in $\succcurlyeq^P$. Thinking of $\succcurlyeq^P$-equivalence classes as degrees (Krantz et al. 1971, Cresswell 1977, etc.), we can see that the ordering $\succcurlyeq^P$ is neither additive not intermediate, but rather *maximal*.

(8) μ is **maximal** iff $\mu(x \sqcup y) = \mathbf{max}(\mu(x), \mu(y))$ whenever $x \sqcup y$ is defined.

The schematic inferences in (1) and (3) are valid if the relevant measures are maximal, just as they are if the measures are intermediate: clearly, $\mu(x) \geq \mu(y) \wedge \mu(x) \geq \mu(z) \vDash \mu(x) \geq \mathbf{max}(\mu(y), \mu(z))$. As far as the system of deontic comparisons in (1) goes, then, this semantics gets the right result.

Kratzer's (1991) closely related but slightly more complex semantics also gets this result, and for essentially the same reason (though (8) does not quite characterize its interaction with the join operation due to lack of connectedness in her version of the $\succcurlyeq$ relation). Problematically, though, Kratzer extends the Lewisian semantics to include epistemic modals as well. As a result her semantics predicts that (1) and (2) should pattern together. Despite the theoretical virtue of modal unification, then, this approach is empirically inadequate: some modifications are in order.

Starting with the (preliminary) typology of measurement/join interactions developed in §2, we can derive an alternative approach that explains the contrast in (1) and (2) using an independently motivated parameter of variation and dovetails with recent proposals in the literature. On analogy with *hot* and *heavy*, we can associate *likely* with an additive measure and *good* with an intermediate measure. This is enough to account for the validity of (1) and the invalidity of (2), for precisely the same reasons we saw above with *hot* and *heavy*.

(9) a. $\mu_{likely}(\phi \vee \psi) = \mu_{likely}(\phi) + \mu_{likely}(\psi)$ when ϕ and ψ are non-overlapping (disjoint).

b. $\mu_{good}(\phi) > \mu_{good}(\phi \vee \psi) > \mu_{good}(\psi)$ when $\mu_{good}(\phi) > \mu_{good}(\psi)$.

Recently several authors have proposed that *likely* is associated with a probability scale (Yalcin 2007,2010; Lassiter 2010, 2011; Klecha 2012). In fact, additivity of disjoint propositions is (along with upper-boundedness) the crucial formal property of probability in its standard measure-based axiomatization (Kolmogorov 1933). The proposal that likelihood is probability thus fits in neatly with the typology of scales developed in §2: according to this proposal likelihood is a measure on propositions formally similar to measurements of volume, weight, and length. The invalidity of (2) does not necessitate the use of probability: other measures fail to validate (2), including all those which allow but do not require additivity. However, the fact that additive structures occur elsewhere in scalar language and reasoning lends weight to the probabilistic proposal.

Goble (1996) and Lassiter (2011) argued that deontic modals such as *good* are associated with an intermediate measure — specifically expected utility, a type of WA-measure (7). As in the case of likelihood, we want to ask two questions about this proposal: does it get the facts right, and does it make sense given independently motivated properties of the semantics? The first question is the subject of current debate, and I will not attempt to resolve it here (see the works cited for positive arguments involving a number of deontic puzzles, and von Fintel ms. for a rebuttal). The second point, however, is one that we can address: in effect, Goble and Lassiter treat goodness as structurally analogous to temperature. This semantics relies on a type of measurement which is needed anyway to capture facts about non-modal adjectives and the inferences that they validate. In contrast, non-modal adjectives do not (to my knowledge) provide any motivation for the existence of maximality (8) in the semantics as a strategy for combining measures.

The new semantics has a significant theoretical advantage, then: we can account for the divergence between epistemic and deontic comparatives illustrated by (1) and (2) by invoking an independently motivated point of variation in the constitution of scales. On this account, the difference between *good* and epistemic *likely* is parametrized by the type of measure employed. The difference between epistemic and deontic adjectives is thus explained without threatening the overall project of a unified modal semantics — at the cost of drawing a closer connection between modality and gradation than has traditionally been acknowledged.

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