Modals, conditionals, and probabilistic generative models

Topic 2: Indicative conditionals – Separating semantics from reasoning

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Overall plan

1. probability, generative models, a bit on epistemic modals
2. indicative conditionals
3. causal models & counterfactuals
4. lazy reasoning about impossibilia

Today: Indicative conditionals

- Finish up sampling demos
- Probabilities of indicative conditionals
- Major theories of indicative conditionals
- The trivalent semantics
- Avoiding triviality proofs
- Conditional restriction

Probabilities of conditionals: The data

The lottery

Mary can choose whether to buy a ticket in a fair lottery with 100 tickets.

What is the probability of (1)?

If Mary buys a ticket, she will win.

Under *(un)likely*

Mary can choose whether to buy a ticket in a fair lottery with 100 tickets.

How likely is it that, if Mary buys
a ticket, she will win?

How likely is it that Mary will win
if she buys a ticket?

It’s unlikely that Mary will win if she buys a ticket.

If Mary buys a ticket it’s unlikely that she will win.

Across speakers

Mary can choose whether to buy a ticket in a fair lottery with 100 tickets.

Person A: If Mary buys a ticket, she will win.

Person B:

- That is unlikely.
- What you said is probably wrong.
  - '… but there's a slight possibility you're right'
- There's only a 1% chance that you're right.

(Makes trouble for the restrictor gambit discussed later)

Varying tense

Since *will* may be a modal, check past tense too:

Mary had to choose whether to buy a ticket;
we don’t know if she did.

Person A: If Mary bought a ticket, she won.

Person B:

- That is unlikely.
- What you said is probably wrong.
  - ‘… but there’s a slight possibility you’re right’
- There’s only a 1% chance that you’re right.

Stalnaker's thesis (1970)

$$P(\text{If } A, C) = P(C \mid A)$$

'The English statement of a conditional probability sounds exactly like that of the probability of a conditional. What is the probability that I throw a six if I throw an even number, if not the probability that: If I throw an even number it will be a six?'

(van Fraassen 1976)

Many experimental studies confirm.

Stalnaker vs Adams

- ‘Adams’ Thesis’ is widely discussed but crucially different – about assertibility/ acceptability, not probability

  (Adams ‘65, ’75)

- Douven & Verbrugge (‘The Adams Family’, *Cognition, 2010)*:
  – Adams’ thesis is empirically incorrect
  – Stalnaker’s thesis holds up

Theories of indicative conditionals & what they predict

The material conditional

$$\begin{aligned} A \Rightarrow C &= A \supset C \\ &= \neg A \vee C \\ &= \neg(A \wedge \neg C) \end{aligned}$$

- Bad predictions about probabilities
  - P(1) depends on how likely she is to buy a ticket
- Lots of other problems

Strict conditional

$$A \Rightarrow C = \forall w \in E : A(w) \supset C(w)$$

Assuming P(E) = 1, this entails

$$\text{If } P(C|A) < 1, \text{ then } P(A \Rightarrow C) = 0$$

$$P(\mathbf{Buy} \Rightarrow \mathbf{Win}) = P(\mathbf{Buy} \Rightarrow \neg\mathbf{Win}) = 0!!$$

Definite description theories make similar predictions (e.g., Schlenker '04)

CP 1 semantics

$$A \Rightarrow C \equiv P(C \mid A) = 1$$

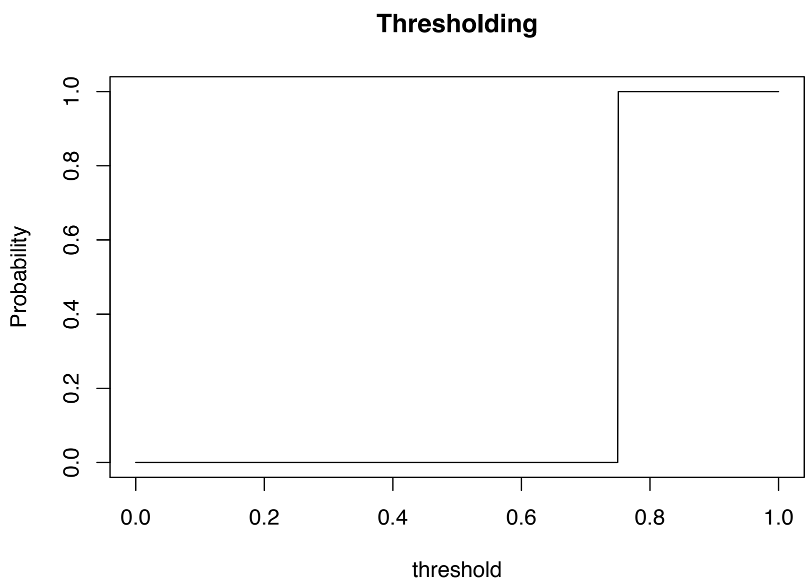
Like strict conditional,

$$\text{If } P(C|A) < 1, \text{ then } P(A \Rightarrow C) = 0$$

$$P(\mathbf{Buy} \Rightarrow \mathbf{Win}) = P(\mathbf{Buy} \Rightarrow \neg\mathbf{Win}) = 0!!$$

Threshold semantics

$$A \Rightarrow C \equiv P(C \mid A) \geq \theta \text{ for some } \theta \in [0, 1]$$



$$\text{If } P(C|A) \geq \theta,\ \text{then } P(A \Rightarrow C) = 1$$

$$\text{If } P(C|A) < \theta,\ \text{then } P(A \Rightarrow C) = 0$$

Lewis/Kratzer semantics

Conditionals restrict overt or covert operators

– Works for *It's likely that if A, C* [modulo syntax]

– Dubious for cross-speaker exx

– Probabilities of bare conditionals? In general,

$$P(\text{If A, must C}) \neq P(\text{If A, C})!$$

under any conceivable interpretation of *must*

Other operators don't fare better

Selection functions

$$A \Rightarrow C \text{ is true at } w \text{ iff } C \text{ is true at } f(A, w)$$

- *f(w)* is the ‘closest’ *A*-world to *w*
- Seems most promising way to get ST
- But Lewis’s proof shows it can’t!

Lewis' bombshell (1976)

No semantic assumptions: just ST, plus

- P is closed under conditionalization
- conditionals behave probabilistically just like any other proposition, so:

$$\begin{aligned} P(A \Rightarrow C) &= P(A \Rightarrow C \wedge B) + P(A \Rightarrow C \wedge \neg B) \\ &= P(A \Rightarrow C \mid B) \times P(B) + P(A \Rightarrow C \mid \neg B) \times P(\neg B) \end{aligned}$$

Lewis' bombshell (1976)

This leads to an unacceptable result:

$$\begin{aligned} P(A \Rightarrow C) &= P(A \Rightarrow C \mid C) \times P(C) + P(A \Rightarrow C \mid \neg C) \times P(\neg C) \\ &= P(C \mid A \wedge C) \times P(C) + P(C \mid A \wedge \neg C) \times P(\neg C) \\ &= 1 \times P(C) + 0 \times P(\neg C) \end{aligned}$$

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Prominent responses:

- Deny Stalnaker's Thesis, despite the evidence
- Deny that conditionals denote propositions

Both seem desperate …

A trivalent, truth-functional approach

The trivalent semantics

Conditionals with false antecedents are undefined.

A	C	$A \Rightarrow C$	$A \supset C$
T	T	T	T
T	F	F	F
F	T	#	T
F	F	#	T

Some key refs: de Finetti ’36, Milne ’97, Cantwell ‘08

Possible-worlds variant

Declarative sentences denote **pairs** of standard bivalent propositions:

$$\begin{aligned} [\![\mathrm{A}]\!] &= \langle TV(A), True(A) \rangle \\ [\![\mathrm{A}]\!]^w &= \#\ \text{unless}\ w \in TV(A) \\ &= 1\ \text{if}\ w \in TV(A) \cap True(A) \\ &= 0\ \text{if}\ w \in TV(A) - True(A) \end{aligned}$$

No TVs at worlds where antecedents is false

$$[\![\text{If } A \text{ then } C]\!] = \langle [\![\mathrm{A}]\!], [\![\mathrm{A}]\!] \cap [\![\mathrm{C}]\!] \rangle$$

$$(\text{for bivalent } A, C)$$

Empirical motivation: Conditional bets

In early 2018, I bet $100 with a bookie on:

If the 2018 Super Bowl is played in Orlando,
the 49ers will be in it.

[Outcome: Minneapolis, with Eagles and Patriots.]

Could the bookie claim that I lost the bet? or won?

Empirical motivation: Conditional bets

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[Outcome: Minneapolis, with Eagles and Patriots.]

Could the bookie claim that I lost the bet? or won?

Experimental participants: the bet is null and void

- See Politzer, Over & Baratgin 2010

Conditional predictions

In early 2018, I predicted:

If the 2018 Super Bowl is played in Orlando,
the 49ers will be in it.

Was my prediction

- true?
- false?
- something else?

Past-tense conditionals

Rip van Winkle has just woken up and asserts

If the 2018 Super Bowl was played in Orlando,
the 49ers were in it.

In our world, what is the truth-value of Rip's claim?

- What sort of facts could make it true?
- What sort of facts could make it false?

Bivalence say we have to make an arbitrary choice ...

Milne '97
Cantwell '06, '08
Rothschild '14

Trivalent probability

Idea: Probability measures ignore # values

What is the probability of 'If Mary buys a ticket she will win'?

w_1: buy, lose	=> F	w_4: not buy, lose	=> #
w_2: buy, lose	=> F	w_5: buy, lose	=> F
w_3: not buy, lose	=> #	w_6: buy, win	=> T

$$P(S) = \frac{P(S \text{ is true})}{P(S \text{ is defined})} = \frac{P(\{w_6\})}{P(\{w_1, w_2, w_5, w_6\})}$$

Trivalent semantics + probability enforces Stalnaker's thesis!

$$[\![\text{If } A \text{ then } C]\!] = \langle [\![\mathrm{A}]\!], [\![\mathrm{A}]\!] \cap [\![\mathrm{C}]\!] \rangle$$

(for bivalent A, C)

$$\begin{aligned} P(A \Rightarrow C) &= \frac{P(A \Rightarrow C \text{ is true})}{P(A \Rightarrow C \text{ is defined})} \\ &= \frac{P(A \wedge C)}{P(A)} \\ &= P(C \mid A) \end{aligned}$$

Conditional sampling is implicitly trivalent

To estimate P(If A then C), repeat *n* times:

1. Sample a possible world from W
2. Check the truth-value of A.
   a. If A is true, return the truth-value of C.
   b. Otherwise, return to step 1.

‘ignore #’

Procedural characterization:

‘Throw out samples where A is false.’

Semantic characterization:

‘Normalize by probability that the sentence is defined.’

Lassiter, in press

Avoiding Lewis' bombshell

The key step –

$$P(A \Rightarrow C) = P(A \Rightarrow C \wedge B) + P(A \Rightarrow C \wedge \neg B)$$
$$= P(A \Rightarrow C \mid B) \times P(B) + P(A \Rightarrow C \mid \neg B) \times P(\neg B)$$

– is invalid in trivalent sem+prob because

$$P(A \Rightarrow C \wedge B) = \frac{P(A \wedge C \wedge B)}{P(A)}$$

non-standard normalization

whereas

$$P(A \Rightarrow C \mid B) \times P(B) = \frac{P(A \wedge C \wedge B)}{P(A \wedge B)} \times P(B)$$

Lassiter, in press

More on triviality proofs

Trivalent semantics avoids eight (8!) other well-known triviality proofs for ST

- usual interpretation: triviality proofs are evidence against ST
- but empirical evidence for ST is overwhelming
- alternative interpretation: triviality proofs are evidence against bivalence

Other ways of getting Stalnaker’s thesis

- Bacon ‘15: bivalent, ‘random-worlds’ interpretation of selection functions
- Khoo t.a.: bivalent, relativize to carefully constructed sequences of worlds
- Stalnaker/Jeffrey/Kaufmann: infinite-valued semantics with similar semantics to Khoo’s

All can be viewed as models of probability **estimation** using **conditional sampling**

  – i.e.: ‘impure’ models that collapse an implicitly trivalent semantics with a sampling procedure

Conditional restriction phenomena

Restriction phenomena

Idea: like probability, natural language operators systematically ignore # values

Consequence: we only ever make comparisons among antecedent-satisfying {individuals, worlds, situations, etc}

Lewis ‘75

Adverbs of quantification

‘Usually if it’s raining my roof leaks’

≠

‘Usually, either it’s not raining or my roof leaks’

and ≠

‘Either it’s not raining, or usually my roof leaks’

but: ~=

‘In most cases where it’s raining, my roof leaks’

Adverbs of quantification

Lewis '75: 'The "if" in "sometimes if", "always if" is on a par with the non-connective "and" in "between … and …", … and with the non-connective "if" in "the probability that … if …". It serves merely to mark an argument-place in a polyadic construction.'

Kratzer '86: 'The history of the conditional is the story of a syntactic mistake. There is no two-place "if … then" connective in the logical forms for natural languages. "If"-clauses are devices for restricting the domains of various operators.'

Huitink '08, ch.5

A purely semantic treatment

'Usually if it's raining my roof leaks'

Relevant worlds:

w_1: rain, leak => T
w_2: rain, leak => T
w_3: no rain, leak => #
w_4: no rain, no leak => #
w_5: no rain, leak => #
w_6: rain, no leak => F

Prediction: $|\{w_1, w_2\}| \,/\, |\{w_1, w_2, w_6\}|$ is high

=

'In most cases where it's raining, my roof leaks'

Huitink '08, ch.5

Restricting modals

'If it's raining my roof must be leaking'

Relevant worlds:

w_1: rain, leak	=> T	w_4: no rain, no leak	=> #
w_2: rain, leak	=> T	w_5: no rain, leak	=> #
w_3: no rain, leak	=> #	w_6: rain, no leak	=> F

Prediction: All worlds in $\{w_1, w_2, w_6\}$ are leak-worlds

=

'In all worlds where it's raining, my roof leaks'

Huitink '08, ch.5

Restricting probability operators

'If it's raining my roof is probably leaking'

<u>Relevant worlds:</u>

w_1: rain, leak	=> T	w_4: no rain, no leak	=> #
w_2: rain, leak	=> T	w_5: no rain, leak	=> #
w_3: no rain, leak	=> #	w_6: rain, no leak	=> F

Prediction: P($w_{1,}$ w_2}) / P{$w_{1,}$ $w_{2,}$ w_6}) is high
= P(**rain** & **leak**) / P(**rain**) is high
'The conditional probability of **leak** given **rain** is high'

Huitink '08,
cf. Belnap '70

Restricting quantifiers

'Most students failed if they goofed off'
MOST[student][{*x*: *x* failed if *x* goofed off}]

<u>Relevant students:</u>

s_1: goof, fail => T
s_2: goof, fail => T
s_3: no goof, fail => #

s_4: no goof, pass => #
s_5: no goof, fail => #
s_6: good, pass => F

Prediction: $|\{s_{1,}\ s_2\}|\ /\ |\{s_{1,}\ s_{2,}\ s_6\}|$ is high
=
'Most students who goofed off failed'

Lewis '75
Kratzer '86

Advantages over restrictor theory

- No unattested readings of *It's likely that q if p*
- Works for cross-sentential cases
- No need to postulate silent operators in bare conditionals
  - 'Whenever there is no explicit operator, we have to posit one. … epistemic modals are candidates for such hidden operators' (Kr. '86)
- Complex syntactic maneuvering unnecessary

(e.g., von Fintel '94)

Some outstanding issues

Conditional commands and questions

If it's raining, take an umbrella!

If it’s raining, will you take an umbrella?

Assertion

A	C	$A \Rightarrow C$	$A \supset C$
T	T	T	T
T	F	F	F
F	T	#	T
F	F	#	T

What is conveyed by

If it's raining, my roof is leaking

? Is it P(C | A) ~= 1? How does this relate to

Either it's not raining, or my roof is leaking?

See Khoo, t.a.

Left-nested conditionals

If the vase broke if it was dropped,
it was fragile

[a certain coin is either double-headed or double-tailed]

If the coin landed heads if it was flipped, it was double-headed

Conjunctions of conditionals

McDermott '96
Bradley '01

If it's not raining, we'll have a picnic; and if it is we won't.

If it's snowing we'll go skiing; or if it's not we'll stay home.

Embeddings

Mary believes that she'll win if she plays.

Is it enough that there be no play-and-don't-win worlds? If so,

$$B_M(\mathbf{play} \Rightarrow \mathbf{win}) \equiv B_M(\mathbf{play} \supset \mathbf{win})$$

Summary

- Trivalent semantics + probability makes sense of evidence for Stalnaker's thesis
  - Conditionals are truth-functional, not epistemic
  - No need for complicated, impure semantics with (e.g.) infinite sequences of worlds
- Conditional restriction comes nearly for free
  - Undercuts motivation for popular restrictor theory
- Promising framework but important empirical, theoretical, philosophical issues remain

Next week (on Wednesday 12/11!)

Counterfactuals & causal models

Counterfactual reasoning as intervention

- connections to Lewis/Stalnaker semantics
- reasons to prefer the causal models approach

Filling a major gap: treatment of complex and quantified antecedents

Thanks!

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