Homework 07

XD

March 7th, 2022

Q1. 9.3: 1, 2, 4b, 5, 6, 7, 10. Q2. 9.4: 1cdef, 2, 6, 10.

Question 1

Problem 1

$7, 6+1, 5+2, 5+1+1, 4+3, 4+2+1, 4+1+1, 3+3+1, 3+2+1+1, 3+1+1+1+1+1, 2+2+2+1, 2+2+1+1+1, 2+1+1+1+1+1, 1+1+1+1+1+1+1$

Problem 2

a.$f(x) = (\frac{1}{1-x^2})(\frac{1}{1-x^4})(\frac{1}{1-x^4})(\frac{1}{1-x^6})...... = \prod_{n=1}^{\infty} \frac{1}{1-x^{2i}}$
b.$f(x) = \prod_{n=1}^{\infty} 1 + x^{2i}$
c. $f(x) = \prod_{n=1}^{\infty} 1 + x^{2i-1}$

Problem 4b

$f(t) = \frac{1}{1-t^2}\frac{t^{12}}{1-t^3}\frac{t^{20}}{1-t^5}\frac{t^{35}}{1-t^7}$

Problem 5

a.$f(t) = 1 + x^2 + x^4 + x^6 + ... = \frac{1}{1-x^2}$
b.Same as part A.

Problem 6

a.Since every summand $n \in N$ cannot appear 5 times in the partition. $f(x) = \prod_{n=1}^{\infty}(1 + x^n + x^{2n} + x^{3n} + x^{4n} + x^{5n}) = \prod_{n=1}^{\infty} \frac{1-x^{6i}}{1-x^i}$
b.Using the answer obtained in part A, we can find case in which summand cannot exceed 12. $f(x) = \prod_{n=1}^{12}(1 + x^n + x^{2n} + x^{3n} + x^{4n} + x^{5n}) = \prod_{n=1}^{12} \frac{1-x^{6i}}{1-x^i}$

Problem 7

$f(x) = \prod_{n=1}^{\infty} \frac{1-x^{3k}}{1-x^k} = \prod_{n \not| 3}^{\infty}(1 - x^l)^{-1}, k, l \in N$
g(x) $=\prod_{n \not| 3}^{\infty}(1 + x^l + x^{2l} + x^{3l} + x^{4l}....) = \prod_{n \not| 3}^{\infty}(1 - x^l)^{-1}$ f(x) = g(x).
Therefore the condition for the problem has been proven true.

Problem 10

Let us consider a Ferrers Graph with partition of 2n into n rows. If we were to remove the first column of the graph, the resulting graph would be a partition for n. This is a one to one correspondence thus confirming the number of partition of n is equal to the partition of 2n into n summands.

Question 2

Problem 1

c.e^{-ax}
d.$e^{a^{2x}}$
e.$ae^{a^{2x}}$
f.xe^{2x}

Problem 2:

a.$3, 3^2, 3^3, 3^4, 3^5, ...$
b.$3, 24, 138, ..., 6(5^n) - 3(2^n)$
c.1,1,3,1,1,1,1,... d.1,9,14,-10,$2^4, 2^5, 2^6, ..$ e.0!,1!,2!,3!... f.4,7,25,145,..,(3n!)$2^n + 1$

Problem 6:

a.i.$(1+x)^2(1+x+(\frac{x^2}{2!}))^2()$
ii. $(1+x)(1+x+(\frac{x^2}{2!}))(1+x+(\frac{x^2}{2!})+(\frac{x^2}{3!})+(\frac{x^2}{4!}))^2$
iii. $(1+x)^3(1+x+\frac{x^2}{2!})^4$
b.$(1+x)\cdot(1+x+\frac{x^2}{2!})\cdot(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!})\cdot(\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!})$

Problem 10:

a. $f(x) = (x+\frac{x^3}{3!}+\frac{x^5}{5!}+..)\cdot(x+\frac{x^2}{2!}+\frac{x^3}{3!}+...)\cdot(e^x)^2 = \frac{1}{2}\cdot(e^x-1)^2\cdot e^{2x} = \frac{1}{2}(e^x-1)\cdot(e^{3x}-e^x) = \frac{1}{2}(e^{4x}-e^{3x}-e^{2x}+e^x)$ Answer is coeff($\frac{x^{20}}{20!}$) $= \frac{1}{2}(4^{20}-3^{20}-2^{20}+1)$
b. $(1+x+\frac{(x^3)}{3!}+\frac{x^4}{4!}+...)^4 = (e^x-\frac{x^2}{2})^4 = e^{4x}-\binom{4}{1}e^{3x}(\frac{x^2}{2}-\binom{4}{2}e^{2x}(\frac{x^2}{2})^2)-\binom{4}{3}e^x(\frac{x^2}{2})^3)+(\frac{x^2}{2})^4$
Answer: $4^{20}-\binom{4}{1}\frac{1}{2}3^{18}(20)(19)+\binom{4}{2}\frac{1}{4}(2^{16})(20)(19)(18)(17)-\binom{4}{3}(\frac{1}{8})(20)(19)(18)(17)(16)(15)$
c.$h(x) = (1+x+(x^3/3!)+(x^4/4!)+....)^4 = (e^x-(x^2/2))^4 = e^{4x}-\binom{4}{1}e^{3x}(x^3)/6+\binom{4}{2}e^{2x}(x^3/6)^2-(x^3/6)^4$
$coeff(\mathrm{x}^{20}\frac{}{20!}) = 4^{20}-\binom{4}{1}(1/6)3^{17}(20)(19)(18)+\binom{4}{2}(1/6)^2(2^{14})(20)(19)(18)(17)(16)()15-\binom{4}{3}(1/6)^3(20!/11!)$
d. $(e^x)^3(1+(x^2/2!)) = e^{3x}+e^{3x}(x^2/2!)$
$coeff = 3^{20}+(1/2)(1/6)(3^{18})(20)(19)$