Chapter 2

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Introduction

- A compiler scans an input of characters and outputs a stream of words labelled by syntatic category
- A microsyntax is used to group words that have meaning within the source language
- Some words such as keywords have special meaning, which makes them reserved
- An example of this would be the *while* and *static* keywords in the Java programming language
- To recognize keywords, the scanner can either use dictionary lookup or encode keywords directly into microsyntax
- The simple lexical structure of programming languages lends itself to efficent scanners

Recognizing Words

- When we are parsing words we can view the parsing process as a series of if-else statements or a state machine
- Transition diagrams often provide a simple means of formalizing the abstractions a compiler may need to implement them
- S is the finite set of states in the recognizer, alongside with error state s_e
- Σ is the finite alphabet recognized by the recognizer
- $\delta(s, c)$ is the transition function, it maps the value of state s and c,into some state
- In state s_i with transition character c, the state makes the following transition $s_i \rightarrow_c \delta(s_i, c)$
- $s_0 \in S$ refers to initial state
- $S_a(S_a \subseteq S)$, is the set of accepting states

Example:

$S = \{s_0, s_1, s_2, s_3, ..., s_10, s_e\}$
$\Sigma = \{e, h, i, l, n, o, t, w\}$
$\delta =$
$\{ s_{\,0} \rightarrow_n s_1, s_0 \rightarrow_w s_6, s_1 \rightarrow_e s_2, s_1 \rightarrow_o s_4, s_2 \rightarrow_w s_3$
$s_4 \rightarrow_t s_5, s_6 \rightarrow_h s_7, s_7 \rightarrow_i s_8, s_8 \rightarrow_l s_9, s_9 \rightarrow_e s_{10}$
$s_0 = s_0$
$S_A = \{s_3, s_5, s_{10}\}$

More complex words:

- For more complex words we can have the state machine accept multiple inputs
- We can vastly simplify state machines by using cycles

Practice Problems:

- Problem 1: A six-character identifier consisting of alphanumeric characters followed by zero to five-alpha numeric characters
  - $S = \{s_0, s_1, s_e\}$
  - $\Sigma = a =$ **set of all-alphabet**, $b =$ **set of all alphanumeric**
  - $s_0 = s_0$
  - $\delta = \{ s_0 \rightarrow_a s_1, s_1 \rightarrow_b s_1$
  - $S_A = s_1$
- Problem 2:
  - $S = \{s_0, s_1, s_2 s_e\}$
  - $\Sigma = (,)$
  - $s_0 = s_0$
  - $S_A = \{s_2\}$
  - $\delta = \{ s_0 \rightarrow_(s_1, s_1 \rightarrow_) s_2, s_2 \rightarrow_(s_1\}$
- Problem 3: A Pascal comment which consists of {, zero or more characters from the alphabet, and closed by }:
  - $S = \{s_0, s_1, s_2\}$
  - $\Sigma = \{\}, \{, a...z, A...Z, 0...9\}$
  - $s_0 = s_0$
  - $S_A = \{S_3\}$
  - $\delta = \{ s_0 \rightarrow_\{ s_1$
    $s_1 \rightarrow_{(a...z,A...Z,0...9)} s_1$
    $s_1 \rightarrow_\{ s_2$

Regular Expression

- The set of all words accepted by a finite automaton, F, forms a language L(F)
- For any FA, we can describe describe the language using regular expression or RE
- The language consists of single world "new" can be described as RE, new

- A language consisting of two words, new or while can be represented as RE new|while
- new or not can be represent by RE, n(ew|ot)
- Let us consider the example of punctuation marks, a REs for punctuation may appear such as: : ; ? = > ( ) []
- Keywords may have an expression such as this: if while this integer instanceof
- more complex RE: 0 |(0|1|2|3|4|5|6|7|8|9)(0|1|2|3|4|5|6|7|8|9)∗
- **The following operator is called a kleen operator and indicates there can be zero or more instances of a RE**

Formalizing notes for regular expressions:

- Given a regular expression r, we can denote the Language it describes as L(r)
- An RE is made up of 3 operations:
- Alternation: The alternation or union of two sets R and S denoted R|S or $\{x|x \in R$ or $x \in S\}$
- Concatenation: The concatentation of two sets RS contains all strings formed by prependong an element of R onto one from S, or
  {xy|$x \in R\, and\, y \in S\}$
- Closure: The kleene closue of a set R, denoted by R^* is $\cup_{i=0}^{\inf} R^i$ is a concantenation of R with itself zero or more times
- Sometimes we can use notation for finite closure if a set is concantenated multiple times:
  $(R|RR|RRR)$
- Positive Closure is Denoted if RR^*
  - If $a \in \Sigma$, then a is also an RE denoting set containing only a
  - If r and s are RES, denoting sets L(r) and L(s) then r | s is a RE denoting the union, or alternation of L(r) and L(s)
    Similarly rs is an RE denoting the concatenation of L(r) and L(s)
    r^* is an RE denoting Kleene closure of L(r)
  - ϵ represents a RE of an empty string
  - Parentheses have the highest precedence, followed by closure, concatenaton, and alternation

Example:

- Imagine a language in keywords start with a letter in the English alphabet and can be then followed by a sequence of alphanumeric character. We can represent the following keyword using RE:
  $([\text{A...Z}]|[a...z])([A..Z]|([a...z])|([0...9])^*$
- Let's consider another example one in which, we are representing unsigned integers:
  $(0|([1...9])([0...9])^*$
- Unsigned real number:
  $(0|([1...9])([1...9])^*)(\epsilon|.[0...9]^*)$
- Quoted strings using complement:
  A quoted String in a programing language is often composed of a " followed by ", in between these two characters any characters can appear. In theory we could write a regular expression that contains all the possible characters but this is impractical. To circumvent this issue we can use the complement operator:
  "(^")"
- Comments can often appear in many forms:
  $(//(\ \hat{}\ \backslash n\ )^*|\ /*(\ \hat{}*\ |\hat{}\ */)^* * /)$

Closure properties of RE:

- Many regular expressions are closed under many operations,i.e if we apply an operation to a RE we get a RE
- Some obvious examples are concatenation, union, and closure
- Imagine we have a collection of regular expressions to describe syntatic categories in a language: $a_0, a_1, ..., a_n$
- To describe all the valid words in a language we can use the RE: $a_0|a_1|a_2|...|a_n$
- Closure under union suggests that any finite language is a regular language and can be arranged in alternation
- Closure under concantentation also allows us to build complex REs from simpler one's by concatenating them
- REs are closed under complements

Practice Problems:

- Chapter 2: pg 42
- Problem 1:

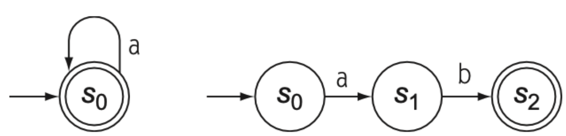
- $a_0 = [A...Z], a_1 = [a...z], a_2 = [0...9]$
  $(a_0|a_1)(a_0a_1a_2)^5$
- Problem 2:
- $a_0 = \hat{}\ ", a_1 = "$
  $a_1(a_0|a_1a_1)^*a_1$

From Regular Expression to Scanner:

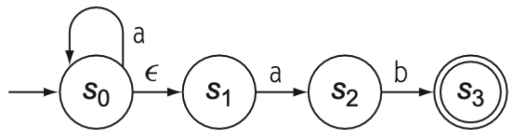
- Up to this point we have covered a decent amount of notation, so what is the point of all this notation
- In this section we will be learning about deterministic FA(DFA) and non-deterministic FA(NFA)

Nondeterministic Finite Automata

- Recall that an empty string ϵ is a RE, but many FAs did not include the empty string within the state diagrams
- So what role does ϵ play in FA?
- The answer lies in using ϵ to build more complex state machines
- For instance, we can use ϵ to represent concatentation of mn using ϵ
  $\rightarrow s_0 \rightarrow_m s_1 \rightarrow_\epsilon s_2 \rightarrow_n s_3$
- ϵ can often complicate how our model work
  - Consider the following two FAs



- Utilizing the ϵ transition we can add complexity to our model to form a^*ab



- The following machine is called a NFA, due to the fact that for a specific character there can be transitions for same state depending on what character follows the input character
- A DFA on the contrary can only have a single transition for a single character for a state

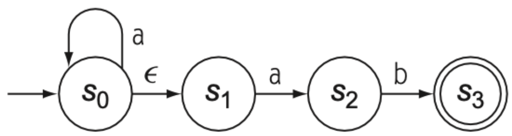
A Set of Models for NFA:

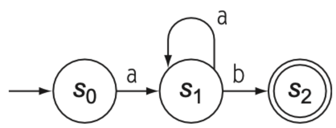
- Each time a NFA makes a nondeterministic choice, it follows the transition that transition that leads to accepting state of the input string
- Each time the NFA makes non-deterministic choice, it replicares itself to pursue each possible translation
- In this state, each possible translation is pursued and the current active state is called the *configuration*

Equivalence of NFAs:

- NFAs and DFAs are equivalent in their expressive power
- Any DFAs is a specical case of an NFA
- Conversely any NFA can be emulated by a DFA
- Let us consider the state of a NFA when it has reached some point in input string
- If there are n states and $|\Sigma|$ character then by rule of product there are $|\Sigma|^n$ configurations
- To simulate the behavior of the NFA, we need a DFA for each configuration of NFA thus have more exponentially more states than NFA
- This can make a DFA expensive from space standpoint

- Examine the two figures below:

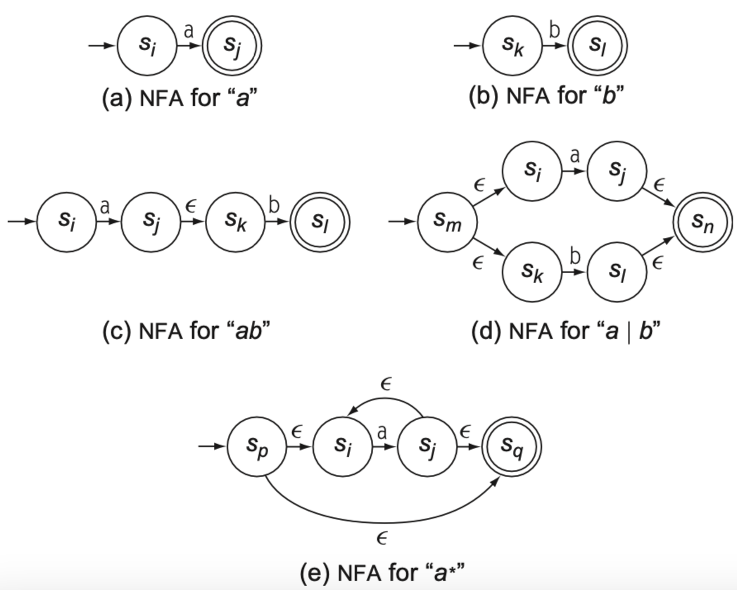




- Notice how both expressions are equivalent but instead of there of being two transition state for same characters one simply moves on to the next state and have self-loop

Regular Expression to NFA: Thompson's Construction

- Thompson's construction can be used to build NFA from RE
- In a sense they are a template for building FAs and can be viewed as a combination of concatentation, alternation, and closure



(a) NFA for "*a*"

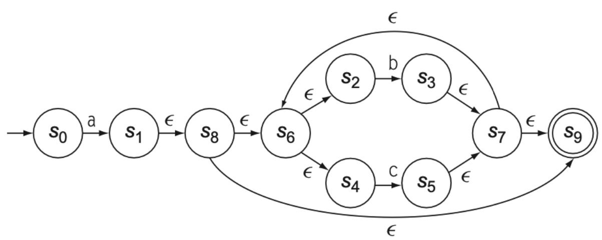
(b) NFA for "*b*"

(c) NFA for "*ab*"

(d) NFA for "*a* | *b*"

(e) NFA for "*a*∗"

- The following FAs above are trivial RE that can be used to build much more complex state machines
- The presence of one starting/accepting state makes the process of building a NFA much simpler



(d) NFA for "$a(b \mid c)^*$"

- The following figure above represents a Thompson construction of $a(b|c)^*$

NFA TO DFA: The Subset Construction

- Thompson Construction produces an NFA to recognize the language specificatons of a RE
- DFA execution is a lot simpler than NFA so having an algorithm to conver a Thompson construction to a DFA is useful
- The resulting DFA are simpler and have several efficent implementations

- Below there shall be the algorithn for DFA construction using a NFA
- The subset takes a NFA,$(N, \Sigma, \delta_N, n_0, N_A)$ and produces a DFA, $(D, \Sigma, \delta_D, d_0, D_A$

```
q0 ← ε-closure({n0});
Q ← q0;
WorkList ← {q0};
while (WorkList≠ ∅ ) do
   remove q from WorkList;
   for each character c ∈ Σ do
      t ← ε-closure(Delta(q,c));
      T[q,c] ← t;
      if t ∉ Q then
         add t to Q and to WorkList;
   end;
end;
```