Ph22.3 N-Body Simulations

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1. The equation of motion for two identical bodies of mass m under the influence of gravity is:

$$\mathbf{a}_{ij} = -\frac{Gm}{|\mathbf{r}_i - \mathbf{r}_j|^3}(\mathbf{r}_i - \mathbf{r}_j)$$

where $\mathbf{a}_{ij}$ is the acceleration experienced by the ith body due to the jth body. Written in first order form, we have:

$$\begin{aligned}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= -\frac{Gm}{\left((x_1-x)^2+(x_2-y)^2\right)^{3/2}}(x_1 - x) \\
\dot{x}_4 &= -\frac{Gm}{\left((x_1-x)^2+(x_2-y)^2\right)^{3/2}}(x_2 - y)
\end{aligned}$$

for the position and velocity of the ith particle under the influence of the jth particle at (x, y).

We implement the symplectic Euler integrator by (taking unit G and masses):

$$\begin{aligned}
x_3(t+\Delta t) &= x_3(t) + \sum_{\text{all other masses}} \frac{1}{\left((x_1-x_j)^2+(x_2-y_j)^2\right)^{3/2}}(x_j - x_1)\Delta t \\
x_1(t+\Delta t) &= x_1(t) + x_3(t+\Delta t)\Delta t \\
x_4(t+\Delta t) &= x_4(t) + \sum_{\text{all other masses}} \frac{1}{\left((x_1-x_j)^2+(x_2-y_j)^2\right)^{3/2}}(y_j - x_2)\Delta t \\
x_2(t+\Delta t) &= x_2(t) + x_4(t+\Delta t)\Delta t
\end{aligned}$$

For animations, run the following codes (one at a time):

runqn1(100,0.001,25,0)
This simulates 100 particles with time steps of 0.001 without force softening.

runqn1(100,0.001,25,0.1)
This simulates 100 particles with time steps of 0.001 with force softening.

Question 2.
runqn2(100,0.001,25,0.1,0.1)
This simulates 100 particles with time steps of 0.001 with force softening of 0.1 and initial velocity of 0.1.

runqn2(100,0.001,25,0.1,0)
This simulates 100 particles with time steps of 0.001 with force softening of 0.1 and initial velocity of 0.