# Ph22.4 N-Body Simulations with Approximations

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**1.** See the following sample run parameters:

runqn1(30,0.01,1,0.1,0.1,1)
This simulation has 30 particles, a timestep of 0.01, $a = 0.1, \theta = 0.1, m = 1$.

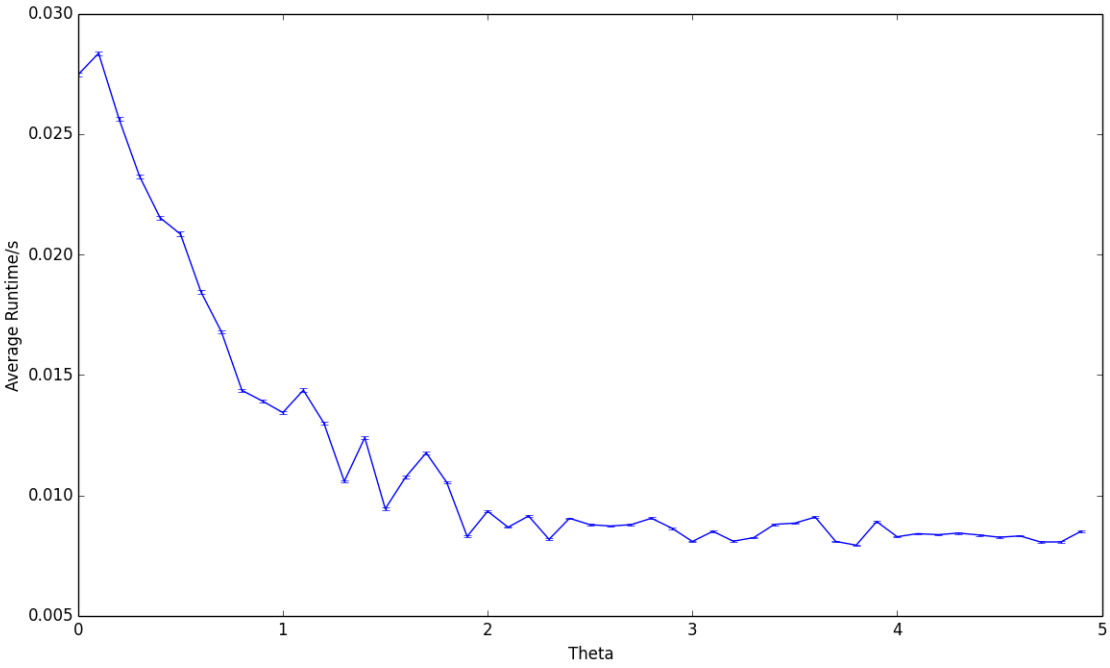
runqn1(100,0.01,1,0.1,0.5,1)
This simulation has 100 particles, a timestep of 0.01, $a = 0.1, \theta = 0.5, m = 1$. This simulation may take about 0.3s per timestep.

**2.** See the following sample run parameters (empirically determined to minimise fly-off)

runqn2(30,0.01,1,0.1,0,0.1,100)
This simulation has 30 particles, a timestep of 0.01, $a = 0.1, \theta = 0.1, m = 100$ and an initial velocity of zero.

For large $\theta$ values, the particles first congregate to the four centre of the biggest subquads. The systems then evolve in four distinct regions marked out by the four subquads. For $\theta \approx 0.1$, the particles behave very similarly to the $\theta = 0$ case.

We also investigate the effect of varying $\theta$ on the runtime of the simulation. We define average runtime to be the time taken for the slow step of the simulation to run one time (i.e. for the calculation of the acceleration to run one time). Taking 20 particles, a time-step of 0.001, and averaging over 1000 time-steps, we plot the average runtime (with $1\sigma$ standard error of the mean) against the value of theta used:

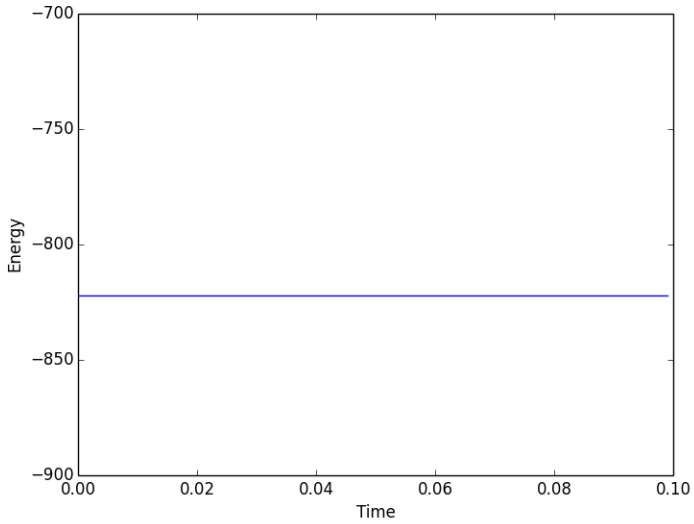



Clearly, a $\theta$ of around 1 is suitable to approximate the asymptotic minimum runtime.

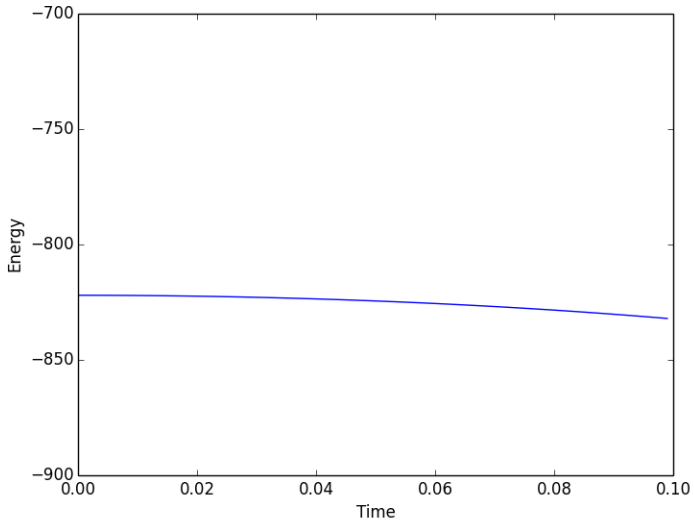
A cluster of 100 particles with $\theta = 0.5$ takes about 0.3 seconds per step, and perhaps represents the largest number of particles that can be simulated (reasonably) smoothly.

**3.** We introduce the total energy: $E = \sum_i \frac{1}{2} v_i^2 + \sum_{i,j,i<j} \frac{-1}{r_{ij}}$ and track it against time. To ensure energy conservation, we take $a = 0$ and investigate the deviation of total energy from the initial value for different values of $\theta$. We pick 30 particles, a timestep of 0.001 and a total of 100 steps. Some graphs are plotted below:

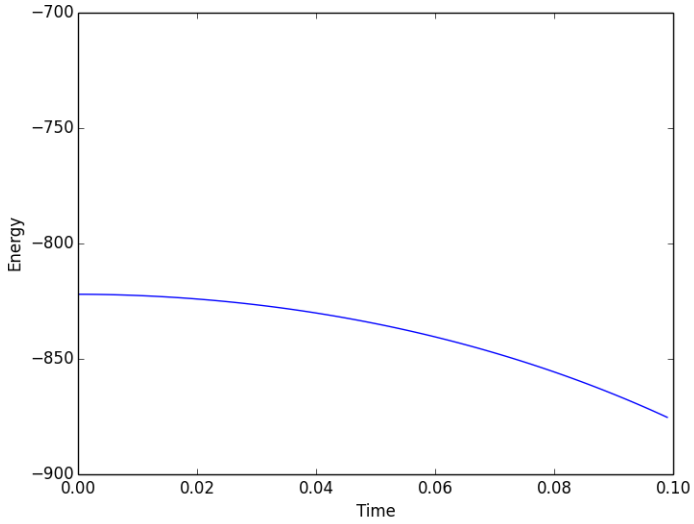
For $\theta = 0$, we have energy conservation.



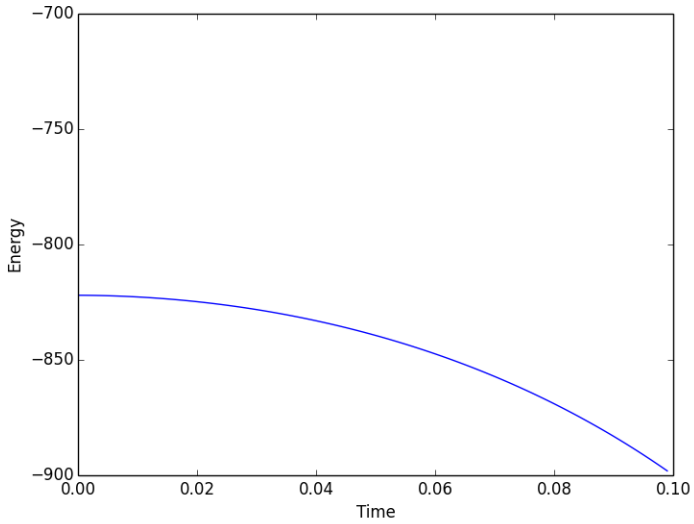

For $\theta = 0.1$,



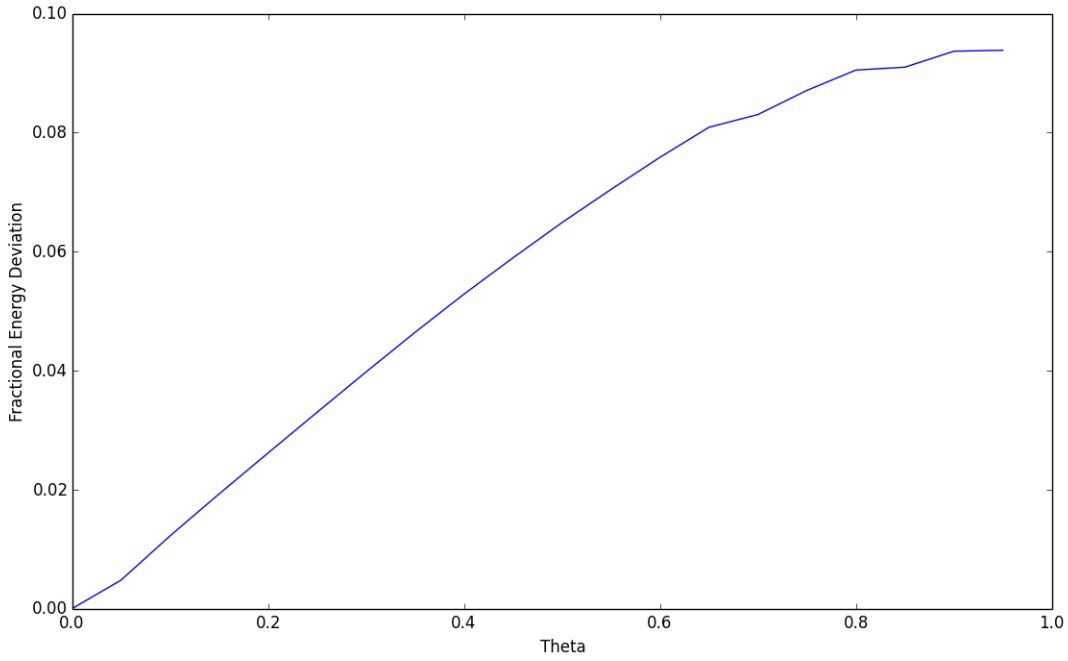

For $\theta = 0.5$,



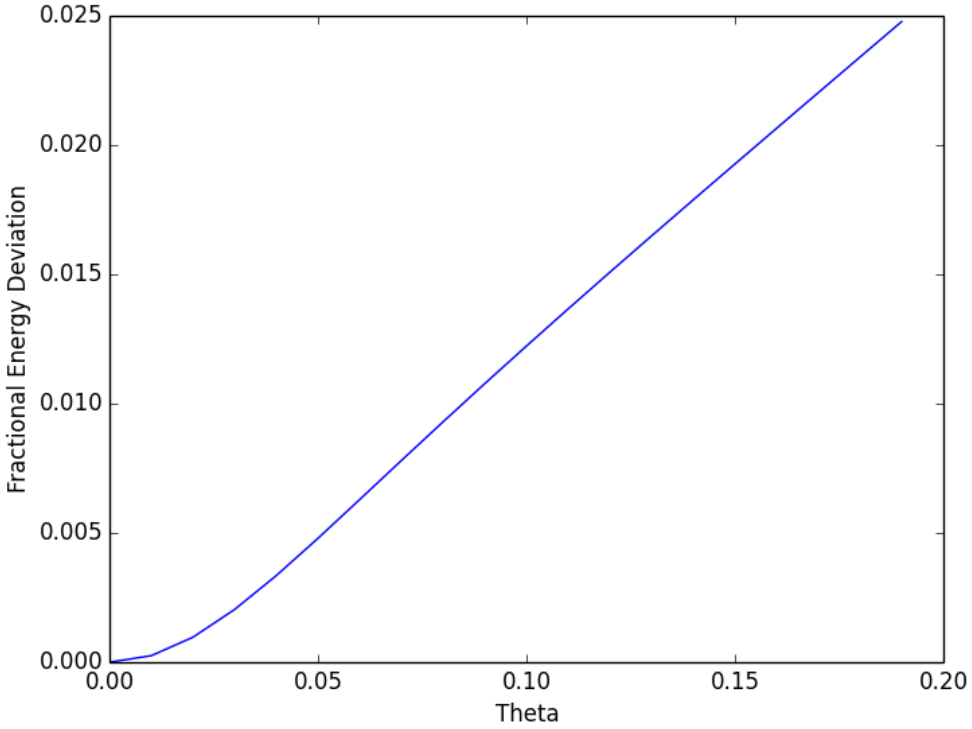

For $\theta = 1.0$,




To investigate the deviation quantitatively, we plot the fractional deviation error = (final energy - initial energy) / initial energy against $\theta$ under the same conditions and same random seed.




We observe that as $\theta$ increases, the fractional energy deviation also increases, as expected. We may zoom into the region between $\theta = 0$ and $\theta = 0.2$ to estimate the $\theta$ value when the fractional error is 1%.




Hence we observe that when $\theta \approx 0.1$, the fractional error is about 1%, justifying the use of $\theta = 0.1$ for the simulations.