Tarea: Producto por bloques y factorizaciones triangulares

Pregunta 1

Calcula la factorización $A = LU$ o $PA = LU$ de las siguientes matrices

a)
$$A_1 = \begin{pmatrix} 1 & 4 & 6 \\ 2 & -1 & 3 \\ 3 & 2 & 5 \end{pmatrix}$$

$$A_1 \sim f_2 - 2f_1 \begin{pmatrix} 1 & 4 & 6 \\ 0 & -9 & -9 \\ 3 & 2 & 5 \end{pmatrix} \sim f_3 - 3f_1 \begin{pmatrix} 1 & 4 & 6 \\ 0 & -9 & -9 \\ 0 & -10 & -13 \end{pmatrix} \sim -\frac{1}{9} f_2 \begin{pmatrix} 1 & 4 & 6 \\ 0 & 1 & 1 \\ 0 & -10 & -13 \end{pmatrix}$$

$$\sim f_3 + 10 f_2 \begin{pmatrix} 1 & 4 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{pmatrix} \sim -\frac{1}{3} f_3 \begin{pmatrix} 1 & 4 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 4 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = F_3(-\tfrac{1}{3}) \cdot F_{32}(10) \cdot F_2(-\tfrac{1}{9}) \cdot F_{31}(-3) \cdot F_{21}(-2) \cdot A_1$$

$$L = F_{21}(2) \cdot F_{31}(3) \cdot F_2(-9) \cdot F_{32}(-10) \cdot F_3(-3)$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -10 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -9 & 0 \\ 3 & -10 & -3 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 4 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A_1 = L \cdot U = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -9 & 0 \\ 3 & -10 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 6 \\ 2 & -1 & 3 \\ 3 & 2 & 5 \end{pmatrix}$$

b) $$A_2 = \begin{pmatrix} 2 & 4 & 2 \\ 1 & 3 & 1 \\ 7 & 5 & 6 \end{pmatrix}$$

$$A_2 \sim \frac{1}{2}f_1 \begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 7 & 5 & 6 \end{pmatrix} \sim f_2 - f_1 \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 7 & 5 & 6 \end{pmatrix} \sim f_3 - 7f_1 \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & -9 & -1 \end{pmatrix}$$

$$\sim f_3 + 9f_2 \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \sim (-1)f_3 \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = F_3(-1) \cdot F_{32}(9) \cdot F_{31}(-7) \cdot F_{21}(-1) \cdot F_1(\tfrac{1}{2}) \cdot A_2$$

$$L = F_1(2) \cdot F_{21}(1) \cdot F_{31}(7) \cdot F_{32}(-9) \cdot F_3(-1)$$

$$L = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -9 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$L = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 7 & -9 & -1 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A_2 = L \cdot U = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 7 & -9 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 4 & 2 \\ 1 & 3 & 1 \\ 7 & 5 & 6 \end{pmatrix}$$

c) $A_3 = \begin{pmatrix} 0 & 0 & 5 \\ -1 & 4 & 1 \\ 2 & -3 & 2 \end{pmatrix}$ $A_3 \sim f_1 \leftrightarrow f_2 \begin{pmatrix} -1 & 4 & 1 \\ 0 & 0 & 5 \\ 2 & -3 & 2 \end{pmatrix} \sim f_2 \leftrightarrow f_3$

$$\begin{pmatrix} -1 & 4 & 1 \\ 2 & -3 & 2 \\ 0 & 0 & 5 \end{pmatrix} \sim (-1) f_1 \begin{pmatrix} 1 & -4 & -1 \\ 2 & -3 & 2 \\ 0 & 0 & 5 \end{pmatrix} \sim f_2 - 2f_1 \begin{pmatrix} 1 & -4 & -1 \\ 0 & 5 & 4 \\ 0 & 0 & 5 \end{pmatrix} \sim \tfrac{1}{5} f_2$$

$$\begin{pmatrix} 1 & -4 & -1 \\ 0 & 1 & 4/5 \\ 0 & 0 & 5 \end{pmatrix} \sim \tfrac{1}{5} f_3 \begin{pmatrix} 1 & -4 & -1 \\ 0 & 1 & 4/5 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & -4 & -1 \\ 0 & 1 & 4/5 \\ 0 & 0 & 1 \end{pmatrix} = F_3\left(\tfrac{1}{5}\right) \cdot F_2\left(\tfrac{1}{5}\right) \cdot F_{21}(-2) \cdot F_1(-1) \cdot P \cdot A$$

$$P = F_{23} \cdot F_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$P \cdot A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 5 \\ -1 & 4 & 1 \\ 2 & -3 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 4 & 1 \\ 2 & -3 & 2 \\ 0 & 0 & 5 \end{pmatrix}$$

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$L = F_1(-1) \cdot F_{21}(2) \cdot F_2(5) \cdot F_3(5)$

$$L = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$$L = \begin{pmatrix} -1 & 0 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$$PA = LU = \begin{pmatrix} -1 & 0 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 & -4 & -1 \\ 0 & 1 & 4/5 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 & 1 \\ 2 & -3 & 2 \\ 0 & 0 & 5 \end{pmatrix}$$

d)

$$A_4 = \begin{pmatrix} 3 & 9 & -2 \\ 6 & -3 & 8 \\ 4 & 6 & 5 \end{pmatrix} \quad A_4 \sim \tfrac{1}{3} f_1 \begin{pmatrix} 1 & 3 & -2/3 \\ 6 & -3 & 8 \\ 4 & 6 & 5 \end{pmatrix} \sim f_2 - 6f_1$$

$$\begin{pmatrix} 1 & 3 & -2/3 \\ 0 & -21 & 12 \\ 4 & 6 & 5 \end{pmatrix} \sim f_3 - 4f_1 \begin{pmatrix} 1 & 3 & -2/3 \\ 0 & -21 & 12 \\ 0 & -6 & 23/3 \end{pmatrix} \sim -\tfrac{1}{21} f_2 \begin{pmatrix} 1 & 3 & -2/3 \\ 0 & 1 & -4/7 \\ 0 & -6 & 23/3 \end{pmatrix} \sim f_3 + 6f_2$$

$5 + \frac{4 \cdot 2}{3}$

$\frac{15}{3} + \frac{8}{3}$ $\quad 5 + \frac{8}{3}$

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$$\begin{pmatrix} 1 & 3 & -2/3 \\ 0 & 1 & -4/7 \\ 0 & 0 & 89/21 \end{pmatrix} \sim \frac{21}{89} f_3 \begin{pmatrix} 1 & 3 & -2/3 \\ 0 & 1 & -4/7 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 3 & -2/3 \\ 0 & 1 & -4/7 \\ 0 & 0 & 1 \end{pmatrix} = F_3\left(\frac{21}{89}\right) \cdot F_{32}(6) \cdot F_2\left(-\frac{1}{21}\right) \cdot F_{31}(-4) \cdot F_{21}(-6) \cdot F_1\left(\frac{1}{3}\right) \cdot A$$

$$L = F_1(3) \cdot F_{21}(6) \cdot F_{31}(4) \cdot F_2(-21) \cdot F_{32}(-6) \cdot F_3\left(\frac{89}{21}\right)$$

$$L = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -21 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -6 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{89}{21} \end{pmatrix}$$

$$L = \begin{pmatrix} 3 & 0 & 0 \\ 6 & -21 & 0 \\ 4 & -6 & \frac{89}{21} \end{pmatrix}$$

$$A = LU = \begin{pmatrix} 3 & 0 & 0 \\ 6 & -21 & 0 \\ 4 & -6 & \frac{89}{21} \end{pmatrix} \begin{pmatrix} 1 & 3 & -2/3 \\ 0 & 1 & -4/7 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 9 & -2 \\ 6 & -3 & 8 \\ 4 & 6 & 5 \end{pmatrix}$$

Pregunta 2

Utiliza las factorizaciones de la pregunta anterior para resolver los sistemas $AX=B$, donde las A_i son las del ejercicio anterior y las B_i correspondientes son las mostradas a continuación

a)
$$A_1=\begin{bmatrix}1&4&6\\2&-1&3\\3&2&5\end{bmatrix}\qquad B_1=\begin{pmatrix}-1\\7\\2\end{pmatrix}$$

$$AX=B$$
$$LUX=B$$
$$UX=Y \qquad LY=B$$

$$\begin{bmatrix}1&0&0\\2&-9&0\\3&-10&-3\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}-1\\7\\2\end{bmatrix}$$

$y_1=-1$

$2y_1-9y_2=7$

$2(-1)-9y_2=7$

$-2-9y_2=7$

$-9y_2=9$

$y_2=-\frac{9}{9}$

$y_2=-1$

$3y_1-10y_2-3y_3=2$

$3(-1)-10(-1)-3y_3=2$

$-3+10-3y_3=2$

$-3y_3=-5$

$y_3=5/3$

$$Y=\begin{bmatrix}-1\\-1\\5/3\end{bmatrix}$$

$$\begin{bmatrix}1&4&6\\0&1&1\\0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-1\\-1\\5/3\end{bmatrix}$$

$x_3=5/3$

$x_2+x_3=-1$

$x_2+5/3=-1$

$x_2=-8/3$

$x_1+4x_2+6x_3=-1$

$x_1+4(-8/3)+6(5/3)=-1$

$x_1+32/3+30/3=-1$

$x_1-2/3=-1$

$x_1=-1/3$

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$$X = \begin{pmatrix} -1/3 \\ -8/3 \\ 5/3 \end{pmatrix}$$

b)

$$A_2 = \begin{pmatrix} 2 & 4 & 2 \\ 1 & 3 & 1 \\ 7 & 5 & 6 \end{pmatrix} \qquad B_2 = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix}$$

$AX = B$

$LUX = B \quad LY = B$

$UX = Y$

$$LY = B \Rightarrow \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 7 & -9 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix} \qquad \begin{aligned} 2y_1 &= 6 \\ y_1 + y_2 &= 1 \\ 7y_1 - 9y_2 - y_3 &= 1 \end{aligned}$$

$2y_1 = 6$

$y_1 = 3$

$y_1 + y_2 = 1$

$3 + y_2 = 1$

$y_2 = 1 - 3$

$y_2 = -2$

$7y_1 - 9y_2 - y_3 = 1$

$7(3) - 9(-2) - y_3 = 1$

$21 + 18 - y_3 = 1$

$y_3 = 39 - 1$

$y_3 = 38$

$$Y = \begin{pmatrix} 3 \\ -2 \\ 38 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 38 \end{pmatrix} \qquad \begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ x_2 &= -2 \\ x_3 &= 38 \end{aligned}$$

$x_3 = 38$

$x_2 = -2$

$x_1 + 2x_2 + x_3 = 3$

$x_1 + 2(-2) + 38 = 3$

$x_1 - 4 + 38 = 3$

$x_1 + 34 = 3$

$x_1 = 3 - 34$

$x_1 = -31$

$$X = \begin{bmatrix} -31 \\ -2 \\ 38 \end{bmatrix}$$

c) $A = \begin{bmatrix} 0 & 0 & 5 \\ -1 & 4 & 1 \\ 2 & -3 & 2 \end{bmatrix}$ $B_3 = \begin{bmatrix} 3 \\ 10 \\ 4 \end{bmatrix}$

$AX = B$

$PAX = PB$

$LUX = PB$

$Y = UX$

$LY = PB$

$$LY = PB \Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 10 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \\ 3 \end{bmatrix}$$

$-y_1 = 10$

$2y_1 + 5y_2 = 4$

$5y_3 = 3$

$-y_1 = 10$

$y_1 = -10$

$2y_1 + 5y_2 = 4$

$2(-10) + 5y_2 = 4$

$-20 + 5y_2 = 4$

$5y_2 = 24$

$y_2 = 24/5$

$5y_3 = 3$

$y_3 = 3/5$

$$Y = \begin{bmatrix} -10 \\ 24/5 \\ 3/5 \end{bmatrix}$$

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$$UX = Y \Rightarrow \begin{pmatrix} 1 & -4 & -1 \\ 0 & 1 & 4/5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -10 \\ 24/5 \\ 3/5 \end{pmatrix}$$

$$x_1 - 4x_2 - x_3 = -10$$
$$x_2 + 4/5\, x_3 = 24/5$$
$$x_3 = 3/5$$

$$x_3 = 3/5$$

$$x_2 + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{24}{5}$$
$$x_2 + \frac{12}{25} = \frac{24}{5}$$
$$x_2 = \frac{24}{5} - \frac{12}{25}$$
$$x_2 = \frac{108}{25}$$

$$x_1 - 4\left(\frac{108}{25}\right) - \frac{3}{5} = -10$$
$$x_1 - \frac{432}{25} - \frac{3}{5} = -10$$
$$x_1 - \frac{447}{25} = -10$$
$$x_1 = \frac{197}{25}$$

$$X = \begin{pmatrix} 197/25 \\ 108/25 \\ 3/5 \end{pmatrix}$$

d)

$$A_4 = \begin{pmatrix} 3 & 9 & -2 \\ 6 & -3 & 8 \\ 4 & 6 & 5 \end{pmatrix} \qquad B_4 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$$

$AX = B$

$LUX = B$ $\qquad LY = B$

$UX = Y$

$$LY = B \Rightarrow \begin{pmatrix} 3 & 0 & 0 \\ 6 & -21 & 0 \\ 4 & -6 & \frac{89}{21} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$$

$$3y_1 = -1$$
$$6y_1 - 21y_2 = 0$$
$$4y_1 - 6y_2 + \frac{89}{21}y_3 = 2$$

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$3y_1 = -1$

$y_1 = -1/3$

$6y_1 - 21y_2 = 0$

$6(-1/3) - 21y_2 = 0$

$-2 - 21y_2 = 0$

$-21y_2 = 2$

$y_2 = -2/21$

$4y_1 - 6y_2 + \frac{89}{21}y_3 = 2$

$4(-1/3) - 6(-2/21) + \frac{89}{21}y_3 = 2$

$-4/3 + \frac{12}{21} + \frac{89}{21}y_3 = 2$

$-\frac{16}{21} + \frac{89}{21}y_3 = 2$

$\frac{89}{21}y_3 = \frac{58}{21}$

$y_3 = \frac{58}{\cancel{21}} \cdot \frac{\cancel{21}}{89} = \frac{58}{89}$

$$Y = \begin{pmatrix} -1/3 \\ -2/21 \\ 58/89 \end{pmatrix}$$

$$UX = Y \Rightarrow \begin{pmatrix} 1 & 3 & -2/3 \\ 0 & 1 & -4/7 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1/3 \\ -2/21 \\ 58/89 \end{pmatrix}$$

$x_3 = \frac{58}{89}$

$x_2 - \frac{4}{7}x_3 = -\frac{2}{21}$

$x_2 - \frac{4}{7}\left(\frac{58}{89}\right) = -\frac{2}{21}$

$x_2 - \frac{232}{623} = -\frac{2}{21}$

$x_2 = \frac{74}{267}$

$x_1 + 3x_2 - \frac{2}{3}x_3 = -1/3$

$x_1 + 3\left(\frac{74}{267}\right) - \frac{2}{3}\left(\frac{58}{89}\right) = -1/3$

$x_1 + \frac{106}{267} = -\frac{1}{3}$

$x_1 = -\frac{65}{89}$

$$X = \begin{pmatrix} -65/89 \\ 74/267 \\ 58/89 \end{pmatrix}$$

Pregunta 3

Calcular la inversa de la matriz por bloques

$$A = \left(\begin{array}{cc|c} 1 & 1 & -2 \\ 1 & 2 & -1 \\ \hline 0 & 0 & 2 \end{array}\right) = \left(\begin{array}{c|c} B & C \\ \hline 0 & D \end{array}\right)$$

$$A^{-1} = \left(\begin{array}{c|c} B^{-1} & -B^{-1}CD^{-1} \\ \hline 0 & D^{-1} \end{array}\right)$$

$$B^{-1} = \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \sim f_2 - f_1 \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right) \sim f_1 - f_2$$

$$B^{-1} = \left(\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 1 \end{array}\right) \qquad B^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$D^{-1} = \left(\frac{1}{2}\right)$$

$$-B^{-1}C = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$-B^{-1}CD^{-1} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}\left(\frac{1}{2}\right) = \begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$$

$$A^{-1} = \left(\begin{array}{cc|c} 2 & -1 & 3/2 \\ -1 & 1 & -1/2 \\ \hline 0 & 0 & 1/2 \end{array}\right)$$