Detecting High Frequency Gravitational Waves

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1 Abstract

Multiple-hour long observations of single pulsars allow us to probe relatively high-frequency regions of the gravitational wave (GW) spectrum typically outside the sensitivity of the pulsar timing array (PTA). Using such single pulsar observations from the North American Nanohertz Observatory for Gravitational Waves (NANOGrav), we applied known techniques for seeking single-source continuous wave (CW) sources. By analyzing simulated data with an injected source, we verified the accuracy of the CW search method for hours-long data sets. Furthermore, by applying an identical analysis to the true data, we found that the data places new upper limits on GW strain in the high frequency regime.

2 Introduction

The search for low frequency gravitational waves in the universe is currently best carried out using Pulsar Timing Arrays. The incredible precision timing of pulsars, along with their relative abundance in the universe, make the PTA a high precision gravitational wave detection instrument with the ability to probe a large range of the gravitational wave parameter space.

The sensitivity of the PTA is due to two factors: the natural precision of millisecond pulsars, and the self-correcting nature of the timing model used to model pulsar behavior. Both of these aspects of the PTA will be covered in more detail in the next section.

The high fidelity of the PTA stems mainly from the fact that there is an abundance of pulsars in the galaxy with many different placements in the sky. Each line of sight to a pulsar provides data on the strength of the gravitational wave passing across the line of sight. Furthermore, each pair of pulsars provides correlation data that can give information on the direction and overall magnitude of the gravitational wave. This is because each line of sight gives information analagous to the projection of the wave onto the line of sight. Thus, to ascertain a two-dimensional signal source position, at least two lines of sight are needed. Furthermore, with additional lines of sight, correlation restrictions can be put on the data to add to the confidence of a detection.

For example, if the data from two lines of sight suggested a source location at some coordinates (θ, ϕ), one would expect any other set of two lines of sight to yield the same result. If a second set of lines of sight confirmed that the source location was at coordinates (θ, ϕ), that would add to the confidence of the detection. However, if another line of sight were to predict different coordinates, that would suggest that the (θ, ϕ) detection in the first pair of pulsars is more likely to be a false positive.

With so many lines of sight, many interesting properties of gravitational waves can be probed, including precise measurements of the location of the source, as well as the strain of the gravitational wave itself. However, the hallmark of such a detector is the ability to detect the stochastic background of gravitational waves. This is described in further detail in the next section.

3 Background

The goal of this section is to provide a comprehensive background of the theory and techniques used in gravitational wave pulsar astrophysics.

It is necessary, then, to first understand what a pulsar is. When a large star reaches the end of its life cycle, it has the potential to go supernova. This occurs when the core of the star runs out of material to fuse, and the fusion reactions begin to slow down. As the rate of fusion decreases, the outward pressure caused by the fusion also decreases. There reaches a point, then, when the pressure due to fusion does not balance the gravitational pressure of the star. At that point, the star collapses on itself in a catastrophic event. After such an event, there are a few possibilities for the outcome of the core remnants of the star. If the gravitational pressure is enough to force the core past its Scwarzshield radius, the core will collapse into a black hole. On the other hand, if the gravitational pressure is relatively weak, the core will remain relatively undisturbed, and will become a white dwarf star. In the intermediary range between these two possibilities lies a third one; if the gravitational pressure is just barely not enough to force the core into a black hole state, it will instead force the formation of a neutron star. Neutron stars are the most extreme objects in the universe. With such intense gravitational pressure forming them, high-energy nuclear processes occur in the atoms that formed the core of the original star. As protons and electrons are pushed together, it becomes energetically favorable for the electrons to undergo reverse beta decay, and fuse into neutrons. With no electrostatic repulsion between the protons and electrons, the core mass–which is almost all neutrons at this point–can collapse further on itself, achieving a density similar to that of an atomic nucleus. At this point, the core mass has reached a stable configuration, and is now a neutron star. More than an entire solar mass of core material forms the star, which is spherical with a diameter of about 10km.

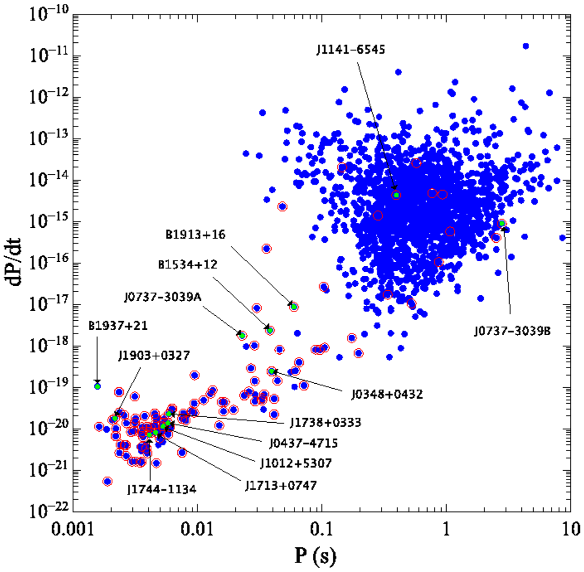
After the formation of the neutron star, there is a possibility that the star will be "spun up" to high angular speeds. If the star was in a binary system, it may accrete material from its companion, and take some angular momentum from it. Since the neutron star is so small, a small change in angular momentum will be magnified to a larger change in the angular velocity of the star.

Another peculiar feature of neutron stars is their intense magnetic fields–up to 1e8 times as much as the earth. For neutron stars that are spinning rapidly, this magnetic field moves very rapidly through space, accelerating free electrons and causing synchrotron radiation. The radiation is then forced out of the poles of the magnetic fields in a beam formation.

A pulsar is exactly this sort of object–a rotating neutron star with beam emissions of electromagnetic waves. Pulsars are particularly bright in radio frequencies, which makes them best studied using radio telescopes. They have very slow spin-down rates, so their periods are very stable in time. As seen in the P-Pdot diagram, there exists a certain class of pulsars with incredibly low periods and spin-down rates. These are known as the millisecond pulsars, called such due to their extremely fast rotational periods. With such amazing stability and cadence, millisecond pulsars function incredibly well as stellar timekeepers–some rivaling even an atomic clock's precision. It is this stability that makes millisecond pulsars the primary data source for gravitational wave searches.

A gravitational wave is a perturbation in spacetime caused by a rotating

Figure 1: The P-Pdot diagram for pulsars. Here rotational periods (x-axis) are plotted against their spindown rates. Millisecond pulsars live in the bottom left quadrant of the plot, and spin down very slowly.



quadropole somewhere in the universe. The most common sources of gravitational waves are black hole binary systems, especially supermassive black hole binaries that occur in galaxy collisions. Such binary systems emit sinusoidal gravitational waves that give rise to the perturbation [GW equation]. This perturbation occurs in the local metric of the space, and represents a change in the space-time intervals between objects. The co-ordinate distances remain fixed, but the practical (read: light travel time) distance is perturbed slightly.

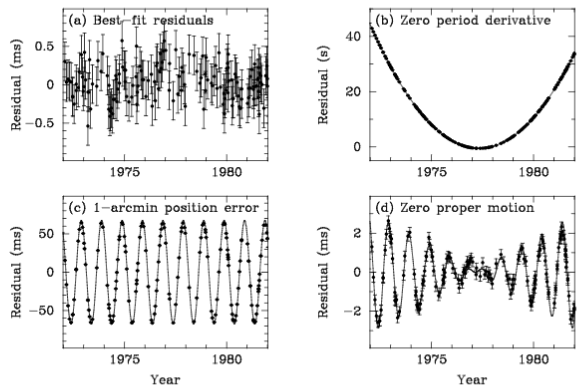
The LIGO detectors recently announced the first detection of gravitational waves. They detected a strain $\frac{\delta L}{L} \approx 10^{-21}$, corresponding to an absolute distance change on the order of a fraction of a proton diameter. The LIGO observatory was able to detect such small changes in distance by using an optimized interferometer. By splitting a laser along two orthogonal paths, reflecting them back to each other, and observing the interference magnitude, an interferometer is able to precisely measure miniscule changes in the differential lengths of their arms. One way of conceptualizing the interferometer is by following the crests of the laser across the device. The crests are evenly spaced in the arm, and without loss of generality will recombine constructively. However, imagine a change in the length of one of the arms. Now, the crests along one arm have more distance to travel, and thus will arrive delayed compared to their companions. Such a delay will induce a slightly destructive interference in the resulting recombination beam, and will be detectable by a light intensity detector. One can think of the stable arm as a "reference beam", and the changing arm as the "detector beam". With this mode of thinking, the interferometer acts as a comparison device between the reference beam and the detector beam.

In a way, pulsar timing functions the same way as interferometers do. The line of sight between us and the pulsar functions as the detector beam, with the pulses originating from the pulsar functioning as the crests. In order to get the equivalent of a reference beam, it is necessary to develop a timing model that describes with great precision the expected time of arrival of the pulse from the pulsar. With such a timing model, it is possible to compare the measured time of arrival and the expected time of arrival to gain information about the distance changes on the line of sight of the pulsar.

However, such a method requires an incredibly accurate timing model for the pulsar that takes into account all known effects between the earth and the pulsar. Such a model has been developed, and is used in the analysis pipeline to accurately calculate the expected time of arrival of a pulse. The timing model takes into account a wide array of effects. First and foremost, the timing model requires a precise measurement of the period of the observed pulsar, as well as the spindown rate, to model the pulse source accurately. Next, the timing model takes into account the motion of the earth relative to the pulsar. The model normalizes the times of arrival to the center of the solar system, and accounts for the movement of the pulsar towards, away, or transverse to the solar system. Next, the model takes into account any effects due to general relativity, including the Shapiro delay effect for binary systems, and precession for orbiting pulsars. The ultimate goal of the timing model is to "zero out" any known effects between us and the pulsar. After the timing model is made, any deviation we observe from the timing model must be due to some unmodelled effect.

At regular intervals, NANOGrav observes the 43 [?] pulsars that are in the pulsar timing array. such observations allow comparisions to be made between the expected times of arrival and the measured times of arrival. One use of these observations is to improve the timing model for the pulsar. Deviations from the timing model that follow known patterns due to incorrect timing model values can give insight on how to improve the timing model

Figure 2: Having an incorrect parameter in the model produces easily detectable and correctable signatures in the residual data.



parameters. Thus, the timing model is "self-correcting" in the sense that observations themselves improve the precision of the timing model.

However, there may be deviations from the timing model that do not correspond to incorrect parameters. These deviations must necessarily, then, come from unmodelled processes. Of the most importance is the deviation corresponding to a sine-like change in the distance between us and the pulsar. Although there could be many potential sources for such a perturbation, a likely candidate is a gravitational wave between the earth and the pulsar. It is possible, however, that such a sine-like signal corresponds to an unmodelled process in the timing model. For example, if the pulsar were in a binary system that was not known beforehand, it would appear as if the distance between us and the pulsar were changing sinusoidally–because it actually is changing sinusoidally due to the orbit of the pulsar around its companion! Many binary systems have been discovered this way, long before the optical telescopes verify the existence of the companion.

To verify whether or not a sine-like signal corresponds to a gravitational wave, the data is searched for a time correlation between pulsars. If the sine wave shows up in multiple pulsars, then it can be concluded that the signal is not originating from an unmodelled process in any individual pulsar. Such a correlated signal must, then. correspond to some unmodelled process in the space between all the pulsars in the galaxy. Using the relative locations of the pulsars, specifics of the sine signal can be probed. Polarity and propagation direction can be ascertained from the relative amplitudes of the signals in the different pulsars in the pulsar timing array.

4 Stochastic Detection

While continuous wave sources are detectable by the PTA, the stochastic background of gravitational waves is what the PTA is best tuned for. The stochastic background of gravitational waves is the sum of all the gravitational waves passing through our galaxy. Dominating the stochastic background is the vast collection of graviational waves from supermassive black hole binaries. Using the pulsar timing array, the properties of the stochastic background can be probed. This is done by looking for correlated signals across the pulsars, in an analysis similar to the continuous wave search method. By looking for these correlated signals in the data, it is possible to ascertain the relative abundance of the frequencies of waves that contribute to the background. This data can be used, then, to look at the various proportions of black hole masses and orbital frequencies in the universe. This gives information on galaxy collision rates, as well as collision durations and galaxy black hole masses.

5 High-Frequency Continuous Wave Detection

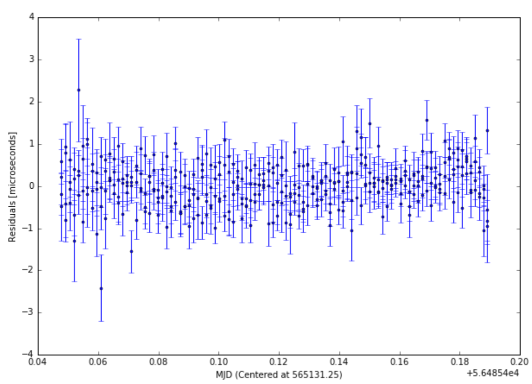
The International Pulsar Timing Array (IPTA) is tuned to probe GW frequencies in the months to years timescale. Observations of individual pulsars are carried out on a biweekly basis, due to the large amount of pulsars needed to be observed. With such infrequent observations, however, GW frequencies on the order of hours remain unexplored. This is due to the intrinsic limit on detection set by the Nyquist frequency, defined as the maximum frequency periodic signal that is detectable for a given sampling frequency. In order to accurately sample a periodic signal, it is necessary to sample at least twice per cycle to avoid introducing errors. Beyond this frequency, the sampling rate is not enough to properly detect a sine wave signal, and any such CW signal would appear as white noise in the data.

Recently, a 24 hour global observation was carried out on pulsar J1713+0747, which resulted in a new set of limits on Continuous Wave (CW) strain in the days to hours frequencies due to the average sampling rate of about 0.5 sam-

ples/min. Limits were set for both CW sources in the direction of J1713, as well as random-direction CW sources.
However, J1713 is not the only pulsar that has been observed for long, continuous observations. The pulsars J2302 and J0613 were each observed for multiple hours at a time, originally for the purpose of measuring the shapiro delay effect for each of the pulsars. Such data sets allow for analysis similar to that done to the J1713 24 hour campaign. These pulsars allow for better direcitonal limits to be placed in the microhertz to millihertz frequency range.

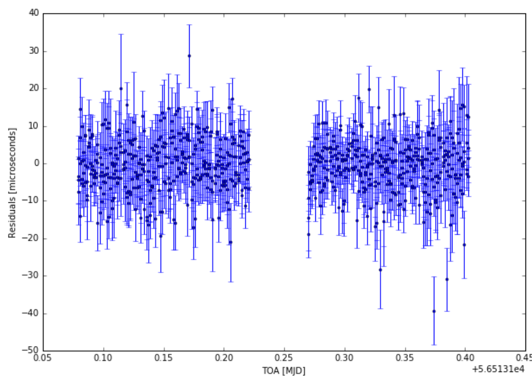
6 Data Used

Figure 3: Residual plot for the three-hour observation of J0613-0200. Integration time is 120s, and error bars are at the 1-sigma level. Data from four separate sub-bands in the 820MHz band are plotted.



In 2013, eight pulsars were observed for extended periods of time for the initial purpose of observing NANOGrav pulsars with significant Shapiro delays, targeted strategically in order to better observe the delays and therefore improve timing models (Pennucci et al. 2015, Fonseca et al. 2016). Of the eight data sets, two were chosen for our analysis. Both J2302+4442 and J0613-0200 had small rms residual values, and long continuous timing tracks. Each pulsar was observed three times over the course of the full observation period. For each pulsar, the longest single observation track was selected for observation. J2302 was observed continuously for over 8 hours, and J0613

Figure 4: Residual plot of the eight-hour observation of J2302+4442. The integration time is 120s, and the error bars are at the 1-sigma level. Data from four separate sub-bands in the 820MHz band are plotted.0



was observed continuously for over three hours. The residuals were generated using the tempo2 software, drawing its parameters from NANOGrav's 9 year data release. The binary orbit periods of each pulsar are of particular interest, as a binary period naturally weakens the sensitivity of the pulsar to continuous wave sources of a similar frequency. The binary period for pulsar J2302 is given as about 126 days, and the binary period for pulsar J0613 is given as about 1.2 days.

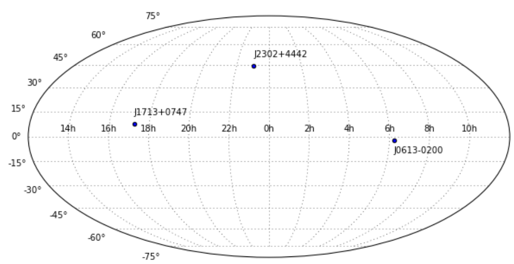
7 Data Analysis

7.1 Noise

For each pulsar, we developed a noise modelto account for any intrinsic or extrinsic sources of noise in the data. The model included both white and red noise estimations. Five parameters were varied: EFAC, EQUAD, Jitter-EQUAD, RN-Amplitude, and RN-Spectral-Index. Using the Markov Chain Monte Carlo method for exploring parameter spaces, we found the maximum a posteriori values for each parameter, and used them to construct the noise covariance matrix.

The five parameters were chosen for a noise model that has been used in previous analysis schemes. The EFAC parameter acts as a constant multiplier

Figure 5: Sky location in RA/DEC of the two pulsars analyzed (J0613-0200 and J2302+4442), as well as the previously-analyzed J1713+0747. Further analysis will put stronger limits on continuous wave strain amplitudes in the directions of J0613-0200 and J2302+4442.



on the TOA uncertainties. The EQUAD parameters represent additional Gaussian white noise added to the EFAC noise. The RN-Amplitude and RN-Spectral-Index parameters describe a red noise variance.

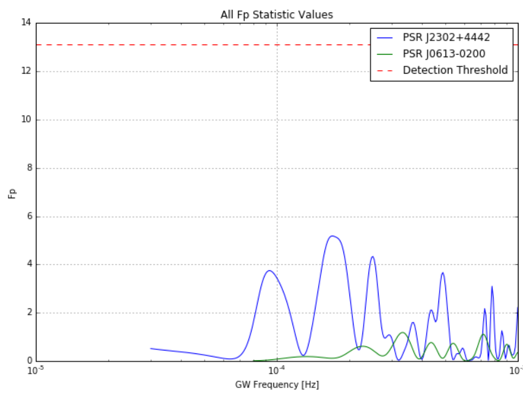
7.2 Detection Statistic

Our main tool for analyzing the sensitivity of a pulsar to continuous wave sources is the F_p statistic, developed by Justin Ellis (Arzoumanian et. al 2014). The F_p statistic is a frequentist detection statistic. It can be thought of as a measure of how much a particular frequency dominates the data. A high F_p statistic is indicative of a potential CW source at that frequency. For each potential CW frequency, we calculate the F_p statistic for the data at that frequency. Our detection threshold of about $F_p \geq 13$ corresponds to a false alarm probability of 10^{-4}.

7.3 Upper Limits and Source Simulation

After creating the noise model for the pulsar, we determine the sensitivity to CW sources at different gravitational wave frequencies. On a high level, this is done by simulating CW sources in random directions with a given strain, and seeing if they can be detected with the given noise model. By ramping up

Figure 6: The Fp statistic calculated from the residuals. The dashed line shows the minimum Fp value needed to claim a detection with a false alarm probability of 1e-4.

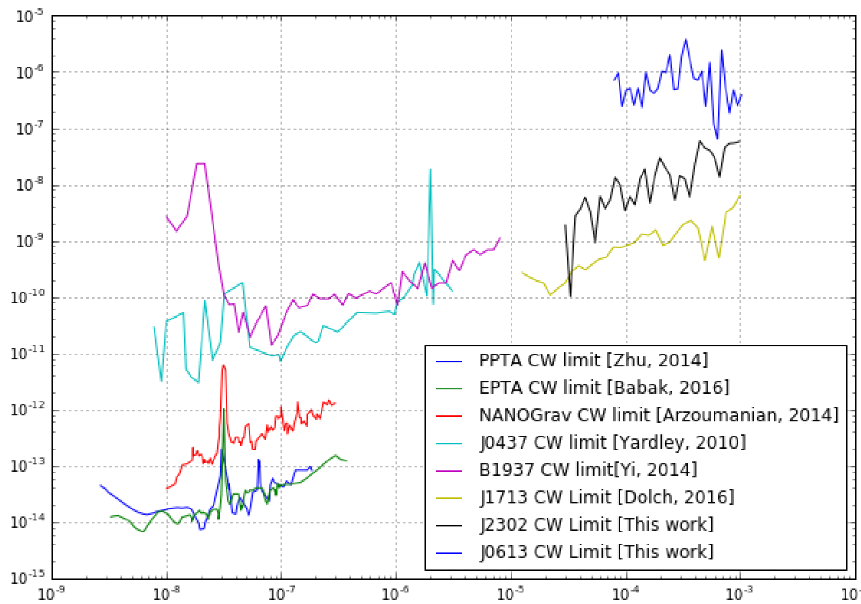


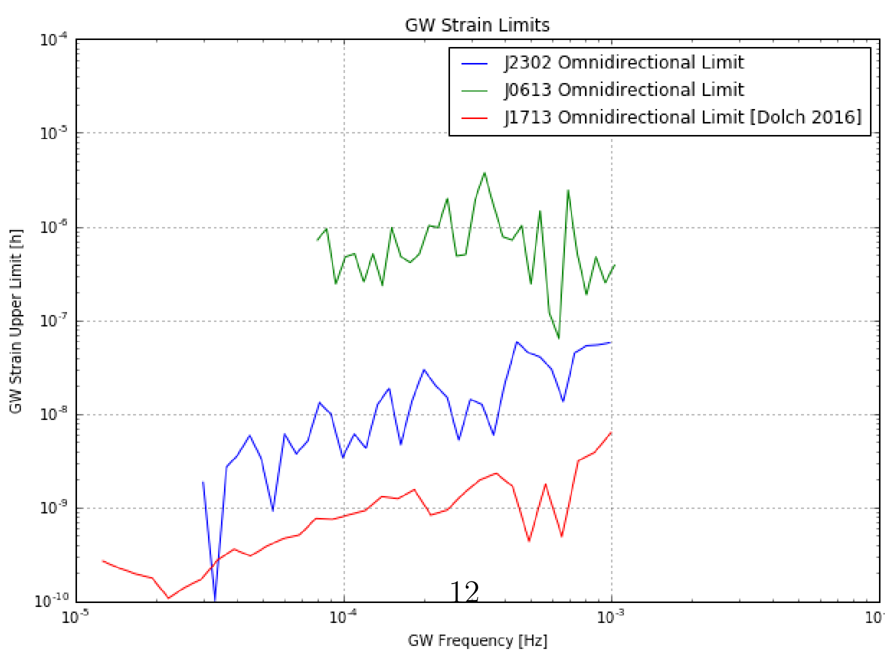
the simulated strain until a detection is made, we obtain the lowest detectable strain–the strain limit–for that GW frequency. To determine whether or not a specific strain can be detected, we simulate 5000 sources sequentially in either random directions or, in the case of a directional search, in the direction of the pulsar. Each source is simulated with a random inclination, and a set distance to keep the strain amplitude constant. Then, for each source, we create simulated residuals based on the CW source and the noise model, and generate an$f_p$statistic. A source is labelled "detectable" if the Fp statistic is higher than the actual data's$f_p$statistic. If more than 95% of the simulated sources are detectable, then it can be concluded that a CW source with a strain equal to the fixed strain is detectable above the noise. Thus, that strain is an upper bound on CW source strains for the particular GW frequency being examined.

8 Results

Applying the f_p statistic algorithm to the two pulsars analyzed, we found there was no evidence for a single source gravitational wave signal. However, using the source simulation algorithm, we were able to place upper limits on the strength of the gravitational waves passing through the galaxy at the frequencies probed. Such limits were in coincidence with previously published

Figure 7: Recent single-source GW limits from the PPTA, EPTA, and NANOGrav, as well as single-pulsar limits from J0437-0745 and B1937+21. Also plotted is the high frequency limits obtained in this analysis from J0613-0200 and J2302+4442, and the previously published limit from J1713+0747.





limits from pulsar J1713+0747. Furthermore, the results coincide with a reasonable extrapolation of the results obtained by NANOGrav and the other PTA consortia.

9 Conclusion

Using our pipeline, we were able to recreate the omnidirecitonal and directional limits from the J1713 24 hour global campaign, which verified the integrity of our analysis algorithm. Using data from the Green Bank Telescope taken to measure shapiro delay effects on PSR J0613-0200 and PSR J2302+4442, we were able to place omnidirectional limits consistent with limits already placed in that GW frequency range. While these limit curves do add confidence to the current strain limit given by analysis of J1713, further analysis will be able to place more strict directional limits on CW strain amplitude in directions that have not been probed before in this frequency range.

Further analysis will attempt to remove statistical fluctuations in the data through using a greater amount of simulations. Furthermore, the noise model can be improved upon by sampling the parameter space of the noise model more precisely. Directional limits can be placed in the direction of both J2302 and J0613. Also, similar analyses can be carried out on each of the other six pulsars observed by the Green Bank Telescope in the shapiro delay project. For each pulsar, strict directional limits on CW strain amplitude can be placed for GW frequencies in the microhertz to millihertz range.

10 Bibliography

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