Problem Set 2

Daniel Halmrast

January 25, 2018

Problem 1

Show that the retract of a contractible space is contractible.

Proof. Let X retract onto A via $r : X \to X$, with X a contractible space. In particular, we have a homotopy equivalence $X \simeq \{\cdot\}$.

Now, we have the commutative diagram

$$\begin{array}{ccccc} A & \xhookrightarrow{i} & X & \xrightarrow{r} & A \\ & \searrow & & \nearrow_{\mathbb{1}_A} & \end{array}$$

Now, since π_1 is a covariant functor from Top^* to Grp, we can apply it to the diagram above to obtain

$$\begin{array}{ccccc} & & 0 & & \\ & & \| & & \\ \pi_1(A) & \xhookrightarrow{i_*} & \pi_1(X) & \xrightarrow{r_*} & \pi_1(A) \\ & \searrow & & \nearrow_{\mathbb{1}_{\pi_1(A)}} & \end{array}$$

and since $\mathbb{1}_{\pi_1(A)}$ factors through zero, it must be that $\pi_1(A) = 0$ and thus A is contractible. $\square$