Problem Set: 2.18, 2.19

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PROBLEM 2.18

Problem: Show the multiplicity for an Einstein solid is as given.

Solution: The multiplicity of an Einstein solid is given as

$$\Omega(N,q) = \frac{N}{q+N}\frac{(q+N)!}{q!N!}$$

Applying Stirling's approximation yields

$$\begin{aligned}
\Omega(N,q) &\approx \frac{N}{q+N}\frac{(q+N)^{q+N}e^{-q-N}\sqrt{2\pi(q+N)}}{q^q e^{-q}\sqrt{2\pi q}N^N e^{-N}\sqrt{2\pi N}} \\
&\approx \frac{N}{q+N}\frac{(q+N)^{q+N}\sqrt{(q+N)}}{q^q\sqrt{q}N^N\sqrt{2\pi N}} \\
&\approx \frac{\frac{q+N}{q}^q \frac{q+N}{N}^N}{\sqrt{2\pi q(q+N)N^{-1}}}
\end{aligned} \tag{0.1}$$

PROBLEM 2.19

Problem: Find the approximate formula for the multiplicity of a two-state paramagnet.

Solution: The full formula for the two-state paramagnet is given as

$$\Omega(N,N_u) = \frac{N!}{N_d!N_u!} \tag{0.2}$$

We will use Stirling's approximation, which states:

$$N! \approx N^N e^{-N} \sqrt{2\pi N}$$

Which simplifies 0.2 to

$$\begin{aligned}
\Omega(N, N_u) &\approx \frac{N^N e^{-N} \sqrt{2\pi N}}{(N_d^{N_d} e^{-N_d} \sqrt{2\pi N_d})(N_u^{N_u} e^{-N_u} \sqrt{2\pi N_u})} \\
&\approx \frac{N^N \sqrt{N}}{N_d^{N_d} N_u^{N_u} \sqrt{2\pi N_d N_u}} \\
&\approx \frac{N^N \sqrt{N}}{N_d^{N_d} (N - N_d)^{N-N_d} \sqrt{2\pi (N - N_d) N_d}} \\
&\approx \left(\frac{N}{N - N_d}\right)^{N-\frac{1}{2}} \left(\frac{(N - N_d)}{N_d}\right)^{N_d} \frac{1}{\sqrt{2\pi N_d}}
\end{aligned} \tag{0.3}$$

Using the approximation $N - N_d \to N$, we get:

$$\Omega(N, N_d) \approx \left(\frac{N}{N_d}\right)^{N_d} \frac{1}{\sqrt{2\pi N_d}}$$

which is about what we expected.