Homework 1

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April 15, 2018

Problem 1

Suppose M is a complete Riemannian manifold of dimension n, and suppose there exist constants $a > 0$ and $c \geq 0$ such that for all pairs of points and all minimizing geodesics $\gamma(s)$ parameterized by arc length s, joining these points, we have

$$R(\gamma'(s)) \geq a + \partial_s f$$

where f is a function of s and $|f(s)| \leq c$ along γ. Prove that M is compact.

Proof. To show M is compact, we just need to show M is bounded. That is, there is some number N such that $d(p, q) < N$ for all $p, q \in M$.

So, suppose for a contradiction that □

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