Electronics Homework 6

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Problem (2.9)**.** *A $1H$ inductor carries a current of $500mA$. The wire breaks, and in $10^{-3}s$, the current goes to zero. What happens?*

[2.9]

Solution (2.9)**.** Since $V_L = L\frac{dI}{dt}$, and $\frac{dI}{dt} \approx \frac{\Delta I}{\Delta t}$

$$\frac{dI}{dt} \approx \frac{500mA}{10^{-3}s}$$
$$V_L \approx 1H\frac{500mA}{10^{-3}s}V_L \quad \approx 50V$$

So, once the wire breaks, there will be a voltage difference across the inductor of $50V$.

Problem (2.10)**.** *Calculate the impedance for a RC circuit in series and in parallel.*

Solution (2.10)**.** In series, the impedances add together and we get

$$\begin{aligned} Z_{tot} &= Z_R + Z_C \\ &= R + \frac{-j}{C\omega} \\ &= R - \frac{1}{C\omega}j \\ &= \sqrt{R^2 + (\frac{1}{Cw})^2 e^{\arctan(\frac{1}{RC\omega})}} \end{aligned}$$

In parallel, the impedances follow the parallel law

$$\begin{aligned}
Z_{tot} &= \frac{Z_R Z_C}{Z_R + Z_C} \\
&= \frac{R\frac{-j}{C\omega}}{R + \frac{-j}{C\omega}} \\
&= \frac{R\frac{-j}{C\omega}}{\sqrt{R^2 + (\frac{1}{Cw})^2} e^{\arctan(\frac{1}{RC\omega})j}} \\
&= \frac{\frac{R}{C\omega}}{\sqrt{R^2+(\frac{1}{Cw})^2}} e^{(\frac{3\pi}{2} - \arctan(\frac{1}{RC\omega}))j} \\
&= \frac{\frac{R}{C\omega}}{\sqrt{R^2+(\frac{1}{Cw})^2}} \Big[\cos(\frac{3\pi}{2} - \arctan(\frac{1}{RC\omega})) + j\sin(\frac{3\pi}{2} - \arctan(\frac{1}{RC\omega}))\Big]
\end{aligned}$$

Problem (2.11)**.** *Calculate the impedance for an LRC series circuit and an RL parallel circuit.*

Solution (2.11)**.** Again, we will use the series law to write

$$\begin{aligned}
Z_{tot} &= Z_R + Z_L + Z_C \\
&= R + (L\omega - \frac{1}{C\omega})j \\
&= \sqrt{R^2 + (L\omega - \frac{1}{Cw})^2} e^{\arctan(\frac{L\omega - \frac{1}{Cw}}{R})}
\end{aligned}$$

For the parallel combination, we see that

$$\begin{aligned}
Z_{tot} &= \frac{Z_R Z_L}{Z_R + Z_L} \\
&= \frac{RL\omega j}{R + L\omega j} \\
&= \frac{RL\omega}{\sqrt{R^2+(L\omega)^2}} e^{(\frac{\pi}{2} - \arctan\frac{L\omega}{R})} \\
&= \frac{RL\omega}{\sqrt{R^2+(L\omega)^2}} \Big[\cos(\frac{\pi}{2} - \arctan\frac{L\omega}{R}) + j\sin(\frac{\pi}{2} - \arctan\frac{L\omega}{R})\Big]
\end{aligned}$$

Problem (2.12)**.** *Calculate the impedence for a $C||(R+L)$ circuit.*

Solution (2.12)**.** The impedance of $R + L$ is

$$\begin{aligned} Z_{RL} &= Z_R + Z_L \\ &= R + L\omega j \\ &= \sqrt{R^2 + (L\omega)^2} e^{\arctan(\frac{L\omega}{R})} \end{aligned}$$

so

$$\begin{aligned} Z_{tot} &= \frac{Z_{RL} Z_C}{Z_{RL} + Z_C} \\ &= \frac{(R + L\omega j)(\frac{-j}{C\omega})}{R + (L\omega - \frac{1}{C\omega})j} \\ &= \frac{\frac{L}{C} - \frac{R}{C\omega} j}{R + (L\omega - \frac{1}{C\omega})j} \\ &= \frac{R}{(C\omega)^2} + \left[\frac{L - R^2 C - CL^2\omega^2}{C^2\omega}\right] j \end{aligned}$$