Problem Set 5: 2.2, 2.3, 2.5ab, 2.6, 2.7

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PROBLEM 2.2

Problem: Suppose you flip 20 fair coins...

PART A

Problem: How many microstates are there?

Solution: The state space is the functions from the outcomes (H,T) to the trials (N=20) which has cardinality 2^{20}

PART B

Problem: What is the probability of getting <specific outcome>?

Solution: This is a particular microstate, with multiplicity one, so the probability of it occurring is $\frac{1}{2^{20}}$.

PART C

Problem: What is the probability of getting twelve heads and eight tails?

Solution: The multiplicity of this state is given as $\binom{n}{k} = \binom{20}{12} = 125970$, so the probability of it being in such a macrostate is $\frac{\Omega(state)}{\Omega(all)} = \frac{125970}{2^{20}} \approx 0.12$.

Problem 2.3

Problem: Suppose you flip 50 fair coins...

Part A

Problem: How many microstates are there?

Solution: The number of ways two outcomes map to 50 events is 2^{50}.

Part B

Problem: How many ways are there of getting exactly 25 heads?

Solution: The multiplicity of this microstate is given as $\binom{50}{25}$.

Part C

Problem: What is the probability of getting 25 heads?

Solution: The probability is given as $\frac{\binom{50}{25}}{2^{50}} \approx 0.112$.

Part D

Problem: What about for 30 heads?

Solution: The probability is $\frac{\binom{50}{30}}{2^{50}} \approx 0.041$.

Part E

Problem: What about 40 heads?

Solution: The probability is $\frac{\binom{50}{40}}{2^{50}} \approx 9.12 \times 10^{-6}$

Part F

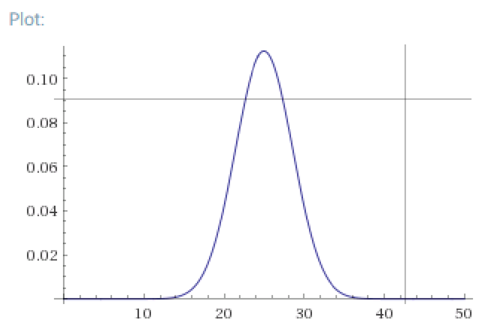
Problem: What about all 50 heads?

Solution: The probability is $\frac{1}{2^{50}} \approx 8.88 \times 10^{-16}$

Part G

Problem: Graph the probability of getting n heads as a function of n.

Figure 0.1: Plot of probability of getting a macrostate with n heads as a function of n.



Problem 2.5

Part A

Problem: List all microstates of an Einstein solid with $N = 3, q = 4$.

Solution: $\Omega(3,4) = \binom{4+3-1}{4} = 15$, so we expect fifteen total solutions, which are:
(4,0,0) (0,4,0) (0,0,4)
(3,1,0) (1,3,0) (3,0,1)
(1,0,3) (0,3,1) (0,1,3)
(2,2,0) (2,0,2) (0,2,2)
(2,1,1) (1,2,1) (1,1,2)

Part B

What about for $N = 3, q = 5$?

Solution: $\Omega(3,5) = \binom{5+3-1}{5} = 21$, so we expect twenty one total solutions, which are:
(5,0,0) (0,5,0) (0,0,5)
(4,1,0) (4,0,1) (0,4,1)
(1,4,0) (1,0,4) (0,1,4)
(3,1,1) (1,3,1) (1,1,3)
(3,2,0) (3,0,2) (0,3,2)
(2,3,0) (2,0,3) (0,2,3)
(2,2,1) (2,1,2) (1,2,2)

Problem 2.6

Problem: Calculate the multiplicity of an Einstein solid with $N = q = 30$.

Solution: The multiplicity is $\Omega(30,30) = \binom{30+30-1}{30} \approx 5.91 \times 10^{16}$.

Problem 2.7

Problem: For an Einstein solid with four oscillators and two units of energy, represent the states as a series of dots and lines.

Solution:

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