THERMAL PHYSICS

Problem Set: 2.42 3.1 3.3 3.5 3.6

Daniel Halmrast

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PROBLEM 2.42

PART A

The dimensions of G are $\frac{m^3}{kgs^2}$, so the units of $\frac{GM}{c^2}$ will be in meters.
For a black hole of one solar mass, the radius will be about $1500m$.

PART B

We know that the entropy for most systems is proportional to the number of particles in the system, the entropy of the black hole will be on the order of the number of particles used to make it.

PART C

We want the number of low-energy photons that sum to the mass of a black hole. That is, the total energy of all photons will be Mc^2.
The energy of an individual photon with wavelength λ is given as $E = \frac{hc}{\lambda}$. Set λ to $\frac{GM}{c^2}$ to get $E = \frac{hc^3}{GM}$. So, divide Mc^2 by the energy per photon to get

$$\begin{aligned} n_\gamma &= \frac{Mc^2}{\frac{hc^3}{GM}} \\ &= \frac{GM^2}{hc} \end{aligned}$$

Then, entropy will be

$$S = Nk = \frac{GM^2}{hc}k$$

Part D

The entropy of a one solar mass black hole is (via wolfram)

$$S = 1.45 \times 10^{54} \frac{J}{K}$$

This is a very large value for entropy, which makes sense as the black hole is the largest-entropy state for a gravitational system.