Problem Set 4: 1.49, 1.50, 1.51, 1.53, 1.55

Daniel Halmrast

January 30, 2017

PROBLEM 1.49

Problem: Combust one mole of H_2 with $\frac{1}{2}$ mole of O_2. How much heat comes from a decrease in internal energy, and how much comes from work done by the atmosphere?

Solution: We know the total amount of heat generated is 286kJ, so we need only calculate one of the two quantities, and subtract it from the total to get the other.
The work done by the atmosphere collapsing on the gas (when it turns to a negligible-volume liquid) is given by

$$\begin{aligned} W &= -\int_{V_i}^{V_f} PdV = -P\Delta V = PV_i = RT \\ &= 2500J \end{aligned}$$

So, the work done by the atmosphere is 2500J, and the decrease in internal energy is

$$\begin{aligned} \Delta U &= \Delta H - W \\ &= 286kJ - 2.5kJ \\ &= 284.5kJ \end{aligned}$$

PROBLEM 1.50

Problem: Consider the combustion reaction

$$CH_4 + 2O_2 \to CO_2 + 2H_2O_{(g)}$$

at 298K and 10^5Pa both before and after the reaction.

Part A

Problem: Imagine the process of converting a mole of methane into its constituents. What is ΔH?

Solution: The book gives $\Delta H_f(CH_4) = -74.81$, so the formation process gives off 74.81kJ of heat, and the decomposition takes 74.81kJ to do.

Part B

Problem: Now form a mole of CO_2 and two moles of $H_2O_{(g)}$, what is ΔH?

Solution: Well, the book gives $\Delta H_f(CO_2) = -393.51$kJ, and $\Delta H_f(H_2O_{(g)}) = -241.82$kJ.

Part C

Problem: What is the total ΔH for the reaction?

Solution: It will take 74.81kJ to break up the methane, but then we will get $393.51 + 241.82 = 635.33$kJ out of the reaction, for a net total of $635.33 - 74.81 = 560.52$kJ emitted from the reaction.

Part D

Problem: If no other work is done, how much heat is given off?

Solution: If no other work is done, then all the energy went into heat, so there was 560.52kJ of heat emitted.

Part E

Problem: What is the change in the energy of the system?

Solution: Since the degrees of freedom and the temperature remain constant, $\Delta U = 0$ for the reaction. If instead the water had condensed to a liquid form, the degrees of freedom would change and the system would either gain or lose energy to make the temperature constant.

Part F

The sun has a mass of 2×10^{30}kg and gives off energy at a rate of 3.9×10^{26} watts. If the source of the sun's energy were ordinary combustion, how long could it last?

Solution: Using wolfram alpha, I find that 2×10^{30}kg of methane is about 1.32×10^{32}mol. To emit 3.9×10^{26}J of energy, it would take $\frac{3.9\times10^{26} J}{5.6\times10^{5} \frac{J}{mol}} = 7.0 \times 10^{30} mol$. So, the sun would burn

7×10^{30} moles per second, lasting a grand total of

$$\frac{1.32 \times 10^{32} mol}{7 \times 10^{30} \frac{mol}{s}} = 19s$$

So, the sun would last about 19 seconds before it burnt through all its methane.

PROBLEM 1.51

Problem: Determine the ΔH for the reaction

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$$

Solution: It will take 1273kJ to break up glucose, and we will gain 393.51kJ for each mol of CO_2 we form, and 285.83kJ for each mol of water we form.
This results in a net enthalpy of

$$\Delta H = 1273 - 6 * (393.51 + 285.83) = -2803kJ$$

PROBLEM 1.53

Problem: Find the enthalpy of formation of atomic hydrogen, and determine the energy needed to dissociate a single H_2 molecule, in eV.

Solution: $\Delta H_f(H) = 217.97$kJ is the energy to dissociate a half mole of H_2, so we divide by one half Avogadro's number to get $\Delta H_{singlemolecule}(H) = 4.52eV$.

PROBLEM 1.55

PART A

Consider two particles orbiting each other. Show the gravitational potential energy is -2 times the kinetic energy.

Solution: For a particle of mass m orbiting a center-of-gravity, orbital mechanics tells us that the potential of a particle is related to its velocity by

$$\frac{\partial V}{\partial r} = \frac{v^2}{r}$$
$$r\frac{\partial V}{\partial r} = v^2$$

Since the potential is a $\frac{1}{r}$ potential, $r\frac{\partial V}{\partial r} = -V$, and thus

$$-V = v^2$$

Therefore, the kinetic energy, given by $KE = \frac{1}{2}mv^2 = \frac{-1}{2}mV = \frac{-1}{2}U_p$.

Part B

Problem: If you add energy to the system, then wait for it to equilibriate, does the average total kinetic energy increase or decrease?

Solution: Since the total energy of the system is $U = KE + V = KE - 2KE = -KE$, adding energy will decrease the kinetic energy.

Part C

Problem: Express the total energy of a star in terms of its average temperature, and calculate the heat capacity.

Solution: Since $KE = \frac{3}{2}kT$, and the total energy of the star is $U = -N * KE$, $U = \frac{-3N}{2}kT$, and thus $C_p = \frac{-3}{2}kN$

Part D

Problem: Argue that the potential energy of the star should be a constant multiple of $\frac{-GM^2}{R}$.

Solution: The gravitational constant G has dimensions that make $V = G\frac{M}{r}$ consistent. Thus, the dimensions of $\frac{-GM^2}{R}$ are dimensions of energy.

Part E

Problem: Estimate the average temperature of the sun, with a mass 2×10^{30}kg with radius 7×10^8m.

Solution: From part d, the total potential energy of the sun will be

$$V = -\frac{GM^2}{R} = -3.814 \times 10^{41} J$$

Which gives a total kinetic energy of $KE = -\frac{1}{2}U_p = 1.9 \times 10^{41} J$ So, the kinetic energy per particle, if there are 1×10^{57} protons in the sun, is $KE_p = 1.6 \times 10^{-16} J$ So, $1.6 \times 10^{-16} = \frac{3}{2}kT$. Thus,the average temperature is $7.7 \times 10^6 K$.