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# Homework 1

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## PROBLEM 1

Express complex multiplication and Hermetian multiplication in terms of the dot product and scalar wedge product.

*Proof.* Observe that the complex product  $zw$  is symmetric, and so we expect its decomposition to have only the symmetric dot product appear (the wedge product would have to be symmetrized, but since its antisymmetric, its symmetric part is trivial) We calculate directly

$$\begin{aligned} zw &= (a_1 + ib_1)(a_2 + ib_2) \\ &= a_1a_2 - b_1b_2 + i(b_1a_2 + a_1b_2) \end{aligned}$$

Now, we can use  $1 = [1, 0]^T$  and  $i = [0, 1]^T$  to get

$$\begin{aligned} a_1 &= z \cdot 1 \\ a_2 &= w \cdot 1 \\ b_1 &= z \cdot i \\ b_2 &= w \cdot i \end{aligned}$$

which leads to the expression

$$zw = (z \cdot 1)(w \cdot 1) - (z \cdot i)(w \cdot i) + ((z \cdot 1)(w \cdot i) + (w \cdot 1)(z \cdot i))i$$

Now, for the Hermetian product, we have a much simpler expression. We calculate

$$\begin{aligned} H(zw) &= (a_1 + b_1i)(a_2 - b_2i) \\ &= a_1a_2 + b_1b_2 + (b_1a_2 - a_1b_2)i \\ &= z \cdot w + i(z \wedge w) \end{aligned}$$

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