Differential Geometry Notes

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Smooth Functions

Definition. *Given a smooth manifold M, a function $f : M \to \mathbb{R}$ is said to be* smooth*(or C^∞) iff:*
$\forall(U, \phi)$ charts on M, $f \circ \phi^{-1} : \mathbb{R}^n \to \mathbb{R}$ is a smooth (C^∞) function.

Remark. *Note that you only have to check smoothness against one atlas, not necessarily all in the smooth structure.*

Smooth Maps

Now, let's generalize this to maps between manifolds.

Definition. *Given two smooth manifolds M and N, a map $f : M \to N$ is said to be* smooth*iff:*
$\forall p \in M$, there exist charts (U, ϕ) on M and (V, τ) on N such that $p \in U$, $f(p) \in V$, and the map $\tau \circ f \circ \phi^{-1} : \mathbb{R}^m \to \mathbb{R}^n$ is smooth.

Basically, f has to be compatible with the coordinate charts at p and $f(p)$.