Problem Set 2: 1.17, 1.18, 1.21

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PROBLEM 1.17

PART A

Problem: For each temperature, calculate the second term in the Virial expansion $\frac{B(T)}{\frac{V}{n}}$ for N_2 at atmospheric pressure.

Solution:

T (K)	**B** ($\frac{cm^3}{mol}$)	$\frac{B(T)}{\frac{V}{n}}$
100	-160	-0.0194
200	-35	-0.00213
300	-4.2	$-1.71 * 10^{-4}$
400	9.0	$2.74 * 10^{-4}$
500	16.9	$4.119 * 10^{-4}$
600	21.3	$4.33 * 10^{-4}$

Since the expansion term is so small, this shows the ideal gas law is quite valid for high temperature N_2.

PART B

As the molecules get close to each other (low temperature), the intermolecular forces will begin to cause attraction between the molecules. This attraction will *lessen* the pressure of the gas compared to the ideal gas law prediction.

Part C

By direct computation:

$$(P+\frac{an^2}{V^2})(V-nb)=nRT$$
$$(P+\frac{an^2}{V^2})(1-\frac{nb}{V})V=nRT$$
$$(P+\frac{an^2}{V^2})V=nRT\left(1-\frac{nb}{V}\right)^{-1}$$
$$(P+\frac{an^2}{V^2})V=nRT\left(1-\frac{nb}{V}+b^2\frac{n^2}{V^2}\right)$$
$$PV+\frac{an^2}{V}=nRT\left(1-\frac{nb}{V}+b^2\frac{n^2}{V^2}\right)$$
$$PV=nRT\left(1-\frac{nb}{V}+b^2\frac{n^2}{V^2}-\frac{an}{VRT}\right)$$
$$PV=nRT\left(1-(b+\frac{a}{RT})\frac{n}{V}+b^2\frac{n^2}{V^2}\right)$$

Part D

Using values of $a=1.31*10^8$ and $b=48.5$ (Found using the data for temperatures 400 and 500 kelvin), the graph of $B(T)$ looks like a $\frac{1}{x}$ graph that reaches valid ($|B(T)|<100$) values past about 100 kelvin, so the ideal gas law will hold quite well for 100k and above tempearatures.

Problem 1.18

We know that the rms speed of a molecule at a temperature is given by the equation

$$\sqrt{\frac{3kT}{m}}$$

Which, for molecular nitrogen (N_2, $m=4.7*10^{-26}kg$), yields

$$v_{rms}=514\frac{m}{s}$$

Problem 1.21

Since the overall speed of the hailstones is $15\frac{m}{s}$, and it hits at a 45° angle, the v_x component of the velocity will be $\frac{15}{\sqrt{2}}\frac{m}{s}\approx 10.6$.

Thus, the momentum imparted to the window pane is $2p=2mv=2*0.002kg*10.6\frac{m}{s}=0.042\frac{kg*m}{s}$. The force imparted to the window is $\frac{\Delta p}{\Delta t}$ where $\frac{1}{\Delta t}$ is the frequency of arrival, at 30 times per second. Thus, $F=0.042*30=1.26N$, and $P=\frac{F}{A}=2.54Pa$.

Compare this to the atmospheric pressure $P_{atm}=1.013*10^5$, and you will see the hailstones exert a miniscule (0.002%) pressure on the window compared to the atmosphere.