QFT

Homework 1

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Problem 1

Show that multiparticle nonrelativistic quantum can be recovered from QFT. Namely, define

$$H = \int d^3x a^\dagger(x) \left(\frac{-\hbar^2}{2m} \nabla^2 + U(x) \right) a(x) + \int d^3x d^3y V(x-y) a^\dagger(x) a^\dagger(y) a(y) a(x)$$

and

$$|\psi, t\rangle = \int d^3x_1 \dots d^3x_n \psi(x_1, \dots, x_n; t) a^\dagger(x_1) \dots a^\dagger(x_n) |0\rangle$$

We will show that $|\psi, t\rangle$ satisfies the abstract Schrodinger equation if and only if ψ satisfies the Schrodinger equation

$$i\partial_t \psi = H\psi$$

for

$$H = \sum_{i=1}^n \frac{-\hbar^2}{2m} \nabla_i^2 + U(x_i) + \sum_{j=1}^n \sum_{i=1}^{j-1} V(x_i - x_j)$$

We calculate $H|\psi, t\rangle$ directly in three parts. That is:

$$\begin{aligned} H|\psi, t\rangle &= \int d^3 a^\dagger(x) \frac{-\hbar^2}{2m} \nabla^2 a(x) |\psi, t\rangle \\ &+ \int d^3x a^\dagger(x) U(x) a(x) |\psi, t\rangle \\ &+ \int d^3x d^3y V(x-y) a^\dagger(x) a^\dagger(y) a(y) a(x) |\psi, t\rangle \end{aligned}$$

For this calculation, we'll use the fact that

$$a(x) a^\dagger(x_1) \dots a^\dagger(x_n) = \sum_{i=1}^n \delta(x - x_i) a^\dagger(x_1) \dots a^\dagger(\hat{x_i}) \dots a^\dagger(x_n) + a^\dagger(x_1) \dots a^\dagger(x_n) a(x)$$

where $a^\dagger(\hat{x_i})$ indicates that the ith creation operator is omitted.