Problem Set 3: 1.23, 1.28, 1.31-34

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PROBLEM 1.23

Problem: Calculate the total thermal energy in a liter of helium at STP. Then, repeat for a liter of air.

Solution: Thermal energy is given as

$$U_{thermal} = Nf\frac{1}{2}kT$$

For a liter of helium at STP, $\frac{PV}{kT} = N = 2.44 * 10^{22}$. Then, $U_{thermal} = 2.44 * 10^{22} * 3 * \frac{1}{2}k * 300k = 152J$.
For a liter of air, $N_{air} = N_{helium}$, but air has 5 DoF compared to helium's 3. Thus, $U_{air} = \frac{5}{3}U_{helium} = 253J$.

PROBLEM 1.28

Problem: Estimate how long it takes to bring a cup of water to boiling in a 600-Watt microwave, where all the energy ends up in the water. Explain why no heat is involved in the process.

Solution: Assuming the water starts at 20 celcius, and ends at 100 celcius, $\Delta T = 80K$. One cup of water weighs approximately 200 grams, so the energy needed to heat the cup of water to boiling is

$$\Delta U = 80K * 200g * 4.186\frac{J}{gK}\Delta U \approx 67000$$

Thus, for a 600W microwave, to deliver 67000 Joules of energy would take 110 seconds.

PROBLEM 1.31

Problem: Imagine a container of helium, initial volume 1 Liter, initial pressure 1 atm, final volume 3 Liters, and the process is linear on a PV diagram.

PART A

Sketch a PV diagram for this process:

PART B

Calculate the work done on the gas during this process:

Solution: Work is defined as $W = -\int_{V-i}^{V_f} P(V)dV$. Furthermore, it is assumed that $P(V) = V$ Thus,

$$\begin{aligned} W &= -\int_1^3 aV dV \\ &= -\frac{1}{2}V^2|_1^3 \\ &= \frac{1}{2}(1-9) \\ &= -4 \end{aligned}$$

Where the work is in atmosphere liters. In Joules, this yields $W = -405J$.

PART C

Calculate the change in the helium's energy content during the process.

Solution: We'll assume that 0.1 mol of helium is in the container, for an initial temperature of 122 kelvin. Then, the final temperature is given as

$$T = \frac{PV}{nR}$$

which, for $P = 3atm$, $V = 3L$, $n = 0.1mol$, yields $T_f = 1097K$, for a $\Delta T = 975$. Thus,

$$\begin{aligned} \Delta U &= (0.1mol) * N_a * f * \frac{1}{2} * k * \Delta T \\ &= 1216J \end{aligned}$$

PART D

Calculate the amount of heat added to the helium during this process.

Solution: Since $\Delta U = Q + W$, $Q = \Delta U - W$, and plugging in values from parts b and c yields

$$Q = 1621J$$

PART E

Describe how you would cause the pressure to rise in this process.

Solution: By heating the gas as it expands at an appropriately high rate, the pressure of the gas will also increase.

PROBLEM 1.32

Sketch a PV diagram of compressing an amount of water to 99% of its initial volume at a pressure of 200atm.

Estimate the work required to compress a liter.

Solution: For an initial pressure of 1atm, initial volume of 1L, final pressure of 200atm, final volume of .99L, and a linear relationship between P and V, $P(V) = 19901 - 19900V$.

$$W = -\int_{V_i}^{V_f} P(V)dV$$
$$W = -\int_{1}^{0.99} (19901 - 19900V)dV$$
$$W = 1.005$$

Which, in freedom units, is $W = 101J$. This is a bit surprising, as 200atm is quite the pressure to reach. However, since the volume barely changes, it makes sense why the energy needed to compress it is (relatively) low.

PROBLEM 1.33

Determine the sign of the value specified in reference to the diagrams on page 23:

Path	**work on gas**	ΔU **gas**	**Q to gas**
A	Negative	Positive	Positive
B	Zero	Positive	Positive
C	Positive	Negative	Negative
Full Cycle	Positive	Zero	Negative

Problem 1.34

There is an ideal diatomic gas with vibrational modes frozen out ($f = 5$). It undergoes a process in a rectangular fasion from P_1, V_1 to P_2, V_2 and back.

Part A

For each path, we will use the integral definition of work, and the equipartition theorem combined with the ideal gas law for $\Delta U = PV\frac{f}{2}$.

Path	**work on gas**	ΔU **gas**	**Q to gas**
A	Zero	$(P_2 - P_1) * V_1 * \frac{5}{2}$	ΔU
B	$-(V_2 - V_1) * P_2$	$(V_2 - V_1) * P_2 * \frac{5}{2}$	$\Delta U - W$
C	Zero	$(P_1 - P_2) * V_2 * \frac{5}{2}$	ΔU
D	$-(V_1 - V_2) * P_2$	$(V_1 - V_2) * P_1 * \frac{5}{2}$	$\Delta U - W$

Part B

In part A, heat is added to the gas to increase its pressure. Then, in part B, the gas is expanded and heat is added to keep pressure constant. C is the reversal of A, and D is the reversal of B.

Part C

Net work is calculated by finding the total area of the rectangle, which is clearly $-(V_2 - V_1)(P_2 - P_1)$ (it's negative, since the higher pressure process expands the gas). The net heat added to the gas will exactly cancel with the net work, since the end state is the same (temperature) as the initial state, so $\Delta U_{net} = 0$.