Population Growth Modeling

Tristan Jai 8941924
Benjamin Mathov
Daniel Davis
Seth Marceno
Kevin Sandoval
Ellie Yuen
Indervir Badial

March 16 2020

1 Introduction

Population growth rate is the measurement of how the size of a population changes over time. It relies on several factors, such as population size, birth rate, and death rate. As long as there is enough sustenance for a population group, the population growth rate will be positive. However, population cannot grow forever, as eventually resources will run out, and thus the carrying capacity is reached. Carrying capacity is the maximum number of individuals an environment can support. Thus, the higher the population is, the more resources will be used, and therefore population growth rate decreases. This is called logistic population growth. We can determine if Earth will reach its carrying capacity by approximating population growth on Earth for the next two centuries, utilizing the Logistic Differential Equation. This equation is given by

$$\frac{dP}{dt} = rP(1 - \frac{P}{K})$$

This equation involves two positive parameters. The first parameter r is the growth parameter, or the maximum per capita growth rate for a population. The second parameter K is the carrying capacity. P is the population.

2 Methods

The Runge-Kutta Method of order four was used to approximate the solutions to the Logistic. Using data of the current world population, Earth's theoretical carrying capacity, and recent average growth rate as references, $P(0) = 7.8 \times 10^9$, $K = 9.5 \times 10^9$, and $r = 0.011$ were used as initial conditions. We now show that this initial value problem satisfies the Lipschitz condition as follows.

$$\frac{dP}{dt} = 0.011P - \frac{0.011P^2}{9.5 \times 10^9}$$

We must show that a Lipschitz constant exists with

$$|f(y_1) - f(y_2)| \leq L|y_1 - y_2|$$

.

$$\frac{d}{dP} = 0.011 - \frac{0.022P}{9.5 \times 10^9} \leq |-0.011| \leq 0.011 = L$$

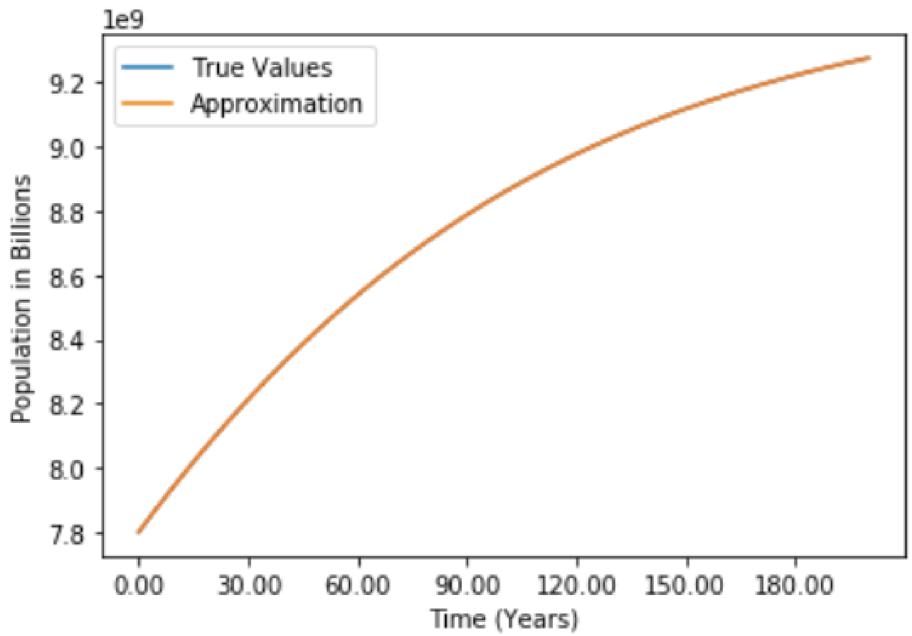
Thus a Lipschitz constant L exists. Due to the equation being continuous, by definition the initial value problem is well-posed.

3 Results

We implemented the Runge-Kutta Method of order four in Python. The results were formulated into a table, comparing the Runge-Kutta approximation and the actual values, as shown below.

Year	Actual	RK approx.	Actual Error
str6	float64	float64	str9
0.00	7800000000.000001	7800000000.0	0.0000010
1.00	7815299498.553896	7815299498.553194	0.0007019
2.00	7830490777.460744	7830490777.459344	0.0014000
3.00	7845574071.564598	7845574071.562503	0.0020952
4.00	7860549625.355695	7860549625.352906	0.0027885
5.00	7875417692.787141	7875417692.783665	0.0034761
6.00	7890178537.091906	7890178537.087744	0.0041618
7.00	7904832430.600152	7904832430.595308	0.0048437
8.00	7919379654.557003	7919379654.551483	0.0055199
9.00	7933820498.940803	7933820498.934608	0.0061941
...	...	...	...
191.00	9253283895.619425	9253283895.57791	0.0415154
192.00	9255913554.640348	9255913554.598982	0.0413666
193.00	9258515916.528805	9258515916.487587	0.0412178
194.00	9261091249.121618	9261091249.080551	0.0410671
195.00	9263639817.957369	9263639817.916452	0.0409164
196.00	9266161886.28896	9266161886.248196	0.0407639
197.00	9268657715.0963	9268657715.05569	0.0406094
198.00	9271127563.099075	9271127563.058622	0.0404530
199.00	9273571686.769602	9273571686.729305	0.0402966
200.00	9275990340.345772	9275990340.305634	0.0401382

The values of the table were also graphed, as shown.



The values of the Runge-Kutta approximation are a near perfect estimate to the actual values of population growth. We see that Earth does not reach the theoretical carrying capacity of 9.5×10^9 within two centuries, albeit very close, at about 9.2×10^9.