NEGATION

Which of the following statements are true?

(i) If $f:\mathbb{R}\to\mathbb{R}$ is an even function and $g:\mathbb{R}\to\mathbb{R}$ is an odd function then $f+g$ is an odd function.

(ii) If $f:\mathbb{R}\to\mathbb{R}$ is an even function and $g:\mathbb{R}\to\mathbb{R}$ is an odd function then fg is an odd function.

(iii) $\exists x\in\mathbb{R}\ (x^2<0)$

(iv) $\exists x\in\mathbb{R}\ \exists y\in\mathbb{R}\ (3x-2y=1 \wedge x+2y=3)$

(v) $\exists x\in\mathbb{R}\ \exists y\in\mathbb{R}\ (x+y=0 \wedge x+y=1)$

(vi) $\exists N\in\mathbb{Z}\ \forall m\in\mathbb{Z}\ (m\leq N)$

(vii) $\forall x\in\mathbb{R}\ \forall y\in\mathbb{R}\ ((x\geq 0 \wedge y\geq 0)\to xy\geq 0)$

Write the negation of each false statement.

Direct Proofs

Prove each of the following propositions.

1. For all $m \in \mathbb{Z}$, m^2+m is even.

2. If $x \geq 10$ then $x^4 \geq 100x$

3. The product of two odd functions is even.

Proofs by Cases

1. If $4 \leq n \leq 13$, then n is the sum of two primes.

2. For all $n \in \mathbb{Z}$, $n^2 - n \geq 0$.

3. It is possible to pay any (integer) number of dollars at least 6 with \$3 and \$4 bills.

Proofs by Contradiction and Contrapositive

Prove each of the following propositions.

1. The square root of an irrational number is irrational.

2. If 6 people need to eat 50 skittles, then someone must eat more than 8 skittles.

3. The function $\sqrt{x}$ is not a rational function.

$\sqrt{4}$ IS IRRATIONAL

Prop. $\sqrt{4}$ is irrational.

Proof. Suppose, for contradiction that $\sqrt{4} = p/q$, in lowest terms.

Then $4 = p^2/q^2$

so $4q^2 = p^2$

so p is even

so p^2 is divis. by 4.

so $p^2 = 4r^2$

so $4q^2 = 4r^2$

so $q^2 = r^2$

so $q = r$

so $p/q = 2r/r$ is not in lowest terms.

This is a contradiction.

MORE PROOFS

Prove or disprove each of the following propositions.

1. No two consecutive integers are prime.

2. $\sqrt{3}$ is irrational.

3. For all $x, y \in \mathbb{R}$, $|x+y| \leq |x| + |y|$.

4. It is possible to tile a chessboard with dominos after two opposite corners have been removed.