

# TRUTH TABLES

Is the following proposition a tautology, a contradiction, or neither?

$$(\neg p \wedge q) \wedge (p \vee \neg q)$$

Can you verify your answer without truth tables?

# DISJUNCTIVE NORMAL FORM

p	q	r	S
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T

Can you find some statement  $S$  with this truth table?

Hint: disjunctive normal form

Can you find a short statement  $S$  with this truth table?

Hint: use  $\rightarrow$

# DISJUNCTIVE NORMAL FORM

Is disjunctive normal form unique? In other words, is it possible to find different disjunctive normal forms that are equivalent?

# BASIC LOGICAL EQUIVALENCES

Idempotence.  $p \vee p \equiv p$

$$p \wedge p \equiv p$$

Commutativity  $p \vee q \equiv q \vee p$

$$p \wedge q \equiv q \wedge p$$

Associativity  $p \vee (q \vee r) \equiv (p \vee q) \vee r$

$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$

Distributivity  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

Double negation.  $\neg(\neg p) \equiv p$

Domination.  $p \vee T \equiv T$

$$p \wedge F \equiv F$$

DeMorgan's Laws.  $\neg(p \vee q) \equiv \neg p \wedge \neg q$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Implications.  $p \rightarrow q \equiv q \vee \neg p$

# LOGICAL EQUIVALENCES

Show the following equivalences:

$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$

$$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$$

$$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$$

# TAUTOLOGIES

Show that the following statements are tautologies.

1.  $(p \wedge q) \rightarrow (p \vee q)$

2.  $\neg p \wedge (p \vee q) \rightarrow q$

# TAUTOLOGIES

Determine whether or not the following statements are tautologies.

$$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$$

$$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$$