Section 7.5
Repetitions

REPETITIONS

QUESTION: How many ways are there to put r identical marbles into n boxes, if you are allowed to put more than one marble per box?

First try 3 marbles into 10 boxes.

What about 10 marbles in 3 boxes?

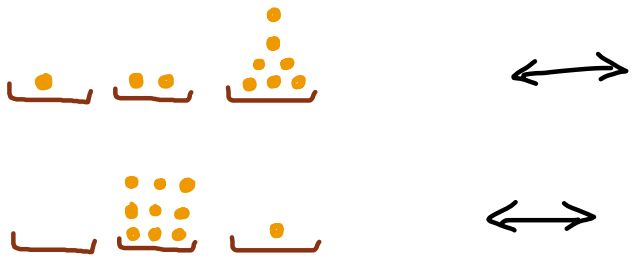
Stars and Bars

Can answer the last question by looking at it the right way:

The number of ways of putting 10 marbles into 3 boxes is the same as:

the number of binary strings with 10 zeros, 2 ones

(or 10 stars, 2 bars)



How many such strings are there?

REPETITIONS

QUESTION: How many ways are there to put r identical marbles into n boxes, if you are allowed to put more than one marble per box?

ANSWER: This is the same as the number of strings with r stars and $n-1$ bars:

Repetitions, Permutations, and Combinations

How many ways to put r marbles in n boxes if...

	the marbles are indistinguishable	the marbles are distinguishable
at most one marble is allowed per box		
any number of marbles is allowed in a box		

REPETITIONS

EXAMPLE: How many ways are there to choose 15 cans of soda from a cooler with (lots of) Coke, Dr. Pepper, Mtn Dew, RC cola, and Mr. Pibb?

FURTHER: What if I insist on at least 3 Cokes and exactly one Mr. Pibb?

REPETITIONS

EXAMPLE. In how many ways can we choose 4 nonnegative integers a, b, c, and d so that $a+b+c+d=100$?

What if a, b, c, and d are natural numbers?

REPETITIONS

EXAMPLE. How many ways are there to choose 4 integers $a, b, c,$ and d so that:

$$a+b+c+d=15$$

$$a \geq -3,\ b \geq 0,\ c \geq -2,\ d \geq -1\ ?$$

Generalized Permutations

Example. How many ways are there to arrange the letters of SYZYGY?

Example. What about MISSISSIPPI?

Generalized Permutations

In general, say we have n objects that fall into k groups, with n_i objects in the i^{th} group. Two objects in the same group are indistinguishable, but objects in different groups are distinguishable. In how many ways can we order the objects?

$$P(n; n_1, \ldots, n_k) =$$

Generalized Permutations

Example. Suppose there are 100 spots in the showroom of a car dealership. There are 15 (identical) sports cars, 25 compact cars, 30 station wagons, and 20 vans. In how many ways can the cars be parked?

Section 7.6
Derangements

A Curious Probability

Question. A professor hands back exams randomly. What is the probability that no student gets their own exam?

Answer.

5 students ~

10 students ~

100 students ~

DERANGEMENTS

A *derangement* of n objects that have some natural order is a rearrangement of the objects so that no object is in its correct position.

QUESTION. How many are there? Call the number D_n.

n	D_n	$P(D_n)$
1		
2		
3		
4		

What is the pattern?

A Formula For D_n

Let A_k be the permutations of n ordered objects with object k in the correct spot.

$D_n =$

$D_4 =$

$D_4 =$

Theorem. $D_n =$

D_N AND e

THEOREM. $D_n =$

Recall: $e^x =$

DERANGEMENTS

PROBLEM. Fifteen people check coats at a party and at the end they are handed back randomly. How likely is it that...

(a) Tim gets his coat back?
(b) Jeremy gets his coat back?
(c) Jeremy and Tim get their coats back?
(d) Jeremy and Tim get their coats back but no one else does?
(e) The members of the Beatles get the right set of coats back (maybe not in the right order)?
(f) Everyone gets their coat back?
(g) Exactly one person gets their coat back?
(h) Nobody gets their own coat back?
(i) At least one person gets their coat back?