CHAPTER 7
PERMUTATIONS AND COMBINATIONS

SECTION 7.1
Permutations

Permutations

The basic question: How many ways are there to make an ordered list?

Example: In a club with 30 people, how many ways are there to choose a president, vice president, and secretary?

A permutation of n objects is an arrangement of those objects in some order.

How many permutations of n objects?

r-Permutations

An r-permutation of n objects is a choice of r of the objects and a permutation of those r objects.

How many r-permutations of n objects?

We define

$$P(n,r) =$$

In other words, $P(n,r)$ is the number of ways to put r distinguishable marbles into n boxes, at most one marble to a box.

Example: In an olympic event with 8 athletes, in how many ways can gold, silver, and bronze be awarded?

PERMUTATION PROBLEMS

1. How many ways are there to give 7 sad puppies away to 20 people, if each person can take at most one sad puppy?

2. A group has n men and n women. In how many ways can they be lined up so that men and women alternate?

PERMUTATION PROBLEMS

3. How many ways are there to seat 6 boys and 4 girls at a round table if no two girls sit together?

Note: A rotation of a configuration is considered the same as the original configuration.

PERMUTATION PROBLEMS

4. Arrange all 26 letters of the alphabet in a row.

a) How many such "words" are there?

b) How many contain HAMLET as a subword, e.g.:
VRPKGCHAMLETBDFIZWJNQOSYUX

c) How many have exactly 4 letters between H and T?

COMBINATIONS

The basic question: How many ways to make an unordered list with n items?

Example: In a club with 30 people, how many ways to choose a committee with 3 members?

Or: How many ways to put 3 indistinguishable marbles in 30 boxes?

MARBLES AND BOXES

Distinguishable marbles: Say we want to put a red, a green, and a blue marble into 5 boxes. How many ways?

Indistinguishable marbles: Say we want to put 3 indistinguishable marbles in 5 boxes. How many ways?

N CHOOSE K

Number of ways to put k indistinguishable marbles in n boxes:

$$\binom{n}{k} = \frac{P(n,k)}{k!} = \frac{n!}{(n-k)!\,k!}$$ "n choose k"

Fact: $\binom{n}{k} = \binom{n}{n-k}$

Proof:

COMBINATION PROBLEMS

1. Five people need a ride. My car holds 4. In how many ways can I choose who gets a ride?

2. If you toss a coin 7 times, in how many ways can you get 4 heads?

3. The House of Representatives has 435 representatives. How many 4-person committees can there be?

MORE PROBLEMS

1. How many bit strings are there with fifteen 0's and six 1's if every 1 is followed by a 0?

Note: Too hard if you think of it as a sequence of 21 tasks.

MORE PROBLEMS

2. How many strings in the letters a, b, and c have length 10 and exactly 4 a's?

Again, don't choose the 10 letters one by one.

MORE PROBLEMS

3. A lottery ticket has six numbers from 1 to 40. How many different tickets are there?

The lottery agency chooses six winning numbers. How many different possible lottery tickets have exactly four winning numbers?

MORE PROBLEMS

4. Determine the number of alphabetic strings of length 5 consisting of distinct (capital) letters that
   (a) do not contain A
   (b) contain A
   (c) start with ABC
   (d) start with A, B, C in any order
   (e) contain A, B, C in that order
   (f) contain A, B, C

5. Determine the number of possible softball teams (= 9 people) can be made from a group of 10 men, 12 women, and 17 children if:
   (a) there are no restrictions
   (b) there must be 3 men, 3 women, 3 children
   (c) the team must be all men, all women, or all children
   (d) the team cannot have both men and women.

MORE PROBLEMS

6. In how many ways can you put 5 indistiguishable red balls and 8 indistinguishable green balls into 20 boxes if
   (a) there can be at most one ball per box
   (b) there can be at most one ball of each color per box.

7. How many poker hands are:
   (a) total
   (b) 4 of a kind
   (c) flush
   (d) straight
   (e) straight flush
   (f) full house
   (g) 3 of a kind
   (h) 2-pair
   (i) pair
   (j) neither flushes straights, full house 3 of a kind, 2 pair, pair