

Announcements April 4

- WebWork 5.3 and 5.5 due Thursday
- Homework 7 due Friday April 8
- Midterm 3 in class **Friday April 8** on **Chapter 5**
- Office Hours Wed 2-3 (and more on Thu).
- LA Office Hours: Scott Mon 12-1, Yashvi Mon 2-3, Shivang Tue 5-6, Baishen Wed 4-5, Matt Thu 3-4
- Math Lab, Clough 280
 - Regular hours: Mon/Wed 11-5 and Tue/Thu 11-5
 - Math 1553 hours: Mon-Thu 5-6 and Tue/Thu 11-12
 - LA hours: Matt Tue 11-12, Scott Tue 5-6, Baishen Thu 11-12, Yashvi/Shivang Thu 5-6

Section 5.5

Complex Eigenvalues

A matrix with an eigenvector

Find the eigenvectors and eigenvalues of:

$$A = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$$

What do complex eigenvalues mean?

Fact. If an $n \times n$ matrix A has a complex eigenvalue there is a 2D plane in \mathbb{R}^n where A is (similar to) the product of a rotation and a dilation.

Here is the actual statement for 2×2 matrices:

Theorem. Let A be a real 2×2 matrix with a complex eigenvalue $\lambda = a - bi$ (where $b \neq 0$) and associated eigenvector v . Then

$$A = PCP^{-1}$$

where

$$P = (\operatorname{Re} v \quad \operatorname{Im} v) \quad \text{and} \quad C = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

C is the composition of a rotation by θ and scaling by r .

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C is the composition of a rotation by θ and scaling by r . Why?

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Example. Find C and P when

$$A = \begin{pmatrix} 5 & -2 \\ 1 & 3 \end{pmatrix}$$

Three pictures

Compute the decomposition PCP^{-1} for the matrix:

$$A = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$$

What about the higher dimensional case?

Theorem. Let A be a real $n \times n$ matrix. Suppose that for each (real or complex) eigenvalue the dimension of the eigenspace equals the algebraic multiplicity. We can find a block diagonalization

$$A = PCP^{-1}$$

where P and C are found as follows:

1. C is *block diagonal* where the blocks are 1×1 blocks containing the real eigenvalues (with multiplicity) or 2×2 blocks containing the matrices $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ for each complex eigenvalue $a - bi$ (with multiplicity).
2. The columns of P form bases for the eigenspaces for the real eigenvectors or come in pairs $(\operatorname{Re} v \quad \operatorname{Im} v)$ for the complex eigenvectors.

A 3×3 example

Find the block diagonalization of:

$$A = \begin{pmatrix} 4 & -3 & 3 \\ 3 & 4 & -2 \\ 0 & 0 & 2 \end{pmatrix}$$

Three pictures

There are three possible pictures for the action on \mathbb{R}^2 of a 2×2 matrix with complex eigenvalues.

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\lambda = 1 + i$$

$$|\lambda| > 1$$

$$A = \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix}$$

$$\lambda = 3/5 + 4/5i$$

$$|\lambda| = 1$$

$$A = \begin{pmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

$$\lambda = 1/2 + 1/2i$$

$$|\lambda| < 1$$

A 3×3 example

What does A do to \mathbb{R}^3 ? Draw a picture!

$$A = \begin{pmatrix} 4 & -3 & 3 \\ 3 & 4 & -2 \\ 0 & 0 & 2 \end{pmatrix}$$