COUNTING SOLDIERS

A general lines her troops in rows of 9, then 10, then 11. Each time, there are leftovers: 1, 2, and 4, respectively.

Can the general tell just from this information exactly how many soldiers she has?

CONGRUENCE

Compute: $1234567 \pmod{10}$
$1027581 \pmod{2}$
$624897 \pmod{3}$
$169 \pmod{24}$

CONGRUENCE

Compute: 101×122 $(\mathrm{mod}\ 3)$
   4^{157} $(\mathrm{mod}\ 3)$
   149728×51 $(\mathrm{mod}\ 3)$

CONGRUENCE

Solve: $x + 7 \equiv 2 \pmod{19}$

Solve: $x \equiv 1 \pmod{3}$
 $x \equiv 3 \pmod{5}$

MULTIPLICATIVE INVERSES

Does every number have a multiplicative inverse
mod n ?

CONGRUENCE

Solve: $2x \equiv 1 \pmod 9$

$7x \equiv 1 \pmod{100}$

$7x \equiv 2 \pmod{100}$

$7x \equiv 3 \pmod{100}$

CHINESE REMAINDER THEOREM

Solve: $x \equiv 1 \pmod 3$
$\qquad x \equiv 3 \pmod 5$

Solve: $x \equiv 1 \pmod 3$
$\qquad x \equiv 3 \pmod 5$
$\qquad x \equiv 2 \pmod 7$

Solve: $x \equiv 1 \pmod 9$ Hint: $6 \cdot 90 - 49 \cdot 11 = 1$
$\qquad x \equiv 2 \pmod{10}$
$\qquad x \equiv 4 \pmod{11}$

CHINESE REMAINDER THEOREM

Solve: $x \equiv 1 \pmod 9$
$x \equiv 2 \pmod{10}$
$x \equiv 4 \pmod{11}$

First solve just the first two, mod 90. Since

$$1 \cdot 10 - 1 \cdot 9 = 1$$

we have: $1 \cdot (1 \cdot 10) - 2(1 \cdot 9) = -8 \equiv 82 \pmod{90}$

Now solve: $x \equiv 82 \pmod{90}$
$x \equiv 4 \pmod{11}$

Since $6 \cdot 90 - 49 \cdot 11 = 1$

we have: $4(6 \cdot 90) - 82(49 \cdot 11) = -42{,}038 \equiv 532 \pmod{990}$