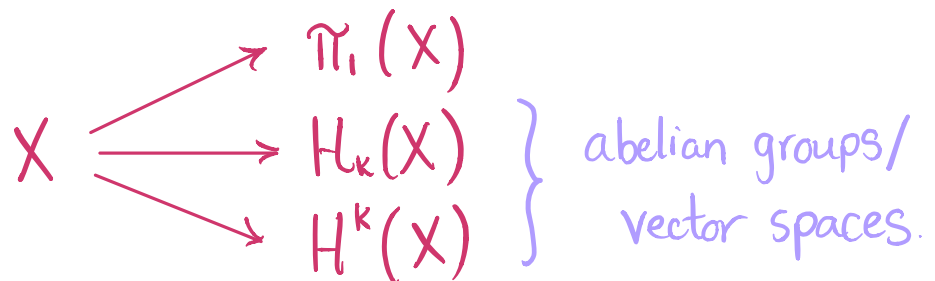
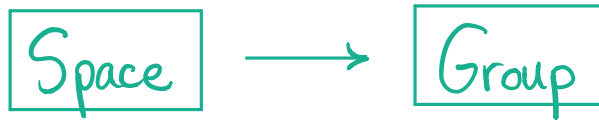


ALGEBRAIC TOPOLOGY

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OVERVIEW

What is algebraic topology?



Because groups and vector spaces are simpler than spaces.
Roughly, the groups encode the holes in X .

What kinds of questions does it answer?

① When are two spaces the same (or not)?

$$\text{e.g. } \mathbb{R}^m \neq \mathbb{R}^n$$

what about

$$\mathbb{R}^3 - \bigcirc \quad \text{vs.} \quad \mathbb{R}^3 - \bigcirc \bigcirc$$

② Embeddings

What is the smallest N so that a given manifold embeds in \mathbb{R}^N ?

e.g. $\mathbb{R}P^2$ embeds in \mathbb{R}^4 but not \mathbb{R}^3
unsolved for $\mathbb{R}P^n$ in general.

③ Fixed point theorems

Brouwer fixed point theorem \Rightarrow PF theorem, ...
Borsuk-Ulam theorem

④ Actions

Which finite groups act freely on S^n ?

known in some cases, e.g. $\mathbb{Z}/n\mathbb{Z} \hookrightarrow S^{2k-1} \quad \forall n, k.$

⑤ Sections

What is the largest K s.t. a given manifold admits a continuously varying K -plane field?

→ Hairy ball theorem

⑥ Group theory

Every subgroup of a free group is free
 $[F_n, F_n]$ is not finitely generated.

⑦ Geometric group theory

Braid groups are torsion free



⑧ Algebra

Fundamental theorem of algebra

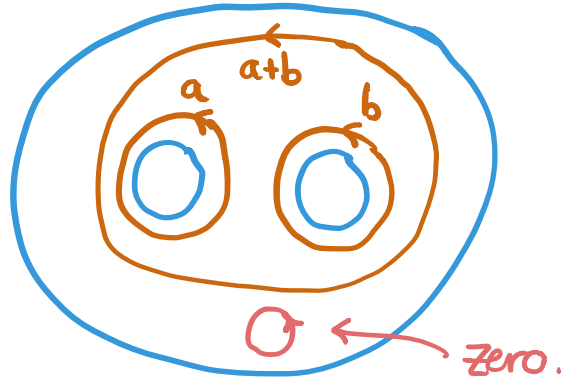
Abel-Ruffini (unsolvability of quintics)

BASIC IDEAS

Homology $H_k(X)$ = abelian group of k -dim holes in X

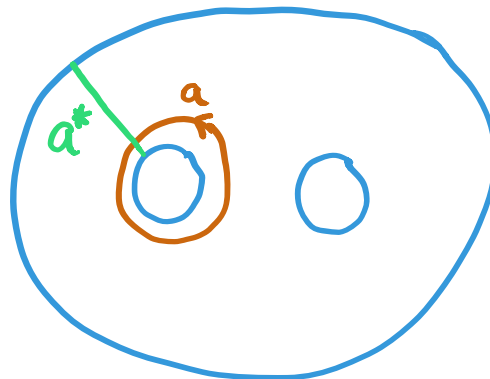
Elements are spheres, etc. computable

example. X = pair of pants
 $H_1(X) = \mathbb{Z}^2$



Cohomology $H^k(X)$ elements are dual to elements of $H_k(X)$

example



a^* counts intersections with this arc.

Poincaré Duality (big goal!)

$$X = n\text{-manifold} \rightsquigarrow H^k(X) \cong H_{n-k}(X)$$

More precisely, elements of H^k are:
intersect with an element of H_{n-k}
(see above example).

Poincaré Conjecture

Original version: $H_*(X) \cong H_*(S^n) \Rightarrow X \cong S^n$

False for $n=3$: Poincaré dodecahedral space
 \rightsquigarrow need something better.

Fundamental Group

$\pi_1(X)$ = group of based loops in X .

π_1 - version of Poincaré conj. is true (Perelman).