HOTEL INFINITY

Hotel Infinity has rooms numbered 1, 2, 3, ...

Today, every room is occupied. Someone walks into the lobby and asks for a room. Can the hotel accommodate her?

HOTEL INFINITY

Hotel Infinity is so successful they open Hotel Infinity 2, just like the first.

There is a fire in Hotel Infinity 2. Can Hotel Infinity accommodate the overflow?

HOTEL INFINITY

Now there is Hotel Infinity 2, Hotel Infinity 3, etc. All the hotels except the first burn to the ground. Can Hotel Infinity accommodate the overflow?

HOTEL INFINITY

Show that the following sets are countable:

$\mathbb{N} \cup \{0\}$

$\mathbb{N} \cup \mathbb{N}$

$\mathbb{N} \cup \mathbb{N} \cup \mathbb{N} \cup \cdots$

INTERVALS

Show that the following pairs of sets have the same cardinalities.

$(0,1)$ and $(1,3)$

$(-\infty,\infty)$ and $(0,\infty)$

CANTOR DIAGONALIZATION

THEOREM. $\mathbb{R}$ is uncountable.

RATIONALS

Is $\mathbb{Q}$ countable?

What about $\mathbb{R} \setminus \mathbb{Q}$?

What about the Cantor set?

LINES AND SQUARES

Show that $|[0,1]| = |[0,1]^2|$.