SECTION 7.3
Elementary Probability

INTUITIVE PROBABILITY

What is the probability that...

a) A flipped coin comes up heads?

b) A rolled die comes up 3?

c) A rolled pair of dice comes up 4?

DEFINITIONS

An experiment is a procedure that yields one of a given set of outcomes.

The sample space of the experiment is the set of possible outcomes.

S = finite set

An event is a subset of the sample space:

$$A \subseteq S$$


Blaise Pascal


Pierre Laplace

The probability of an event A, assuming each outcome of the experiment is equally likely, is:

$$P(A) = |A|/|S|$$

EXAMPLES OF EXPERIMENTS

Experiment	Sample space S	Outcome A	Probability P(A)
Flipping a coin			$1/2$
Rolling a die			$1/6$
Rolling a pair of dice			$3/36 = 1/12$

MORE EXAMPLES

1. You toss a coin 5 times. What is the probability of getting 4 heads?

2. What is the probability of correctly guessing the winners in a 64-team single elimination tournament?
(Assume every team has a 50% chance of winning each game.

MORE EXAMPLES

3. An urn has 4 red balls, 3 green balls. You pull one ball at random. What is the probability of pulling a green ball?

Suppose you pull one ball, replace it, then pull another ball. What is the probability of pulling two balls of the same color?

MORE EXAMPLES

Same urn (4 red, 3 green). Now suppose you pull one ball, don't replace it, and pull another ball. What is the probability of getting two balls of the same color?

MORE EXAMPLES

4. In poker, what is the probability of dealing a 4-of-a-kind?

What about a full house?

SOME PROBABILITY RULES

THEOREM: Let S be the sample space of some experiment. Let A and B be events.

(i) $0 \le P(A) \le 1$
$P(\emptyset) = 0, \quad P(S) = 1$

(ii) $P(A^c) = 1 - P(A)$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

More generally:

$$P(A_1 \cup \cdots \cup A_n) = \sum_i P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \cdots$$

Can rephrase all counting rules as probability rules.

APPLYING PROBABILITY RULES

EXAMPLE: A number from 1 to 100 is chosen at random.
What is the probability it is...

a) divisible by 2, 3, or 5?
b) divisible by 2 and 3, but not 5?
c) divisible by 3 but not 2 or 5?
d) divisible by at most two of 2, 3, and 5?

MUTUAL EXCLUSIVITY

Two events A and B are mutually exclusive if $A \cap B = \emptyset$.

Events $A_1, \ldots, A_n$ are pairwise mutually exclusive if $A_i \cap A_j = \emptyset$ whenever $i \neq j$.

A special case of the last theorem:

If $A_1, \ldots, A_n$ are pairwise mutually exclusive events, then

$$P(A_1 \cup \cdots \cup A_n) = P(A_1) + \cdots + P(A_n)$$ (addition rule)

EXAMPLE: A number from 1 to 100 is chosen at random. What is the probability that the number is divisible by 7 or 30?

APPLYING PROBABILITY RULES

1. What is the probability that a length 10 bit string (chosen at random) has at least one zero? at least two zeros?

2. What is the probability that a poker hand (dealt at random) is a flush? a straight? royal flush?

Note: A,2,3,4,5 and 10,J,Q,K,A are both straights.