Scores: 1 2 3 4 5 6 7 8 9 10 Name Prof. M

Section K__

Mathematics 2602

Midterm 1
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1. Write T if the sentence is a true statement, F is the sentence is a false statement, and N if the sentence is not a statement.

Solve for x: $e^x = 1$.

N

If $1+1=2$ or $1+1=3$ then $1+2=4$ and $1+1=2$.

F

$\forall x \exists y \ (y^2 = x) \quad x, y \in \mathbb{R}$

F

2. Complete the following truth table.

p	q	$\neg p$	$p \to q$	$\neg p \wedge (p \to q)$
T	T	F	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

From the truth table, we can conclude $\neg p \wedge (p \to q)$ is equivalent to what other statement?

$\neg p$

3. Show that $p \to (q \to p)$ is a tautology by using the rules for logical equivalence.

$$\begin{aligned}
p \to (q \to p) &\equiv (q \to p) \vee \neg p \\
&\equiv (p \vee \neg q) \vee \neg p \\
&\equiv (p \vee \neg p) \vee \neg q \\
&\equiv T \vee \neg q \\
&\equiv T
\end{aligned}$$

4. Let n be some fixed integer. Consider the statement:

If n is prime then n is odd.

Write the converse of the statement.

If n is odd then
n is prime.

Write the contrapositive of the statement.

If n is even then
n is not prime.

Write the negation of the statement without using a conditional.

n is prime and even.

5. For each of the following functions $f : \mathbb{R} \to \mathbb{R}$, state whether the function is one-to-one, onto, both, or neither. Explain your answer.

$f(x) = |x|$

Not 1-1 since $f(1) = f(-1)$.

Not onto since $f(x) \geq 0$

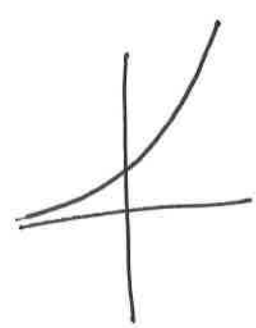
$f(x) = 1$

Not 1-1 since $f(0) = f(1)$

Not onto since $f(x)$ only equals 1.

$f(x) = e^x$

Is 1-1 since e^x is increasing



Not onto since $e^x > 0$.

6. What does it mean for a set A to be countably infinite?

A. The set A has the same cardinality as $\mathbb{N}$.

B. The set A is an infinite subset of $\mathbb{Q}$.

C. There is a one-to-one and onto function $\mathbb{R} \to A$.

D. The set A is a subset of $\mathbb{N}$.

Which of the following sets are countably infinite? Select all that apply.

A. $\mathbb{R}$

B. $\mathbb{Q}$

C. $\mathbb{Z}$

D. $(0, 1)$

E. $(0, \infty)$

F. $\{1, \ldots, 100\}$

Show that $(0, \infty)$ to $(1, \infty)$ have the same cardinality.

Need a 1-1 correspondence.

$f : (0, \infty) \longrightarrow (1, \infty)$

$f(x) = x + 1.$

7. For each of the following binary relations on $\mathbb{R}$, state whether or not the relation is an equivalence relation. If it is an equivalence relation, describe the set of equivalence classes. If it is not an equivalence relation, explain why not.

$x \sim y$ if and only if $|x - y| < 1$

Not transitive.

$x \sim y$ if and only if $x = y + 1$

Not reflexive.

$x \sim y$ if and only if $x - y \in \mathbb{Z}$

$[0, 1)$

8. Prove that if numbers a and b have $a + b = n$, then $a \geq n/2$ or $b \geq n/2$.

Proof by contraposition:

$a > n/2$ and $b > n/2$

$\rightarrow a+b > n/2 + n/2$

$\rightarrow a+b > n$

$\rightarrow a+b \neq n$

9. Prove that $\sqrt{2}+1$ is irrational. You can use the fact that $\sqrt{2}$ is irrational.

Proof by contradiction

$\sqrt{2}+1$ rational

$\rightarrow \sqrt{2}+1 = p/q \qquad p \in \mathbb{Z}, q \in \mathbb{Z} \setminus \{0\}.$

$\rightarrow \sqrt{2} = \frac{p}{q} - 1$

$\rightarrow \sqrt{2} = \frac{p-q}{q}$

$\rightarrow \sqrt{2}$ rational, which is F.

10. Prove or disprove that the product of any two irrational numbers is irrational.

Counterexample: $\sqrt{2}\sqrt{2} = 2$.