Math 8803: MCG

- Primer, Farb-M
- Flipped / Just in Time
- ~ 1 chapter / week
- Midterm: Read & summarize a paper on MCG

  Target: Oct 5
- Final: Attempt research

  Proposal Nov 2

  Target Nov 23

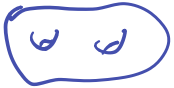
  Groups ok.
- Participation

  Teams: Q's for class

  Open q's.

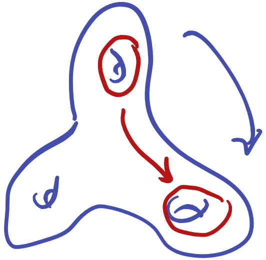
Also: This Wed 11:15 start.

Mapping Class Gps

$S =$ 

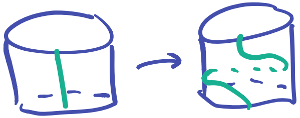
$$\mathrm{Mod}(S) = \pi_0 \, \mathrm{Homeo}^+(S)$$
$$= \mathrm{Homeo}^+(S) / \text{isotopy}$$

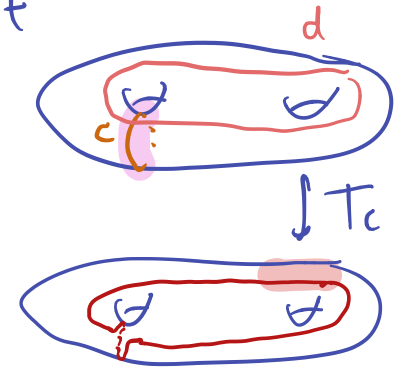
Sample elements

① 

S_3

② Dehn twist





3 Reasons

① Alg geometry.

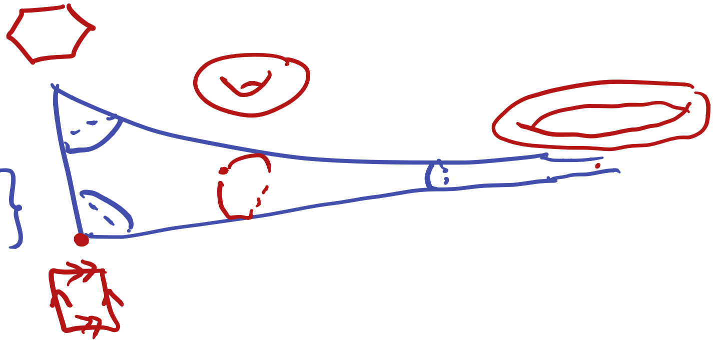
$\mathrm{Mod}(S_g) = $ "π_1" $\mathcal{M}_g$

$\mathcal{M}_g$ = moduli space of alg. curves of genus g.

$\mathcal{M}_1$

$=$

{Eucl. unit area tori}



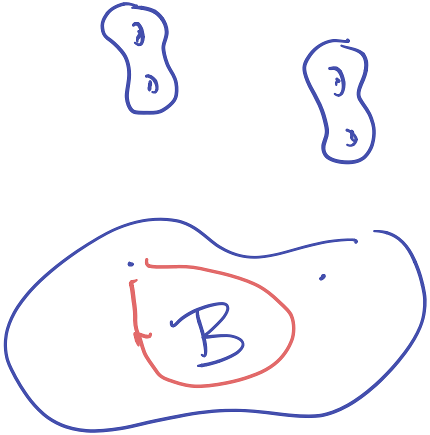
② Topology

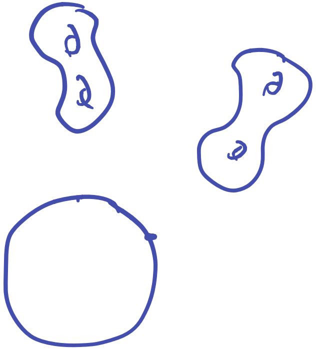
$$\{ S\text{-bundles over } B \} \longleftrightarrow \{ \pi_1 B \to \mathrm{Mod}^{\pm}(S) \}$$

"monodromy"

Mobius band also

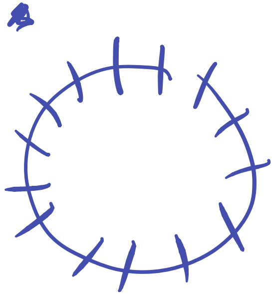
Annulus: $[0,1]$-bund over S^1





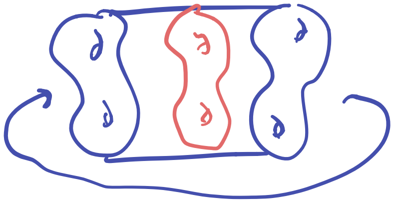
$B = S^1$

e.g. $S \times S^1$



Agol, Wise, Perelman, Thurston....

Essentially all 3-manifolds arise this way.



$f \in \mathrm{Mod}(S)$

$$S \times [0,1] \big/ (x,1) \sim (\varphi(x),0)$$

$$[\varphi] = f$$

Donaldson:

All symplectic 4-mans arise essentially this way.

Also: Contact topology: open bks

③ Geometric Group Thy

$\mathrm{Out}(G) \cong \mathrm{Aut}(G)/\mathrm{Inn}(G)$

Dehn-Nielsen-Baer thm

$$\mathrm{Mod}^{\pm}(S_g) \cong \mathrm{Out}\,\pi_1(S_g)$$

Topology. Algebra

Related topics

Number thy.

Group theory

Grap cohom.

Rep thy

Complex anal.

Graph thy

Hyp. geom

Dynamics

Alg top.

Combinatorics

Part I

Overview of Book/Class

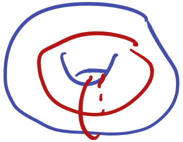
① Curves on surfaces – Wed.

homeos : linear maps ::

curves : vectors

② MCG basics

$\mathrm{Mod}(T^2) \xrightarrow{\cong} SL_2\mathbb{Z}$



Alexander Method

③ Dehn twists

Prop. $a \neq b$

alg. $T_a T_b T_a = T_b T_a T_b$

$\Longleftrightarrow i(a,b) = 1$. topol.

④ Generating MCG

Dehn: $\mathrm{Mod}(S_g) = \langle T_c \rangle$

5. Presentations of MCG

$$H_1(\mathrm{Mod}(S_g)) = 0$$

$$H_2(\mathrm{Mod}(S_g)) \cong \mathbb{Z}$$

$H_k(\mathrm{Mod}(S_g)) \longleftrightarrow$ characteristic classes for S_g-bundles

Super duper mysterious.

6. Symplectic rep.

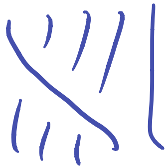
7. Torsion

In $\mathrm{Mod}(S_g)$, elements of order

1, 2, 3, 4, 6, 7, 8, 9, 12, 14.

8. DNB (see above)

9. Braid gps



Parts II & III

Nielsen-Thurston Classification Thm: Any f in $\mathrm{Mod}(S)$ has a rep. φ that is

1. finite order
2. reducible: fixes a collection of disjoint curves.
3. pseudo-Anosov: like $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \curvearrowright \mathbb{R}^2$