

Practice Midterm 2

This is a preview of the draft version of the quiz

Started: Oct 9 at 6:02pm

Quiz Instructions

Question 1

1 pts

Answer the following three true/false questions.

(a) There is a 5×4 matrix whose rank is 2 and whose nullity is 2.

[Select] **True**

(b) There is a 4×6 matrix whose rank is 5 and whose nullity is 1.

[Select] **False**

(max 4 pivots)

(c) If the column vectors of a 3×3 matrix A span \mathbb{R}^3 , then A has 3 pivots.

[Select] **True**

Question 2

1 pts

Consider the subset $V = \{(x, y) \text{ in } \mathbb{R}^2 \mid |x| + |y| = 1\}$ of \mathbb{R}^2 .

Does V contain the 0 vector?

[Select]

NO

$0+0 \neq 1$

Is V closed under addition?

[Select]

NO

$(0, 0) \in V, (1, 1) \notin V$

Is V closed under scalar multiplication?

[Select]

NO

$(1, 0) \in V, (-1, 0) \notin V$

(3) EV

Is V a subspace of \mathbb{R}^2 ? [Select]

N D

Question 3

1 pts

Which of the following matrix transformations has as its range the xz -plane in \mathbb{R}^3 ?

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 0 \\ 3 & 4 & 0 \\ 1 & 5 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Question 4

1 pts

Suppose that A is a 11×3 matrix, B is a 3×4 matrix, and C is a 4×11 matrix. Which of the following matrix multiplications is allowed? Select all that apply.

AB

$$\underline{11 \times 3} \quad 3 \times 4$$

 AC

$$\underline{11 \times 3} \quad 4 \times 11$$

 BC

$$3 \times \underline{4} \quad 4 \times 11$$

 CA

$$4 \times \underline{11} \quad \underline{11} \times 3$$

Question 5

1 pts

Which of the following statements are true for all 2×2 matrices A and B .

(a) $AB=BA$

[Select]

False(b) $AB=0$ (the zero matrix) implies that either A or B is 0.

[Select]

False(c) $A+B=B+A$

[Select]

True**Question 6**

1 pts

Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the transformation given by counterclockwise rotation by 30 degrees, and let A be the standard matrix for T .

Which of the following statements must be true about A ?

rotations are invertible A is invertible A is not invertible $\text{rank}(A)=1$ Nullity(A)=0

Question 7

1 pts

If u , v , and w form a basis of subspace W , then $u + v$, v , and w also form a basis for W .

 True False**Question 8**

1 pts

Find the value of k so that the matrix transformation for the following matrix is not onto.

$$\begin{pmatrix} 2 & 3 & 4 \\ 6 & 9 & k \end{pmatrix}$$

When $k=12$, the matrix
only has 1 pivot

$$k = 12$$

Question 9

1 pts

Suppose that A is 10×9 matrix in row echelon form with 3 pivots. What is the dimension of $Nul(A)$?

$$6$$

$$9 - 3 = 6$$

Question 10

1 pts

Solve for the matrix X if $(AX + D)(BX + E)^{-1} = C$. Assume that all the matrices that arise in the solution are invertible.

- (A - CB) $^{-1}$ (CE - D)
- (A - BC)(CE - D) $^{-1}$
- (A - B) $^{-1}$ (C - D)
- (CE - D)(A - BC) $^{-1}$

$$\begin{aligned} AX + D &= C(BX + E) \\ AX + D &= CBX + CE \\ AX - CBX &= CE - D \\ (A - CB)X &= CE - D \\ X &= (A - CB)^{-1}(CE - D) \end{aligned}$$

Question 11

1 pts

Suppose that A and B are $n \times n$ matrices and AB is not invertible. Which of the following must be true?

- B is not invertible
- A is not invertible
- Both A and B are not invertible
- None of these must be true

either A or B must be not invertible

Question 12

1 pts

Let $A = \begin{pmatrix} 1 & 0 & -3 & 5 & 0 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$. Which of the following is a basis for $\text{Nul}(A)$?

(must be 2 vectors in \mathbb{R}^5 since 2 f.v. & 5 columns)

$\left\{ \begin{pmatrix} -3 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$

$$x_1 = 3x_3 - 5x_4$$

$$x_2 = -2x_3 - 4x_4$$

$$x_5 = 0$$

~~$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$~~

$$\underline{\underline{x}} = \begin{pmatrix} 3x_3 - 5x_4 \\ -2x_3 - 4x_4 \\ x_3 \\ x_4 \\ 0 \end{pmatrix}$$

$\left\{ \begin{pmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ -4 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$

$$= x_3 \begin{pmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -5 \\ -4 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

~~$\left\{ \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} \right\}$~~

Question 13

1 pts

Suppose A is a 7×4 matrix such that the associated linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^7$ is one-to-one. What is $\text{rank}(A)$?

4

pivot in every column = 4 pivots

Question 14

1 pts

Which of the following linear transformations are invertible? Select all that apply.

- Projection onto the x-axis, followed by dilation by a factor of 2
- Rotation by -47 degrees, followed by reflection across the y-axis
- Dilation by a factor of -0.001
- Dilation by a factor of 3, followed by projection onto the line $y = 2x$

Question 15**1 pts**

Consider the matrix

$$A = \begin{pmatrix} 3 & 3 & 2 & -8 \\ -1 & -1 & 5 & -3 \\ 0 & 0 & 1 & -1 \end{pmatrix},$$

which can be row reduced to the following matrix

$$\begin{pmatrix} 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

- (a) Is the vector $\begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix}$ in $\text{Nul}(A)$? [Select] **No**

The dimension is not correct

- (b) Is the vector $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ in $\text{Nul}(A)$? [Select] **Yes**

$$A \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Question 16**1 pts**

Consider the matrix

$$A = \begin{pmatrix} 3 & 0 & -3 & 1 \\ 0 & -2 & 6 & 0 \\ 0 & 0 & 0 & 8 \end{pmatrix}.$$

Let T be the matrix transformation $T(v) = Av$.

The domain of T is \mathbb{R}^n , where $n =$ 4

The codomain of T is \mathbb{R}^m , where $m =$ 3

The range of T has dimension 3

There are 3 columns with pivots

Question 17

1 pts

For which of the following matrices A is the range of the associated matrix transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ a line? Select all that apply.

$A = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$ Pivots in 2 columns - range is a plane

$A = \begin{pmatrix} 1 & -2 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ Pivot in 1 column - range is a line

$A = \begin{pmatrix} -2 & 4 & 1 \\ 0 & -3 & 9 \\ 0 & 0 & 0 \end{pmatrix}$ Pivots in 2 columns - range is a plane

$A = \begin{pmatrix} 2 & -1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ Pivot in 1 column - range is a line

Question 18**1 pts**

Suppose that T is a linear transformation and

$$T \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } T \begin{pmatrix} 10 \\ 11 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

What is $T \begin{pmatrix} 12 \\ 13 \end{pmatrix}$? $= T \begin{pmatrix} 10 \\ 11 \end{pmatrix} + 2T \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

Question 19**1 pts**

Suppose that $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is reflection about the line $y=7x$ and A is the standard matrix. What is A^2 ?

Reflection any vector twice will return to original position. So the map for A^2 is identity.

•

Question 20**1 pts**

Suppose that A is an $n \times n$ matrix and $T(v)=Av$. Answer the following three questions.

If the columns of A add up to 0, is it possible that T is onto?

[Select] No

Adding to 0 means the cols. are dependent. So, the dimension of the range is smaller than n.

If the row echelon form of A has no row of zeros, is it possible for $Ax=b$ to have

infinitely many solutions? [Select] No

If the row echelon form has no row of zeros, there will be a pivot in every row (and col. as A is n-by-n) So, A is invertible and $Ax=b$ has a unique solution.

If $Ax=b$ is consistent for all b, can $Ax=0$ have infinitely many solutions?

[Select] No

" $Ax=b$ consistent for all b" means "T is surjective". Since A is n-by-n, surjective also implies injective. So, $Ax=0$ has only the trivial solution.

No new data to save. Last checked at 6:06pm

Submit Quiz