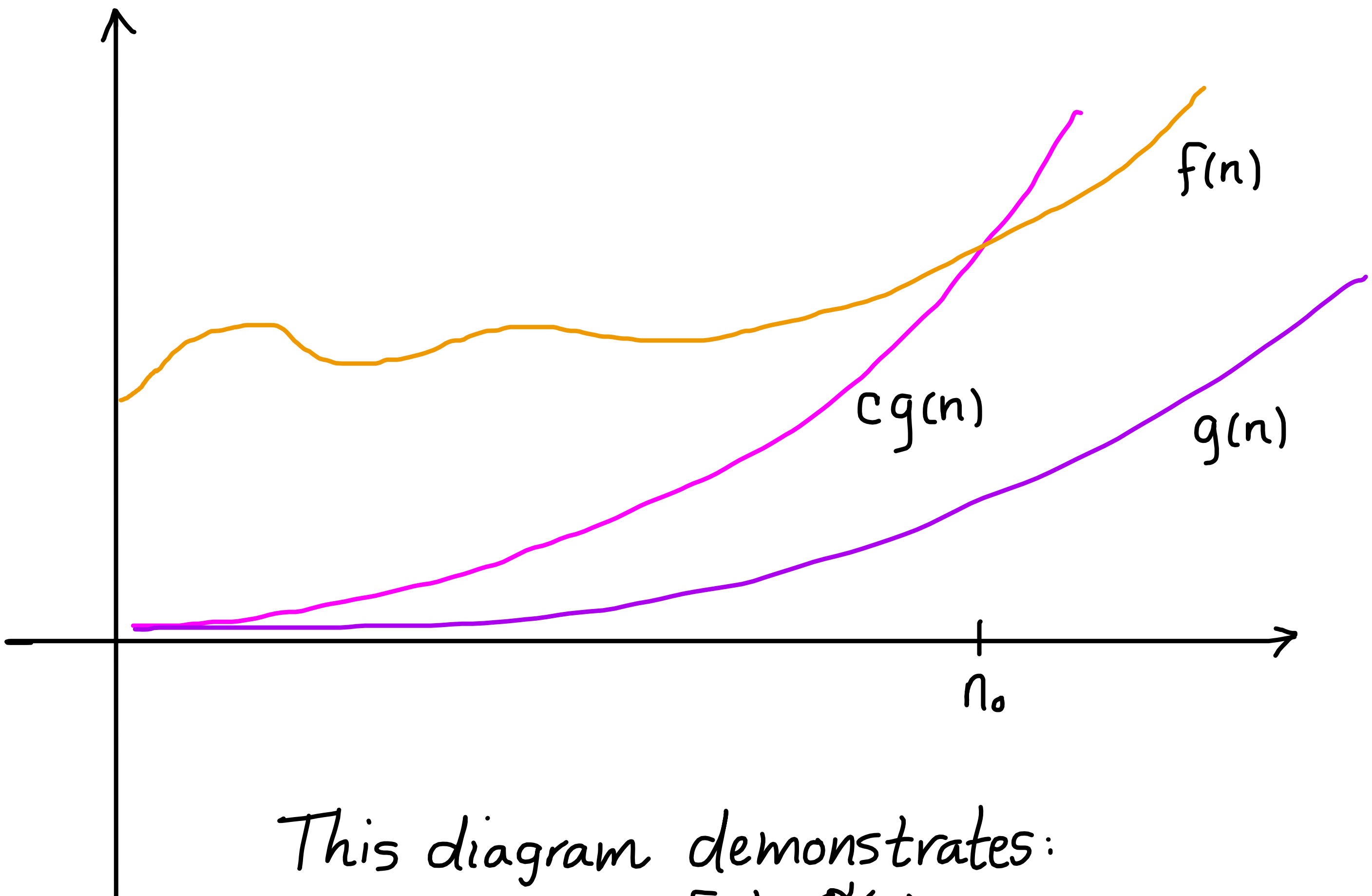
BIG O



This diagram demonstrates:

(a) f is $O(g)$

(b) g is $O(f)$

(c) both

BIG O

We say that "f is big O of g" and write

$$f = O(g) \quad \text{or} \quad f \in O(g)$$

if there is a natural number n_0 and a positive real number c such that

$$|f(n)| \leq c\,|g(n)|$$

for $n \geq n_0$.

First examples: ① $f(n) = n^2, \quad g(n) = 7n^2$

② $f(n) = 4n+2, \quad g(n) = n$

③ $f(n) = n^2, \quad g(n) = n^2 + 2n + 1$

④ $f(n) = n, \quad g(n) = \sqrt{n}$

LIMIT THEOREM

THEOREM: Let f, g be functions $\mathbb{N} \to [0, \infty)$

(a) If $\lim_{n\to\infty} f(n)/g(n) = 0$, then $f \prec g$

(b) If $\lim_{n\to\infty} f(n)/g(n) = \infty$, then $g \prec f$

(c) If $\lim_{n\to\infty} f(n)/g(n) = L \neq 0$, then $f \asymp g$

MORE EXAMPLES

① Compare $n!$ & n^n

② Compare $n!$ & 2^n

COMBINING FUNCTIONS

Theorem: Let f, g be functions $\mathbb{N} \to \mathbb{R}$.

(a) If $f \in O(F)$, then $f + F \in O(F)$

(b) If $f \in O(F)$ and $g \in O(G)$ then $fg \in O(FG)$.

POLYNOMIALS

Theorem: Let $f(n) = a_d n^d + \cdots a_1 n + a_0$ be a degree d polynomial ($a_d \neq 0$). Then $f(n) \asymp n^d$.

MORE COMPARISONS

Theorem: (a) If $k < \ell$, then $n^k < n^\ell$

(b) If $k > 1$, then $\log_k n < n$

(c) If $k > 0$, then $n^k < 2^k$

HIERARCHY

$$1 < \log n < n < n^k < k^n < n! < n^n$$

const < log < linear < poly < exp < fact < tower

MORE DETAILED HIERARCHY

$$1 < \log n < \sqrt{n} < n/\log n < n < n\log n < n^{3/2}$$

$$< n^2 < n^3 < \cdots$$

$$< 2^n < 3^n < \cdots$$

$$< n!$$

$$< n^n < n^{n^n} < \cdots$$

COMPARING DIFFERENT ORDERS

	10	50	100	300	1000
$5n$	50	250	500	1500	5,000
$n \log n$	33	282	665	2469	9966
n^2	100	2500	10,000	90,000	1,000,000
n^3	1,000	125,000	1 mil	27 mil	1 bil
2^n	10^{24}	16 digits	31 dig.	91 dig.	302 dig.
$n!$	3.6 mil	65 dig.	161 dig.	623 dig.	unimaginable
n^n	10 bil.	85 dig.	201 dig.	744 dig.	unimaginable

μsecs since big bang: $\sim 10^{24}$

protons in the known universe: $\sim 10^{126}$

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COMPARING DIFFERENT ORDERS

How long would it take at 1 step per μsec?

	10	20	50	100	300
n^2	1/10,000 sec.	1/2500 sec.	1/400 sec	1/100 sec.	9/100 sec.
n^5	1/10 sec.	3.2 sec	5.2 min	2.8 hr	28.1 days
2^n	1/1,000 sec	1 sec	35.7 yr	400 trillion cent.	75 digit # of centuries
n^n	2.8 hr	3.3 trillion yr	70 digit # of centuries	185 digit # of centuries	728 digit # of centuries.

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