PHONE NUMBERS

Are there two students at Georgia Tech with the same last 4 digits of their phone number?

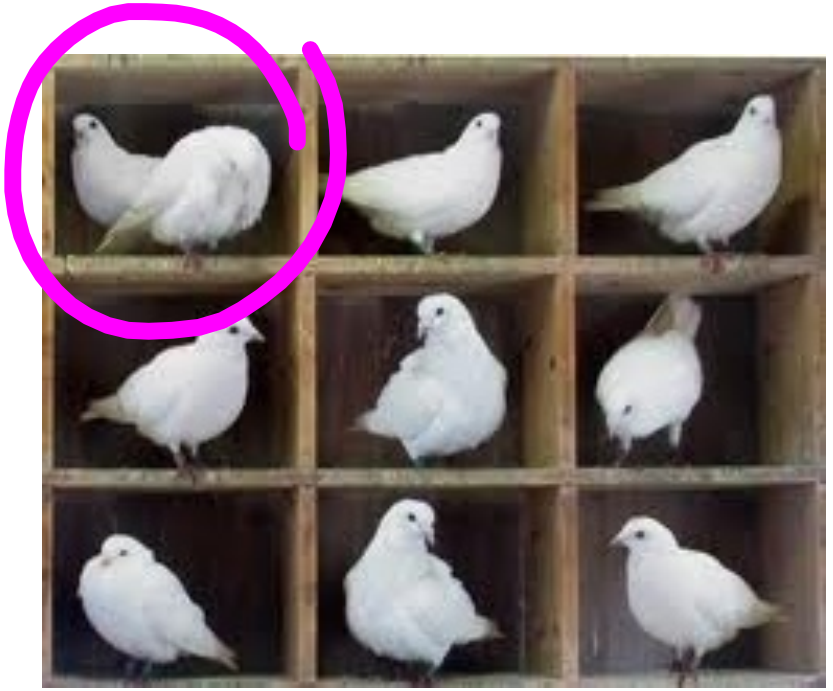
HAIR

Are there two non-bald people in Atlanta with the same number of hairs on their heads?

THE PIGEONHOLE PRINCIPLE

If n objects are put into m boxes, and $n > m$, then at least one box will have multiple objects.

Johann Peter Gustav Lejeune Dirichlet



PIGEONHOLE PROBLEMS

1. Show that, given 5 points in a unit square, there are two points within $\sqrt{2}/2$ of each other.

2. Show that, given any 11 integers, there is a pair of numbers whose difference is divisible by 10.

3. Show that, at any party, there are always two people with the same number of friends.

PIGEONHOLE PROBLEMS

4. Take a chessboard with two opposite corners removed. Can you cover it with dominos?

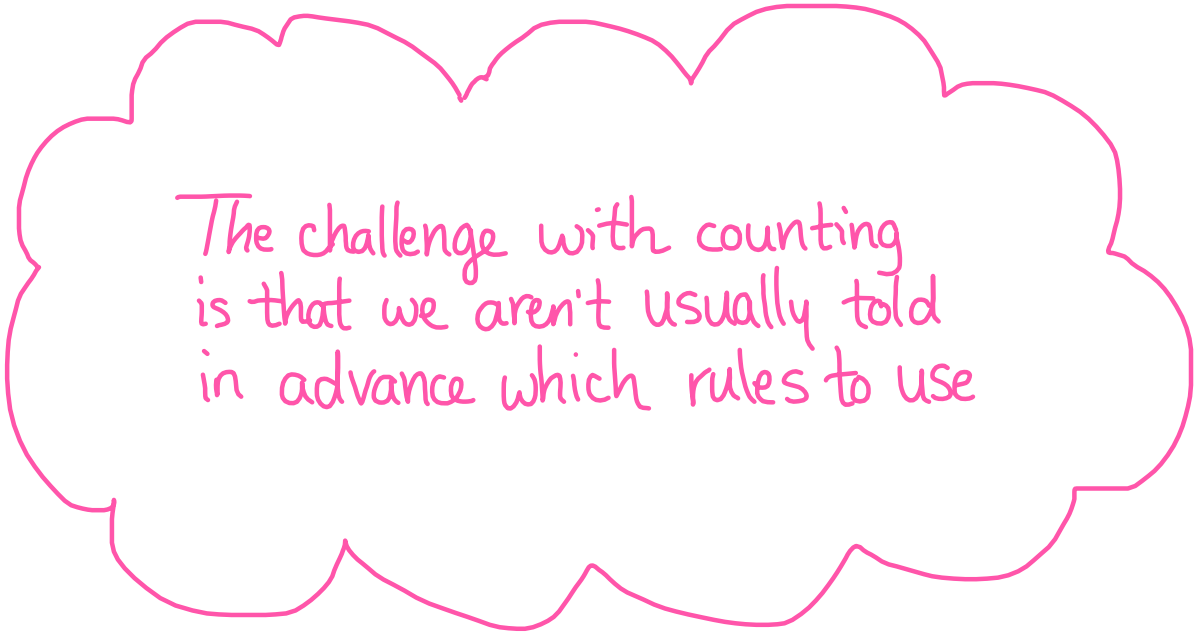
   Hint: The dominos give a bijection between black squares and white squares.

5. On a 5×5 chessboard, there is one flea in each square. Each flea jumps to an adjacent square. Are there now two fleas in the same square?

6. Arrange the numbers $1, \ldots, 10$ on a circle in any order. Show that there are 3 consecutive numbers that add to 17 or more.

STRONG PIGEONHOLE

Our class has 68 students. What is the biggest N so that we know that some month has N birthdays?


The challenge with counting
is that we aren't usually told
in advance which rules to use

MORE PROBLEMS

1. How many 3 digit numbers are there?
2. How many 3 digit numbers are there with no repeated digits?
3. How many 3 digit numbers are there with the i^{th} digit equal to i for some i.

MORE PROBLEMS

4. How many functions are there $A \to B$ if $|A|=m, |B|=n$?

5. How many injective functions are there $A \to B$ if $|A|=m, |B|=n$?

6. How many subsets of A are there if $|A|=n$?

MORE PROBLEMS

7. How many even 4 digit numbers are there with no repeated digits?

8. How many odd 4 digit numbers are there with no repeated digits? (Harder!)

9. How many ways are there to place a domino on a chessboard?

MORE PROBLEMS

10. How many bit strings are there that have length n and begin and/or end with a 1?

11. How many different dominos are there?

12. How many arrangements are there of 6 men and 4 women at a round table if no women sit together?

MORE PROBLEMS

13. Given 20 integers, show there is a pair whose difference is divisible by 19.

14. If we want to label the chairs in a room by one letter and one number from 1 to 100, how many labels are there?

15. How many distinct alphanumeric passcodes are there if each passcode has 6-8 characters and at least one digit?

16. In how many ways can a best-of-5 series go down?

17. Given 5 points on a sphere, how many necessarily lie on the same hemisphere?