Mathematics 8803: Characteristic Classes for Vector Bundles and Surface Bundles

Georgia Institute of Technology
Fall 2013

Professor

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Class Meetings

Mondays, Wednesdays, and Fridays from 1:05 until 1:55 in Skiles 271.

Web Site

http://www.math.gatech.edu/~margalit/classes/math8803

Textbooks

Characteristic Classes, John W. Milnor and James D. Stasheff.

Vector Bundles and K-Theory, Allen Hatcher.

Geometry of Characteristic Classes, Shigeyuki Morita.

Office Hours

After class and by appointment.

Course Description

In the first part of the course, we will cover the theory of characteristic classes for vector bundles, in particular those named after Stiefel and Whitney, Euler, Pontryagin, and Chern. In the second part, we will study the analogous characteristic classes for surface bundles. Towards the end of the course we will prove Morita's theorem that the mapping class group of a surface S does not act on S by diffeomorphisms.

Homework

Optional homework will be assigned throughout the semester. Graduate students will also be expected to give one lecture on a related topic in the Geometry-Topology Student Seminar.

Grades

Grades will be based on attendance, homework, and seminar presentations.

Honor Code

All students are expected to abide by the student honor code: *http://www.honor.gatech.edu*

Class Outline

There are 16 weeks in the semester, with the 4th, 9th, and 15th compromised by holidays. The rough, tentative schedule is as follows:

Week 1. Introduction & overview, Vector bundles
Week 2. Classifying vector bundles
Week 3. Leray-Hirsch theorem, Stiefel-Whitney and Chern classes via axioms
Week 4. Cohomology of the Grassmannian
Week 5. Applications of Stiefel-Whitney classes
Week 6. Thom isomorphism and Gysin sequence
Week 7. Euler class
Week 8. Characteristic classes of 4-manifolds

Week 9. Surface bundles, Contractibility of Diff
Week 10. Classifying spaces for surface bundles
Week 11. Construction of Morita-Mumford-Miller classes, Mumford conjecture
Week 12. Hirzebruch Signature theorem and Pontryagin classes
Week 13. Nontriviality of the first Morita-Mumford-Miller class
Week 14. Mapping class groups do not act by diffeomorphisms
Week 15. Odd Morita-Mumford-Miller classes are geometric
Week 16. Finishing up