A Sample Problem

Among you, your buddy, two mothers, and two sisters, some people hug. There are no hugs between buddies, mothers, or sisters. The other 5 people tell you they all hugged different numbers of people. How many people did you hug?

Four Problems

The Bridges of Konigsberg

Three House-Three Utility

Four Color

Traveling Salesman

Graphs

A graph is a pair of sets V and E, where $V \neq \emptyset$ and each element of E is a pair of elements of V.

Write $G = G(V, E)$.

The elements of V and E are called vertices and edges.

Example. V = Facebook users
E = Friendships

The Handshaking Lemma

Proposition. The sum of the degrees of the vertices of a pseudograph is an even number. Specifically:

$$\sum_{v \in V} \deg v = 2|E|$$

Leonhard Euler

Handshaking Lemma. The number of odd degree vertices of a pseudograph is even.

Proof.

Revisit the hugging problem.

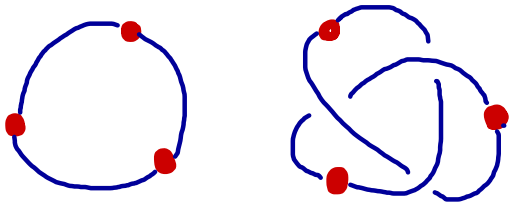
The Handshaking Lemma

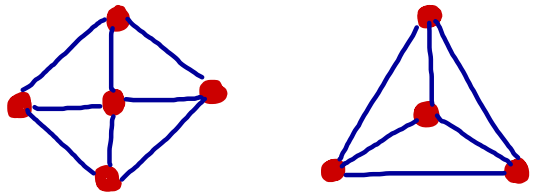
Problem. A graph has 50 edges, 4 vertices of degree 2, 6 of degree 5, 8 of degree 4, all other vertices have degree 6. How many vertices does the graph have?

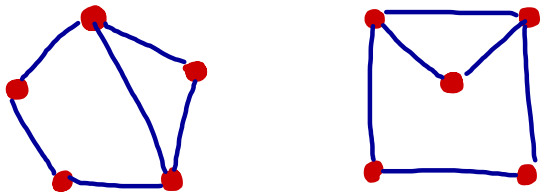
Problem. Out of 24 curling players, 78 pairs have played on the same team. Show that one has played on the same team as 7 others. Show that one has played on the same team with no more than 6 others.

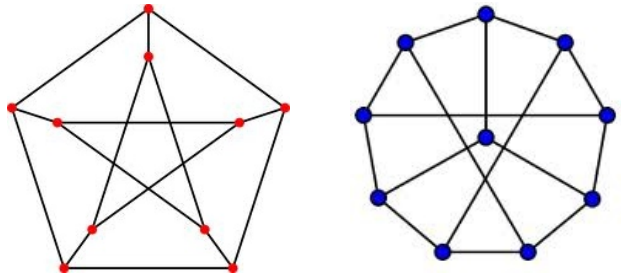
Graph Isomorphism

Which of the following pairs are isomorphic?

a) 

b) 

c) 

d) 

Invariants of Graphs

We can use the following "fingerprints" of graphs in order to tell if two graphs are different:

(i) Number of vertices
(ii) Number of edges
(iii) Degree sequence
etc.

It is possible for two graphs to have the same degree sequence and be nonisomorphic:

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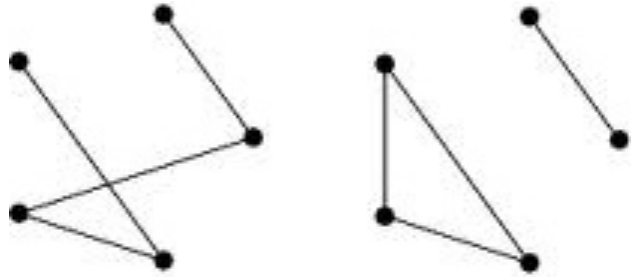
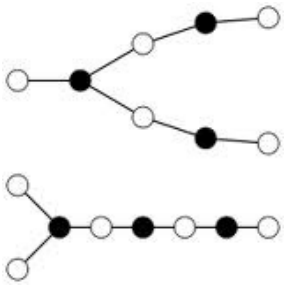
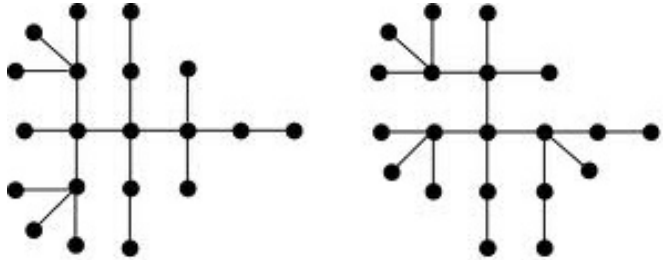


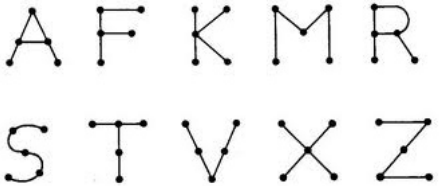
{2, 2, 2, 1, 1} {2, 2, 2, 1, 1}

EXAMPLES

Which of the following graphs are isomorphic?

a) 

b) 

c) 

d)