Ian Runnels seminar @ 2

Writing assignment Dec 9.

Chap 14. pA Theory.

pseudo-Anosov

$\varphi \cdot \mathcal{F}_u = \lambda \mathcal{F}_u$

$\varphi \cdot \mathcal{F}_s = \lambda^{-1} \mathcal{F}_s$

NTC. $f \in Mod(S)$

① periodic
② reducible
③ pA

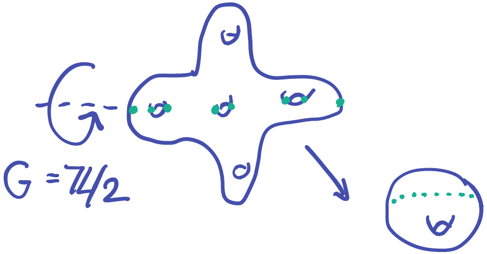
Today: constructions.

Construction #1 Branched covers.

$p: M \to N$ is a branched cover if it is a cover over $N \setminus B$, B small.

For surfaces: B = finite set.

Example. $G \curvearrowright S_g$ $|G| < \infty$.



$p: S_g \to X$ branched cover.

Assume $X \approx (T^2, B)$.

Take $\varphi: T^2 \to T^2$ Anosov.

Up to power, isotopy

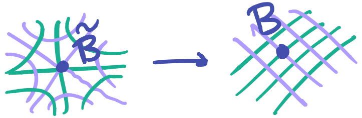
φ fixes B. (periodic pts dense)

Further power: φ lifts to S_g.

(lifting criterion)

The lift is pA. with

$\mathcal{F}_s$, $\mathcal{F}_u$ lifts of foliations

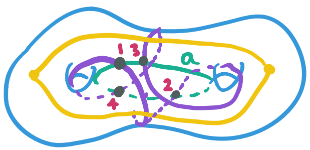
in T^2. 

Note: All resulting stretch factors are quadratic integers.

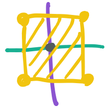
Construction #2 Thurston's construction

Thm $a, b \subseteq S_g$ filling.

$\exists$ sing Eucl. structure and



$$\rho : \langle T_a, T_b \rangle \longrightarrow PSL_2\mathbb{R}$$



$$f \longmapsto Df$$

$$T_a \longmapsto \begin{pmatrix} 1 & -i(a,b) \\ 0 & 1 \end{pmatrix}$$

$$T_b \longmapsto \begin{pmatrix} 1 & 0 \\ i(a,b) & 1 \end{pmatrix}$$

With:

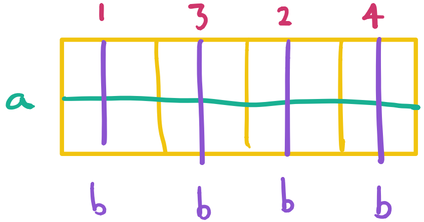
$\rho(f)$ elliptic $\Longleftrightarrow$ f periodic

$\rho(f)$ parabolic $\Longleftrightarrow$ f reducible

$\rho(f)$ hyperbolic $\Longleftrightarrow$ f pA e.g. $T_a T_b^{-1}$

Cor: $\exists$ pA's in $\mathcal{I}(S_g)$. (take a, b sep)

Pf. From $a, b \rightsquigarrow X$ = dual square complex.



T_a acts on Eucl. structure by $\begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix}$

Similar for T_b.

If $\rho(f)$ hyperbolic $\rightsquigarrow$ eigenvals λ, λ^{-1}
2 foliations

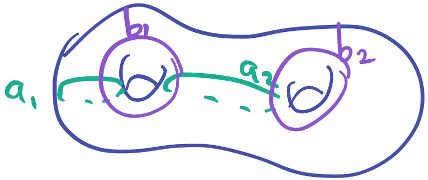
Those are stretch factor, foliations for f.

- There is a version with multicurves A, B.
- All resulting stretch factors totally real.

Penner's construction

Thm. $A = \{a_1, \dots, a_m\}$
$B = \{b_1, \dots, b_n\}$
filling multicurves.

Any
f = product of pos. powers of T_{a_i} & neg powers of T_{b_i} s.t. each a_i, b_i appears at least once.
is pA.



$$T_{a_2}^{15} T_{b_2}^{-7} T_{b_1}^{-1} T_{a_1}^{100}$$

Penner: Do all pA have a power coming from this construction?

Shin-Strenner: No. The Galois conjugates of Penner stretch factors all on S^1.

Construction # 3 Homological criterion.

$A \in Sp_{2g}\mathbb{Z} \Rightarrow$ char poly is monic & palindromic

Why? roots come in pairs λ, λ^{-1}

So do sub: $x^g P(\frac{1}{x})$

Why? $A^T J A = J \Rightarrow A^T \sim A^{-1}$

Thm (Casson-Bleiler, M-Spallone w/ Bestvina)

If char. poly of $\psi(f)$ satisfies:

① symplectically irred

② not cyclotomic

③ not poly in t^k, $k>1$.

Then f is pA.

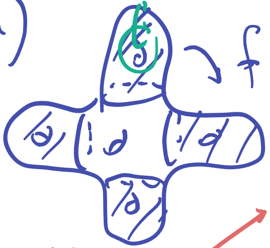
Pf. Suppose f not pA.

f periodic $\Rightarrow$ $\psi(f)$ has root of 1 as eigenval.

$\Rightarrow$ cyclotomic factor, violates 1 or 2.

f reducible, a power fixing nonsep $\Rightarrow$ as above.

f reducible, a power fixes a sep curve



$$\psi(f) = \left(\begin{array}{ccc|c} I & & B & \\ & I & & 0 \\ & & I & \\ \hline & 0 & & C \end{array}\right)$$

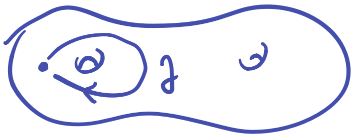
If C there, violate ① — not 0×0.

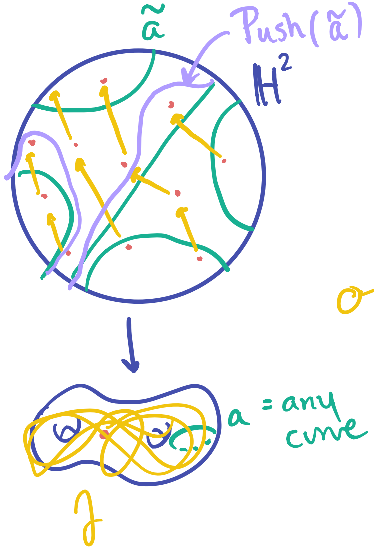
If the I's are +, violate 3

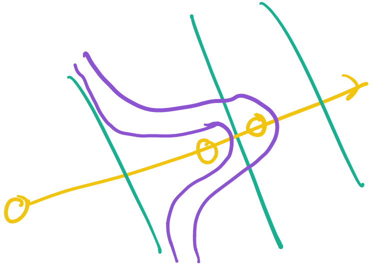
action on H_1(middle)

Construction #4 Kra's construction.

$\text{Push} : \pi_1(S_g) \longrightarrow \text{Mod}(S_{g,1})$





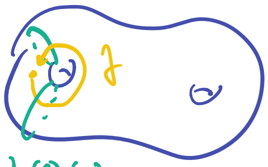


Thm. $\text{Push}(\gamma)$ is pA

$\iff \gamma$ filling

Pf. Enough to show:

γ filling $\Longrightarrow \text{Push}(\gamma)$ does not fix any curve.



Suppose $\text{Push}(a) \simeq a$ ← homotopic.

Then could lift homotopy

cf. Dowdall's thesis.