Matrix Algebra

(1) Solve for X:

$$(A - AX)^{-1} = X^{-1}B$$

(2) If B is the inverse of A^3, then what is the inverse of A?

(3) What is the inverse of this matrix?

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(4) Suppose that A and B are $n \times n$ matrices and that B and AB are invertible. Is A invertible?

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Cryptography

- Encode letters by numbers:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

- Choose a matrix, A, say $n \times n$.
- Break messages into blocks of size n, which gives us (a set of) vectors.
- Apply A to each vector to get encrypted message.

Example. $A = \begin{pmatrix} 2 & 3 & 1 \\ 2 & 0 & 1 \\ 0 & 0 & 6 \end{pmatrix}$, and the encoded message is $\begin{pmatrix} 112 \\ 52 \\ 36 \end{pmatrix}$

What is the encoded message?

We can also ask:

- After decoding one message, can you use the same matrix to decode other messages?
- Can you decode $(76, 37, 42)$? $(81, 36, 72)$?

The Invertible Matrix Theorem

True/False

Are the following statements always true or sometimes false? Explain your answer.

1. If A has two identical columns then A is not invertible.
2. If A is an invertible $n \times n$ matrix then the columns of A^{-1} span $\mathbb{R}^n$.
3. If $Ax = b$ is consistent for all b in $\mathbb{R}^n$ then $Ax = 0$ has exactly one solution.
4. If $Ax = 0$ has only the trivial solution then A is invertible.

Linear Transformations and Inverses

Which of the following linear transformations of $\mathbb{R}^3$ have invertible standard matrices?

- projection to xy-plane
- rotation about z-axis by π
- reflection through xy-plane

The Invertible Matrix Theorem

Which of the following are equivalent to the statement that A is invertible?

m) rows of A span $\mathbb{R}^n$

n) rows of A are linearly independent

o) $Ax = b$ has exactly one solution for all b in $\mathbb{R}^n$

p) $\det(A) \neq 0$, where $\det(A)$ is the volume of the parallelepiped formed by the columns of A

q) A^3 is invertible