Johnson I Computation for S_3

Tuesday, March 13, 2018 11:27 AM

Following Johnson's notation and convention:

1. A chain (or subchain) of curves

   $c_{i_1}, \dots, c_{i_k}$ is denoted $ch(i_1 \dots i_k)$

2. The bounding pair map determined by

   a chain is denoted $[i_1 \dots i_k \, i_{k+1}]$

3. The operation $*$ is conjugation.

   i.e. $g \, \mathrm{Mod}(S_3)$. $g * f = g f g^{-1}$

4. Convention for Dehn twists:

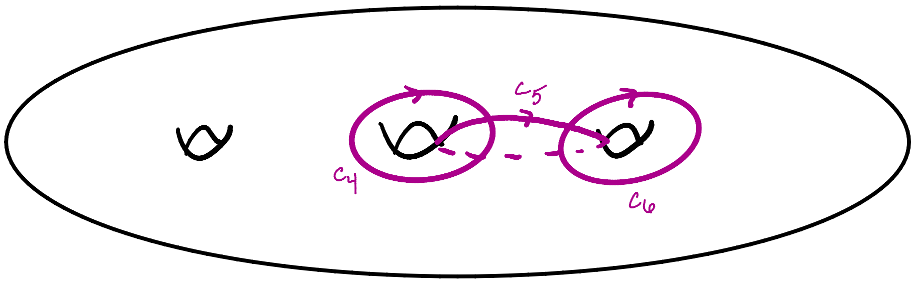
   positive $\rightsquigarrow$ turn right

The map $[4\,5\,6\,7] \in \mathcal{J}$ = "Johnson generators"

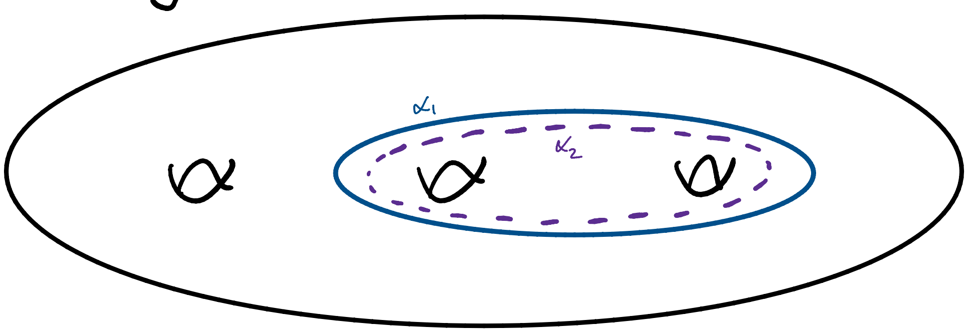
We want to show $T_b * [4567] \in \mathcal{J}$

Corollary 2 in Johnson I shows $T_b * [4567] = [\beta 567]$

The chain $ch(456)$:



The bounding pair map $[4567] = T_{\alpha_1} T_{\alpha_2}^{-1}$



There are two ways to consider $T_b * [4567]$

① Apply T_b to each curve in the chain $ch(456)$

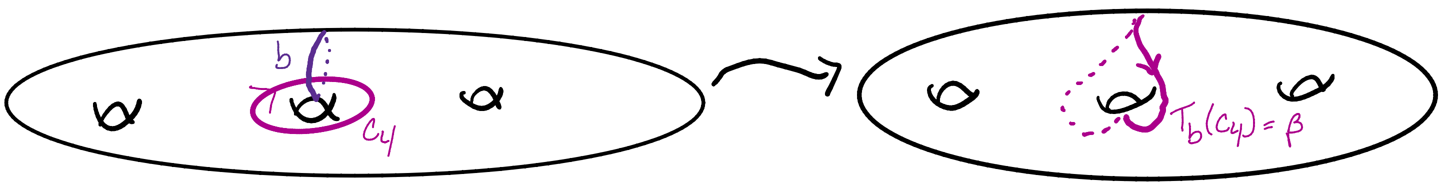
Then the new map is the bounding pair

given by $[T_b(c_4), T_b(c_5), T_b(c_6), 7]$

Note: $\hat{\iota}(b, c_5) = 0$ and $\hat{\iota}(b, c_6) = 0$

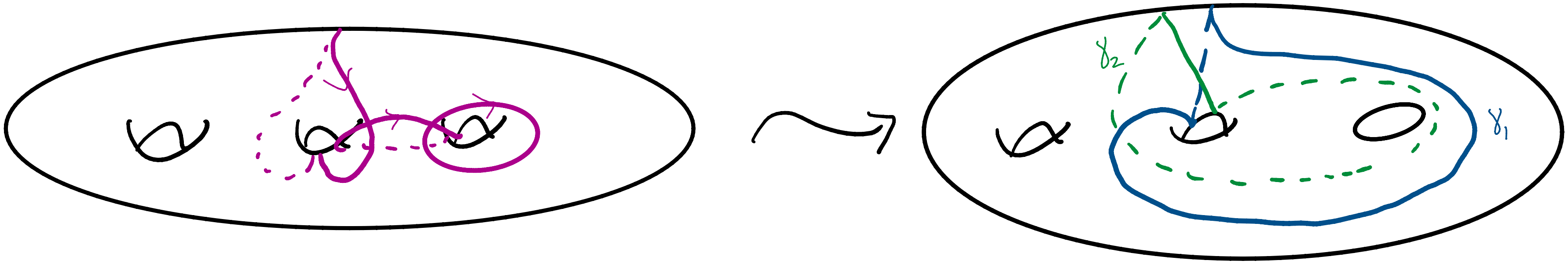
So T_b fixes these curves.

For $T_b(c_4)$:



Thus we get $T_b * [4567] = [\beta\, 567] \in \mathcal{J}$

(β-subchains are defined as Johnson generators)



$ch(\beta 5 6)$

$[\beta 5 6 7] = T_{\gamma_1} T_{\gamma_2}^{-1}$

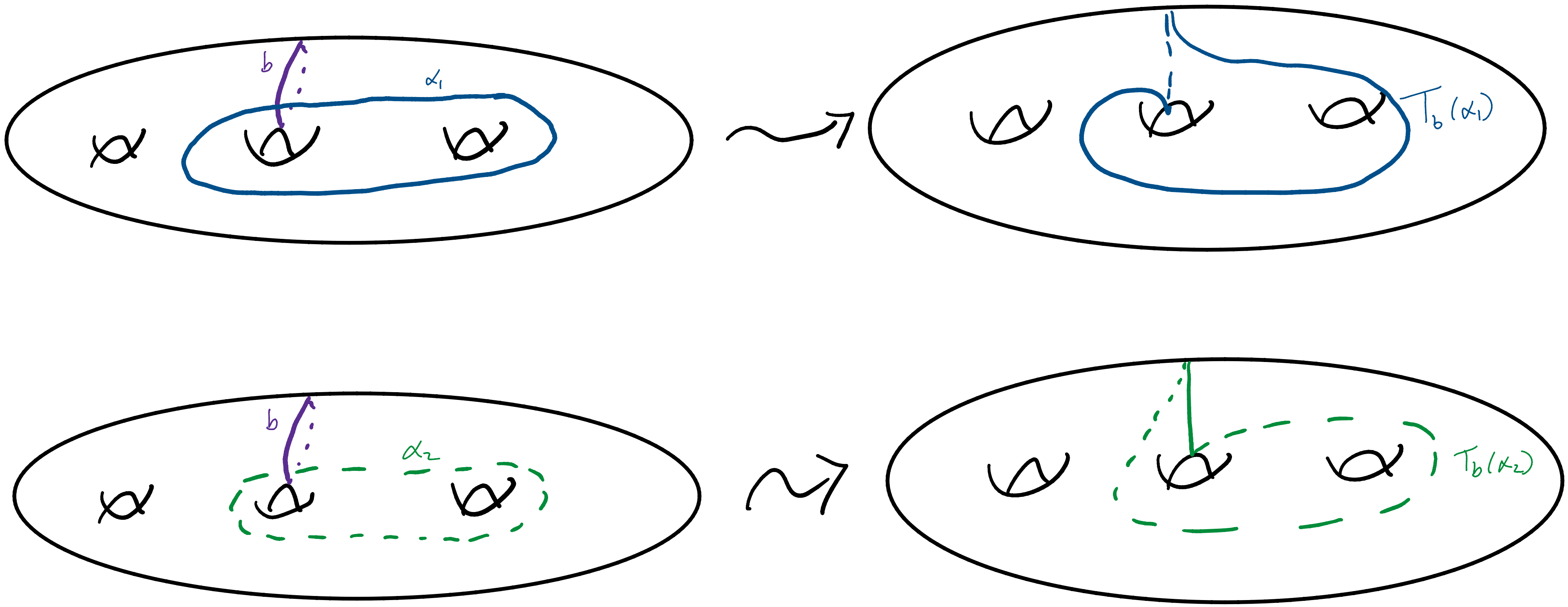
② The second way to see $T_b * [4567] \in J$:

$$T_b * [4567] = T_b * \left(T_{\alpha_1} T_{\alpha_2}^{-1} \right)$$

$$= T_b \, T_{\alpha_1} T_{\alpha_2}^{-1} T_b^{-1}$$

$$= T_{T_b(\alpha_1)} T_{T_b(\alpha_2)}^{-1}$$

i.e. we obtain a new bounding pair map by applying T_b to the curves α_1, α_2:



Note that these pictures show

$T_b(\alpha_1) = \gamma_1$ and $T_b(\alpha_2) = \gamma_2$

So $T_b * [4567] = T_{\gamma_1} T_{\gamma_2}^{-1} = [\beta 567] \in J$