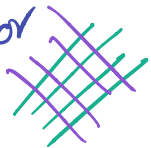# Chap 15. Thurston's Proof

Reference. Thurston's Work on Surfaces.

Fathi, Laudenbach, Poénaru

NTC. $f \in \text{Mod}(S)$ is

① periodic
② reducible
③ pseudo-Anosov

Setup. $\mathcal{S} = \{ \text{s.c. curves in } S \} / \text{isotopy}$

$$\text{Teich}(S) \hookrightarrow \mathbb{P} \mathbb{R}^{\mathcal{S}}$$

fns $\mathcal{S} \to \mathbb{R}$

$$\mathcal{P} \text{MF}(S) \hookrightarrow \mathbb{P} \mathbb{R}^{\mathcal{S}}$$

Thm. $\text{PMF}(S) \cong S^{\dim \text{Teich}(S) - 1}$

$\text{Teich}(S) \cup \text{PMF}(S)$ is a closed ball, on which $\text{Mod}(S)$ acts continuously.

Thm. $PMF(S) \cong S^{\dim \text{Teich}(S) - 1}$

     $\text{Teich}(S) \cup PMF(S)$ is

        a closed ball, on which

    $\text{Mod}(S)$ acts continuously.

Pf of NTC. Brouwer $\Rightarrow$ f fixes

some X in the ball.

    $X \in \text{Teich}(S) \Rightarrow f$ periodic.

    $X \in PMF$ & X has closed leaf $\Rightarrow$ reducible.

        & X has no closed leaf

            & $\lambda = 1 \Rightarrow$ periodic

            & $\lambda > 1 \Rightarrow pA$ $\square$

Torus case:

$PMF(T^2)$

$\text{Teich}(T^2)$

$\leftarrow$ rat'l pts

$\leftrightarrow$ curves

$PMF(S)$

$\text{Teich}(S)$

$PMF(S)$ $\mathbb{R}^d$