Math 1553 Worksheet §1.3

If you don't have a computer, find someone who does.

1. Let
$$v_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$
 $v_2 = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix}$ $w = \begin{pmatrix} 2 \\ -4 \\ 8 \end{pmatrix}$.

Question: Is w in Span $\{v_1, v_2\}$?

- a) Formulate this question as a vector equation.
- **b)** Formulate this question as a system of linear equations.
- c) Formulate this question as an augmented matrix.
- **d)** Answer the question using the interactive demo.
- e) Answer the question using row reduction.

2. Consider the system of linear equations

$$x + 2y = 7$$

$$2x + y = -2$$

$$-x - y = 4$$

Question: Does this system have a solution? If so, what is the solution set?

- a) Formulate this question as an augmented matrix.
- **b)** Formulate this question as a vector equation.
- c) What does this mean in terms of spans?
- d) Answer the question using the interactive demo.
- e) Answer the question using row reduction.

3. Consider the vector equation

$$x \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + y \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} + z \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ -2 \end{pmatrix}.$$

Question: Is there a solution? If so, what is the solution set?

- a) Formulate this question as an augmented matrix.
- **b)** Formulate this question as a system of linear equations.
- c) What does this mean in terms of spans?
- d) Answer the question using the interactive demo.
- **e)** Answer the question using row reduction.

4. Consider the augmented matrix

$$\begin{pmatrix}
2 & -2 & 2 & 0 \\
1 & -3 & -4 & -9 \\
3 & -1 & 8 & 9
\end{pmatrix}$$

Question: Does the corresponding linear system have a solution? If so, what is the solution set?

- a) Formulate this question as a vector equation.
- **b)** Formulate this question as a system of linear equations.
- c) What does this mean in terms of spans?
- **d)** Answer the question using the interactive demo.
- e) Answer the question using row reduction.
- f) Find a different solution in parts (e) and (d).
- **5.** Decide if each of the following statements is true or false. If it is true, prove it; if it is false, provide a counterexample.
 - a) Every set of four or more vectors in \mathbb{R}^3 will span \mathbb{R}^3 .
 - **b)** The span of any set contains the zero vector.