

GENERATING TORELLI

Goal: $I(S_g)$ is gen. by BP maps (and Dehn twists about sep curves)

Original proof: 1971 Birman gives presentation for $Sp_{2g}(\mathbb{Z})$
1978 Powell interprets relations
1980 Johnson, lantern relation

Want a proof analogous to $\text{Mod}(S_g)$ case.

Complex of homologous curves

Fix (primitive) $x \in H_1(S_g; \mathbb{Z})$

$C_x(S_g)$ = subgraph of $C(S_g)$ spanned by
(unoriented) reps of x .

goal: connected.

“borrowing complex”
↙

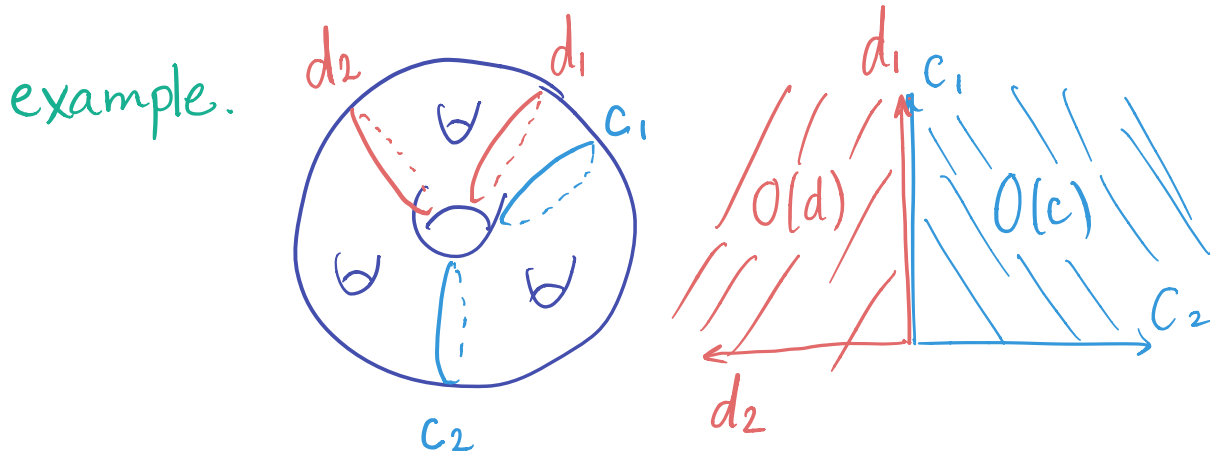
Will use auxilliary complex $B_x(S_g)$, the complex of cycles. Points of $B_x(S_g)$ are simple, irredundant reps of x .

The Complex of Cycles

C = oriented multicurve, n components

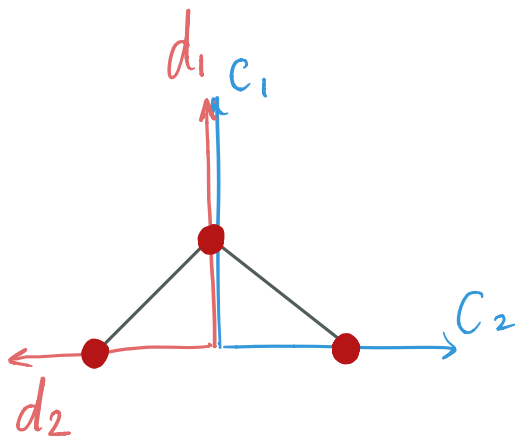
$$\leadsto [0, \infty)^n \rightarrow H_1(S_g; \mathbb{Z}) \quad \text{orthant } O(c)$$

$$A(S_g) = \bigsqcup_c O(c) / \sim$$

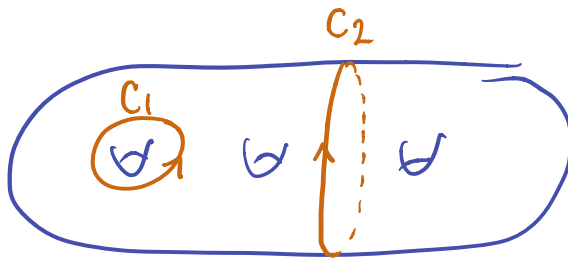


$$A_x(S_g) \subseteq A(S_g) \quad \text{reps of } x.$$

Say $x = [c_1]$

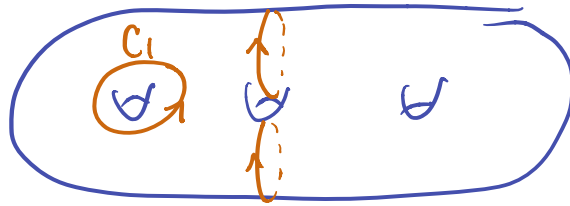


The cells of $A_x(S_g)$ are not necessarily compact:



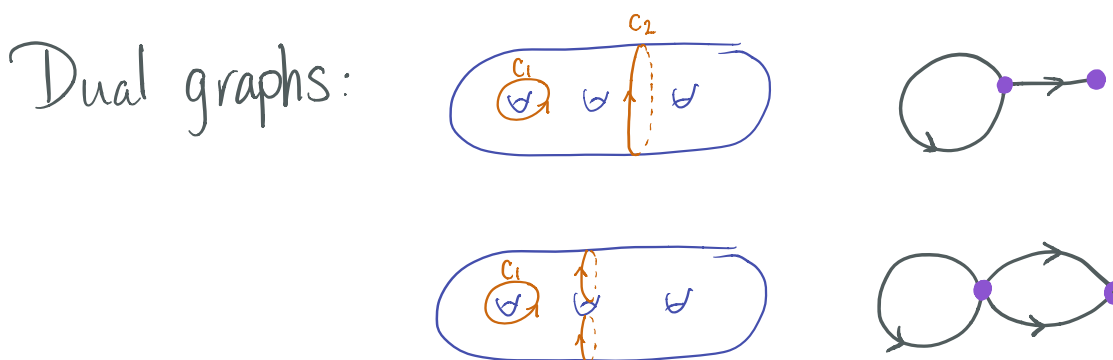
If $[c_1] = x$ then $[c_1 + bc_2] = x \quad \forall b \in \mathbb{R}$

Or:



An oriented multicurve is reduced if

- (1) the corresponding cell is compact
- \iff (2) it has no homologically trivial subset
- \iff (3) the dual directed graph is recurrent

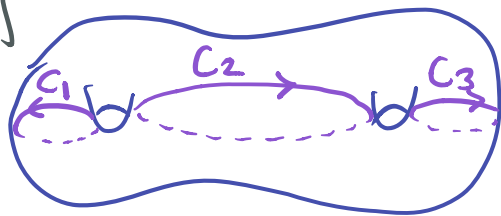


The complex of cycles $B_x(S_g)$ is the subcomplex of $A_x(S_g)$ whose cells correspond to reduced oriented multicurves.

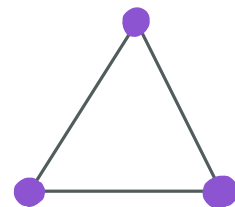
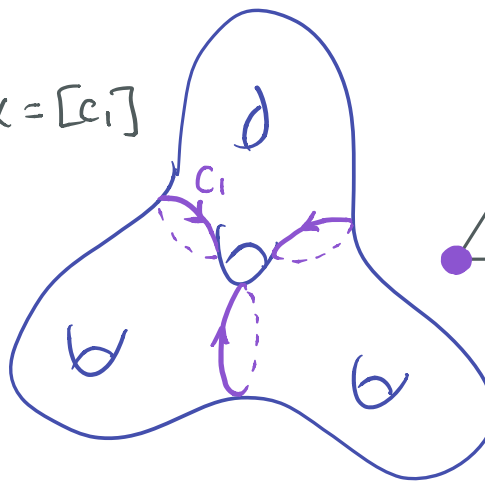
We'll show $B_x(S_g)$ is contractible.

Examples of cells

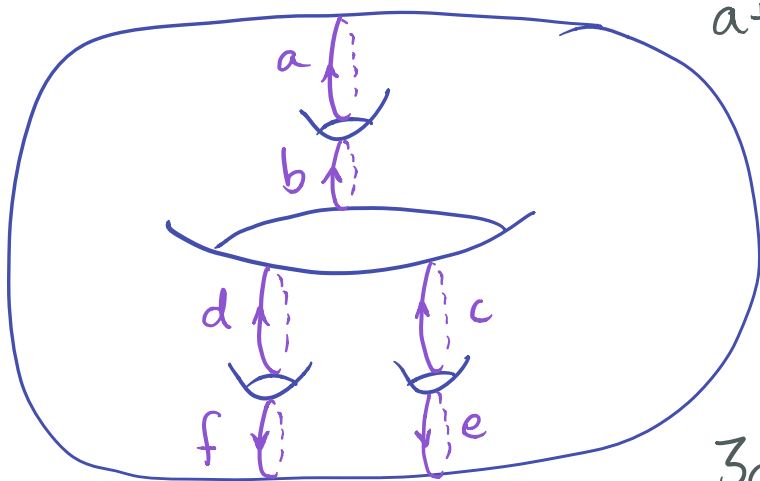
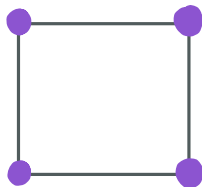
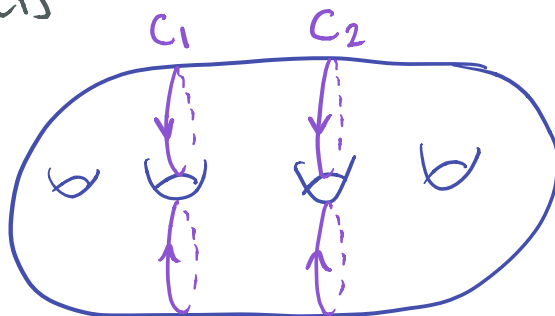
$$x = [c_1]$$



$$x = [c_1]$$



$$x = [c_1] + [c_2]$$



$$a+b+2c+2f$$

$$c+e+2f$$

$$d+e+2f$$

$$3a+3b+2c+2d$$

$$a+b+2d+2e$$

Q. Which polytopes arise?

Properties of Cells

Prop. The dim. of a cell = # compl. comp.'s - 1.

Pf. Defn of homology.

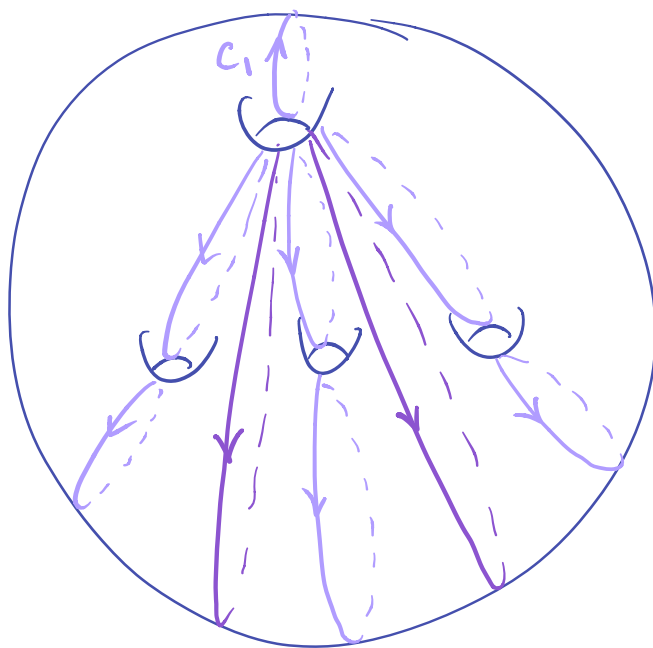
\Rightarrow vertices \longleftrightarrow nonsep. multicurves.

Prop. Vertices of $B_X(S_g)$ are oriented multicurves with integral weights.

Pf. Given a vertex, consider a loop intersecting in one point.

Prop. $\dim B_X(S_g) = 2g - 3$.

Pf.



$x = [c_1]$.