Mathematics 6441

Midterm

Prof. Margalit

Due March 13 2022

This is a take-home exam due March 13th at 11:59 pm on Gradescope. You may use the textbook, the lecture notes, your own notes, and your homework. You may not use any other sources or discuss with anyone else. From the time you first look at the questions to the time you turn in your assignment, you should not look at any outside sources.

1. Let $X = \mathbb{R}P^2 \vee S^1$. Describe all covering spaces for X that have degree 2 or 3.

2. Determine all homotopy classes of maps $\mathbb{R}P^2 \to S^1$. Determine all homotopy classes of maps $S^1 \to \mathbb{R}P^2$.

3. Describe the covering space of $S^1 \vee S^1$ corresponding to the commutator subgroup

$$[F_2, F_2] \leqslant F_2 \cong \pi_1(S^1 \vee S^1)$$

What familiar group is the group of deck transformations isomorphic to? Describe the isomorphism explicitly. Use the covering space to deduce that $[F_2, F_2] \cong F_\infty$.

4. Construct a 2-dimensional CW-complex whose fundamental group is $\mathbb{Z} \times \mathbb{Z}/2$ (and prove it).

5. Consider the space X obtained from S^2 by identifying the north and south poles. Explicitly describe a Δ-complex structure on X and use it to compute the simplicial homology groups of X directly from the definitions.