INDUCTION

Prove the following statements by induction.

(1) $\sum_{i=1}^{n} i = n(n+1)/2 \qquad n \geq 1$

(2) $\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6 \qquad n \geq 1$

(3) $\sum_{i=1}^{n} (2i-1) = ?? \qquad n \geq 1$

TOWERS OF HANOI

Use induction to show that it is possible to solve the Towers of Hanoi puzzle with n disks.

INDUCTION

Prove the following statements by induction.

(1) $7^n - 1$ is divisible by 6 for all $n \geq 0$

(2) $n^2 + 2n$ is divisible by 3 for all $n \geq 0$

(3) $(2n)!$ is divisible by 2^n for $n \geq 0$.

INDUCTION

Prove the following statements by induction.

(1) $n! > 2^n \qquad n \geq 4$

(2) $\frac{1}{n+1} + \cdots + \frac{1}{2n} \geq \frac{1}{2} \qquad n \geq 1$

(3) $(1 + 1/2)^n \geq 1 + n/2 \qquad n \geq 0.$

(4) $(1 + x)^n \geq 1 + nx \qquad n \geq 0$

INDUCTION

Prove the following statements by induction.

(1) The interior angle sum of a convex n-gon is $(n-2)\pi$.

(2) If n lines in $\mathbb{R}^2$ have no triple intersections then they divide the plane into $n+1$ regions.

(3) $F_1 + F_3 + \cdots + F_{2n-1} = F_{2n}$

OTHER INDUCTIONS

Which of the following are correct forms of induction?

(1) If $P(n_0)$ is true and $P(k+1)$ is true whenever $P(k)$ and $P(k-1)$ are true $(k > n_0)$ then $P(n)$ is true for $n \geq n_0$.

(2) If $P(n_0)$ is true and $P(k)$ is true whenever $P(n_0), \ldots, P(k-1)$ are true $(k > n_0)$ then $P(n)$ is true for all $n \geq n_0$

(3) If $P(5)$ is true and $P(k)$ is true whenever $P(k-1)$ is true then $P(n)$ is true for all $n \geq 5$.

MORE INDUCTION

Prove the following statements by induction.

(1) Every natural number has a prime factorization.

(2) In a convex n-gon one can draw at most $n-2$ non-intersecting diagonals.

(3) The number of ways of breaking a $2 \times n$ candy bar into 2×1 pieces is F_{n+1}