Announcements April 1

- Homework 8 due now
- Quiz 8 on 5.2 and 5.3 in class today
- WebWork 5.3 and 5.5 due Thursday
- Homework 7 due Friday April 8
- Midterm 3 in class Friday April 8 on Chapter 5
- Office Hours next week to be decided soon.
- LA Office Hours: Scott Mon 12-1, Yashvi Mon 2-3, Shivang Tue 5-6, Baishen Wed 4-5, Matt Thu 3-4
- Math Lab, Clough 280
  - Regular hours: Mon/Wed 11-5 and Tue/Thu 11-5
  - Math 1553 hours: Mon-Thu 5-6 and Tue/Thu 11-12
  - LA hours: Matt Tue 11-12, Scott Tue 5-6, Baishen Thu 11-12, Yashvi/Shivang Thu 5-6

Population dynamics

Suppose that there are two cities A and B. Every year 70% of the people from A move to B and 30% stay. Likewise, 10% of the people from B move to A and 90% stay.

Say the initial populations are (a_0, b_0). What happens after n years?

The largest eigenvalue is... .

So the populations in the two cities approach this eigenspace. What is it?

See: http://setosa.io/ev/eigenvectors-and-eigenvalues/

Application: Business

Say your car rental company has 3 locations. Make a matrix M whose ij entry is the probability that a car at location i ends at location j. For example,

$$M = \begin{pmatrix} .3 & .4 & .5 \\ .3 & .4 & .3 \\ .4 & .2 & .2 \end{pmatrix}$$

Note the columns sum to 1 (again).

The largest eigenvalue is 1.

All other vectors get pulled towards the 1–eigenvector:

This eigenvector is the steady-state.

Application: Google Pagerank

Say the internet has pages $P_1, \ldots, P_N$

Denote the importance of P_i by $\mathrm{imp}(P_i)$.

Then:

$$\mathrm{imp}(P_i) = \sum_j \left(\frac{\#\text{outgoing links from } P_j \text{ to } P_i}{\#\text{outgoing links from } P_j} \right) \mathrm{imp}(P_j)$$

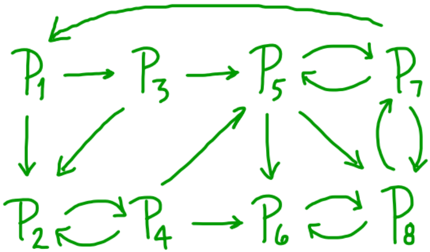
To find the importance of each page, find the 1–eigenvector for the matrix with ij-entry equal to:

$$\frac{\#\text{outgoing links from } P_j \text{ to } P_i}{\#\text{outgoing links from } P_j}$$

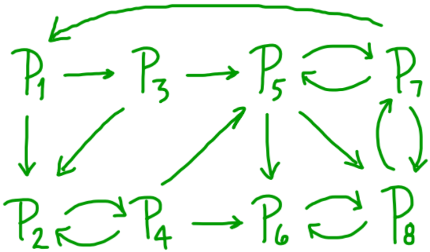
This is just like the previous two problems! Imagine everyone on the internet starts at a random page and clicks once per second. After many clicks, the most important pages will have the most users at that page.

For Google, the matrix is of size 30 trillion(!). Can't compute eigenvectors algebraically. Use the iterative method.

Application: Google Pagerank



Application: Google Pagerank



$$H = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/3 & 0 \end{pmatrix} \rightsquigarrow I = \begin{pmatrix} .06 \\ .07 \\ .03 \\ .06 \\ .09 \\ .20 \\ .18 \\ .31 \end{pmatrix}$$

Eigenvectors, eigenvalues, and diagonalization

Answer yes / no / maybe.

1. A 3×3 matrix has real entries. Can its eigenvalues be 3, 5, and $2 + i$?

2. A 3×3 matrix has one eigenvalue with algebraic multiplicity three. Is it diagonalizable?

3. A 3×3 matrix has two distinct eigenvalues. Is it diagonalizable?

4. A 3×3 matrix has three distinct eigenvalues. Is it invertible?

5. If the 3×3 matrices A and B both have the eigenvalues $-1, 0, 1$, then A is similar to B.

6. If the 3×3 matrices A and B have the same eigenvalues, then A is similar to B.

Diagonalization

Diagonalize the following matrix or explain why it cannot be diagonalized:

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

Complex eigenvalues

Find the (complex) eigenvalues and eigenvectors:

$$\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$$