Eigenvectors and eigenvalues

Find the eigenvalues and bases for each corresponding eigenspace:

$$\begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Find char poly: $\det(A - \lambda I) \rightsquigarrow \pm\lambda^3 + \_\lambda^2 + \_\lambda + \_$

Strategies

1. Don't multiply out: maybe factor out something!

   maybe: $(\lambda - 1)(\text{stuff}) \rightsquigarrow \lambda = 1 \ldots$

2. Multiply it out

   maybe: $\lambda^3 + 5\lambda^2 + \lambda \rightsquigarrow \lambda = 0 \ldots$

3. Guess roots: try $\lambda = 0, 1, -1, 2, -2$

   factor out λ, $\lambda - 1$, $\lambda + 1$, etc...

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$$\begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\det \begin{pmatrix} 2-\lambda & 3 & 1 \\ 3 & 2-\lambda & 4 \\ \boxed{0} & \boxed{0} & -1-\lambda \end{pmatrix} = (-1-\lambda)\big((2-\lambda)(2-\lambda)-9\big)$$

$\lambda = -1$ $\qquad\qquad$ $\lambda = -1, 5$

alg multip. of -1 is 2.

To find bases for eigenspaces: $\boxed{\begin{aligned} &\mathrm{Nul}(A+I) \\ \&\ &\mathrm{Nul}(A-5I) \end{aligned}}$

$$\boxed{\begin{aligned} &(-1-\lambda)(\text{stuff}) + (\text{stuff})(-1-\lambda) \\ &\rightsquigarrow (-1-\lambda)(\text{stuff}+\text{stuff}) \end{aligned}}$$

Eigenvectors and eigenvalues

Find the eigenvalues and bases for each corresponding eigenspace:

$$\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Eigenvectors and eigenvalues

Find the eigenvalues and bases for each corresponding eigenspace:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Eigenvectors and eigenvalues

Find the eigenvalues and bases for each corresponding eigenspace:

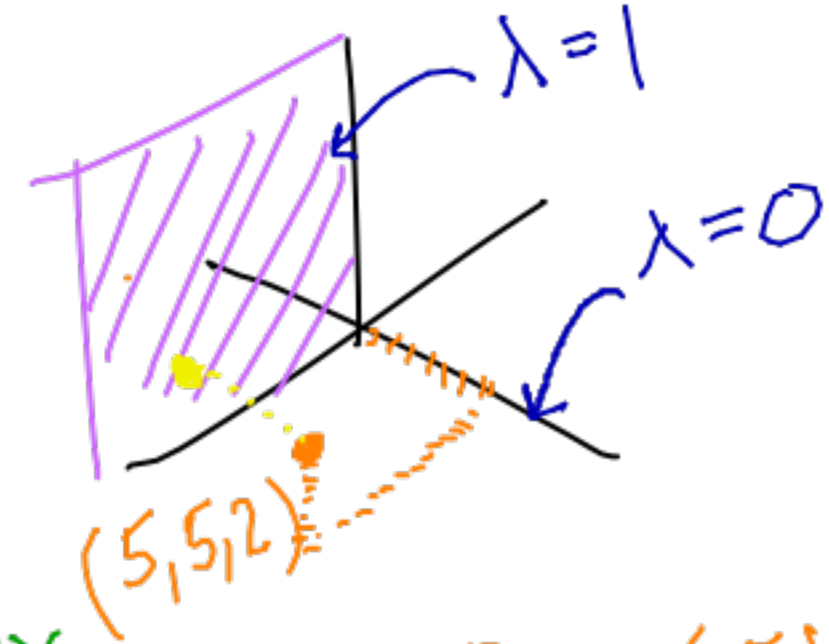
$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Eigenvectors and eigenvalues
Linear transformations

Find the eigenvectors/eigenvalues for A without doing any matrix calculations.

- $\boxed{T_A = \text{identity transformation of } \mathbb{R}^3}$
- $T_A =$ orthogonal projection to xz-plane in $\mathbb{R}^3$
- $T_A =$ counterclockwise rotation by $\pi/4$ in $\mathbb{R}^2$
- $T_A =$ reflection about $y = 2x$

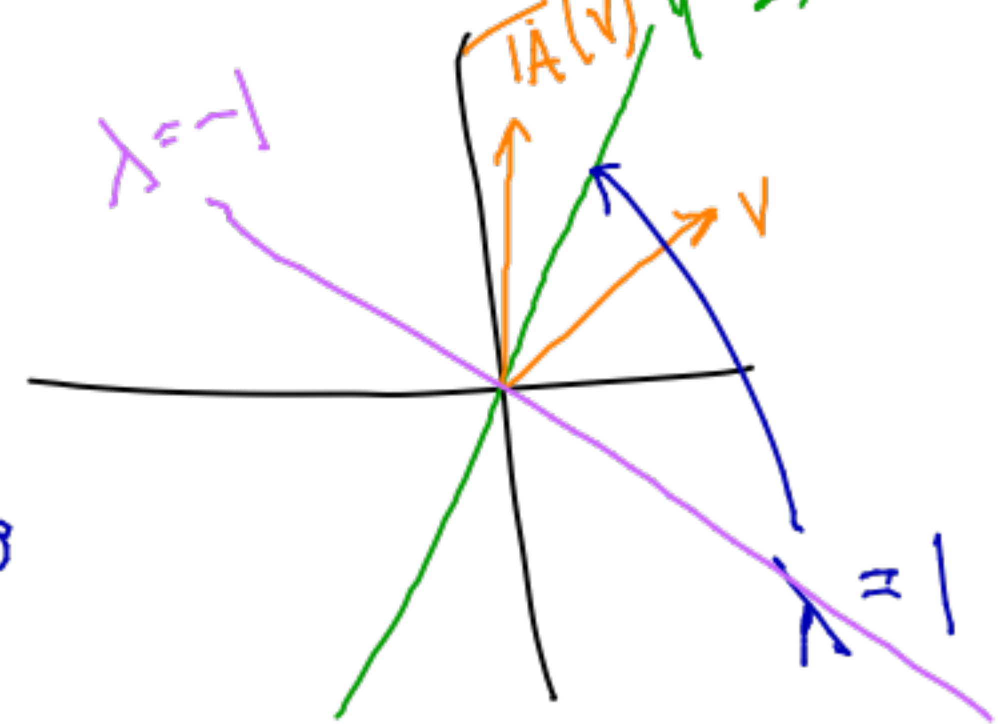
no eigenvector



$$T_A\begin{pmatrix}5\\5\\2\end{pmatrix} = \begin{pmatrix}5\\0\\2\end{pmatrix}$$

$$T_A\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x\\0\\z\end{pmatrix}$$

0



Find the eigenvectors/eigenvalues for A without doing any matrix calculations.

- T_A = identity transformation of $\mathbb{R}^3$

$$T_A(\boxed{v}) = \boxed{v}$$

All nonzero vectors are eigenvectors with eigenvalue 1.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Eigenvectors and difference equations

Say we want to solve

$$x_{k+1} = Ax_k$$

In other words, we need a sequence $x_0, x_1, x_2, \ldots$ with

$$x_1 = Ax_0, \quad x_2 = Ax_1, \quad \text{etc.}$$

Example. $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a+b \end{pmatrix}.$

Structural engineering
Column buckling

Say we have a column with a compressive force. How will the column buckle?

Approximate column with finite number of points, say

$$(0,0),(0,1),(0,2),\ \ldots\ (0,5),(0,6)$$

Buckling leads to (roughly)

$$(0,0),(x_1,1),(x_2,2),\ \ldots\ (x_5,5),(0,6)$$

Engineers use a difference equation to model this

$$x_{i-1} - 2x_i + x_{i+1} + \lambda x_i = 0$$

Structural engineering

Column buckling

Difference equation for column buckling:

$$x_{i-1} - 2x_i + x_{i+1} + \lambda x_i = 0$$

We solve this difference equation as above. We need the eigenvector of

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

For most λ, the only eigenvector is 0, which corresponds to no buckling. This matrix has three eigenvalues, 1, 2, and 3.