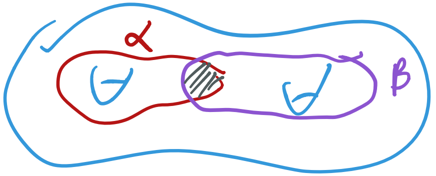
Chapter 1 Highlights

① Geometric int

$$i(\alpha,\beta) = \min_{\substack{\alpha' \sim \alpha \\ \beta' \sim \beta}} |\alpha' \cap \beta'|$$



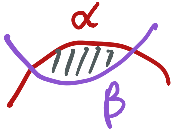
$i(\alpha,\beta)=0$

$\leftarrow$ function on pairs of homotopy classes.

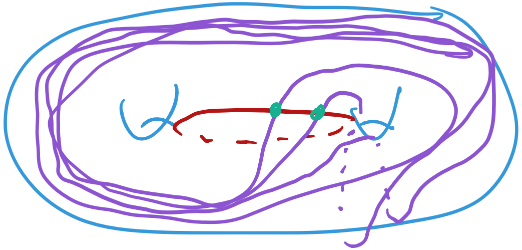
② Bigon criterion

α, β are in minimal position (they realize $i(\alpha,\beta)$)

$\Longleftrightarrow$ they do not form a bigon

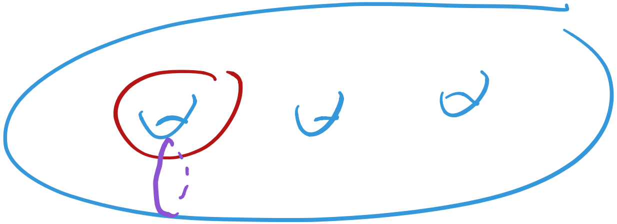


embedded closed disk



③ Change of coordinates principle.

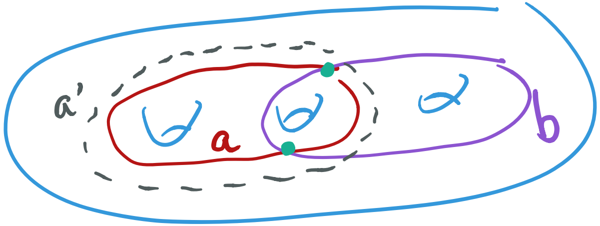
Example. if $i(a,b) = 1$ then it's this pic



$T_a T_b T_a$
$= T_b T_a T_b$

Geometric intersection number

Observ. 1 $i(a,b) \neq |\hat{i}(a,b)|$



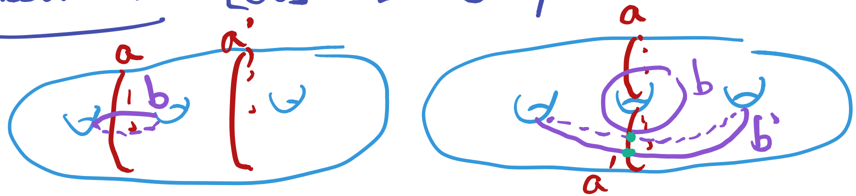
$i(a,b) = 2$

$\hat{i}(a,b) = 0$

Bigon crit $\Longrightarrow$ min. pos.

homology class

Observ. 2 $[a] = [a'] \not\Longrightarrow i(a,b) = i(a',b)$

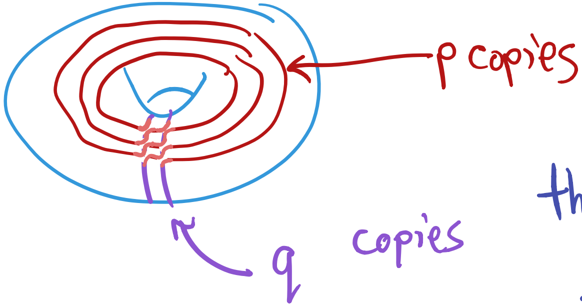


Fact. On T^2: $\left\{ \begin{matrix}\text{hom. classes of} \\ \text{simple closed curves}\end{matrix} \right\} \longleftrightarrow$ primitive elts of $\mathbb{Z}^2 / \pm$

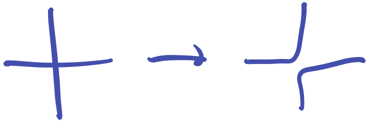
(not an integer multiple, so $(5,10)$ not prim.)

The map $\longleftarrow$ is:

$(p,q) \longrightarrow$



then surger:



Fact. $i((p,q),(r,s)) = \left| \begin{matrix} p & r \\ q & s \end{matrix} \right|$

Pf. First check for $(p,q)=(1,0)$.

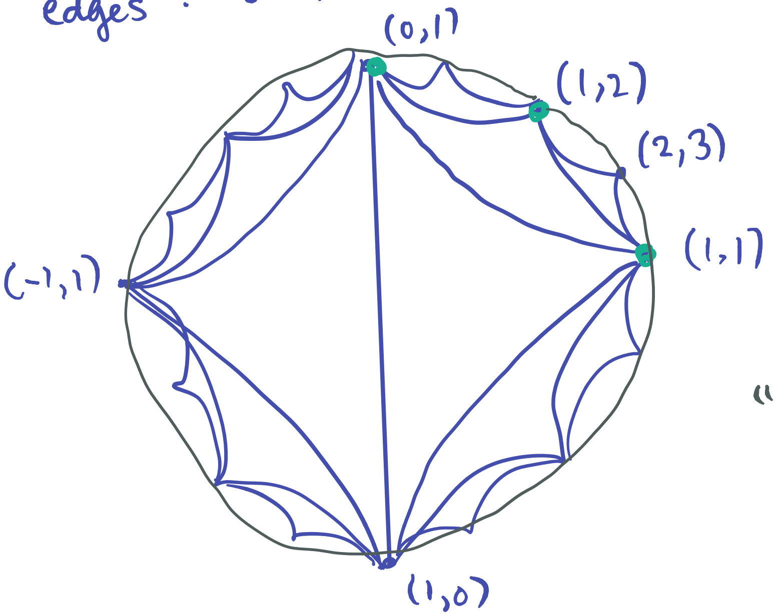
General case: apply $A \in SL_2\mathbb{Z}$ s.t. $A\binom{p}{q} = \binom{1}{0}$.

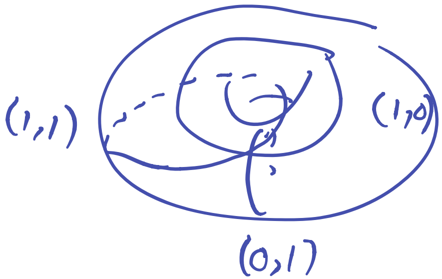
$\uparrow$ lin. map of T^2

Farey graph

vertices : s.c.c. on T^2 (hom. classes)

edges : $i = 1$



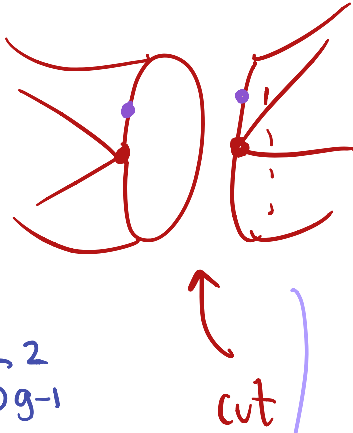


"complex of curves for T^2"

③ Change of Coords Principle

First example: $\alpha, \beta \subseteq S_g$ nonsep.

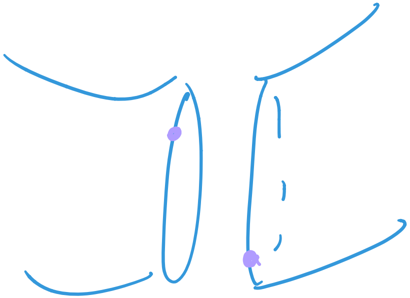
$\exists\ h \in \mathrm{Homeo}(S_g)$ st $h(\alpha) = \beta$.

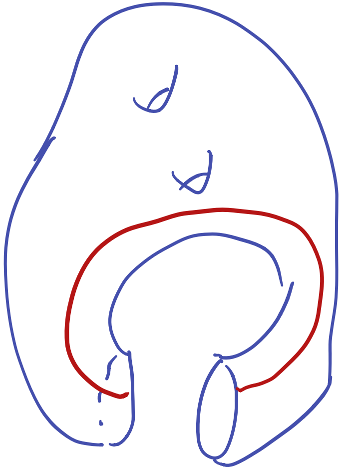


Pf. $S_g \searrow \alpha$ & $S_g \searrow \beta$ are both $\underline{\underline{S_{g-1}^2}}$ ← cut

Class. of surf's $\rightsquigarrow h_0 : S_g \searrow \alpha \longrightarrow S_g \searrow \beta$

$\rightsquigarrow h$.





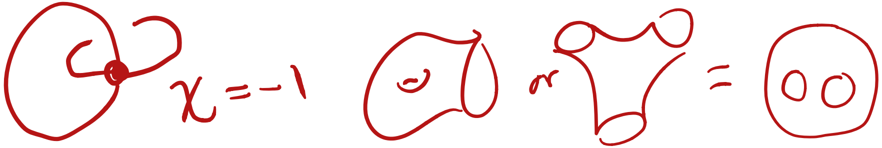
Example If $i(\alpha,\beta)=1$ & $i(\gamma,\delta)=1$

then $\exists\, h \in \mathrm{Homeo}(S_g)$ s.t. $h(\alpha,\beta)=(\gamma,\delta)$

Same proof: Cut, use class. of surf.

$$S_g \searrow (\alpha \cup \beta) = S_{g-1}^1$$

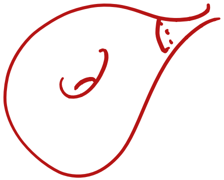
$$\chi(S_g) = 2-2g \qquad \chi(S_{g-1}^1) = 2-2(g-1)-1$$
$$= 3-2g$$



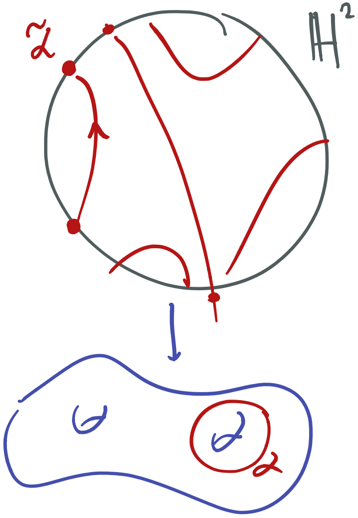
Extra time

Fact. $1 \neq \alpha \in \pi_1(S_g)$ $\quad g > 1$

$$\Rightarrow C(\alpha) \cong \mathbb{Z} = \langle \alpha_0 \rangle \qquad \alpha_0 = \text{root of } \alpha.$$



Pf.



Alg. top: $\pi_1(S) \longrightarrow \mathrm{Homeo}(\tilde{S})$
(deck trans)

Here: $\pi_1(S) \longrightarrow \mathrm{Isom}^+(\mathbb{H}^2)$
image discrete.

Fact 1 $\alpha \longrightarrow$ hyp/lox isometry
(i.e. translates along axis)

Fact 2 In $\mathrm{Isom}^+(\mathbb{H}^2)$
$C(\text{hyp isom}) \cong \mathbb{R}$ = translation along axis.

Classifi. of $\mathrm{Isom}^+\mathbb{H}^2$