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Name Soln

Section H J
Subsection left center right

Mathematics 1553

Written Homework 10
Prof. Margalit
22 April 2016

1. To goal of this assignment is to find the equation of the best-fit plane $z = C + Dx + Ey$ for the values $b = (0, 1, 3, 4)$ observed at the corners of a square:

$$(x_1, y_1) = (1, 0)$$
$$(x_2, y_2) = (0, 1)$$
$$(x_3, y_3) = (-1, 0)$$
$$(x_4, y_4) = (0, -1).$$

The four equations $C + Dx_i + Ey_i = b_i$ give a matrix equation $Ax = b$. What is A?

The four equations are

$$\begin{cases} 0 = C + 1\cdot D + 0\cdot E \\ 1 = C + 0\cdot D + 1\cdot E \\ 3 = C + (-1)\cdot D + 0\cdot E \\ 4 = C + 0\cdot D + (-1)\cdot E \end{cases} \Rightarrow \quad \underset{A}{\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}} \underset{\vec{x}}{\begin{pmatrix} C \\ D \\ E \end{pmatrix}} = \underset{\vec{b}}{\begin{pmatrix} 0 \\ 1 \\ 3 \\ 4 \end{pmatrix}}$$

So $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$

Solve for $x = (C, D, E)$ to find the equation of the plane.

$$A^TA\vec{x} = A^T\vec{b}$$

$$A^TA = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\vec{x} = (A^TA)^{-1}\vec{b} = \begin{pmatrix} 2 \\ -\frac{3}{2} \\ -\frac{3}{2} \end{pmatrix} \Rightarrow \begin{cases} C = 2 \\ D = -\frac{3}{2} \\ E = -\frac{3}{2} \end{cases}$$

$$A^T\vec{b} = \begin{pmatrix} 8 \\ -3 \\ -3 \end{pmatrix}$$

$$\Rightarrow \boxed{z = 2 - \tfrac{3}{2}x - \tfrac{3}{2}y}$$

Check that the height of your plane at $(x, y) = (0, 0)$ is the average of the four entries of b.

(YES) NO