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Section H J
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Mathematics 1553

Written Homework 8
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1. The goal of this assignment is to find a formula for the nth Fibonacci number. You might have seen the Fibonacci numbers before:

$$0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

After the first two numbers, each number in the sequence is the sum of the previous two. In other words we have a *recursion relation*:

$$\begin{aligned} f_0 &= 0 \\ f_1 &= 1 \\ f_n &= f_{n-1} + f_{n-2} \quad n \geq 2 \end{aligned}$$

Problem. Find a formula for the nth Fibonacci number f_n.

The first thing we'll show is that we can get all the Fibonacci numbers from the matrix

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

and the vector corresponding to the first two Fibonacci numbers

$$e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

What is Ae_2?

What is A^2e_2?

What is A^3e_2?

What is A^ne_2? Your answer should be in terms of the Fibonacci numbers $f_0, f_1, f_2, \ldots$.

Based on the answer to the last question, you can find a formula for the nth Fibonacci number if you can find the powers of A. To do this we will want to diagonalize A.

Find the eigenvalues of A.

You can make your calculations later easier by naming the larger eigenvalue a and the smaller one $-1/a$ (it so happens that for this matrix the eigenvalues are negative reciprocals). The number a is a famous number called the golden ratio.

Find an eigenvector for $\lambda = a$. Your answer will be in terms of a. *Hint: You know that the eigenspace must be 1-dimensional. So the second row of $A - \lambda I$ is a multiple of the first and you can replace the second row with a row of zeros.*

$$A - \lambda I = \begin{pmatrix} -a & 1 \\ 1 & 1-a \end{pmatrix} \rightsquigarrow \begin{pmatrix} -a & 1 \\ 0 & 0 \end{pmatrix}$$

Find an eigenvector for $\lambda = -1/a$. Your answer will again be in terms of a.

Diagonalize A. Your answers on this page will again be in terms of a.

Use your diagonalization of A to find a formula for the nth power of A.

Use your formula for A^n to find a formula for f_n (*hint: multiply* A^n *by* e_2).