Section H J Subsection left center right

Mathematics 1553 Written Homework 8 Prof. Margalit 1 April 2016

1. The goal of this assignment is to find a formula for the nth Fibonacci number. You might have seen the Fibonacci numbers before:

$$0, 1, 1, 2, 3, 5, 8, 13, 21, \dots$$

After the first two numbers, each number in the sequence is the sum of the previous two. In other words we have a recursion relation:

$$f_0 = 0$$

 $f_1 = 1$
 $f_n = f_{n-1} + f_{n-2} \quad n \ge 2$

Problem. Find a formula for the nth Fibonacci number f_n .

The first thing we'll show is that we can get all the Fibonacci numbers from the matrix

$$A = \left(\begin{array}{cc} 0 & 1\\ 1 & 1 \end{array}\right)$$

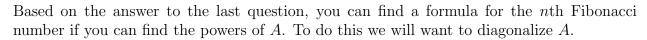
and the vector corresponding to the first two Fibonacci numbers

$$e_2 = \left(\begin{array}{c} 0\\1 \end{array}\right).$$

What is Ae_2 ?

What is A^2e_2 ?

What is A^3e_2 ?



Find the eigenvalues of A.

You can make your calculations later easier by naming the larger eigenvalue a and the smaller one -1/a (it so happens that for this matrix the eigenvalues are negative reciprocals). The number a is a famous number called the golden ratio.

Find an eigenvector for $\lambda = a$. Your answer will be in terms of a. Hint: You know that the eigenspace must be 1-dimensional. So the second row of $A - \lambda I$ is a multiple of the first and you can replace the second row with a row of zeros.

$$A - \lambda I = \begin{pmatrix} -a & 1 \\ 1 & 1 - a \end{pmatrix} \leadsto \begin{pmatrix} -a & 1 \\ 0 & 0 \end{pmatrix}$$

Find an eigenvector for $\lambda = -1/a$. Your answer will again be in terms of a.

