Announcements Feb 29

- WebWork 2.8 and 2.9 due Thursday
- Homework 6 due Friday
- Quiz 6 on 2.8 and 2.9 in class Friday
- Midterm 2 in class Friday Mar 11 on Chapters 2 & 3
- Office Hours Tuesday and Wednesday 2-3, after class, and by appt in Skiles 244 or 236
- LA Office Hours: Scott Mon 12-1, Yashvi Mon 2-3, Shivang Tue 5-6, Baishen Wed 4-5, Matt Thu 3-4
- Math Lab, Clough 280
    - Regular hours: Mon/Wed 11-5 and Tue/Thu 11-5
    - Math 1553 hours: Mon-Thu 5-6 and Tue/Thu 11-12
    - LA hours: Matt Tue 11-12, Scott Tue 5-6, Baishen Thu 11-12, Yashvi/Shivang Thu 5-6

Section 2.9

Dimension and Rank

Bases as Coordinate Systems

$V =$ subspace of $\mathbb{R}^n$

$B = \{b_1, b_2, \ldots, b_k\}$ is a basis for V

x a vector in V

Then we can write x uniquely as

We write

$$[x]_B = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

These are the B-coordinates of x.

Bases as Coordinate Systems

Example

Say $b_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $b_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$B = \{b_1, b_2\}$

$V = \mathrm{Span}\{b_1, b_2\}$.

Q. Verify that B is a basis for V and find the B-coordinates of $x = \begin{pmatrix} 5 \\ 3 \\ 5 \end{pmatrix}$

Bases as Coordinate Systems

Example

Say $v_1 = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$, $v_2 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 2 \\ 8 \\ 6 \end{pmatrix}$

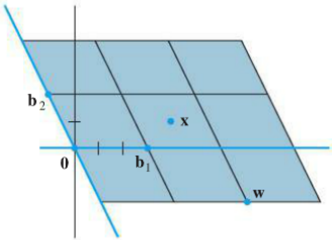
$V = \mathrm{Span}\{v_1, v_2, v_3\}$.

Q. Find a basis for V and find the B-coordinates of $x = \begin{pmatrix} 4 \\ 11 \\ 8 \end{pmatrix}$

Bases as Coordinate Systems

Consider the following basis for $\mathbb{R}^2$:

$$B = \left\{ \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \end{pmatrix} \right\}$$



Use the figure to estimate the B-coordinates of

$$w = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \text{ and } x = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Rank Theorem

Define:

$$\begin{aligned} \textcolor{red}{\operatorname{rank}(A)} &= \dim \operatorname{Col}(A) = \\ &\dim \operatorname{Nul}(A) = \end{aligned}$$

Rank Theorem
If A is an $m \times n$ matrix, then $\operatorname{rank}(A) + \dim \operatorname{Nul}(A) =$

Example.
If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, then $\operatorname{rank}(A) =$ and $\dim \operatorname{Nul}(A) =$

Poll

Poll

If A and B are 3×3 matrices, and $\mathrm{rank}(A) = \mathrm{rank}(B) = 2$ then what are the possible values of $\mathrm{rank}(AB)$?

a) 0

b) 1

c) 2

d) 3

e) 4

Two More Theorems

Basis Theorem
If V is a k-dimensional subspace of $\mathbb{R}^n$, then

- any k linearly independent vectors of V form a basis for V
- any k vectors that span V form a basis for V

In other words if a set has two of these three properties, it is a basis:

spans V, linearly independent, k vectors

Two More Theorems

Invertible Matrix Theorem

(a) A is invertible

$\vdots$

(m) cols of A form a basis for $\mathbb{R}^n$

(n) $\mathrm{Col}(A) = \mathbb{R}^n$

(o) $\dim \mathrm{Col}(A) = n$

(p) $\mathrm{rank}(A) = n$

(q) $\mathrm{Nul}(A) = \{0\}$

(r) $\dim \mathrm{Nul}(A) = 0$