Eigenvectors and eigenvalues

Find the eigenvalues and bases for each corresponding eigenspace:

$$\begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

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Eigenvectors and eigenvalues

Linear transformations

Find the eigenvectors/eigenvalues for A without doing any matrix calculations.

- $T_A = \text{identity transformation of } \mathbb{R}^3$
- $T_A =$ orthogonal projection to xz-plane in $\mathbb{R}^3$
- $T_A =$ counterclockwise rotation by $\pi/4$ in $\mathbb{R}^2$
- $T_A =$ reflection about $y = 2x$

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Eigenvectors and difference equations

Say we want to solve

$$x_{k+1} = Ax_k$$

In other words, we need a sequence $x_0, x_1, x_2, \ldots$ with

$$x_1 = Ax_0, \quad x_2 = Ax_1, \quad \text{etc.}$$

Example. $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a+b \end{pmatrix}.$

Structural engineering

Column buckling

Say we have a column with a compressive force. How will the column buckle?

Approximate column with finite number of points, say

$$(0,0),(0,1),(0,2),\ \dots\ (0,5),(0,6)$$

Buckling leads to (roughly)

$$(0,0),(x_1,1),(x_2,2),\ \dots\ (x_5,5),(0,6)$$

Engineers use a difference equation to model this

$$x_{i-1}-2x_i+x_{i+1}+\lambda x_i=0$$

Difference equation for column buckling:

$$x_{i-1} - 2x_i + x_{i+1} + \lambda x_i = 0$$

We solve this difference equation as above. We need the eigenvector of

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

For most λ, the only eigenvector is 0, which corresponds to no buckling. This matrix has three eigenvalues, 1, 2, and 3.