Scores: 1 2 3 4 5 E

Name ________________________

Section HP ___

Mathematics 1553

Practice Midterm 2

Prof. Margalit

1. Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 2 & 7 \end{pmatrix}$$

Is A invertible?

What is the dimension of the column space of A?

What is the dimension of the null space of A?

Find a basis for the column space of A.

Find a basis for the null space of A.

Let T_A be the linear transformation associated to A. What properties does it have? Select all that apply.

(a) one-to-one

(b) onto

(c) invertible

2. Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Find an LU factorization of A.

Use your LU factorization from the last page to solve

$$Ax = \begin{pmatrix} 7 \\ 12 \\ 8 \\ 3 \end{pmatrix}$$

3. Show that the following matrix has only one LU factorization:

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

What about the other 2×2 matrices?

Describe in your own words how LU factorizations apply to electrical circuits.

Suppose that three shunt circuits are connected in series and that the resistances are R_1, R_2, and R_3. Show that the resulting transfer matrix does not depend on the order in which the three shunt circuits are placed.

4. Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Use your inverse to solve

$$Ax = \begin{pmatrix} 7 \\ 12 \\ 8 \\ 3 \end{pmatrix}$$

5. Find two invertible 2×2 matrices A and B so that $A + B$ is not invertible.

Suppose that A, B, and C are square matrices and that ABC is equal to an invertible matrix M. Explain why A, B, and C are all invertible and find a formula for B^{-1} in terms of A, C, and M.

For which numbers c is the following matrix invertible? Why?

$$A = \begin{pmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{pmatrix}$$

True / False. Every unit lower triangular matrix is invertible. Explain your answer.

True / False. If E and F are $n \times n$ matrices with $EF = I_n$ then E and F commute. Explain your answer.

Suppose that an $m \times n$ matrix A has k pivots. What is the dimension of the set of solutions to $Ax = 0$?

Choose a basis B for $\mathbb{R}^3$ where no vector has a zero coordinate. Choose some nonzero vector x in $\mathbb{R}^3$ and find its B–coordinates $[x]_B$.

Consider the set of vectors (a, b, c, d) in $\mathbb{R}^4$ with $a + d = 0$. Do these vectors form a subspace of $\mathbb{R}^4$? Why or why not?