Name KEY

Mathematics 1553

Quiz 4

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1. Let A be the matrix

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and let T be the associated matrix transformation.

What is $T(v)$ if $v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$?

$$T(v) = Av = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

What is the range of T?

$$\text{range}(T) = \text{span}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right)$$

Which phrase best describes the action of T?

(a) projection to the x-axis

(b) reflection through the xy-plane

(c) projection to the yz-plane

(d) rotation about the z-axis

(e) none of the above

2. State the rank theorem.

For any $m \times n$ matrix A, $\underbrace{\text{rank}(A)}_{\dim(\text{col}(A))} + \underbrace{\text{nullity}(A)}_{\dim(\text{nul}(A))} = \underbrace{n}_{\#\text{ of columns of } A}$.

Turn the page over!

2. Consider the following matrix A and its reduced row echelon form:

$$A = \begin{pmatrix} 3 & 6 & -1 & 1 & 13 \\ 4 & 8 & -1 & 1 & 16 \\ 4 & 8 & -2 & 1 & 15 \\ 1 & 2 & 0 & 0 & 3 \end{pmatrix} \rightsquigarrow \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{pmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Find a basis for the column space of A.

Pivots in 1^{st}, 3^{rd}, 4^{th} columns $\Rightarrow$

$$\text{basis for } \operatorname{col}(A) = \left\{ \begin{pmatrix} 3 \\ 4 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\}$$

Find a basis for the null space of A.

x_2, x_5 free $\Rightarrow$

$$\begin{aligned} x_1 &= -2x_2 - 3x_5 \\ x_2 &= x_2 \\ x_3 &= -x_5 \\ x_4 &= -5x_5 \\ x_5 &= x_5 \end{aligned} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -3 \\ 0 \\ -1 \\ -5 \\ 1 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \text{basis for } \operatorname{nul}(A) = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ -1 \\ -5 \\ 1 \end{pmatrix} \right\}$$