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# Mathematics 1553

Written Homework 6
Prof. Margalit
4 March 2016

1. We defined the dimension of a subspace $V$ to be the number of vectors in a basis for $V$. There's one problem: we haven't shown that all bases have the same number of vectors! The goal of this exercise is to explain why any two bases for $V$ must have the same number of vectors.

Suppose $\{b_1, \ldots, b_k\}$ is a basis for the subspace $V$ of $\mathbb{R}^n$. Let $\{a_1, \ldots, a_\ell\}$ be a set of vectors in $V$ with $\ell > k$. We want to show that $\{a_1, \ldots, a_\ell\}$ is not a basis for $V$ and we will do this by showing that $\{a_1, \ldots, a_\ell\}$ is linearly dependent.

Let $A$ be the matrix $(a_1 \cdots a_\ell)$ and let $B$ be the matrix $(b_1 \cdots b_k)$.

*Step 1.* For each $a_i$ in $A$, explain why there is a vector $c_i$ in $\mathbb{R}^k$ so that $Bc_i = a_i$. *Hint: think about converting vector equations to matrix equations.*

Now let $C$ be the matrix $(c_1 \cdots c_\ell)$.

*Step 2.* Explain why $Cx = 0$ has a nonzero solution. *Hint: use the fact that $k < \ell$.*

Now let $u = (u_1, \ldots, u_\ell)$ be a nonzero solution to $Cx = 0$.

*Step 3.* Show that $Au = 0$. *Hint: Write $Au$ as a linear combination of the $a_i$ and then replace each $a_i$ in the vector equation with $Bc_i$ and then factor out the $B$.*

*Step 4.* Conclude that $\{a_1, \ldots, a_\ell\}$ is linearly dependent and that any two bases for $V$ have the same number of elements.