Name _PROF. M_____

Mathematics 2602
Quiz 4
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1. Use the definition of Big O to show $x \ln x$ is $\mathcal{O}(x^2 + x)$.

$$\ln x < x$$

so $\quad x \ln x < x^2 < x^2 + x$

so $\quad$ can take $c = 1$, $x_0 = 1$:

$$x \ln x \leq 1 \cdot (x^2 + x) \qquad x \geq x_0$$

Use the definition of Big O to show $n!$ is not $\mathcal{O}(3^n)$.

Proof by contradiction.
Suppose $\exists c \, \exists n_0$ so

$$n! \leq c 3^n \qquad n \geq n_0$$

or $\quad n!/3^n \leq c \qquad n \geq n_0$

This is a contradiction because

$$\frac{n!}{3^n} = \frac{n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1}{\underbrace{3 \cdot 3 \cdot 3 \cdots 3 \cdot 3 \cdot 3}_{\geq 1}} \geq \frac{2}{27} n$$

So $\quad \lim_{n \to \infty} n!/3^n = \infty$.