Algebra 2 Notes AII.12 Permutations/Combinations Mrs. Grieser

Name: _____ Date: _____ Block: _____

Permutations

Fundamental Counting Theorem

If an event has ***m*** possible outcomes and another independent event has ***n*** possible outcomes, then there are ***m· n*** possible outcomes for the two events together.

- Can be extended to more than 2 events.
  - If three events can occur in **m**, **n**, and **p** ways, then the number of ways all three events can occur is **m·n·p**.
- Examples:
- For a college interview, Robert has to choose what to wear from the following: 4 slacks, 3 shirts, 2 shoes and 5 ties.
  - How many possible outfits does he have to choose from?_____
- The standard configuration for a Texas license plate is 1 letter followed by 2 digits followed by 3 letters.
  - How many different license plates are possible if letters and digits **can** be repeated? _____
  - How many different license plates are possible if letters and digits **cannot** be repeated?_____

Permutations are orderings of n objects.

*** **ORDER MATTERS** in permutations! ***

- Example: Students sitting in a row:
  - Jack, Bob, Alice is a different permutation than Alice, Jack, Bob
  - How many permutations can we make if three students, Jack, Bob, and Alice are sitting in the same row??'
  - List them:
  - How many permutations? _____
  - Use the fundamental counting principle _____
  - How many permutations are there of the letters ABCD? _____

Factorial

Notice that ordering any pattern one at a time can be found by using the pattern

n*(n-1)*(n-2)*…*1

FACTORIAL is an operator on non-negative integers defined as:

$n! = n*(n-1)*(n-2)*...* 1$

$0! = 1$

Find:

$4!$ _____ $3!$ _____ $6!$ _____ $\frac{5!}{3!}$ _____

What if you want permutations for more than one object at a time?

- A television news director has 8 news stories to present on the evening news.
  - How many different ways can the stories be presented?
  - If only 3 of the stories can be presented, how many possible ways can just those 3 stories be presented?
  - Can we generalize how we found the second question?

$\frac{8!}{5!} = \frac{8!}{(8-3)!} = \frac{n!}{(n-r)!}$ where r is the number of objects in the permutation

Permutation Formula

$nPr = \frac{n!}{(n-r)!}$ where $0 \le r \le n$

- Example: ${}_5P_3$ = _____
- Practice: Find…

${}_4P_4$ ${}_4P_1$ ${}_8P_5$ ${}_{12}P_7$

Example:

A combination lock will open when the right choice of three numbers (from 1 to 30, inclusive) is selected.

How many different lock combinations are possible assuming no number is repeated?

Example:

From a club of 24 members, a President, Vice President, Secretary, Treasurer and Historian are to be elected.

In how many ways can the offices be filled?

Permutations with Repetition:

- Find the number of permutations with letters EYE.

Permutations with Repetition

The number of distinguishable permutations of n objects where one object is repeated s_1 times, another object is repeated s_2 times, and so on, is:

$$\frac{n!}{s_1!\bullet s_2!\bullet,,,\bullet s_k!}$$ where k is the number of repeating objects

Find the number of distinguishable permutations of the letters of the following words:

a) MIAMI b) VIRGINIA c) TALLAHASSEE

You try…

1) How many different license plates can be created if the license plate has 4 letters followed by 2 digits, and…

   a) items can be repeated and b) items cannot be repeated

2) Find the number of permutations:

   a) ${}_5P_5$ b) ${}_6P_2$ c) ${}_{10}P_1$ d) ${}_9P_2$

3) Find the number of distinguishable permutations of the letters in the word:

   a) YELLOW b) PANAMA c) HONOLULU d) MISSISSIPPI

4) A Spanish club is electing a president, vice president, and secretary. The club has 9 members who are eligible for these offices. How many different ways can the 3 offices be filled?

5) The window of a music store has 8 stands in fixed positions where instruments can be displayed. In how many ways can 3 identical guitars, 2 identical keyboards, and 3 identical violins be displayed?

Combinations

Example:

A standard deck of 52 cards has 4 suits and 14 different cards in each suit. If the order in which cards are dealt is not important, how many different 5-card hands are possible?

- If order did matter, we would use _____
- However, order does NOT matter, so remove duplicates
- There are 5! repeats – divide them out _____
- Why are the number of combinations less than the number of permutations?

A **combination** is an arrangement of items in which order does not matter.

*** **ORDER DOES NOT MATTER**! ***

Formula:

$$nCr = \frac{n!}{r!(n-r)!} \text{ where } 0 \le r \le n$$

Also written as: $\binom{n}{r}$

Example: ${}_5C_3$ = _____

- Practice: Find…

${}_8C_3$ ${}_{10}C_6$ ${}_7C_2$ $\binom{14}{5}$

Building upon previous example with the deck of cards, in how may 5-card hands are all 5 cards the same color?

Example: A student must answer 4 out of 6 essay questions on a test. In how many different ways can the student select the questions?

Does order matter in this situation? _____

What is the combination? _____ Compute: _____

Example: A basketball team consists of two centers, five forwards, and four guards. In how many ways can the coach select a starting line up of one center, two forwards, and two guards?

Does order matter in this situation? _____

Solve:

You try:

1) There are 12 boys and 14 girls in Mrs. Schultzkie's math class. Find the number of ways Mrs. Schultzkie can select a team of 3 students from the class to work on a group project. The team is to consist of 1 girl and 2 boys.

2) A basketball team consists of two centers, five forwards, and four guards. In how many ways can the coach select a starting line up of one center, two forwards, and two guards?

3) There are fourteen juniors and twenty-three seniors in the Service Club. The club is to send four representatives to the State Conference.

   a) How many different ways are there to select a group of four students to attend the conference?

   b) If the members of the club decide to send two juniors and two seniors, how many different groupings are possible?

Permutation or Combination?

- Picking three team members from a group.
- Picking a team captain, pitcher, and shortstop from a group.
- Picking first, second and third place winners.
- Selecting three students to attend a conference in New York.
- Selecting a lead and an understudy for a school play.
- Assigning students to their seats on the first day of school.
- Selecting 3 numbers for a combination lock.