Game Theory in Soccer

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Overview

Probability Definitions

Conditional Probability

- Given events A and B, $\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)}$.

Bayes Rule

- Given events A and B, $\Pr(B_i|A) = \frac{\Pr(A|B_i)\Pr(B_i)}{\Pr(A)}$.

Game Theory Definitions

Strictly Dominant Strategy

- Let a and b be strategies in a game. We say that a strictly dominates b if the payoff for using strategy a is higher than the payoff for using strategy b. We will denote this as $A(a) > B(b)$ for teams A and B using strategies a and b, respectively.

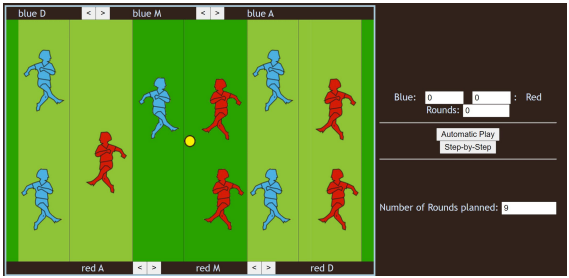
Weakly Dominant Strategy

- Let a and b be strategies in a game. Then we say that a weakly dominates b if the payoff for a is greater than or equal to the payoff of b. This looks like $A(a) \geq B(b)$.

Nash Equillibrium

- The Nash Equilibrium is the scenario where both sides of a game choose their best strategy, regardless of the opponent's strategy. We will denote this as $N(A, B) = (a, b)$, where Team A's best strategy is a and Team B's best strategy is b.

The Game of DMA Soccer



- 2 Teams: Red and Blue
- 5 Players for each team
- 3 Zones
  - No movement between zones by players.
  - Each player has an equal chance to get the ball
- Game is played over a given number of rounds

Calculating Probabilities for DMA Soccer

- First Letter - Position
  - D - Defense
  - M - Midfield
  - A - Attack
- Second Letter - Team
  - Team A - (Blue)
    - 2 Defenders
    - 1 Midfielder
    - 2 Attackers
  - Team B - (Red)
    - 2 Defenders
    - 2 Midfielders
    - 1 Attacker

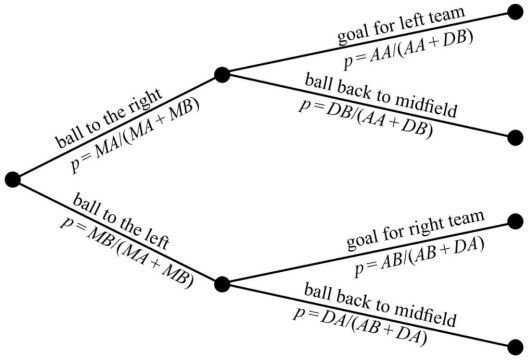


Figure 14.1. The probability tree

Expected Goals

Going back to the game theory terms, it is important to discuss what is meant by the payoff for different strategies. In most economics examples we would expect payoff to be profit. In DMA soccer, expected goals is the payoff.

Expected Goals

In DMA soccer, the expected goals for Team A in one round is the probability that Team A scores a goal in that round. The formula for this is $E(A) = \Pr(A_{goal}) = \frac{MA}{MA+MB} * \frac{AA}{AA+DB}$.

Expected Goal Differential

To compare two different strategies, we will look at the expected goal differential. To calculate this we use the expected goal total for each team. So $E(X) = E(A) - E(B)$. If E(X) is positive, then Team A has a higher payoff and therefore a strategy that strictly dominates Team B's strategy.

Expected Goals

Probabilities PA, PB, P0 for a goal for A, goal for B, and no goal, respectively, in one round of DMA SOCCER

PA

DB	0	0	0	0	0	0	1	1	1	1	1	2	2	2	2	3	3	3	4	4	5
MB	0	1	2	3	4	5	0	1	2	3	4	0	1	2	3	0	1	2	0	1	0
AB	5	4	3	2	1	0	4	3	2	1	0	3	2	1	0	2	1	0	1	0	0

DA	MA	AA		0,0	0,1	0,2	0,3	0,4	0,5	1,0	1,1	1,2	1,3	1,4	2,0	2,1	2,2	2,3	3,0	3,1	3,2	4,0	4,1	5,0
0	0	5	0,0	0.50	0.00	0.00	0.00	0.00	0.00	0.42	0.00	0.00	0.00	0.00	0.36	0.00	0.00	0.00	0.31	0.00	0.00	0.28	0.00	0.25
0	1	4	0,1	1.00	0.50	0.33	0.25	0.20	0.17	0.80	0.40	0.27	0.20	0.16	0.67	0.33	0.22	0.17	0.57	0.29	0.19	0.50	0.25	0.44
0	2	3	0,2	1.00	0.67	0.50	0.40	0.33	0.29	0.75	0.50	0.38	0.30	0.25	0.60	0.40	0.30	0.24	0.50	0.33	0.25	0.43	0.29	0.38
0	3	2	0,3	1.00	0.75	0.60	0.50	0.43	0.38	0.67	0.50	0.40	0.33	0.29	0.50	0.38	0.30	0.25	0.40	0.30	0.24	0.33	0.25	0.29
0	4	1	0,4	1.00	0.80	0.67	0.57	0.50	0.44	0.50	0.40	0.33	0.29	0.25	0.33	0.27	0.22	0.19	0.25	0.20	0.17	0.20	0.16	0.17
0	5	0	0,5	0.50	0.42	0.36	0.31	0.28	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0	4	1,0	0.50	0.00	0.00	0.00	0.00	0.00	0.40	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.00	0.29	0.00	0.00	0.25	0.00	0.22
1	1	3	1,1	1.00	0.50	0.33	0.25	0.20	0.17	0.75	0.38	0.25	0.19	0.15	0.60	0.30	0.20	0.15	0.50	0.25	0.17	0.43	0.21	0.38
1	2	2	1,2	1.00	0.67	0.50	0.40	0.33	0.29	0.67	0.44	0.33	0.27	0.22	0.50	0.33	0.25	0.20	0.40	0.27	0.20	0.33	0.22	0.29
1	3	1	1,3	1.00	0.75	0.60	0.50	0.43	0.38	0.50	0.38	0.30	0.25	0.21	0.33	0.25	0.20	0.17	0.25	0.19	0.15	0.20	0.15	0.17
1	4	0	1,4	0.50	0.40	0.33	0.29	0.25	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0	3	2,0	0.50	0.00	0.00	0.00	0.00	0.00	0.38	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.25	0.00	0.00	0.21	0.00	0.19
2	1	2	2,1	1.00	0.50	0.33	0.25	0.20	0.17	0.67	0.33	0.22	0.17	0.13	0.50	0.25	0.17	0.13	0.40	0.20	0.13	0.33	0.17	0.29
2	2	1	2,2	1.00	0.67	0.50	0.40	0.33	0.29	0.50	0.33	0.25	0.20	0.17	0.33	0.22	0.17	0.13	0.25	0.17	0.13	0.20	0.13	0.17
2	3	0	2,3	0.50	0.38	0.30	0.25	0.21	0.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0	2	3,0	0.50	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.00	0.00	0.25	0.00	0.00	0.00	0.20	0.00	0.00	0.17	0.00	0.14
3	1	1	3,1	1.00	0.50	0.33	0.25	0.20	0.17	0.50	0.25	0.17	0.13	0.10	0.33	0.17	0.11	0.08	0.25	0.13	0.08	0.20	0.10	0.17
3	2	0	3,2	0.50	0.33	0.25	0.20	0.17	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0	1	4,0	0.50	0.00	0.00	0.00	0.00	0.00	0.25	0.00	0.00	0.00	0.00	0.17	0.00	0.00	0.00	0.13	0.00	0.00	0.10	0.00	0.08
4	1	0	4,1	0.50	0.25	0.17	0.13	0.10	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0	0	5,0	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Expected Goal Differential

Expected goal difference in one round of DMA SOCCER

The yellow cells below can be changed if needed

The formula is PA - PB on the other sheet.

			DB	0	0	0	0	0	0	1	1	1	1	1	2	2	2	2	3	3	3	4	4	5	
			MB	0	1	2	3	4	5	0	1	2	3	4	0	1	2	3	0	1	2	0	1	0	
			AB	5	4	3	2	1	0	4	3	2	1	0	3	2	1	0	2	1	0	1	0	0	
DA	MA	AA		0,0	0,1	0,2	0,3	0,4	0,5	1,0	1,1	1,2	1,3	1,4	2,0	2,1	2,2	2,3	3,0	3,1	3,2	4,0	4,1	5,0	0.00
0	0	5	0,0	0.00	-1.00	-1.00	-1.00	-1.00	-0.50	-0.08	-1.00	-1.00	-1.00	-0.50	-0.14	-1.00	-1.00	-0.50	-0.19	-1.00	-0.50	-0.22	-0.50	0.00	-1.00
0	1	4	0,1	1.00	0.00	-0.33	-0.50	-0.60	-0.25	0.80	-0.10	-0.40	-0.55	-0.24	0.67	-0.17	-0.44	-0.21	0.57	-0.21	-0.14	0.50	0.00	0.44	-0.60
0	2	3	0,2	1.00	0.33	0.00	-0.20	-0.33	-0.07	0.75	0.17	-0.13	-0.30	-0.08	0.60	0.07	-0.20	-0.06	0.50	0.00	0.00	0.43	0.12	0.38	-0.33
0	3	2	0,3	1.00	0.50	0.20	0.00	-0.14	0.06	0.67	0.25	0.00	-0.17	0.00	0.50	0.13	-0.10	0.00	0.40	0.05	0.04	0.33	0.13	0.29	-0.17
0	4	1	0,4	1.00	0.60	0.33	0.14	0.00	0.17	0.50	0.20	0.00	-0.14	0.00	0.33	0.07	-0.11	-0.02	0.25	0.00	0.00	0.20	0.06	0.17	-0.14
0	5	0	0,5	0.50	0.25	0.07	-0.06	-0.17	0.00	0.00	-0.17	-0.29	-0.38	-0.22	0.00	-0.17	-0.29	-0.19	0.00	-0.17	-0.14	0.00	-0.08	0.00	-0.38
1	0	4	1,0	0.08	-0.80	-0.75	-0.67	-0.50	0.00	0.00	-0.75	-0.67	-0.50	0.00	-0.04	-0.67	-0.50	0.00	-0.05	-0.50	0.00	0.00	0.00	0.22	-0.80
1	1	3	1,1	1.00	0.10	-0.17	-0.25	-0.20	0.17	0.75	0.00	-0.19	-0.19	0.15	0.60	-0.03	-0.13	0.15	0.50	0.00	0.17	0.43	0.21	0.38	-0.25
1	2	2	1,2	1.00	0.40	0.13	0.00	0.00	0.29	0.67	0.19	0.00	-0.03	0.22	0.50	0.11	0.00	0.20	0.40	0.10	0.20	0.33	0.22	0.29	-0.03
1	3	1	1,3	1.00	0.55	0.30	0.17	0.14	0.38	0.50	0.19	0.03	0.00	0.21	0.33	0.08	0.00	0.17	0.25	0.06	0.15	0.20	0.15	0.17	0.00
1	4	0	1,4	0.50	0.24	0.08	0.00	0.00	0.22	0.00	-0.15	-0.22	-0.21	0.00	0.00	-0.13	-0.17	0.00	0.00	-0.10	0.00	0.00	0.00	0.00	-0.22
2	0	3	2,0	0.14	-0.67	-0.60	-0.50	-0.33	0.00	0.04	-0.60	-0.50	-0.33	0.00	0.00	-0.50	-0.33	0.00	0.00	-0.33	0.00	0.05	0.00	0.19	-0.67
2	1	2	2,1	1.00	0.17	-0.07	-0.13	-0.07	0.17	0.67	0.03	-0.11	-0.08	0.13	0.50	0.00	-0.06	0.13	0.40	0.03	0.13	0.33	0.17	0.29	-0.13
2	2	1	2,2	1.00	0.44	0.20	0.10	0.11	0.29	0.50	0.13	0.00	0.00	0.17	0.33	0.06	0.00	0.13	0.25	0.06	0.13	0.20	0.13	0.17	0.00
2	3	0	2,3	0.50	0.21	0.06	0.00	0.02	0.19	0.00	-0.15	-0.20	-0.17	0.00	0.00	-0.13	-0.13	0.00	0.00	-0.08	0.00	0.00	0.00	0.00	-0.20
3	0	2	3,0	0.19	-0.57	-0.50	-0.40	-0.25	0.00	0.05	-0.50	-0.40	-0.25	0.00	0.00	-0.40	-0.25	0.00	0.00	-0.25	0.00	0.04	0.00	0.14	-0.57
3	1	1	3,1	1.00	0.21	0.00	-0.05	0.00	0.17	0.50	0.00	-0.10	-0.06	0.10	0.33	-0.03	-0.06	0.08	0.25	0.00	0.08	0.20	0.10	0.17	-0.10
3	2	0	3,2	0.50	0.14	0.00	-0.04	0.00	0.14	0.00	-0.17	-0.20	-0.15	0.00	0.00	-0.13	-0.13	0.00	0.00	-0.08	0.00	0.00	0.00	0.00	-0.20
4	0	1	4,0	0.22	-0.50	-0.43	-0.33	-0.20	0.00	0.00	-0.43	-0.33	-0.20	0.00	-0.05	-0.33	-0.20	0.00	-0.04	-0.20	0.00	0.00	0.00	0.08	-0.50
4	1	0	4,1	0.50	0.00	-0.12	-0.13	-0.06	0.08	0.00	-0.21	-0.22	-0.15	0.00	0.00	-0.17	-0.13	0.00	0.00	-0.10	0.00	0.00	0.00	0.00	-0.22
5	0	0	5,0	0.00	-0.44	-0.38	-0.29	-0.17	0.00	-0.22	-0.38	-0.29	-0.17	0.00	-0.19	-0.29	-0.17	0.00	-0.14	-0.17	0.00	-0.08	0.00	0.00	-0.44
			0.00	1.00	0.60	0.33	0.17	0.14	0.38	0.80	0.25	0.03	0.00	0.22	0.67	0.13	0.00	0.20	0.57	0.10	0.20	0.50	0.22	0.44	

One Round Game Strategies

- (A,B)
  - A is the number of defensive players
  - B is the number of midfield players
  - We know that there are 5 total players, so the number of attacking players is $5 - (A + B)$
- The arrows point to the strategy that is weakly dominant
  - Examples
    - (0,5) is weakly dominated by (1,3)
    - (1,3) is weakly dominated by (2,2), but also weakly dominates (2,2)
    - Equilibrium at (1,3) and (2,2)
    - Note: (2,2) weakly dominates all other strategies, but is a relatively conservative strategy.

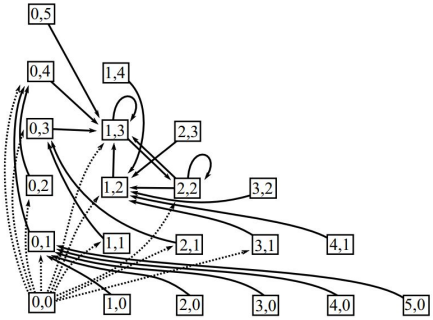


Figure 14.2. Best response digraph for the goal difference

What Strategies Lead to Success?

- Strictly based on expected goal differential, we see that the Nash equilibrium for 5 players per team is a 1-3-1 formation and a 2-2-1 formation.
- Since DMA soccer could be a non-zero-sum game, this could change.
- A zero-sum game is a game where the expected payoff between both strategies sums to zero.
- Baseball is a zero-sum-game. Implicitly, a win is equivalent to 0.5 points and a loss is equivalent to -0.5 points.

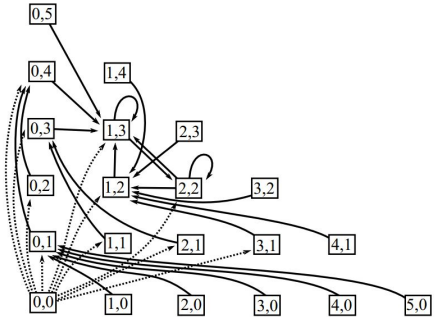


Figure 14.2. Best response digraph for the goal difference

Non-Zero-Sum Games

- Hockey and Soccer are not zero-sum games.
  - In hockey, 2 points are awarded for a win, 1 for a loss in overtime (or a shootout), and 0 points for a loss in regulation time.
  - In soccer, 3 points are awarded for a win, 1 for a tie, and 0 for a loss.
- How does this effect the Nash Equilibrium?

Example for 3-Round Game

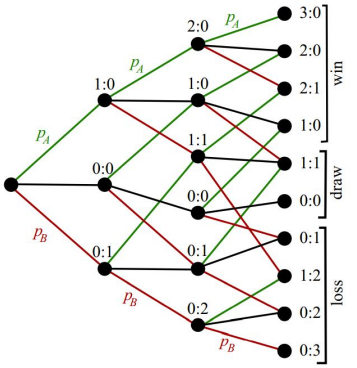


Figure 14.3. The probability tree for three-round DMA soccer

- From the diagram we can see all of the possible scores in a 3-round game.
- To calculate the probability of a win, we calculate each possible outcome using Bayes Rule and add up every score outcome that results in a win.
- If we look at non-zero-sum point systems, the strategies will react differently.
- Consider 3-1 (3 points for a win, 1 for a draw) and 3-2 point systems.

Calculations

- Similar to the calculations for expected goals, the calculation is now expected points.
- $E(P_A) = w \Pr(W) + d \Pr(D) + l \Pr(L)$, where w, d, and l are the the points for a win, draw, and loss respectively, and $\Pr(W)$, $\Pr(D)$, and $\Pr(L)$ are the probabilities of a win, draw, and loss respectively.
- In the 3-1 point system, $w = 3$, $d = 1$, and $l = 0$.
- In the 3-2 point system, $w = 3$, $d = 2$, and $l = 0$.

3-1 and 3-2 Point Systems

- Left: 3-1 Point System
  - New equilibrium of (1,3)
- Right: 3-2 Point System
  - New equilibrium of (2,2)

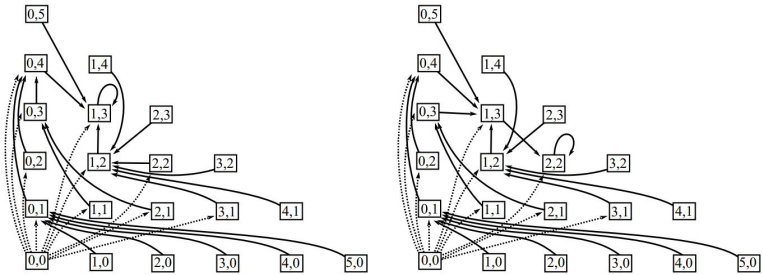
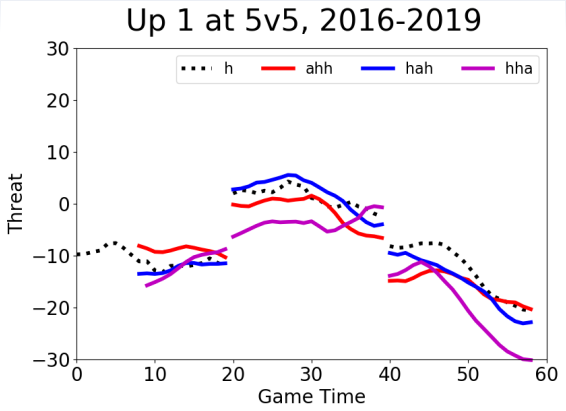


Figure 14.5. Best response digraph for 3 rounds with 3-2 rule

Conclusion

Why does this matter?

- Coaches should take into account the point system when deciding where to put their players on the field.
- If sports leagues want to increase scoring, they should consider changing the point system.
  - Example: National Hockey League
  - The NHL has a 2-1 point system.



https://hockeyviz.com/txt/scoreSeq by Micah Blake McCurdy

The End