Verification of Quantum Superiority

QuIVER 2018

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The University of Edinburgh

Content

Quantum Superiority

Superiority Hypothesis

The set of samples I have in my posetion were drawn from a distribution produced by a classical computer [1] [2]

[1] In a reasonable amount of time

[2] Disproving this null hypothesis would demonstrate quantum superiority [1]

A Recipe

Ingredients:

- A computational problem [3]
- A reason to believe there is a separation between the classical and quantum runtime
- A method of verifying the outcome

Cooking time: polynomial

Serves: you right extended Church-Turing thesis

[3] Not necessarily of practical interest

Factoring [2] as an Instance of our Recipe

- A computational problem:
    - Factoring

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- A computational problem:
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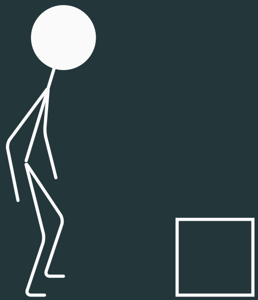
- A reason to believe there is a separation between the classical and quantum runtime
  - Well... we've tried our best for a while now
- A method of verifying the outcome
  - We can multiply the factors

Superiority by Factoring Soon Becomes Daunting [3]



Factoring[4]

[4] Of a 2048 bit number, which is basically impossible for a classcal computer

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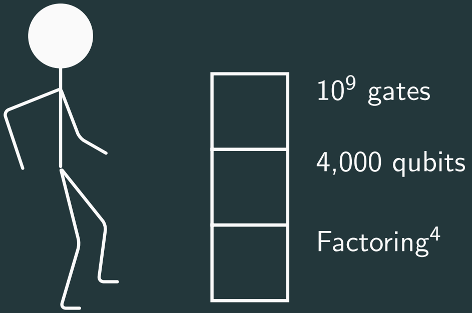

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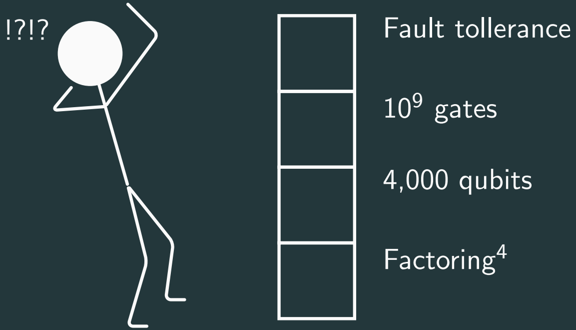

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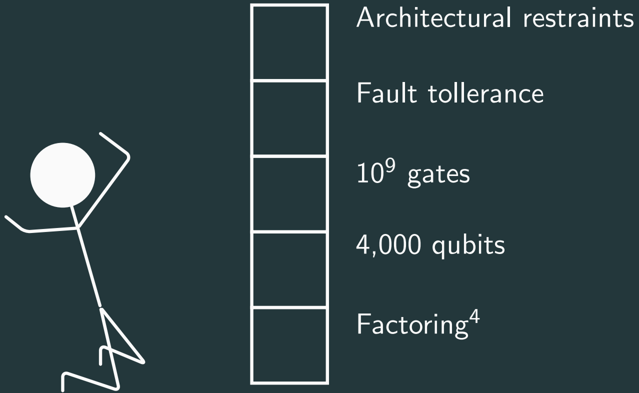

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A New Ingrediant

Ingredients:

- A computational problem [5]
- A reason to believe there is a separation between the classical and quantum runtime
- A method of verifying the outcome
- An implementation on a near-term device

[5] Not necessarily of practical interest

Simpler Quantum Computers

Linear optical network:

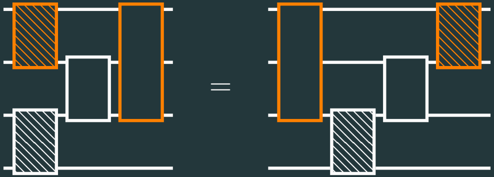
Photons are counted at the end

Boson Sampling Chalenges

- Randomised single photon source has inherently poor scaling
    - Scattershot boson sampling?
- Lossy systems
- Some way to go
    - Can implement ~ 5 photons, ~ 10 modes
    - Can simulate ~ 30 photons ... on a laptop [5]

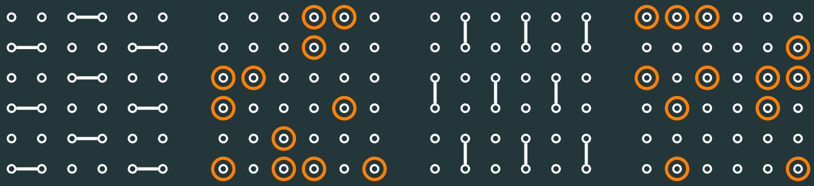
Instantaneous Quantum Polytime [6, 7]

Commuting gates:



Random Quantum Circuits [8]

Alternating entanglement patterns and random gates:



Hardness Results

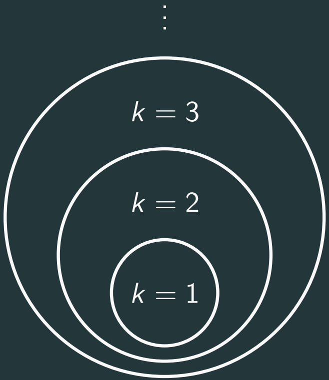
Polynomial Hierarchy

- $f(x) \in \text{NP} \implies f(x) = \vee_y g(x, y)$
- k^{th} level of PH has k alternating quantuifers
  - $f(x) = \vee_{y_1} \wedge_{y_2} \ldots \wedge_{y_k} g(x, y_1, \ldots, y_k)$
- It is conjectured k^{th} and $k+1^{th}$ level of PH are not equal
  - If it is then there is a colapse to k^{th} level - " it's the k^{th} level all the way down"

Post-Selection

- A computation takes input strings x and outputs strings y and z
- we condition on z and output y
- Allowing post selection on exponentially unlikely outcomes is very powerful

What is the Layout?



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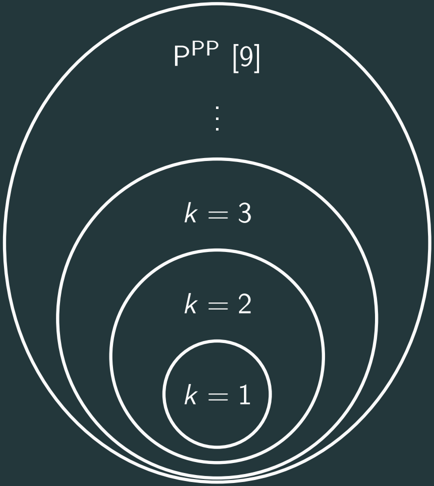

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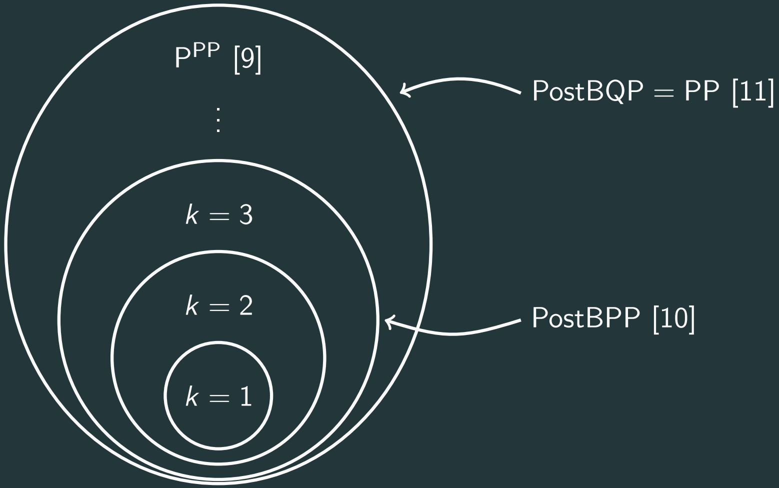
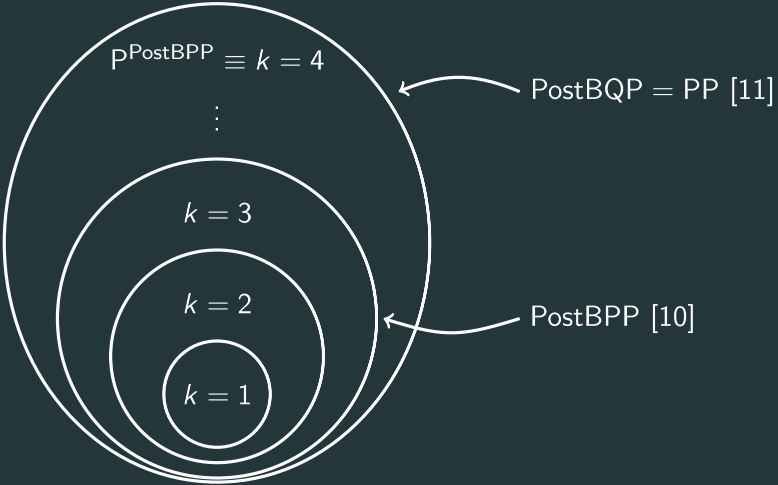


What if PostBQP = PostBPP**?**



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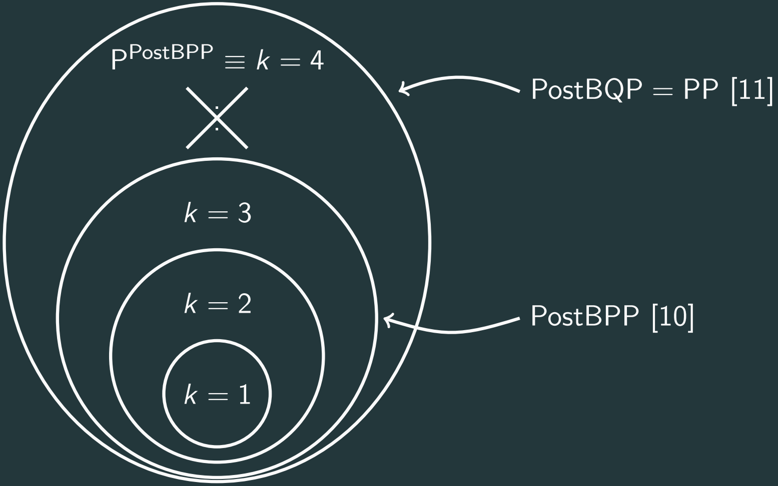

Problem with Complexity Theory

- Asymptotic complexity results tell us little about near term implementations!
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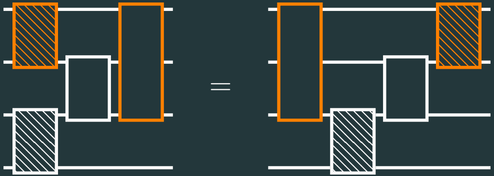
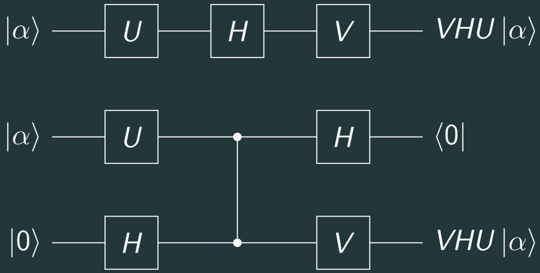
- Worst case results teach us nothing about which computation implements to use
    - We have some average case hardness results based on stronger conjectures
- $\mathrm{BPP} = \mathrm{BQP} \not\Rightarrow \mathrm{PostBQP} = \mathrm{PostBPP}$

Instantaneous Quantum Polytime [6, 7]

Commuting gates:



IQP Superiority [13]



Multiplicative vs Additive Error

$$(1-\epsilon)\, q\left(0^n\right) \leq p\left(0^n\right) \leq (1+\epsilon)\, q\left(0^n\right)$$

vs

$$\sum_z \left|p\left(z\right) - q\left(z\right)\right| \leq \epsilon$$

IQP Additive Superiority [14]

- For two classes of problems, a classical sampler, acurate up to good additive error in the worst case, must be acurate in multiplicative error in the average case.

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- Can use Stockmeyer to estimate individual output probabilities up to small multiplicatie error.
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- This gives an algorithm for computing multiplicative approximation to large fraction of class.
- This causes a collapse of PH , assuming some conjectures about the two classes. [7]

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IQP Superiority

- Arbitrarily small constant noise on each qubit at the end of IQP circuit makes [15] easy up to additive error.

Random Circuit Superiority: 3 Main Arguments

1. No known simulation using reasonable amount of memory
2. IQP-esque complexity results giving asymptotic hardness
3. Circuits have properties we expect of hard distributions

Intuative Initial Arguments

$$
\begin{aligned}
\text{Close to Porter-Thomas} &\implies \text{Behaves like chaotic system} \\
&\implies \text{Small perturbation} = \text{large divergence} \\
&\implies \text{Must store full state} \\
&\implies \text{Hard to simulate}
\end{aligned}
$$

Verification

Verification

Options:

1. Direct certification
2. Classically simulate small instances
3. Statistical test of some properties we expect.

Verification Using HOG [16]

Problem
HOG - Heavey Output Generation

Given as input a random quantum circuit C, generate output strings $x_1, ..., x_k$ at least a $\frac{2}{3}$ fraction of which have greater than median probability in C's output distribution.

Verification Using HOG [16]

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Given as input a random quantum circuit C, generate output strings $x_1, ..., x_k$ at least a $\frac{2}{3}$ fraction of which have greater than median probability in C's output distribution.

Conjecture
QUATH - QUantum THreshold assumption

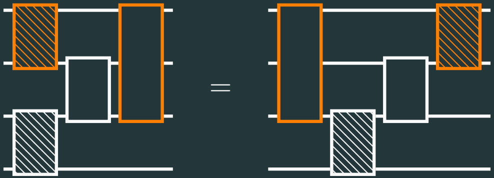
There is no polynomial-time classical algorithm that takes as input a description of a random quantum circuit C, and which guesses whether $|\langle 0^n| C |0^n\rangle|^2$ is greater than or less than the median of all 2^n of the $|\langle 0^n| C |x\rangle|^2$

Verification of Random Circuits Using Entropy Benchmarking

- Measures closeness of output to perfect circuit
- Takes exponential time classically
  - Maybe that's okay?

Instantaneous Quantum Polytime Machine [6]

Commuting gates:



In particular:

$$\exp i\theta \bigotimes_{i:q_i=1} X_i$$

where $q \in \{0,1\}^{n_p}$, $\theta \in [0, 2\pi]$.

Instantaneous Quantum Polytime Machine [6]

$$\exp i\theta \bigotimes_{i:q_i=1} X_i$$

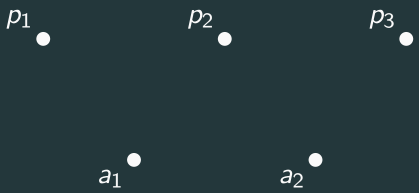
An IQP program may consist of many of these gates, and so many different q. Hence we may represent the whole computation by, for example:

$$\mathbf{Q} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

where, in this case, we have two gates defined by $q = (101)$ and $q = (010)$.

The input is $|0^{n_p}\rangle$ and the output is the resulting state measured in the computational basis.

IQP in MBQC



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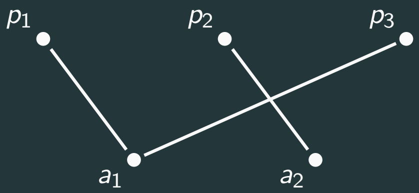

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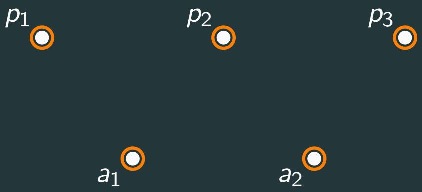

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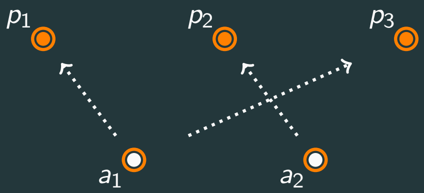

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Bridge and Break [17]

$cZ_{1,2}cZ_{2,3}\,|0/1\rangle \otimes |\phi\rangle$

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$$Z_1^{0/1} Z_3^{0/1} \left|0/1\right\rangle \otimes \left|\phi\right\rangle$$

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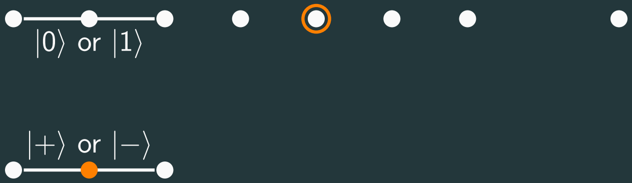

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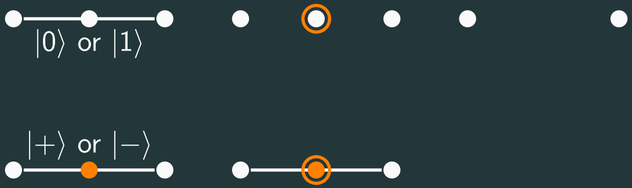

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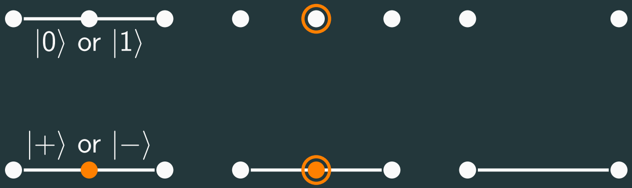
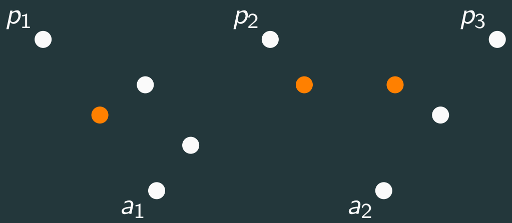


$$S_1^{f(+/-,s)} S_3^{f(+/-,s)} Z_{1,3} |\phi\rangle$$

IQP By Bridge and Break



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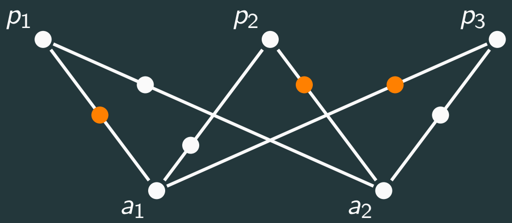

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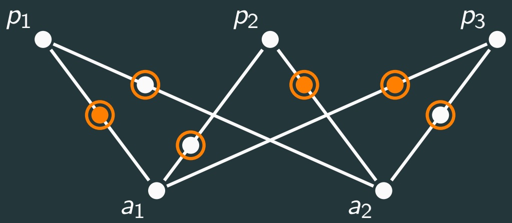

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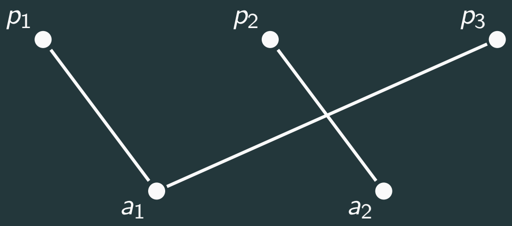

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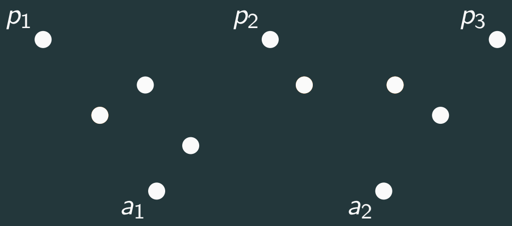

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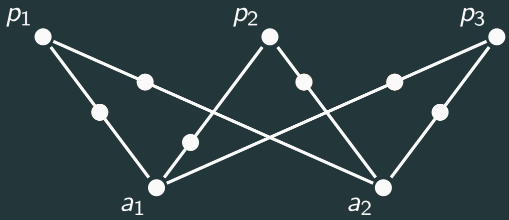

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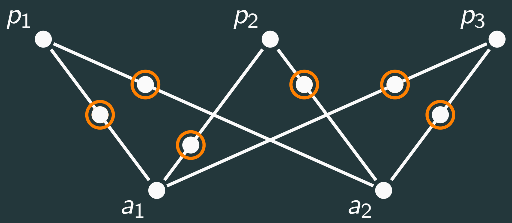

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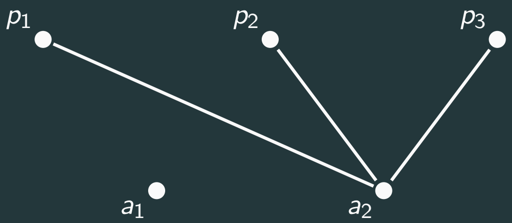

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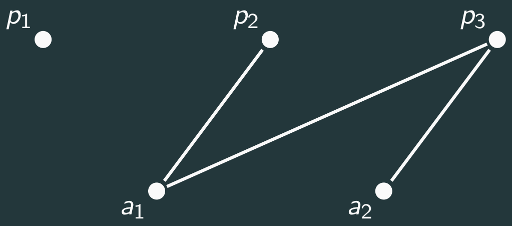

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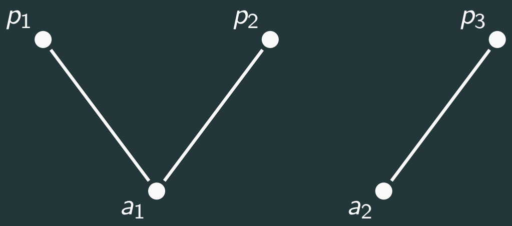


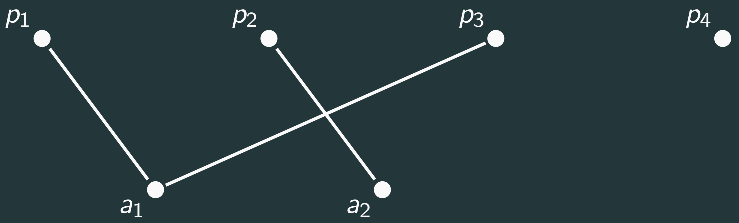
Hypothesis Test

Bias of a random variable, $X \in \{0,1\}^{n_p}$, in a direction $s \in \{0,1\}^{n_p}$.

$$\mathbb{P}\left(X \cdot s^T = 0\right) = \mathit{Bias}\left(X, s\right)$$

Can be easily calculated, for some special IQP computations (depending on s), if one knows s [6].

Hypothesis Test



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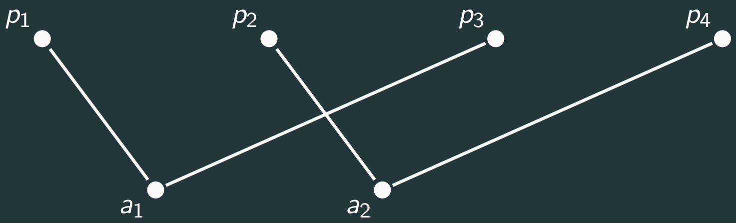
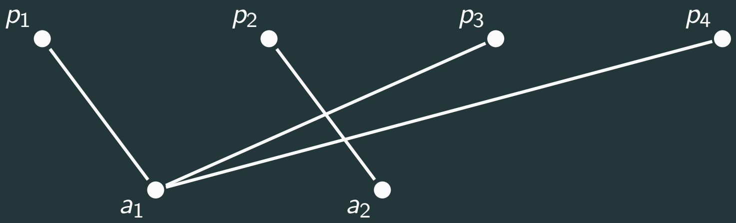


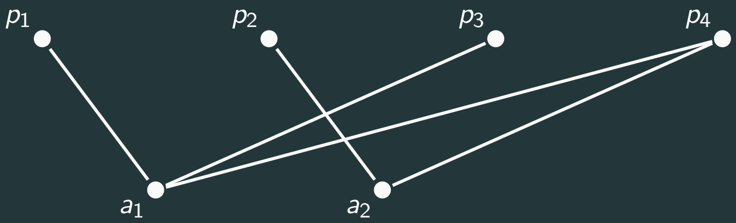
$$Bias\,(X, s_1) = p$$

Hypothesis Test



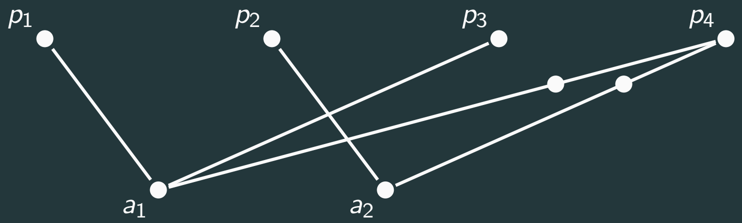
$$Bias\left(X, s_2\right) = p$$

Hypothesis Test



$$Bias\,(X, s_3) = p$$

Hypothesis Test



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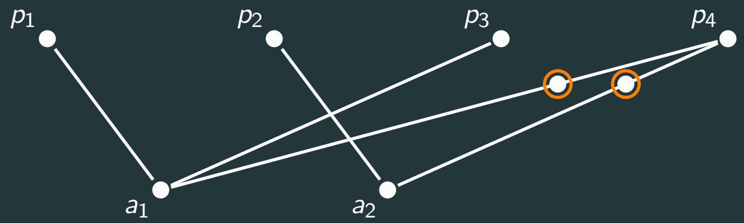

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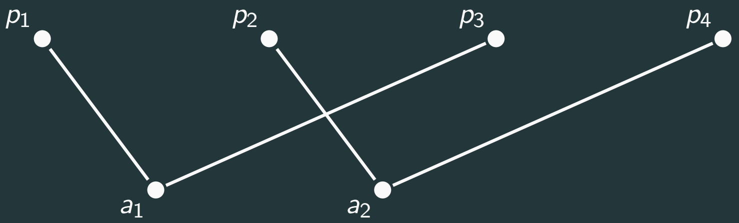

The Hypothesis Test Outline

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Three conditions for a successful hypothesis test:

- The Server must complete a hard computations
    - Computation bias calculation is hard
- The Client knows a secret property allowing them to check the outcome
    - The Client knows the direction s
- The Server hides the secret property
    - Using blind IQP

Conclusion

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- VERIFICATION OF SOME PROPERTY (BUT NOT THE WHOLE THING) IS INTERESTING!

References

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