Consumption Tax in HANK

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1 One-Asset HANK as in Auclert et al. (2021)

In this model, τ_t is a *tax*, proportional to households' labor productivity e_{it}: $\tau_t \bar{\tau}(e_{it})$.

Households Continuum of households i. Utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu} \right\}.$$

Budget constraint and borrowing constraint

$$\begin{aligned} & c_{it} + b_{it} = w_t e_{it} n_{it} + (1+r_t) b_{it-1} - \tau_t \bar{\tau}(e_{it}) + d_t \bar{d}(e_{it}), \\ & b_{it} \geq \underline{b}. \end{aligned}$$

Lagrangian

$$\mathcal{L}_i = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu} + \lambda_{it} \left[w_t e_{it} n_{it} + (1+r_t) b_{it-1} - \tau_t \bar{\tau}(e_{it}) + d_t \bar{d}(e_{it}) - c_{it} - b_{it} \right] \right\}.$$

Optimal labor supply and consumption-saving decisions for unconstrained households

$$\begin{aligned} n_{it} &= \left(\frac{w_t e_{it}}{\varphi c_{it}^{\sigma}} \right)^{\frac{1}{\nu}}. \\ c_{it}^{-\sigma} &= \beta E_t (1+r_{t+1}) c_{it+1}^{-\sigma}. \end{aligned}$$

When the borrowing constraint binds, $b_{it} = \underline{b}$, the budget constraint rewrites as

$$c_{it} + \underline{b} = w_t e_{it} n_{it} + (1+r_t) b_{it-1} - \tau_t \bar{\tau}(e_{it}) + d_t \bar{d}(e_{it}).$$

Plug in the labor supply condition

$$c_{it} + \underline{b} = w_t e_{it} \left(\frac{w_t e_{it}}{\varphi c_{it}^{\sigma}} \right)^{\frac{1}{\nu}} + (1+r_t) b_{it-1} - \tau_t \bar{\tau}(e_{it}) + d_t \bar{d}(e_{it}).$$

Firms Continuum of intermediate firms j. Production function

$$y_{jt} = Z_t n_{jt}.$$

Cost minimization $-w_t n_{jt} + mc_{jt}[Z_t n_{jt} - y_{jt}]$ yields labor demand

$$mc_{jt} = mc_t = \frac{w_t}{Z_t}.$$

Quadratic adjustment cost

$$\psi(p_{jt}, p_{jt-1}) = \frac{\mu}{\mu - 1} \frac{1}{2\kappa} \left[\log \left(\frac{p_{jt}}{p_{jt-1}} \right) \right]^2 Y_t.$$

Profit

$$d_{jt} = \frac{p_{jt}}{P_t} y_{jt} - w_t n_{jt} - \frac{\mu}{\mu - 1} \frac{1}{2\kappa} \left[\log \left(\frac{p_{jt}}{p_{jt-1}} \right) \right]^2 Y_t.$$

Profit maximization, where $M_t \equiv \beta^t c_t^{-\sigma}$

$$\max_{p_{jt}} E_0 \sum_{t=0}^{\infty} M_t P_t \left[\frac{p_{jt}}{P_t} y_{jt} - w_t n_{jt} - \psi(p_{jt}, p_{jt-1}) \right] \quad \text{s.t.} \quad y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\frac{\mu}{\mu-1}} Y_t.$$

Plug in the demand function

$$\max_{p_{jt}} E_0 \sum_{t=0}^{\infty} P_t M_t \left[\frac{p_{jt}}{P_t} \left(\frac{p_{jt}}{P_t} \right)^{-\frac{\mu}{\mu-1}} Y_t - w_t \left(\frac{p_{jt}}{P_t} \right)^{-\frac{\mu}{\mu-1}} \frac{Y_t}{Z_t} - \frac{\mu}{\mu - 1} \frac{1}{2\kappa} \left[\log \left(\frac{p_{jt}}{p_{jt-1}} \right) \right]^2 Y_t \right]$$

Phillips curve (ie FOC with respect to p_{jt}, after imposing symmetric equilibrium $p_{jt} = P_t$, defining aggregate inflation $1 + \pi_t \equiv P_t / P_{t-1}$, and aggregate discount factor $M_{t+1}/M_t = (1 + r_{t+1})^{-1}$) I FIND

$$(1 + \pi_t) \log(1 + \pi_t) = \kappa \left(\frac{w_t}{Z_t} - \frac{1}{\mu} \right) + \frac{1}{1 + r_{t+1}} \frac{Y_{t+1}}{Y_t} (1 + \pi_{t+1})^2 \log(1 + \pi_{t+1}).$$

Phillips curve AUCLERT ET AL.

$$\log(1 + \pi_t) = \kappa \left(\frac{w_t}{Z_t} - \frac{1}{\mu} \right) + \frac{1}{1 + r_{t+1}} \frac{Y_{t+1}}{Y_t} \log(1 + \pi_{t+1}).$$

Aggregate dividends

$$d_t = Y_t - w_t N_t - \psi_t.$$

Government Government budget constraint: the tax finances interest payment on bonds

$$\tau_t = r_t B.$$

Monetary policy

$$i_t = r_t^* + \phi \pi_t.$$

Fisher equation

$$1 + r_t = \frac{1 + i_{t-1}}{1 + \pi_t}.$$

Market clearing Aggregate production function and resource constraint

$$Y_t = Z_t N_t; \quad Y_t = C_t + \psi_t \quad \text{where } C_t \int_i c_{it} di.$$

Clearing in the bond and labor markets

$$B_t = \int_i b_{it} di; \quad N_t = \int_i e_{it} n_{it} di.$$

2 One-Asset HANK with Exogenous Uniform Cash Transfers

Same model as in Section 1, except that τ_t is no longer a proportional *tax* but instead becomes an exogenous, lump-sum *transfer*. Government debt B becomes a time-varying endogenous variable, B_t.

Households Continuum of households i. Utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu} \right\}.$$

Budget constraint and borrowing constraint

$$\begin{aligned} & c_{it} + b_{it} = w_t e_{it} n_{it} + (1+r_t) b_{it-1} + \tau_t + d_t \bar{d}(e_{it}), \\ & b_{it} \geq \underline{b}. \end{aligned}$$

Same FOCs as before

$$\begin{aligned} n_{it} &= \left(\frac{w_t e_{it}}{\varphi c_{it}^{\sigma}} \right)^{\frac{1}{\nu}}. \\ c_{it}^{-\sigma} &= \beta E_t (1+r_{t+1}) c_{it+1}^{-\sigma}. \end{aligned}$$

Firms Same as before

Government Government budget constraint: τ_t is now a transfer

$$\tau_t + (1+r_t) B_{t-1} = B_t.$$

In steady state, this implies $\tau = -rB$.

Market clearing Same as before

3 One-Asset HANK with Transfers and Taxes

Same model as in Section 2, but now we add exogenous consumption and labor taxes.

Households Continuum of households i. Utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu} \right\}.$$

Budget constraint and borrowing constraint

$$\begin{aligned} & (1+\tau_{ct}) c_{it} + b_{it} = (1-\tau_n) w_t e_{it} n_{it} + (1+r_t) b_{it-1} + \tau_t + d_t \bar{d}(e_{it}), \\ & b_{it} \geq \underline{b}. \end{aligned}$$

Optimal labor supply and consumption-saving decisions

$$\begin{aligned} n_{it} &= \left[\frac{(1-\tau_n) w_t e_{it}}{(1+\tau_{ct}) \varphi c_{it}^{\sigma}} \right]^{\frac{1}{\nu}}. \\ c_{it}^{-\sigma} &= \beta E_t \frac{1+\tau_{ct}}{1+\tau_{ct+1}} (1+r_{t+1}) c_{it+1}^{-\sigma}. \end{aligned}$$

Firms Same as before

Government Government budget constraint

$$\tau_t + (1+r_t)B_{t-1} = \tau_{ct}C_t + \tau_n W_t N_t + B_t.$$

Same monetary policy and Fisher equation

$$i_t = r_t^* + \phi\pi_t; \quad 1 + r_t = \frac{1+i_{t-1}}{1+\pi_t}.$$

Market clearing Same as before

4 One-Asset HANK with Sticky Wage

Same model, except that the labor supply equation is replaced by a wage Phillips curve.

Unions Continuum of unions k. Every household supplies every labor types, so each union represents all households. Quadratic adjustment costs

$$\psi_w(W_{kt}, W_{kt-1}) = \frac{\mu_w}{\mu_w - 1}\frac{1}{2\kappa_w}\left[\log\left(\frac{W_{kt}}{W_{kt-1}}\right)\right]^2.$$

Wage-setting problem

$$\max_{W_{kt}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{-\varphi\frac{N_{kt}^{1+\nu}}{1+\nu} + \lambda_{kt}\left[(1-\tau_n)W_{kt}N_{kt} - \psi_w(W_{kt}, W_{kt-1})\right]\right\} \text{ s.t. } N_{kt} = \left(\frac{W_{kt}}{W_t}\right)^{-\frac{\mu_w}{\mu_w-1}} N_t.$$

Plug in the labor demand function

$$\begin{aligned}\max_{W_{kt}} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & -\varphi\frac{N_t^{1+\nu}}{1+\nu}\left(\frac{W_{kt}}{W_t}\right)^{-\frac{\mu_w(1+\nu)}{\mu_w-1}} \\ & + P_t\lambda_{kt}\left[(1-\tau_n)W_{kt}\left(\frac{W_{kt}}{W_t}\right)^{-\frac{\mu_w}{\mu_w-1}} N_t - \frac{\mu_w}{\mu_w-1}\frac{1}{2\kappa_w}\left[\log\left(\frac{W_{kt}}{W_{kt-1}}\right)\right]^2\right]\Bigg\}.\end{aligned}$$

FOC wage

$$\log\left(\frac{w_t}{w_{t-1}}\right)\frac{w_t}{w_{t-1}} = \kappa_w\left(\varphi N_t^{1+\nu} - \frac{(1-\tau_n)w_t N_t}{\mu_w}\int_i e_{it}c_{it}^{-\sigma}di\right) + \beta\log\left(\frac{w_{t+1}}{w_t}\right)\frac{w_{t+1}}{w_t}.$$

Wage Phillips curve, where aggregate wage inflation is $1 + \pi_t^w = (1+\pi_t)w_t/w_{t-1}$

$$\log(1+\pi_t^w) = \kappa_w\left(\varphi N_t^{1+\nu} - \frac{(1-\tau_n)w_t N_t}{\mu_w}\int_i e_{it}c_{it}^{-\sigma}di\right) + \beta\log(1+\pi_{t+1}^w).$$