Consumption Tax in HANK

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1 One-Asset HANK as in Auclert et al. (2021)

In this model, τ_t is a *tax*, proportional to households' labor productivity e_{it}: $\tau_t \bar{\tau}(e_{it})$.

Households Continuum of households i. Utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu} \right\}.$$

Budget constraint and borrowing constraint

$$\begin{aligned} &c_{it} + b_{it} = w_t e_{it} n_{it} + (1+r_t) b_{it-1} - \tau_t \bar{\tau}(e_{it}) + d_t \bar{d}(e_{it}), \\ &b_{it} \geq \underline{b}. \end{aligned}$$

Lagrangian

$$\mathcal{L}_i = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu} + \lambda_{it} \left[w_t e_{it} n_{it} + (1+r_t) b_{it-1} - \tau_t \bar{\tau}(e_{it}) + d_t \bar{d}(e_{it}) - c_{it} - b_{it} \right] \right\}.$$

Optimal labor supply and consumption-saving decisions for unconstrained households

$$\begin{aligned} n_{it} &= \left(\frac{w_t e_{it}}{\varphi c_{it}^{\sigma}} \right)^{\frac{1}{\nu}}. \\ c_{it}^{-\sigma} &= \beta E_t (1+r_{t+1}) c_{it+1}^{-\sigma}. \end{aligned}$$

When the borrowing constraint binds, $b_{it} = \underline{b}$, the budget constraint rewrites as

$$c_{it} + \underline{b} = w_t e_{it} n_{it} + (1+r_t) b_{it-1} - \tau_t \bar{\tau}(e_{it}) + d_t \bar{d}(e_{it}).$$

Plug in the labor supply condition

$$c_{it} + \underline{b} = w_t e_{it} \left(\frac{w_t e_{it}}{\varphi c_{it}^{\sigma}} \right)^{\frac{1}{\nu}} + (1+r_t) b_{it-1} - \tau_t \bar{\tau}(e_{it}) + d_t \bar{d}(e_{it}).$$

Firms Continuum of intermediate firms j. Production function

$$y_{jt} = Z_t n_{jt}.$$

Cost minimization $-w_t n_{jt} + mc_{jt}[Z_t n_{jt} - y_{jt}]$ yields labor demand

$$mc_{jt} = mc_t = \frac{w_t}{Z_t}.$$

Quadratic adjustment cost

$$\psi(p_{jt}, p_{jt-1}) = \frac{\mu}{\mu - 1}\frac{1}{2\kappa}\left[\log\left(\frac{p_{jt}}{p_{jt-1}}\right)\right]^2 Y_t.$$

Profit

$$d_{jt} = \frac{p_{jt}}{P_t}y_{jt} - w_t n_{jt} - \frac{\mu}{\mu - 1}\frac{1}{2\kappa}\left[\log\left(\frac{p_{jt}}{p_{jt-1}}\right)\right]^2 Y_t.$$

Profit maximization, where $M_t \equiv \beta^t c_t^{-\sigma}$

$$\max_{p_{jt}} E_0 \sum_{t=0}^{\infty} M_t \left[\frac{p_{jt}}{P_t}y_{jt} - w_t n_{jt} - \psi(p_{jt}, p_{jt-1})\right]$$

subject to $\quad y_{jt} = Z_t n_{jt},\ y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\mu}{\mu-1}} Y_t,$ and $\psi(p_{jt}, p_{jt-1}) = \frac{\mu}{\mu - 1}\frac{1}{2\kappa}\left[\log\left(\frac{p_{jt}}{p_{jt-1}}\right)\right]^2 Y_t.$

Plug the constraints into the objective

$$\max_{p_{jt}} E_0 \sum_{t=0}^{\infty} M_t \left[\left(\frac{p_{jt}}{P_t}\right)^{-\frac{1}{\mu-1}} Y_t - w_t \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\mu}{\mu-1}} \frac{Y_t}{Z_t} - \frac{\mu}{\mu - 1}\frac{1}{2\kappa}\left[\log(p_{jt}) - \log(p_{jt-1})\right]^2 Y_t\right].$$

Phillips curve (ie FOC with respect to p_{jt}, after imposing symmetric equilibrium $p_{jt} = P_t$, defining inflation $1 + \pi_t \equiv P_t/P_{t-1}$, and aggregate discount factor $M_{t+1}/M_t = (1 + r_{t+1})^{-1}$)

$$\log(1 + \pi_t) = \kappa\left(\frac{w_t}{Z_t} - \frac{1}{\mu}\right) + E_t \frac{1}{1 + r_{t+1}}\frac{Y_{t+1}}{Y_t}\log(1 + \pi_{t+1}).$$

Aggregate dividends

$$d_t = Y_t - w_t N_t - \psi_t.$$

Government Government budget constraint: the tax finances interest payment on bonds

$$\tau_t = r_t B.$$

Monetary policy

$$i_t = r_t^* + \phi_\pi \pi_t.$$

Fisher equation

$$1 + r_t = \frac{1 + i_{t-1}}{1 + \pi_t}.$$

Market clearing Aggregate production function and resource constraint

$$Y_t = Z_t N_t; \quad Y_t = C_t + \psi_t \quad \text{where } C_t = \int_i c_{it} di.$$

Clearing in the bond and labor markets

$$B_t = \int_i b_{it} di; \quad N_t = \int_i e_{it} n_{it} di.$$

2 One-Asset HANK with Exogenous Uniform Cash Transfers

Same model as in Section 1, except that τ_t is no longer a proportional *tax* but instead becomes an exogenous, lump-sum *transfer*. Government debt B becomes a time-varying endogenous variable, B_t.

Households Same utility function, borrowing constraint, and FOCs as before. Only transfers in the budget constraint change

$$c_{it} + b_{it} = w_t e_{it} n_{it} + (1 + r_t) b_{it-1} + \tau_t + d_t \bar{d}(e_{it}).$$

Firms Same as before

Government Government budget constraint: τ_t is now a transfer

$$\tau_t + (1 + r_t) B_{t-1} = B_t.$$

In steady state, this implies $\tau = -rB$.

Market clearing Same as before

3 One-Asset HANK with Transfers and Taxes

Same model as in Section 2, but now we add exogenous consumption and labor taxes.

Households Same utility function as before. Budget and borrowing constraints

$$\begin{aligned}
&(1 + \tau_{ct}) c_{it} + b_{it} = (1 - \tau_n) w_t e_{it} n_{it} + (1 + r_t) b_{it-1} + \tau_t + d_t \bar{d}(e_{it}), \\
&b_{it} \geq \underline{b}.
\end{aligned}$$

Optimal labor supply and consumption-saving decisions

$$\begin{aligned}
n_{it} &= \left[\frac{(1 - \tau_n) w_t e_{it}}{(1 + \tau_{ct}) \varphi c_{it}^{\sigma}} \right]^{\frac{1}{\nu}}. \\
c_{it}^{-\sigma} &= \beta E_t \frac{1 + \tau_{ct}}{1 + \tau_{ct+1}} (1 + r_{t+1}) c_{it+1}^{-\sigma}.
\end{aligned}$$

Budget constraint when borrowing constraint binds

$$(1 + \tau_{ct}) c_{it} + \underline{b} = (1 - \tau_n) w_t e_{it} n_{it} + (1 + r_t) b_{it-1} + \tau_t + d_t \bar{d}(e_{it}).$$

Plug in the labor supply condition

$$(1 + \tau_{ct}) c_{it} + \underline{b} = (1 - \tau_n) w_t e_{it} \left[\frac{(1 - \tau_n) w_t e_{it}}{(1 + \tau_{ct}) \varphi c_{it}^{\sigma}} \right]^{\frac{1}{\nu}} + (1 + r_t) b_{it-1} + \tau_t + d_t \bar{d}(e_{it}).$$

Firms Same as before

Government Government budget constraint

$$\tau_t + (1 + r_t)B_{t-1} = \tau_{ct}C_t + \tau_n W_t N_t + B_t.$$

In steady state, this implies $\tau = -rB + \tau_c C + \tau_n WN$.

Market clearing Same as before

4 One-Asset HANK with Sticky Wage

Same model as in section 2, except that the labor supply equation is replaced by a wage Phillips curve.

Unions Continuum of unions k. Every household supplies every labor types, so each union represents all households. Union k chooses a wage W_{kt} to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_i \left[\frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu} \right] di - \psi_w(W_{kt}, W_{kt-1}) \right\}$$

$$\text{subject to} \quad N_{kt} = \left(\frac{W_{kt}}{W_t}\right)^{-\frac{\mu_w}{\mu_w - 1}} N_t \quad \text{and} \quad \psi_w(W_{kt}, W_{kt-1}) = \frac{\mu_w}{\mu_w - 1} \frac{1}{2\kappa_w} \left[\log\left(\frac{W_{kt}}{W_{kt-1}}\right) \right]^2.$$

Household i's total real earnings are

$$\begin{aligned} z_{it} &= (1-\tau_n) \frac{W_t}{P_t} e_{it} n_{it} \lambda \\ &= (1-\tau_n) \left(\frac{1}{P_t} \int_0^1 W_{k,t} e_{it} n_{ikt} dk \right) \\ &= (1-\tau_n) \left[\frac{1}{P_t} \int_0^1 W_{k,t} e_{it} \left(\frac{W_{kt}}{W_t}\right)^{-\frac{\mu_w}{\mu_w-1}} N_t dk \right]. \end{aligned}$$

The envelope theorem implies that we can evaluate indirect utility as if all income from the union wage change is consumed. That means $\frac{\partial c_{it}}{\partial W_{kt}} = \frac{\partial z_{it}}{\partial W_{kt}}$, where

$$\frac{\partial z_{it}}{\partial W_{kt}} = (1-\tau_n) \frac{e_{it}}{P_t} \left(1 - \frac{\mu_w}{\mu_w - 1}\right) N_{kt}.$$

Household i's total hours worked are

$$n_{it} = \int_0^1 \left(\frac{W_{kt}}{W_t}\right)^{-\frac{\mu_w}{\mu_w-1}} N_t dk.$$

Hours fall when W_{kt} increases

$$\frac{\partial n_{it}}{\partial W_{kt}} = -\frac{\mu_w}{\mu_w - 1} \frac{N_{kt}}{W_{kt}}.$$

Combining everything, the first-order condition of the union with respect to W_{kt} is

$$\begin{aligned} 0 = &\int_i N_{kt} \left\{ (1-\tau_n) \frac{e_{it}}{P_t} \left(1 - \frac{\mu_w}{\mu_w - 1}\right) c_{it}^{-\sigma} + \varphi \frac{\mu_w}{\mu_w - 1} \frac{1}{W_{kt}} n_{it}^{\nu} \right\} di \\ &- \frac{\mu_w}{\mu_w - 1} \frac{1}{\kappa_w} \log\left(\frac{W_{kt}}{W_{kt-1}}\right) \frac{1}{W_{kt}} + \beta \frac{\mu_w}{\mu_w - 1} \frac{1}{\kappa_w} E_t \log\left(\frac{W_{kt+1}}{W_{kt}}\right) \frac{1}{W_{kt}}. \end{aligned}$$

In a symmetric equilibrium, all unions set the same wage, so $W_{kt} = W_t$ and $n_{it} = N_{kt} = N_t$. Define wage inflation $1 + \pi_t^w = \frac{w_t}{w_{t-1}}$ and obtain the aggregate wage Phillips curve

$$\log(1 + \pi_t^w) = \kappa_w \left(\varphi N_t^{1+\nu} - \frac{(1-\tau_n) w_t N_t}{\mu_w} \int_i e_{it} c_{it}^{-\sigma} di \right) + \beta E_t \log(1 + \pi_{t+1}^w).$$

In steady state, the wage Phillips curve gives us, solving for φ

$$\varphi = \frac{(1-\tau_n) w N^{-\nu}}{\mu_w} \int_i e_i c_i^{-\sigma} di.$$

5 One-Asset HANK with Sticky Wage and Consumption Tax TO CHECK

Same model as in section 3, except that the labor supply equation is replaced by a wage Phillips curve in which we have consumption taxes.

Unions Continuum of unions k. Every household supplies every labor types, so each union represents all households. Union k chooses a wage W_{kt} to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_i \left[\frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu} \right] di - \psi_w(W_{kt}, W_{kt-1}) \right\}$$

subject to $N_{kt} = \left(\frac{W_{kt}}{W_t} \right)^{-\frac{\mu_w}{\mu_w - 1}} N_t$ and $\psi_w(W_{kt}, W_{kt-1}) = \frac{\mu_w}{\mu_w - 1} \frac{1}{2\kappa_w} \left[\log \left(\frac{W_{kt}}{W_{kt-1}} \right) \right]^2$.

Household i's total real earnings are

$$\begin{aligned}
z_{it} &= \frac{1-\tau_n}{1+\tau_{ct}} \frac{W_t}{P_t} e_{it} n_{it} \lambda \\
&= \frac{1-\tau_n}{1+\tau_{ct}} \left(\frac{1}{P_t} \int_0^1 W_{k,t} e_{it} n_{ikt} dk \right) \\
&= \frac{1-\tau_n}{1+\tau_{ct}} \left[\frac{1}{P_t} \int_0^1 W_{k,t} e_{it} \left(\frac{W_{kt}}{W_t} \right)^{-\frac{\mu_w}{\mu_w - 1}} N_t dk \right].
\end{aligned}$$

The envelope theorem implies that we can evaluate indirect utility as if all income from the union wage change is consumed. That means $\frac{\partial c_{it}}{\partial W_{kt}} = \frac{\partial z_{it}}{\partial W_{kt}}$, where

$$\frac{\partial z_{it}}{\partial W_{kt}} = \frac{1-\tau_n}{1+\tau_{ct}} \frac{e_{it}}{P_t} \left(1 - \frac{\mu_w}{\mu_w - 1} \right) N_{kt}.$$

Household i's total hours worked are

$$n_{it} = \int_0^1 \left(\frac{W_{kt}}{W_t} \right)^{-\frac{\mu_w}{\mu_w - 1}} N_t dk.$$

Hours fall when W_{kt} increases

$$\frac{\partial n_{it}}{\partial W_{kt}} = -\frac{\mu_w}{\mu_w - 1} \frac{N_{kt}}{W_{kt}}.$$

Combining everything, the first-order condition of the union with respect to W_{kt} is

$$
\begin{aligned}
0 = &\int_i N_{kt} \left\{ \frac{1-\tau_n}{1+\tau_{ct}} \frac{e_{it}}{P_t} \left(1 - \frac{\mu_w}{\mu_w - 1}\right) c_{it}^{-\sigma} + \varphi \frac{\mu_w}{\mu_w - 1} \frac{1}{W_{kt}} n_{it}^{\nu} \right\} di \\
&- \frac{\mu_w}{\mu_w - 1} \frac{1}{\kappa_w} \log\left(\frac{W_{kt}}{W_{kt-1}}\right) \frac{1}{W_{kt}} + \beta \frac{\mu_w}{\mu_w - 1} \frac{1}{\kappa_w} E_t \log\left(\frac{W_{kt+1}}{W_{kt}}\right) \frac{1}{W_{kt}}.
\end{aligned}
$$

In a symmetric equilibrium, all unions set the same wage, so $W_{kt} = W_t$ and $n_{it} = N_{kt} = N_t$. Define wage inflation $1 + \pi_t^w = \frac{w_t}{w_{t-1}}$ and obtain the aggregate wage Phillips curve

$$
\log(1 + \pi_t^w) = \kappa_w \left(\varphi N_t^{1+\nu} - \frac{(1-\tau_n) w_t N_t}{(1+\tau_{ct})\mu_w} \int_i e_{it} c_{it}^{-\sigma} di \right) + \beta E_t \log(1 + \pi_{t+1}^w).
$$

In steady state, the wage Phillips curve gives us, solving for φ

$$
\varphi = \frac{(1-\tau_n) w N^{-\nu}}{(1+\tau_c)\mu_w} \int_i e_i c_i^{-\sigma} di.
$$