The Natural and Unnatural Histories of Covid-19 Contagion

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We study the epidemiology of covid-19 using an overlapping-generations method (OLG) in which cohorts of infectives are temporarily contagious. OLG is less sensitive to measurement error than SIR. We use OLG to estimate R0 and Rt for covid-19 in various countries and over time using data on confirmed morbidity and estimates of unconfirmed morbidity. We use these data to study the effect of mitigation policy on Rt. We show that even in the absence of mitigation policy Rt tends to decrease by week 3 of the epidemic. Mitigation policy causes Rt to decrease more rapidly.

The natural history of contagion is expressed by R0, which is the number of susceptibles that infectives are expected to infect until they cease to be contagious. However, the natural history of contagion is rarely observed because it is concealed by mitigating action undertaken by individuals and governments, such as quarantine, social distancing, travel restrictions and lock down policy. The unnatural history of contagion, which is measured by the effective rate of contagion (Rt), is observed instead, which is inevitably smaller than its natural counterpart. Furthermore, infectives are imperfectly observed because some take time to be diagnosed, while others are not diagnosed at all.

The outbreak of covid-19 in early December 2019 in the Chinese city of Wuhan in Hubei prefecture presents a rare opportunity to study the natural history of contagion because mitigating action was not taken until January 24 when the government imposed a policy of lock down across the whole of China. So, do countries such as Sweden and Japan, where until recently their governments did not undertake mitigation. By contrast, a large number of countries provide opportunities to study the unnatural history of contagion for covid-19 because their governments undertook a variety of different mitigation policies including travel restrictions, quarantine, national and local lock downs, and mass testing.

We use data for a large number of countries to estimate the dynamics of R0 and Rt using an overlapping generations method (OLG), which is used to calculate R in real time. Since many carriers are undetected, we propose a simple method for inferring the latter from data on diagnosed carriers. We study the effect of the scale and timing of mitigation policy on Rt by different countries. Finally, we use the latter to project Rt in Israel. Our approach may be applied in other countries.

We draw three main conclusions. First, evidence from over 30 Chinese prefectures indicates that stringent mitigation policy reduced R to zero by mid March. Second, R tends to decrease even in the absence of mitigation policy. Third, mitigation policy accelerates the decrease in R. In many respects China serves as a crystal ball for other countries such as Israel. However, if mitigation is less stringent than in China, it will take somewhat longer to confine the epidemic into more manageable proportions.

Studies such as Ferguson et al (2020) are largely informed by experience with other infectious diseases under the implicit assumption that the epidemiology of covid-19 is similar. Using Chinese data for January they infer that covid-19 is highly contagious, and that eventually 90 percent of the world's population will be infected. They conclude that although mitigation policy will not change this conclusion, it will "flatten the curve", thereby saving lives by relieving pressure on the health services. We argue that the international evidence on covid-19 up to the end of March does not support this view in three respects. First, covid-19 does not appear to be as contagious as assumed by Ferguson et al. Second, isolating diagnosed infectives and mass testing have reduced rates of contagion. Third, evidence-based policy analysis reveals that lock downs have had a limited effect on contagion.

As of April 5, 2020, global prevalence for covid-19 was 0.01 percent. The highest prevalence occurred in Iceland (0.33 percent) and Luxembourg (0.31 percent). Prevalence in Italy, Spain and Switzerland are of the order of 0.2 percent. In many countries such as Japan and China prevalence is even smaller. Prevalence for Israel was 0.056 percent. These prevalence rates do not appear to be consistent with the hypothesis that covid-19 is a major pandemic. According to CDC 21 million Americans contracted swine flu during the first 6 months of the epidemic. So far (April 11) 533,088 Americans have contracted coronavirus.

The rest of the paper is organized as follows. In section 1 we introduce a new methodology for calculating contagion in real time, which we apply to a large number of countries and to Chinese provinces. We explain why we think this methodology is more reliable than the SEIR methodology used widely by epidemiologists. We also propose a methodology for inferring the number of covid-19 carriers, which is unknown, using data on diagnosed carriers. In section 2 we use international panel data to study the causal effect of mitigation policies on the rates of contagion estimated in section 1.

1. Methods

Overlapping Generations Model

Suppose, to begin with, that the duration (τ) of the infective period measured in days is known, and that infectives are diagnosed immediately. On day 0 the first infectives are diagnosed. We denote the true number of ever-infectives by C and days by t, hence C_0 is the initial number of infectives. We assume that the daily rate of contagion is $R0/\tau$ so that after τ days infectives infect R0 susceptibles, i.e. the temporal distribution of infection is uniform. The number of new infectives on day 1 is C_0R0/τ, so that the number of infectives at the end of day 1 is $C_1 = C_0 + C_0R0/\tau$. Until $t = \tau$ the number of infectives is $C_t = (1 + R0/\tau)C_{t-1}$. Repeated backward substitution implies that $C_t = C_0(1 + R0/\tau)^t$ and $C_\tau = C_0(1 + R0/\tau)^\tau$.

We distinguish between daily cohorts and generations of infectives. The first generation covers the first τ days, and comprises τ cohorts. The first cohort comprises C_0, the second comprises $\Delta C_1 = C_0R0/\tau$, and the last cohort comprises $\Delta C_{\tau-1}$. After $t = \tau$ cohorts, the first generation of infectives ceases to be contagious. During day $\tau + 1$ the first cohort of generation 1 ceases to be contagious. The next day the second cohort ceases to be contagious, and so on. By day 2τ all members of generation 1 cease to be contagious.

The number of ever-infected by the end of day $\tau + 1$ is $C_{\tau+1} = (1 + R0/\tau)(C_\tau – C_0)$ where the last term is the number of infectives on day τ, which equals the ever-infected minus the initial number of infectives (C_0) who ceased to be infective by day $\tau + 1$. More generally, $C_{\tau+j} = (1+ R0/\tau)(C_{\tau+j-1} - \Delta C_{j-1})$, which implies that by day 2τ the number of ever-infected is:

$$C_{2\tau} = C_\tau\left(1+\frac{R0}{\tau}\right)^{\tau} - \frac{R0}{\tau}\left(1+\frac{R0}{\tau}\right)^{\tau-1} C_0 \qquad (1)$$

Suppose initially there is only one infective ($C_0 = 1$), infectives are contagious for 14 days ($\tau = 14$), and on day 14 there are 70 infectives ($C_\tau = 70$), the implicit value for R0 during the first generation is 4.9636, which is the average rate of infection during the first 14 days of the epidemic. If this rate of infection applies over the next 14 days equation (1) implies that the number of ever-infected by day 28 of the epidemic will be 4881.7. If the number of ever-infected is less than this, R0 during the second generation is less than 4.9636. For example, if the number is 4000, equation (1) implies that R0 during days 14 – 28 is 4.6968 instead of 4.9636.

$R0/\tau$ refers to the daily rate of contagion (ROD), which varies directly with R0 and inversely with the duration of contagion (τ). The number of ever-infected obviously varies directly with the daily rate of contagion since:

$$\frac{\partial C_t}{\partial ROD} = \ln(1+ROD)\,[C_t - (1+ROD)C_{t-\tau-1}] > 0 \qquad (2)$$

Where t = τ + j. Equation (2) is positive because the term in square brackets is positive. Given ROD, the number of ever-infected varies directly, but perhaps less obviously, with the duration of contagion since:

$$\frac{\partial C_t}{\partial \tau} = R0Dln(1 + R0D)C_{t-\tau-1} \quad > 0 \qquad (3)$$

Finally, when ROD is not given, the separate effects of R0 and are determined by:

$$dC_t = \ln(1 + R0D)\,(C_t - C_{t-\tau-1})\frac{1}{\tau}(dR0 - R0Dd\tau) \qquad (4)$$

The number of ever-infected varies directly with R0 and inversely with the duration of contagion (τ). The reason for this apparent paradox is that when τ increases, the daily rate of contagion (R0D) decreases. Therefore, what matters is not R0, but the daily rate of contagion and the duration of contagion.

The counterpart of equation (1) for generation g is:

$$C_{g.\tau} = C_{g-1.\tau}\left(1 + \frac{R0}{\tau}\right)^{\tau} - \frac{R0}{\tau}\left(1 + \frac{R0}{\tau}\right)^{\tau-1} C_{(g-2)0} \qquad (5)$$

Where at the end of generation g, gτ = t, g-1.τ = t - τ and g-2.1 = t - 2τ. In summary, this is a τ- order overlapping generations (OLG) model in which infectives are contagious for τ days. The implicit value of R0 is "sharp"; it has no variance. These implicit values may vary over time, as indicated, and may be calculated for further generations of infectives as the epidemic continues. In particular, the OLG method may be used to calculate how Rt changes after mitigating actions such as travel bans, shut-downs etc. We have ignored the fact that the number of susceptibles decreases over time because in the early stages of epidemics the ever-infected constitute a tiny fraction of the population.

It may be show that the "doubling time" (the time in which C doubles) is equal to $ln2/\ln\,(1 + R0D)$. For example, if R = 3 and τ = 14 days, the doubling time is 3.23 days. The doubling time varies inversely with R and directly with τ.

Comparison of OLG with MCMC

The OLG method is conceptually different to the SIR methodology, which is widely used to estimate R0. Whereas OLG directly calculates R0 from the data, SIR infers

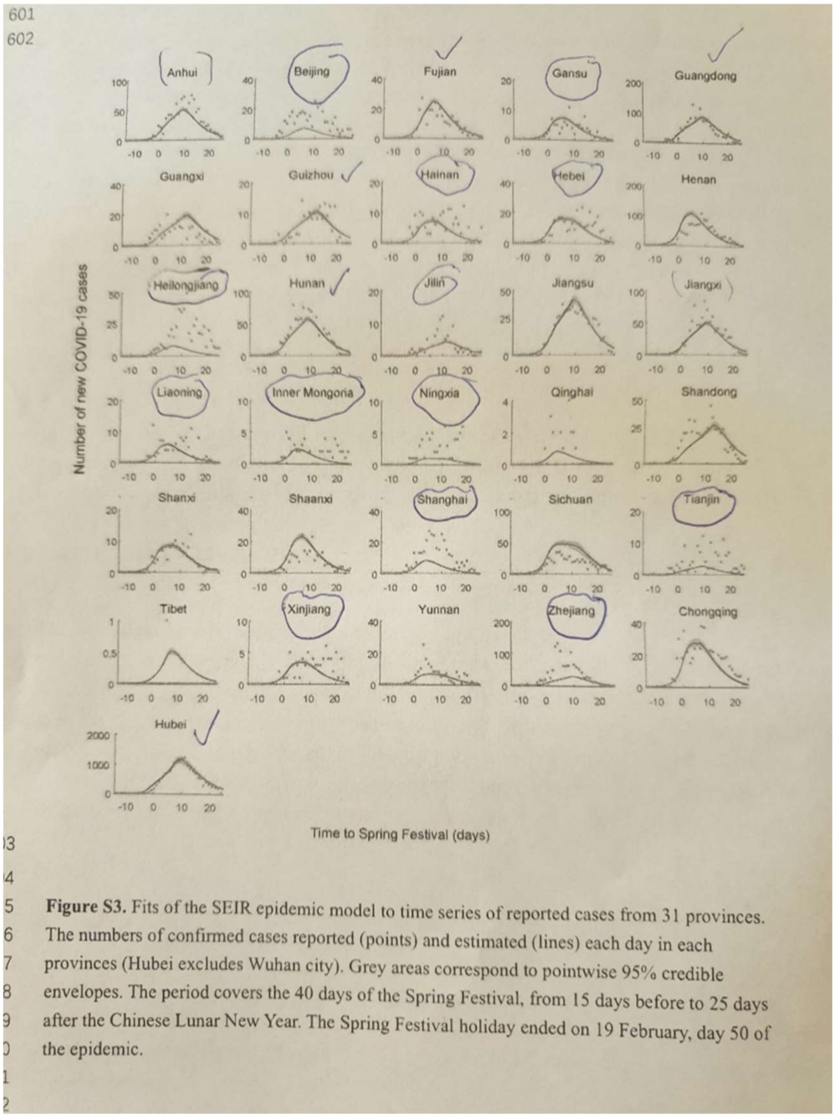
R0 by estimating a statistical model in which the entry rate into infection and the exit rate from infection are assumed to be exponential. The SIR estimate of R0 is the ratio of the entry rate to the exit rate. There are two methodological problems with this. If the entry and exit rates are not exponential, the estimate of R0 will be biased. Exponential exit implies that some patients recover almost instantaneously, while others recover extremely slowly. Evidence on recovery from covid-19 suggests that recovery rates are not exponential. Second, exit rates are estimated by regressing recoveries on lagged infectives. Since the latter generally contain measurement error, the estimated exit rate is affected by attenuation bias (see below). The SIR estimate of R0 involves both types of bias. A third problem, which arises during the mature stage of epidemics is that susceptibility to covid-19 is expected to decrease if the least resistant succumb first while the most resistant may not succumb at all. SIR usually ignores the effect of heterogeneity in resistance.

By contrast OLG does not involve statistical methods for inference. Hence, it is less sensitive to measurement error than SIR (see below). On the other hand, OLG makes assumptions about τ. However, the nature of the bias is known and equals the ratio of τ to its true value. For example, in the numerical illustration above, if the true value of τ is 20 days instead of 14 days, or 43 percent larger, R0 would have been 43 percent larger, i.e. 7.09 instead of 4.9636. By contrast the bias generated by SIR is entirely unknown.

A further methodological criticism of SIR concerns the widespread use of the Metropolis – Hastings algorithm used to estimate Bayesian Markov-Chain Monte Carlo (MC^2) models. The starting point is that the model is correct, but its parameters are unknown and need to be estimated using data. The method is Bayesian in the sense that priors are chosen for these parameters, such as R0. It is first order Markov because, for example, recoveries from covid-19 today are assumed to be strictly proportional to the number of yesterday's infected. Hence the day before yesterday etc does not matter. It is Monte Carlo based because random shocks to exits and entries have to be sampled (using the Gibbs sampler). Parameter estimates for exit and entry rates, and their standard deviations, are represented by their posterior distributions. The latter are generally influenced by their priors and their anterior distributions.

If the priors are chosen badly, so might their MC^2 estimates. Since, recovery from covid-19 is not exponential, a higher order Markov chain is required. Most importantly, since the model is assumed to be correct, it is never tested empirically. For example, Tian et al (2020) estimate R0 and other parameters in a SIR-type model by MC^2. However, their model poorly fits the data for covid-19 incidence in almost all provinces in China (see Figure 1). The posterior estimates of the entry and exit rates may be statistically significant to Bayesians but not to classical, frequentist statisticians.

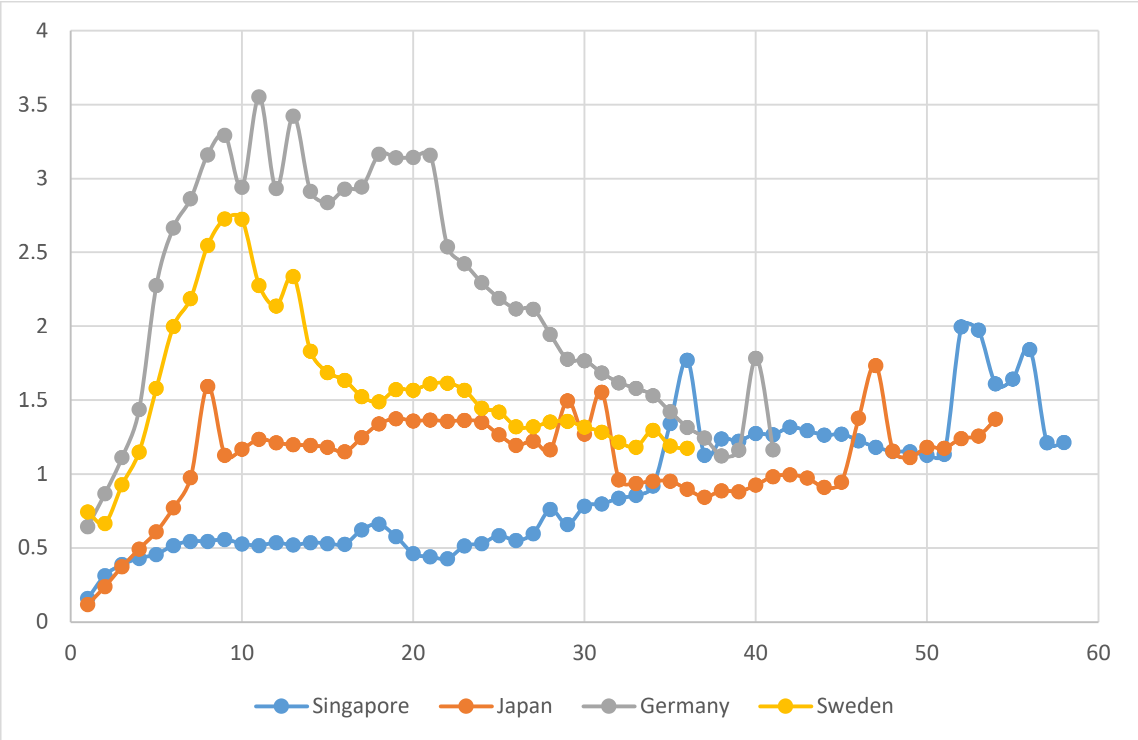
Figure 1 Example of Badness-of-fit of SEIR Models



OLG Results using Diagnosed Infectives

Figures 2 plots rolling daily OLG estimates for R in countries where according to the Oxford Covid-19 Government Response Tracker (OCGRT) mitigation policy was minimal. Note that the x axis is measured in "corona days" where day 1 occurs when the number of diagnosed infectives in large countries exceeds 100, to avoid "imported" infectives, and the threshold is reduced for less populated countries. The time series for R in Sweden and Germany follow a similar pattern; R increases and eventually decreases. This pattern does not, of course, result from mitigation policy. By contrast, in Japan and Singapore R has been relatively low. It has remained stable in Japan, but has increased in Singapore.

Figure 2 Rates of Contagion in Countries with Minimal Mitigation



Perhaps the most remarkable feature of Figure 2 is the low rates of contagion when mitigation is minimal. With the passage of corona time, Figure 2 suggests that after 5 weeks R may tend to a number close to 1. When R is less than 1 the contagious population decreases towards zero. When R is zero, the ever-infected population ceases to increase.

Despite the initial sharp increases in R in Sweden and Germany, these countries resisted mitigation policy until recently. Sweden began to mitigate during the second half of March and Germany towards the end of March. On March 23 Japan began a relatively stringent mitigation policy. We have included Singapore in Figure 2 despite the gradual increase in the stringency of mitigation policy, because the level of mitigation was and remains relatively low.

Mitigation scores for some countries are plotted in Figure 3 where the x axis is measured in calendar time. Italy has mitigated most, but has recently been overtaken by Israel, while Sweden has mitigated least. Mitigation has tended to become globally more stringent globally time. Indeed, stringency scores have tended to converge in excess of 80 points.

Figure 3 OCGRT Stringency Scores for Mitigation Policy

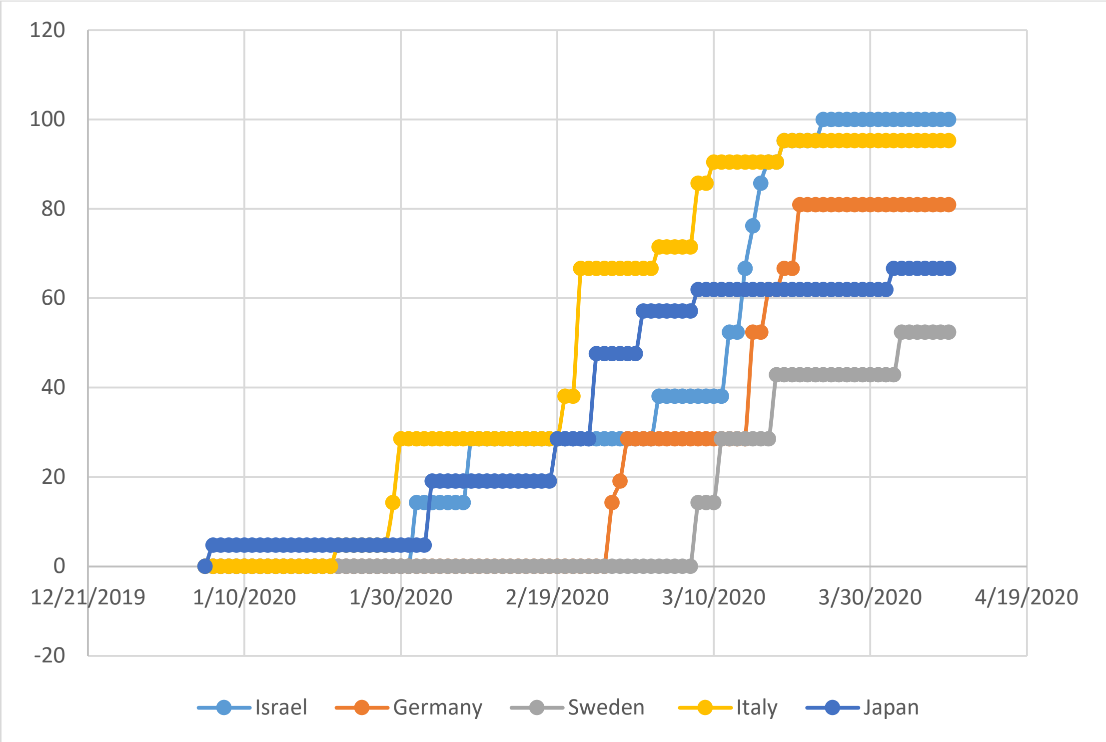
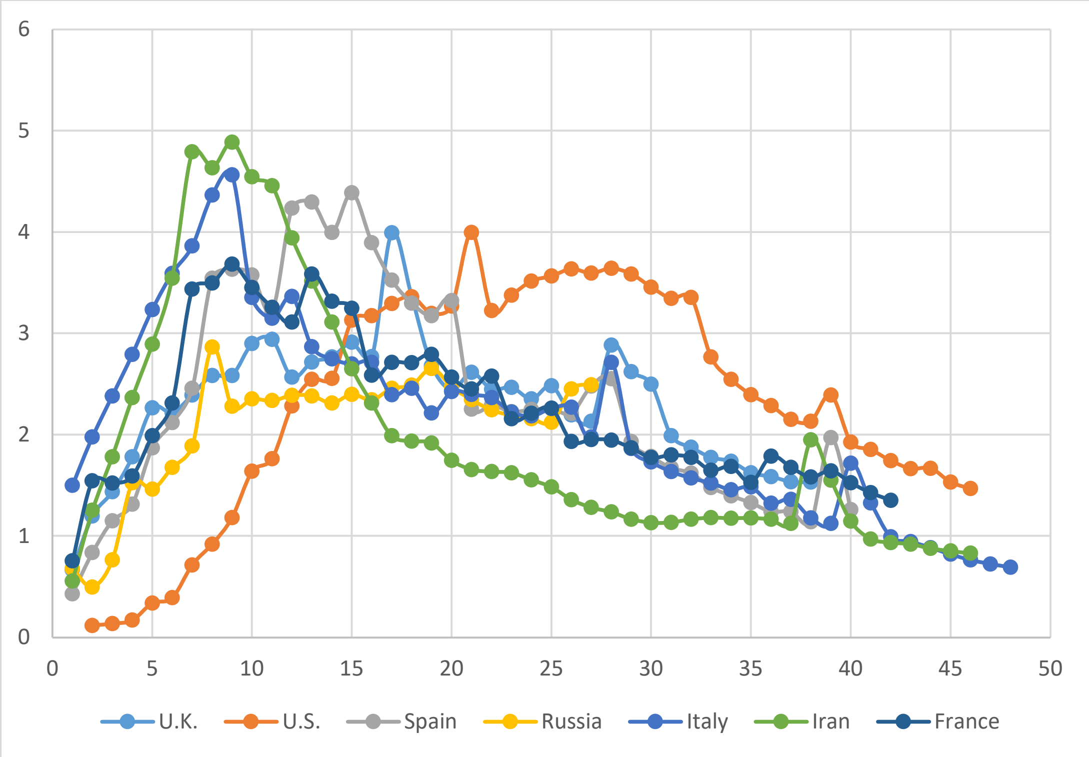


Figure 4 repeats Figure 2 for some other countries. It resembles Figure 2 insofar as R initially increases before it decreases. For example, R peaked in Italy at 4.6 before decreasing to 0.7. The most prominent peaks are for Iran, Italy and Spain where R peaked in excess of 4.

Figure 4 Rates of Contagion in Some Countries

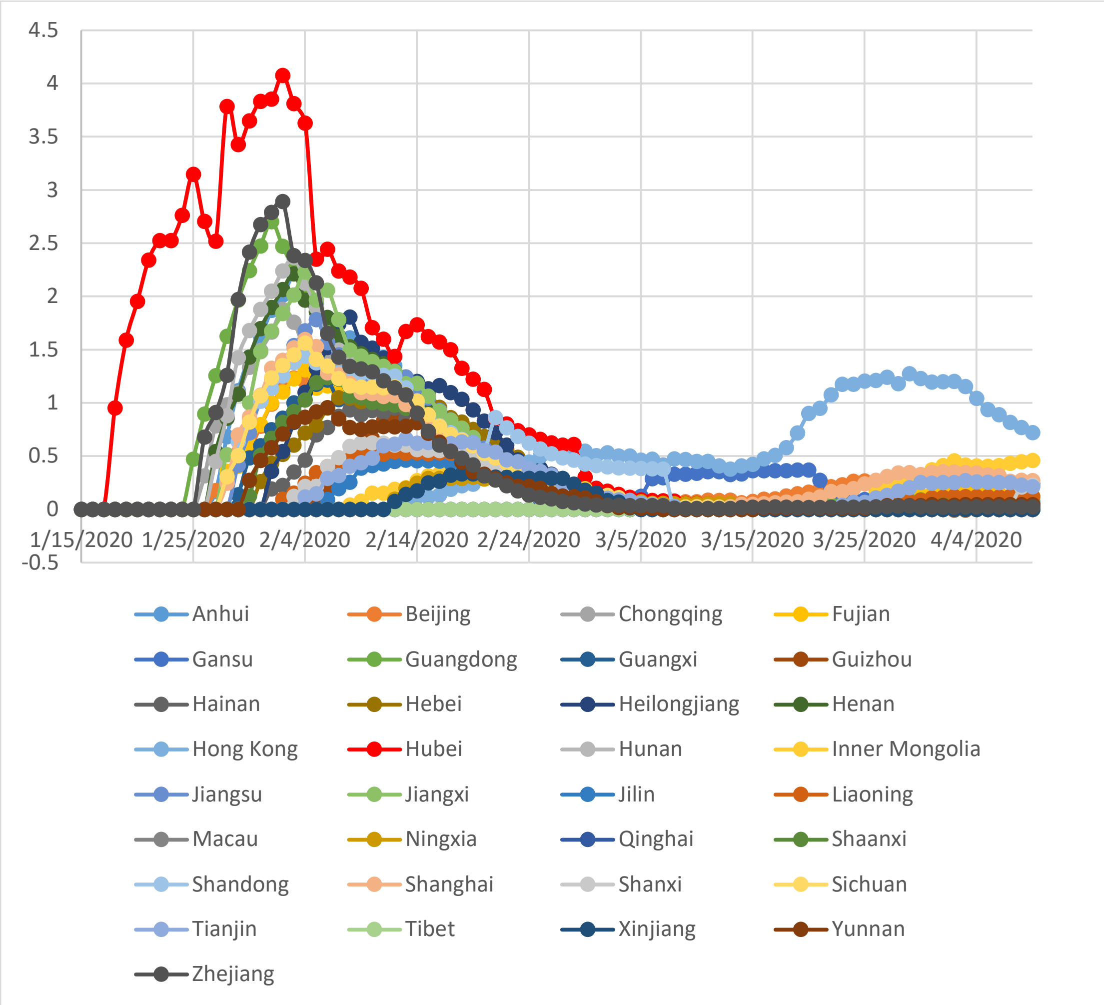


Finally, Figure 5 plots rates of contagion in Chinese provinces where days are measured by calendar time. On January 24/5 the entirety of China was locked down, by which time corona began to be diagnosed in other provinces apart from Hubei (where Wuhan is located). Note that following the lock down R appeared to peak at 3, before surging again to 4.1. Subsequently R decreased across China as a whole, but at different rates. Figure 5 indicates that R is heterogeneous both in terms of scale and dynamics. The strongest impressions from Figure 5 is the way in which R increased before it decreased across all Chinese provinces, and tended to zero during March.

During the last week of March, the government began to ease the lock down in all provinces apart from Hubei. Lock down was eased in Hubei during early April. Figure 5 indicates that since mid-March R began to increase again, especially in Hong Kong. However, in the majority of provinces R has been approximately zero. Some of the increase in R has been imported with the easement of international travel restrictions. It is too early to judge whether the increases in R in Tianjin and Inner Mongolia are related to the relaxation of restrictions.

We suggest below that developments in China may serve as a "crystal ball" for other countries because it has had the longest experience of coronavirus. This not only applies to the convergence of R towards zero, it also applies to what might be expected when restrictions are lifted. What happens to R during April is of critical importance.

Figure 5 Rates of Contagion in China



Diagnostic Delay and Hidden Infectives

Suppose infectives are not diagnosed immediately so that the true number of ever-infectives (C) is greater than the number of diagnosed infectives denoted by D. We assume that a proportion ϕ of infectives are never diagnosed; they remain hidden or ghosts, and that on each day a proportion θ of non-hidden infectives are diagnosed:

$$\Delta D_t = \theta[(1-\phi)C_{t-1} - D_{t-1}] \qquad (6)$$

The terms in square brackets is the backlog of cases to be diagnosed. Note that θ will depend on testing capacity of the health authorities.

The general solution for the number of diagnosed cases is:

$$D_t = A(1-\theta)^t + \theta(1-\phi)\sum_{j=0}^{t-1}(1-\theta)^j C_{t-1-j} \qquad (7)$$

Where A is an arbitrary constant determined by initial conditions D_0, and C_t is determined as in equation (1). Equation (7) states the number of diagnosed cases lags behind the number of carriers, of whom some will never be diagnosed. If, for expositional simplicity there are C infectives, equation (7) simplifies to:

$$D_t = A(1-\theta)^t + (1-\phi)C \qquad (8)$$

Where $A = D_0 - (1 - \phi)C$. Since $1 - \theta$ is a positive fraction and $\phi - 1$ is a negative fraction, equation (8) states that the number of diagnosed cases converges from below, as expected, to the number of non-hidden infectives. It implies that if x percent of them are diagnosed within d days θ would be:

$$\theta = 1 - \left[\frac{(1-\phi)(x-1)C}{A}\right]^{1/d} \qquad (9)$$

If according to the Director of CDC 25 percent of infectives are hidden ($\phi = 0.25$), there are a hundred infectives ($C = 100$) of which 5 have been diagnosed ($D_0 = 5$), 90 percent ($x = 0.9$) are diagnosed within 10 days if $\theta = 0.2$. This implies that the daily rate of diagnosis is 20 percent.

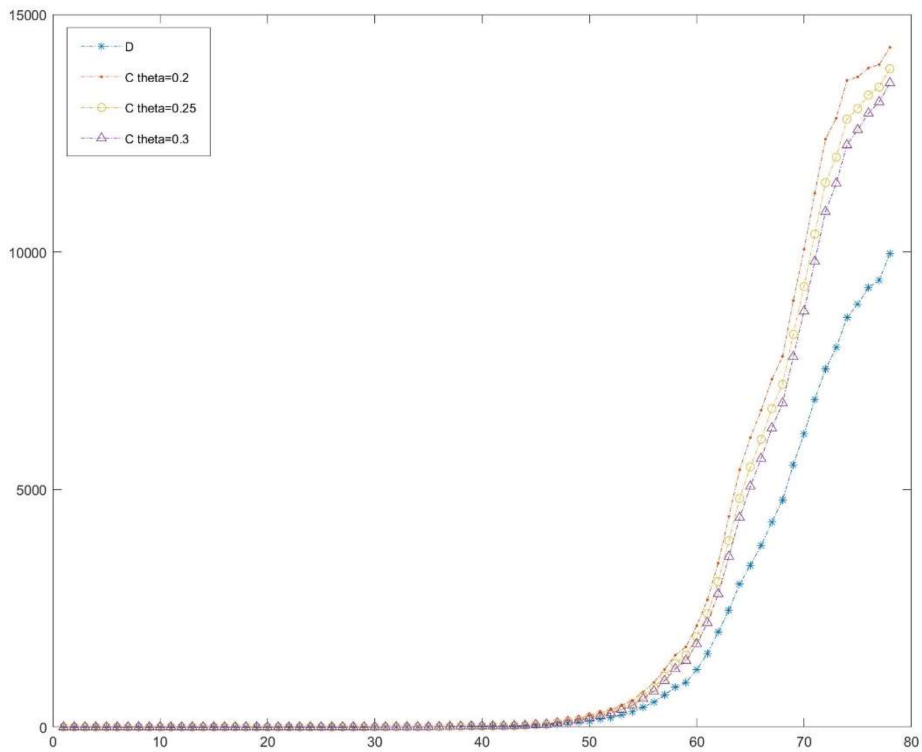
By reverse engineering, equation (6) implies that the true number of infectives is:

$$C_{t-1} = \frac{1}{1-\phi}\left(\frac{\Delta D_t}{\theta} + D_{t-1}\right) \qquad (10)$$

For example, if $\phi = 0.25$, $\theta = 0.2$, the number of diagnosed cases yesterday was 10,000 (D_{t-1}) and there are 800 newly diagnosed cases today (ΔD_t), the true number of infectives yesterday is 18,666 (C_{t-1}). In principle ϕ may be estimating by randomized testing for covid-19 in the population. Icelandic test data (see Table 1), which are not randomized, suggest that 0.43 percent of the population are asymptomatic carriers of covid-19. Without supplementary data on the proportion of carriers who eventually became symptomatic, randomized testing provides at best an upper bound for ϕ.

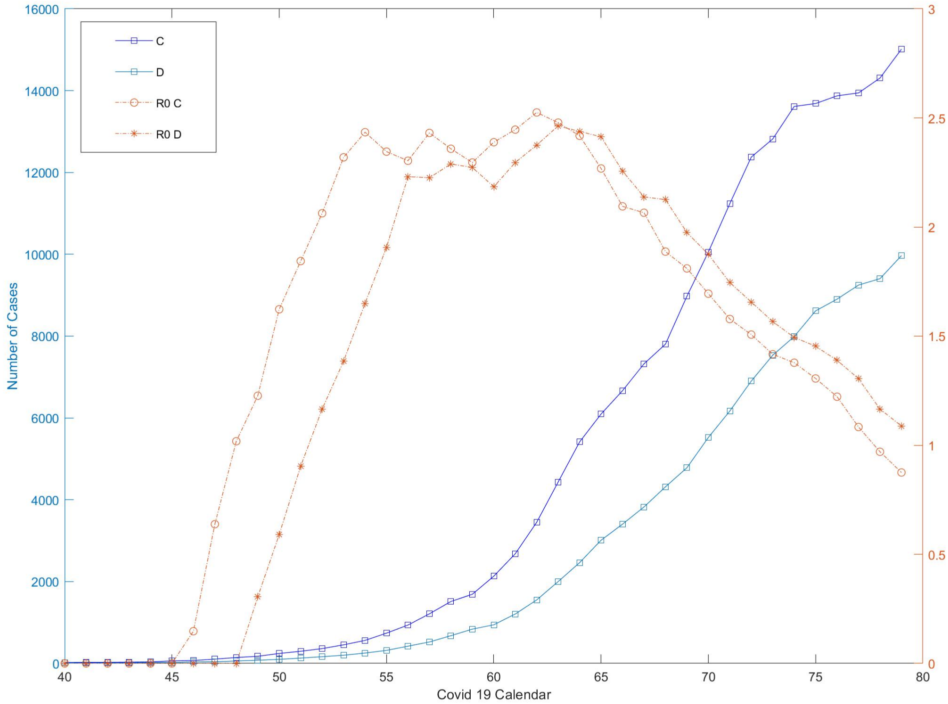
In Figure 6A equation (10) is used to calculate C using Israeli data for D setting ϕ = 0.25 for different values of θ. For these purposes a moving average of ΔD is specified to ensure that C cannot decrease. The difference between D and C naturally varies inversely with θ. By the end of the period (April 10) the number of diagnosed cases under-estimated the number of undiagnosed cases by about 29 percent. By construction 25 percent were corona zombies (since ϕ = 0.25) and 4 percent would be diagnosed eventually.

Figure 6A Diagnosed and Undiagnosed Covid-19 Carriers in Israel



In Figure 6B we report OLG estimates of R using the imputed values for C from Figure 6A, which are compared with their counterparts using the number of diagnosed cases. As expected, R is initially larger for C than it is for D. The former precedes the latter by about 5 days. However, the trends are similar, and both measures peak at the same level of R. Subsequently, R based on C is less than R based on D. This means that just as conventional estimates of R understate the rate of contagion when R is increasing, they overstate contagion when R decreases.

Figure 6B OLG Estimates of R including Asymptomatics (Israel)



Testing

In the vast majority of countries testing rates have been very small. Table 1 reports locations where testing rates have been relatively large. With the exception of S. Korea these locations are small, as expected. However, not all small locations, such as Andorra, have high rates of testing. Although Iceland has the highest testing rate per capita, as of April 1 only 5.36 percent of the population were tested. Testing naturally increases the prevalence of covid-19 because it identifies asymptomatic carriers. On the other hand, it reduces R because it prevents the latter from infecting susceptibles. Also, mortality rates among the infected tend to be relatively low.

Table 1 Locations with High Rates of Testing

	Test Rate %	Prevalence %	Mortality %	R
S. Korea	0.83	0.02	1.53	
Iceland	5.36	0.38	0.31	1.08
Faeroe Islands	3.85	0.36	0	1.19
Liechtenstein	2.32	0.40	0	2.71
Slovenia	1.07	0.04	1.85	
Isle of Man	1.25	0.11	1.07	

Notes: Data refer to late March and early April 2020.

Quarantine

Let q denote the proportion of infectives in quarantine. Because quarantine shortens the period during which infectives are contagious, the OLG model becomes:

$$C_t = [1 + (1 - q_{t-1})R0D](C_{t-1} - \Delta C_{t-\tau-1}) \qquad (11)$$

It is obvious that quarantine mitigates the propagation of coronavirus because it reduces the daily rate of contagion. Since only diagnosed infectives can be quarantined, the proportion of infectives in quarantine at the beginning of day t (the end of day t-1) is assumed to be $q_{t-1} = D_{t-1}/C_{t-1}$. This assumes that once diagnosed, infectives are quarantined.

We modify equation (6) by defining D in terms of contagious diagnosed instead of ever-diagnosed by subtracting patients who recovered or died:

$$\Delta D_t = \theta[(1 - \phi)C_{t-1} - D_{t-1}] - rD_{t-1} \qquad (12)$$

Where the recovery/mortality rate is denoted by r. In the early stages of the epidemic D is zero, because it takes time for infectives to be diagnosed, in which case q is initially zero. Subsequently, q becomes positive, which reduces the propagation of the epidemic through equation (11). Substituting for q_{t-1} equation (11) may be rewritten as:

$$\Delta C_t = R0D(C_{t-1} - D_{t-1}) - \Delta C_{t-\tau-1} - \frac{C_{t-1} - D_{t-1}}{C_{t-1}}\Delta C_{t-\tau-1} \qquad (13)$$

Equations (12) and (13) are simultaneous difference equations, which solve for infectives (C) and diagnosed infectives (D). Because equation (12) is linear, but equation (13) is nonlinear, we are unable to obtain analytical solutions. Nevertheless, they obviously imply that q grows over time from zero to less than one if R0D is positive. This means that OLG estimates of R0 tend to decrease after q becomes positive.

Regression Methods and Measurement Error

Suppose that instead of estimating R0D using OLG, we used regression methods. Denoting 1 + R0D by β, equation (1) suggests that β may be estimated by regression:

$$C_t = \beta(C_{t-1} - C_{t-1-\tau}) + u_t \qquad (14)$$

Where u ~ iiN(0, σ^2) denotes a residual error. Since C is unknown, β is estimated using data for D, which contains measurement error (m):

$$D_t = (1-\phi)C_t + m_t \qquad (15a)$$

$$m_t = \rho m_{t-1} + e_t \qquad (15b)$$

Where e is distributed iiN with zero mean. Equations (6) implies that measurement error will be autocorrelated as in equation (15b). Since the unconditional expectation of m is zero, equations (15) imply that diagnosed cases (D) lag behind their true counterparts (C), of which only 1 - ϕ percent of them are eventually diagnosed. Substituting equation (15a) into equation (13) implies that the regression model for D is:

$$D_t = \beta(D_{t-1} - D_{t-1-\tau}) + w_t \qquad (16a)$$

$$w_t = (1-\phi)u_t + m_t - \lambda(m_{t-1} - m_{t-1-\tau}) \qquad (16b)$$

From equation (15b) the sign of the covariance between D_{t-1} and m_{t-1} is ρ - λ and the sign of the covariance between $D_{t-1-\tau}$ and $m_{t-1-\tau}$ is negative. Hence, least squares estimates of β will be attenuated (biased downwards) especially if ρ is less than λ. This problem arises with other estimation methods including MC^2. Since estimates of β are attenuated, so are estimates of the daily rate of contagion (R0D).

Nevertheless, regression methods may yield unbiased estimates of β provided that the measurement error model is taken into consideration. Substituting equation (6) with iid residual error d into equation (11), implies the following autoregressive model for D:

$$\Delta D_t = -\theta D_{t-1} + \beta\Delta_\tau \Delta D_{t-1} + \beta\theta\Delta_\tau D_{t-2} + \omega_t \qquad (17a)$$

$$\omega_t = \theta(1-\phi)u_{t-1} + d_t - \beta\Delta_\tau d_{t-1} \qquad (17b)$$

Where Δ_τ denotes the "seasonal" difference operator ($\Delta_2 X_t = X_t - X_{t-2}$) . Since u and v are iid so is ω. Therefore, constrained least squares estimates of β and θ are unbiased. Note that equation (17a) delivers unbiased estimates of β and θ. Hence, it identifies that daily rate of contagion (R0D) and the rate at which covid-19 carriers are diagnosed (θ). Furthermore, it almost identifies the proportion of covid-19 ghosts (ϕ) since the variance of ω is:

$$\sigma_\omega^2 = \theta^2(1-\phi)^2\sigma_u^2 + (1+2\beta)\sigma_d^2 \qquad (17c)$$

Using estimates of θ and β from equation (17a) and σ_ω^2, combinations of ϕ and the variances of u and d are identified.

2 The Effect of Mitigation Policy on Contagion

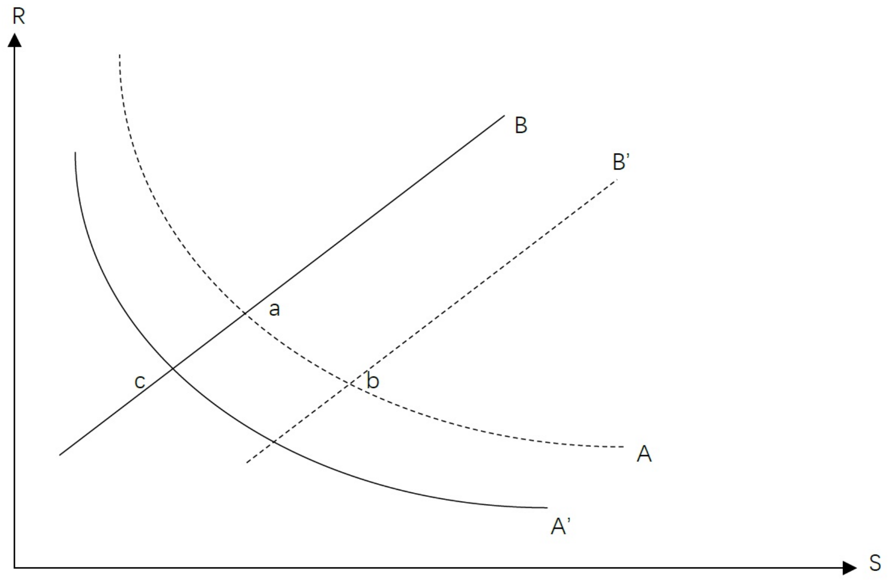
Ideally, the study of infectious diseases should take place after they have occurred. Although four months have passed since coronavirus broke out in China, most countries have short histories of coronavirus (Figures 1 and 3). These histories get longer by the day, but they are arguably too short to study the treatment effects of mitigation policy on contagion especially because it took time before governments took mitigating action, and because it takes time for mitigation to affect morbidity through its effect on Rt. Several countries ramped up their mitigation policies during the second half of March (Figure 3), so it is too early to judge their treatment effects. However, other countries, such as Spain and Italy undertook such policies earlier.

Consequently, the empirical results we report on the efficacy of mitigation policy are inevitably reported with reservation. On the other hand, mitigation policy in China seems to have stopped coronavirus after two months (Figure 5). Within five weeks Rt began to decrease towards 1 in all Chinese provinces, and within ten weeks it continued to decrease towards zero. We therefore look for early signs that the same is happening in other countries. However, whereas in China mitigation policy was very stringent, policy elsewhere was more variegated.

Corona Equilibrium and the Identification of Treatment Effects for Mitigation

We use the stringency index (S) developed by Oxford Covid-19 Government Response Tracker to measure mitigation policy in different countries. Our purpose is to identify the causal effect of mitigation policy on R when there may be reverse causality from contagion to mitigation policy. Indeed, in countries such as Japan and Singapore mitigation policy has been minimal because the prevalence of covid-19 was relatively small. By contrast, in China, Italy and Spain mitigation policy has been maximal because prevalence has been relatively large.

Figure 7 Corona Equilibrium



The identification problem is illustrated in Figure 7 where R is measured on the vertical axis and S on the horizontal. Schedule A plots the hypothesized negative causal effect of mitigation policy on R. The size of the treatment effect varies directly with its slope. Schedule B plots the policy response of mitigation to the rate of contagion. It expresses the idea that if morbidity and mortality are small, governments will mitigate less if at all. Governments more politically prone to mitigation will have flatter B schedules. The data for R and S are jointly determined at point a, where schedules A and B intersect. We refer to this as the "corona equilibrium"

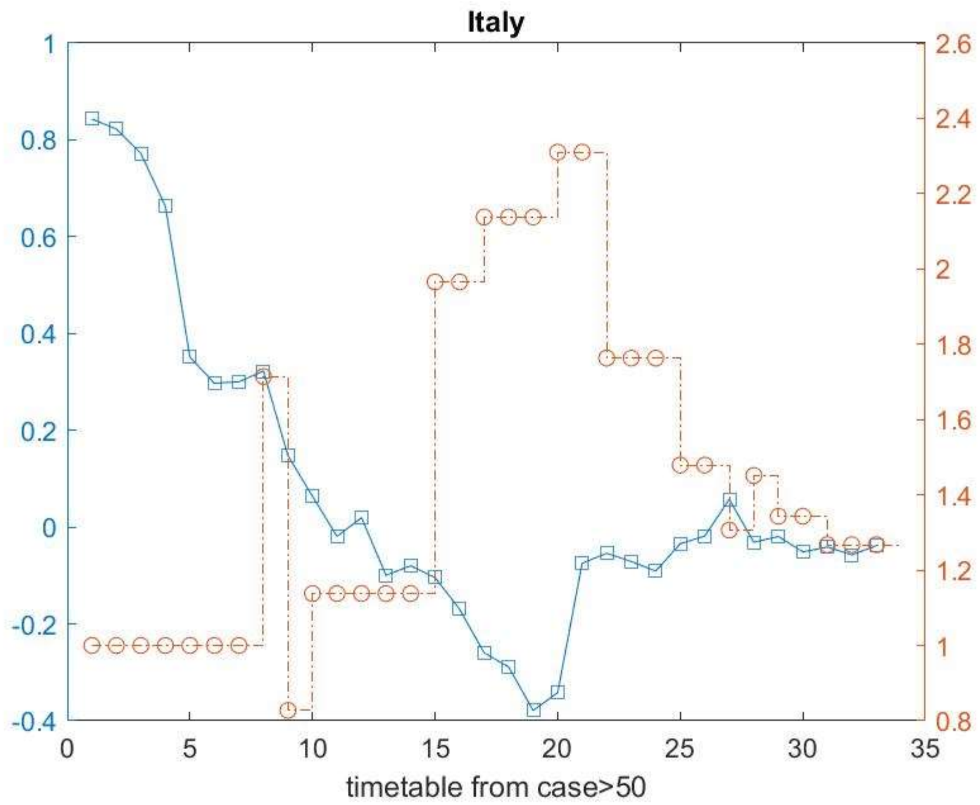
Since the treatment effect of mitigation is reflected in the slope of schedule A, we ideally wish to apply an autonomous increase in S such that schedule B shifts to the right to B' in which the new corona equilibrium would be point b. For example, countries with fewer ICUs (Italy?) may resort to more stringent mitigation as in schedule B. The slope of schedule A between points a and b is the local average treatment effect (LATE).

Suppose the population in another country is less susceptible to corona e.g. because of indigenous social distancing as in schedule A', i.e. R is smaller given S. The "corona equilibrium" will be determined at point c instead of point a, at which both R and S are smaller (Sweden?). The unwitting might think that R is smaller because S is smaller, but this confuses cause and effect. The corona equilibrium in countries with low susceptibility (A') and plentiful ICUs (B) may involve zero mitigation.

Triple Differences-in-Differences

We carry out three empirical exercises to identify the causal effect of mitigation policy on Rt. First, we use a triple differences-in-differences (3DID) strategy in which the first two differences refer to the changes in Rt before and after the change in mitigation policy, and the third difference consists of a comparison with R in a country, which did not mitigate altogether. For example, Italy adopted a stringent mitigation policy after which R decreased. But so did R decrease in Sweden despite the fact that it refrained from mitigation. If, however, the decrease in R in Italy was greater than in Sweden, the difference may be attributed to the fact that Italy mitigated whereas Sweden did not.

Figure 8 Event Analysis: Italy v Sweden



In Figure 8 the left-hand vertical axis measures the logarithm of the ratio of R in Italy to R in Sweden. The right-hand vertical measures the ratio between stringency in Italy and Sweden. The horizontal axis measures corona time in days. Initially R was much higher in Italy than in Sweden (blue). However, by day 19 it was considerably smaller in Italy, and towards the end R in Italy and Sweden were similar. On day 8 Italy began to mitigate, which was intensified on day 15 (brown schedule). On day 22 Sweden began to mitigate but by less than in Italy, hence the decrease in the brown schedule. Did the decrease in relative R result from the increase in relative S, and did the stabilization in relative R result from the subsequent decrease in relative S? Sweden's partial *volte face* over its non-mitigation policy most probably came too late to affect relative R (blue).

Since relative S increased by about 0.9 and the log of relative R decreased by about 0.5, the semi-elasticity of relative R to relative S was about -0.56 for Italy. The semi-elastic model implies:

$$lnR_A = lnR_B + lnR_A^* - lnR_B^* + \gamma\left[\left(\frac{S_A}{S_A^*}\right) - \left(\frac{S_B}{S_B^*}\right)\right] \qquad (18)$$

Where subscripts A and B refer to after and before treatment, * refers here to Sweden, and γ denotes the semi-elasticity. Semi-elasticities relative to Sweden for other countries are reported in Table 2. The absence of countries such as the UK is because mitigation policy came too late to calculate γ. However, in most countries such Netherlands γ appeared to be zero. The results reported in Table 2 are obviously provisional. With the passage of time it may be possible to affirm them. However, because countries are increasingly adopting similar treatments in terms of more stringent mitigation policies, Sweden included, it naturally becomes more difficult to estimate treatment effects in this way.

Table 2 3DID Estimates of Treatment Effects for Mitigation Policy

Country	**Semi-elasticity (γ)**
Austria	-1.20
Denmark	-2.00
France	-0.45
Germany	-1.60
Israel	-0.78
Italy	-0.56

Norway	-1.8
Spain	-1.00
Switzerland	-0.43

Equation (18) and the results in Table 2 imply that when Israel ramped-up its mitigation policy in March, R subsequently decreased by 23 percent (approximately 0.64).

Vector Autoregressions

Corona equilibrium theory implies that contagion and policy mitigation are dynamically interdependent. If R increases S tends to increase subsequently, which in turn may reduce R in the future. Vector autoregression (VAR) models capture this dynamic interaction, and their impulse response functions shed light on how autonomous shocks to mitigation policy influence, or Granger-cause R. VAR models also shed light on how autonomous shocks to R Granger-cause mitigation policy. However, it is the former, which concerns us here.

The VAR model for Israel is:

$$R_t = \underbrace{0.531}_{11.05} + \underbrace{1.031}_{29.58} R_{t-1} - \underbrace{0.00659}_{-6.96} S_{t-2} + r_t \qquad (19a)$$

$R^2(\text{adj}) = 0.984 \quad \text{ADF} = -3.0$

$$S_t = \underbrace{10.577}_{4.24} + \underbrace{0.777}_{14.46} S_{t-1} + \underbrace{5.746}_{3.06} R_{t-1} + s_t \qquad (19b)$$

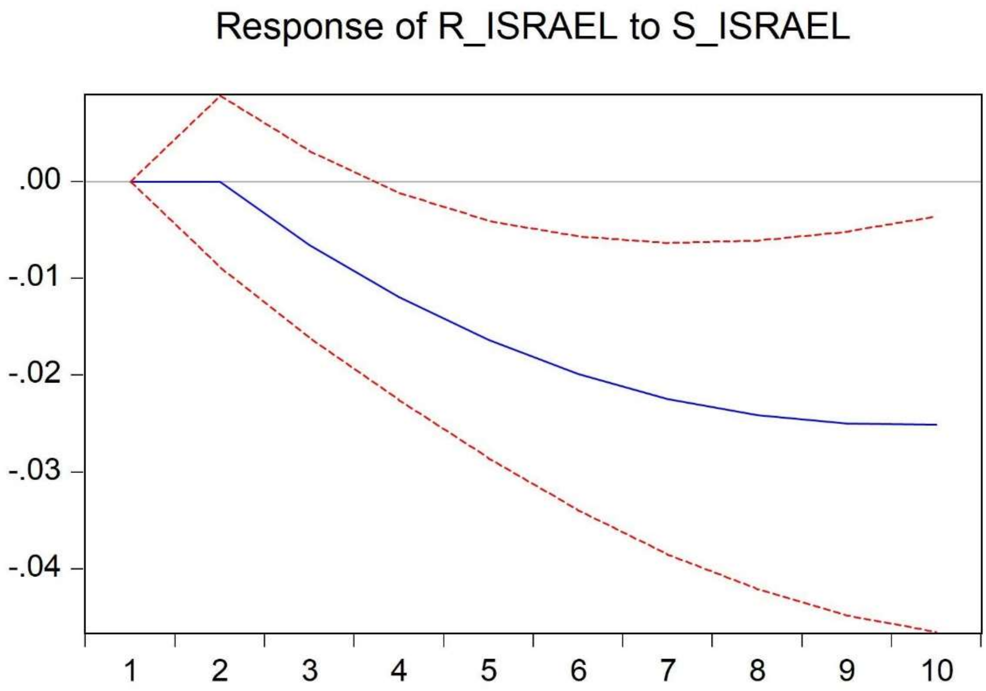
$R^2(\text{adj}) = 0.9747 \quad \text{ADF} = -2.375 \quad \text{se(s)} = 5.206$

Where t statistics are reported below their respective parameter estimates, r and s denote innovations, and ADF is their augmented Dickey – Fuller statistics, which test for their stationarity. R and S Granger-cause each other. As expected, R Granger-causes S positively and S Granger-causes R negatively.

Figure 9 plots the impulse responses for R of an s-shock of two standard deviations (equivalent to $\Delta S = 10.412$). After 10 days the cumulative impulse response is -0.022, i.e. R decreases by 0.022. This implies that when Israel ramped-up its mitigation stringency by 60, it reduced R by 0.127 within 10 days, which is less than what was implied by Table 2.

Note that Granger-causality is equivalent to causality when S_{t-2} in equation (19a) and R_{t-1} in equation (19b) are weakly exogenous, necessary conditions for which are that the VAR innovations, r and s, are serially independent and independent of each other.

Figure 9 Impulse Responses for Mitigation Shocks on R



Chinese Crystal Ball

Whereas Sweden served as the comparator in the 3DID estimates reported in Table 2, here we use China as a comparator. Whereas Sweden undertook minimal mitigation until recently, the opposite was true in China. Moreover, Chinese exposure to corona is much longer than in Sweden. As noted in Figure 5 R converged to zero in all Chinese provinces. If a country adopted Chinese style mitigation, it might be plausible to assume that the temporal diffusion profile of R might be similar to China's. If so, what happened in China to R might serve as a crystal ball for countries elsewhere. Since in most other countries mitigation policy was less stringent than in China, we test the crystal ball hypothesis using data for Israel.

This hypothesis predicts that R in Israel varies directly in corona time with R in China, and it varies inversely with the stringency of mitigation policy in Israel. In principle, what matters is relative stringency as in the triple difference method, but since stringency in China was maximal (until recently), its effect is absorbed into the intercept. Note that the crystal ball hypothesis does not imply that had Israel mitigated maximally as in China, R in Israel would have been the same as R in China. It means, instead, that the temporal diffusion of R in Israel is correlated with its Chinese counterpart. Moreover, the relation between R in Israel and R in China is not instantaneous. Instead, it is dynamically related through an error correction model.

We report the following result for Hubei province, where t refers to corona time in Israel and Hubei. In terms of calendar time Hubei is 53 days ahead of Israel.

$$\ln(1+R_t) = \underbrace{1.462}_{6.70} + \underbrace{0.915}_{27.86}\ln(1+R_{t-1}) + \underbrace{0.05}_{2.37}\ln(1+H_{t-1})$$

$$+\underbrace{0.058}_{2.16}\ln\,(1+H_{t-2}) - \underbrace{0.0152}_{-6.407}S_{t-7} \qquad (20)$$

R(adj) = 0.9961 DW = 2.043

Where H denotes R in Hubei. Since H = 0 the model is specified in terms of 1 + R and 1 + H. Equation (20) implies that in the long run the elasticity of R in Israel with respect to its counterpart in Hubei is $1.27\frac{1+R}{1+H}\frac{H}{R}$, which tends to zero as H tends to zero, and tends to 1.27 when H = R. The semi-elasticity with respect to stringency is $-0.18\frac{1+R}{1+H}\frac{H}{R}$. This constitutes the largest estimate of the treatment effect for mitigation policy. If H = R, a 10 percentage point increase in Israel's stringency score eventually reduces R by about 20 percent.

The equation implies that with the passage of corona time, R in Israel follows the diffusion profile for R in Hubei, but the level of R in Israel varies inversely with stringency in Israel with a lag of a week. Because the time series in equation (20) are nonstationary and cointegrated, the parameter estimates are super-consistent. Consequently, the causal effect of mitigation policy in Israel on R is identified.

Figure 10 projects R in Israel using equation (20) until the end of April. R peaked on March 25 and by April 7 it had decreased to 1.5. By the end of April, it is projected to decrease to 0.6.

Figure 10A Projection of R in Israel Until April 30: Current Policy

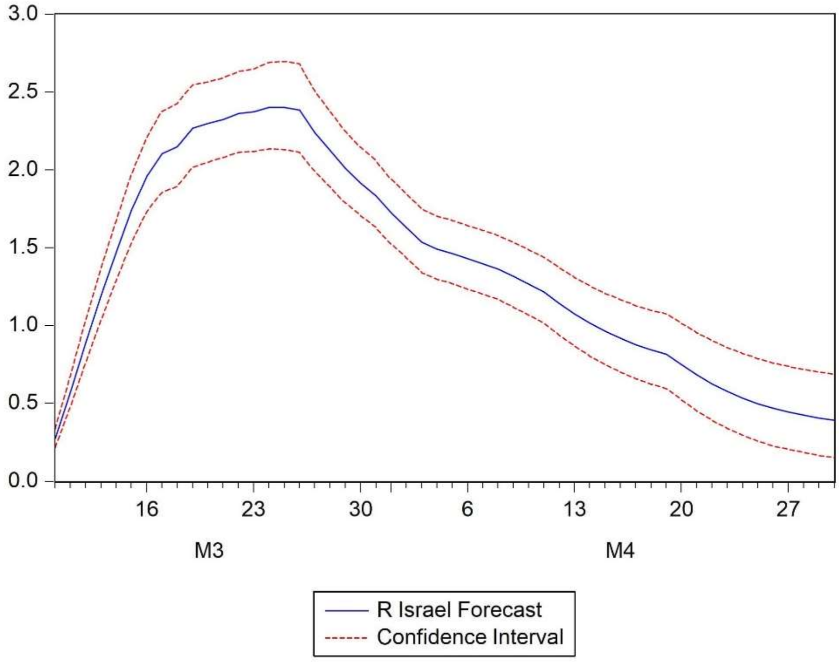
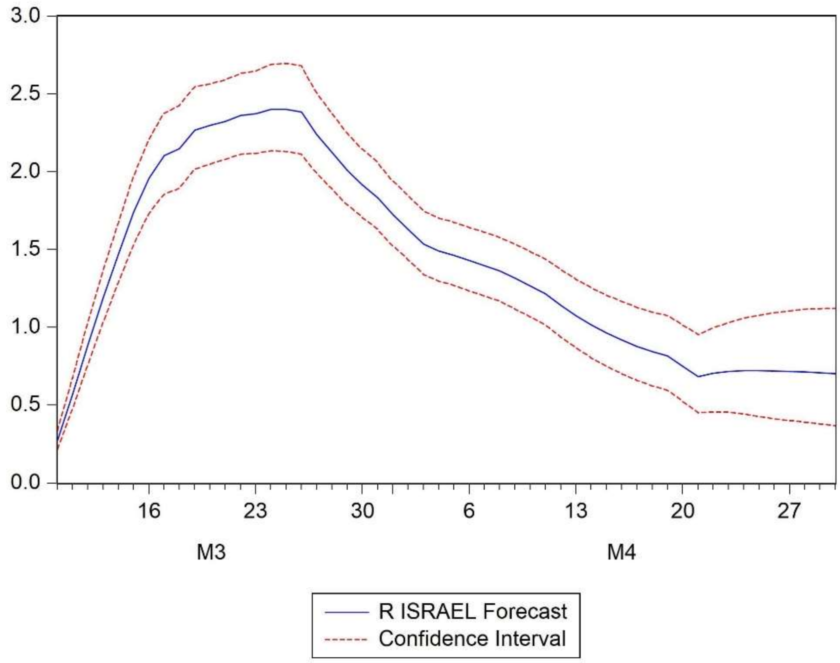


Figure 10B Partial Mitigation from April 19



In Figure 10B it is assumed that on April 19 the government partially relaxes its mitigation policy (by 50 points), as a result of which R stabilizes at 0.75 instead of decreasing as in Figure 10A. Finally, Figure 10C is a counterfactual simulation in which the increase in mitigation stringency since mid March did not occur. In the absence of lock down, R would have increased towards 4 instead of stabilizing and decreasing. However, by the end of April R would have been only slightly larger than in Figure 10A. The lock down simply brought forward a reduction in R that would have happened sooner or later.

Figure 10C No Lock Down since mid March