【 Lista 3 】

técnica de perturbação. $h = \lfloor \log_2 n \rfloor$

$$\sum_{i=0}^{h} (h-i)2^i$$

$$\begin{aligned}(h-0)\,2^0\\(h-1)\cdot 2^1\\(h-2)\cdot 2^2\\(h-3)\cdot 2^3\end{aligned}$$

$$T(n) = \sum_{i=0}^{h=\log_2^n} (h-i)\cdot 2^i = \sum_{i=0}^{h=\log_2^n} (2^i h - 2^i\cdot i) = \boxed{\sum_{i=0}^{h=\log_2^n} 2^i\cdot h} - \sum_{i=0}^{h=\log^n} 2^i\cdot i =$$

$$T(n) = h\cdot\sum_{i=0}^{h=\log_2^n} 2^i - \sum_{i=0}^{h=\log_2^n} 2^i\cdot i = \underbrace{h\cdot(2^{h+1}-1)}_{(*)} - \underbrace{\sum_{i=0}^{h=\log_2^h} 2^i\cdot i}_{} =$$

$$\sum_{i=0}^{h} 2^i\cdot i + (h+1)\cdot 2^{h+1} = \cancel{0}\cdot 2^0 + \sum_{i=0}^{h}(i+1)\cdot 2^{i+1} =$$

$$\sum_{i=0}^{h}(i+1)\cdot 2^i\cdot 2 = \sum_{i=0}^{h} 2i + 2 + 2\cdot i + 2^i =$$

$$\sum_{i=0}^{h} 2i\cdot 2^i + \sum_{i=0}^{h} 2^i$$

$$\sum_{i=0}^{h} 2^i\cdot i + (h+1)\cdot 2^{h+1} = \sum_{i=0}^{h} 2i\cdot 2^i + \sum_{i=0}^{h} 2^i$$

$$(h+1)\cdot 2^{h+1} - (2^{h+1}-1) = \sum_{i=0}^{h} i\cdot 2^i$$

$$h\cdot 2^{h+1} + 2^{h+1} - 2^{h+1} - 1 = \sum_{i=0}^{h} i\cdot 2^i$$

$$h\cdot 2^{h+1} - 1 = \sum_{i=0}^{h} i\cdot 2^i \quad (*) \quad //$$

$\Downarrow$

$$T(n) = h\cdot(2^{h+1}-1) - \sum_{i=0}^{h} i\cdot 2^i = h\cdot 2^{h+1} - h - h\cdot 2^{h+1} + 1$$

$$\boxed{T(n) = -h + 1}\ //$$

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