Complexidade de Algoritmos

Ex 3

Ⓐ $\begin{cases} T(n) = T(n/2) + n \\ T(1) = 1 \end{cases}$

$$T(n) = T(n/2) + n = \left[T(n/2^2) + n/2\right] + n$$

$$T(n) = \left[T\left(n/2^2\right) + \frac{n}{2}\right] + n = \left[T\left(n/2^3\right) + n/2^2 + n/2\right] + n$$

[...]

$$T(n) = T\left(n/2^k\right) + n/2^{k-1} + n/2^{k-2} + \dots + n/2 + n$$

Assumindo $\frac{n}{2^k} = 1$, então temos $n = 2^k$ e $k = \log n$

$$T(n) = T(1) + n\left[\frac{1}{2^{k-1}} + \frac{1}{2^{k-2}} + \dots + \frac{1}{2} + 1\right] = 1 + n\left[1 + 1\right]$$

$$\boxed{T(n) = 1 + 2n} = O(n)$$

Ⓑ $\begin{cases} T(n) = 2T(n-1) + n \\ T(1) = 1 \end{cases}$

$$T(n) = 2\left(2T(n-2) + n-1\right) + n = 4T(n-2) + 2(n-1) + n$$

$$T(n) = 8T(n-3) + 4(n-2) + 2(n-1) + n$$

[...]

$$T(n) = 2^{n-1} \underbrace{T(1)}_{1} + \sum_{i=0}^{n-2} 2^i (n-i) = 2^{n-1} + \sum_{i=0}^{n-2} 2^i (n-i)$$

SOMA GEOMETRICA!

$$T(n) = 2^{n-1} + \left(n \sum_{i=0}^{n-2} 2^i - \sum_{i=0}^{n-2} i \cdot 2^i\right) = 2^{n-1} + \left(n \cdot (2^{n-1} - 1) - \frac{n \cdot 2^n - 3 \cdot 2^n + 4}{2}\right)$$

$$T(n) = \left(n(2^{n-1} - 1) - (n \cdot 2^{n-1} - 3 \cdot 2^{n-1} + 2)\right) + 2^{n-1}$$

$$T(n) = \underline{(3 \cdot 2^{n-1}} - n - 2) \underline{+ 2^{n-1}} = 2 \cdot 2^{n-1} - n - 2 = 2^n - n - 2$$

$$\boxed{T(n) = 2^n - n - 2}$$

D $\begin{cases} T(n) = T(n/2) + \log_2 n = k \rightsquigarrow n = 2^k \\ T(1) = 1 \end{cases}$

SUBSTITUINDO...

$T(2^k) = T(2^k/2) + k$ → supondo $S(k) = T(2^k)$...

$S(k) = S(k-1) + k = S(k-2) + (k-1) + k = \dots =$

$S(k) = S(0) + 1 + 2 + \dots + k = S(0) + O(k^2)$, $S(0) = 1$

$S(k) = O(k^2) = T(2^k)$

$\rightsquigarrow S(k) = T(n)$ ∴ $T(n) = O(k^2)$

$$\boxed{T(n) = \log_2^2 n}$$

c $\begin{cases} T(n) = 4T(n/2) + n \\ T(1) = 1 \end{cases}$

$T(n) = 4T(n/2) + n$ $\times 4$ e $n = n/2$

$4T(n/2) = 4^2T(n/2^2) + 4 \cdot n/2$ $\times 4$ e $n = n/2^2$

$4^2T(n/2^2) = 4^3T(n/2^3) + 4^2 \cdot n/2^2$ ⟶ $T(n) = n + 4(n/2) + 4^2 n/2^2 + \dots + \frac{4^{k-1} n}{2^{k-1}} + 1$

$T(n) = \sum_{i=0}^{n} \frac{4^i n}{2^i} = \sum_{i=0}^{n} 2^i n = n \cdot \sum_{i=0}^{n} 2^i = n \cdot (2^{n+1} - 1)$

$\boxed{2^n = n}$

$T(n) = n(2n-1) \Rightarrow \boxed{T(n) = 2n^2 - n}$