Taller Problemas 1

1.4.5

#3. $e^{i'} \cdot \hat{e}_j = \delta^{i'}_j \longrightarrow \hat{e}_K \cdot \hat{e}_j = \delta^K_j$

$a^{i} = A^{i}_{j'}\, a^{j'} \longrightarrow \hat{e}_K = A^K_{i'}\, \hat{e}_{i'}$

$$\longrightarrow \cancel{\hat{e}_K \cdot \hat{e}_j} = A^K_{i'}\, \hat{e}_{i'} \cdot A^j_{m'}\, \hat{e}_{m'} = A^K_{i'} A^j_{m'} (\cancel{\hat{e}_{i'} \cdot \hat{e}_{m'}})$$

$$\delta^K_j = A^K_{i'} A^j_{m'}\, \delta^{i'}_{m'} = A^K_{i'} A^j_{i'} \;//$$

#4. a. $(x, y) \longrightarrow (-y, x)$

$x^{i'} = A^{i'}_j x^j = \left(\frac{\partial x^{i'}}{\partial x^j}\right) x^j \qquad x^j = \tilde{A}^j_{i'} x^{i'} = \left(\frac{\partial x^j}{\partial x^{i'}}\right) x^{i'}$

$$\longrightarrow \begin{cases} x^1 = -\tilde{x}^2 \\ x_2 = \tilde{x}^1 \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix} \;;\; \begin{pmatrix} -y \\ x \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$A^{i'}_K \tilde{A}^j_{i'} = \delta^j_K \longrightarrow \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ sí cumple

b. $(x, y) \longrightarrow (x, -y)$

$$\longrightarrow \begin{cases} x^1 = \tilde{x}^1 \\ x^2 = -\tilde{x}^2 \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix} \;;\; \begin{pmatrix} x \\ -y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ sí cumple

c. $(x, y) \longrightarrow (x - y, x + y)$

$$\longrightarrow \begin{cases} x^1 = \tilde{x}^1 - \tilde{x}^2 \\ x^2 = \tilde{x}^1 + \tilde{x}^2 \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} x - y \\ x + y \end{pmatrix} \;;\; \begin{pmatrix} x - y \\ x + y \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I$ No cumple

d. $(x, y) \longrightarrow (x + y, x - y)$

$$\longrightarrow \begin{cases} x^1 = \tilde{x}^1 + \tilde{x}^2 \\ x^2 = \tilde{x}^1 - \tilde{x}^2 \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} x + y \\ x - y \end{pmatrix} \;;\; \begin{pmatrix} x + y \\ x - y \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I$ No cumple

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1.5.7

#2. a. $\nabla(\Phi\Psi) = \partial_i(\Phi\Psi) = \partial_i(\Psi)\Phi + \partial_i(\Phi)\Psi = \Phi\nabla\Psi + \Psi\Delta\Phi$ //

d. $\nabla\cdot(\nabla\times a) = \partial_i\cdot(\partial_j \times a_k) = \partial_i\cdot(\varepsilon_{ijk}\partial_j a_k)$

$= \partial_i\varepsilon_{ijk}\partial_j a_k \rightarrow \varepsilon_{ijk}\partial_i\partial_j a_k = \underline{\varepsilon_{ijk}}\partial_i\partial_i a_k = 0$ // $\quad i=j$

$\nabla\times(\nabla\cdot a) = \varepsilon_{ijk}\partial_j\partial_m a_m$

La operación no funciona al usar producto cruz con un escalar

E. $\nabla\times(\nabla\times a) = \partial_k\times(\varepsilon_{ijk}\partial_j a_k) = \varepsilon_{imn}\partial_m\varepsilon_{mjk}\partial_j a_k$

$= \varepsilon_{imn}\varepsilon_{mjk}\partial_m\partial_j a_k = (\delta_{ij}\delta_{mk} - \delta_{ik}\delta_{jm})\partial_m\partial_j a_k$

$= \delta_{ij}\delta_{mk}\partial_m\partial_j a_k - \delta_{ik}\delta_{jm}\partial_m\partial_j a_k$

$= \partial_i(\partial_k a_k) - \partial_j\partial_j a_i = \nabla(\nabla\cdot a) - \nabla^2 a$ //

1.6.5

#2. $e^{i\theta} = \cos\theta + i\,\text{sen}\,\theta \longrightarrow \theta = 3\alpha$

$Z = \cos(3\alpha) + i\,\text{sen}(3\alpha) = e^{i(3\alpha)} = (e^{i\alpha})^3$

$= (\cos\alpha + i\,\text{sen}\,\alpha)^3$

$= \underbrace{\cos^3\alpha}_{Re(z)} + \underbrace{3\cos^2\alpha\, i\,\text{sen}\,\alpha}_{Im(z)} \underbrace{- 3\cos\alpha\,\text{sen}^2\alpha}_{Re(z)} \underbrace{- i\,\text{sen}^3\alpha}_{Im(z)}$

$Re(z): \cos(3\alpha) = \cos^3\alpha - 3\cos\alpha\,\text{sen}^2\alpha$ //

$Im(z): \text{sen}(3\alpha) = 3\cos^2\alpha\,\text{sen}\,\alpha - \text{sen}^3\alpha$ //

#5. a. $\sqrt{2i} \longrightarrow z = 2i \longrightarrow z^{1/2} = (2i)^{1/2} \qquad \theta = \pi/2$

$z^{1/2} = (2e^{i\pi/2})^{1/2} = \sqrt{2}\left(\cos\left(\frac{\pi/2 + 2k\pi}{2}\right) + i\,\text{sen}\left(\frac{\pi/2 + 2k\pi}{2}\right)\right)$

$K = 0, 1 \longrightarrow z^{1/2} = \sqrt{2}\left(\cos\frac{\pi}{4} + i\,\text{sen}\frac{\pi}{4}\right) = 1 + i$ //

$= \sqrt{2}\left(\cos\frac{5\pi}{4} + i\,\text{sen}\frac{5\pi}{4}\right) = -(1+i)$ //

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b. $\sqrt{1-\sqrt{3}i} \rightarrow z = 1-\sqrt{3}i \rightarrow z^{1/2} = (1-\sqrt{3}i)^{1/2}$ $\quad |z| = 2$

$\tan\theta = -\sqrt{3}/1 \rightarrow \theta = \tan^{-1}(-\sqrt{3}) = -\pi/3$

$z^{1/2} = 2^{1/2}\left(\cos\left(\frac{-\pi/3+2k\pi}{2}\right) + i\,sen\left(\frac{-\pi/3+2k\pi}{2}\right)\right)$

$k=0,1 \rightarrow z^{1/2} = \sqrt{2}\left(\cos\left(\frac{-\pi}{6}\right) + i\,sen\left(\frac{-\pi}{6}\right)\right) = \frac{\sqrt{2}}{2}(\sqrt{3}-i)$ //

$= \sqrt{2}\left(\cos\frac{5\pi}{6} + i\,sen\frac{5\pi}{6}\right) = \frac{\sqrt{2}}{2}(-\sqrt{3}+i)$ //

c. $(-1)^{1/3} \rightarrow z = -1 \rightarrow z^{1/3} = (-1)^{1/3}$

$\tan\theta = 0/-1 \rightarrow \theta = \tan^{-1}(0) = \pi$

$z^{1/3} = 1^{1/3}\left(\cos\left(\frac{\pi+2k\pi}{3}\right) + i\,sen\left(\frac{\pi+2k\pi}{3}\right)\right)$

$k=0,1,2 \rightarrow z^{1/3} = \cos\frac{\pi}{3} + i\,sen\frac{\pi}{3} = \frac{1}{2}(1+i\sqrt{3})$ //

$= \cos\pi + i\,sen\,\pi = -1$ //

$= \cos\frac{5\pi}{3} + i\,sen\frac{5\pi}{3} = \frac{1}{2}(1-i\sqrt{3})$ //

d. $8^{1/6} \rightarrow z = 8 \rightarrow z^{1/6} = 8^{1/6}$ $\quad \tan\theta = 0/1 \rightarrow \theta = 0$

$z^{1/6} = 8^{1/6}\left(\cos\left(\frac{2\pi k}{6}\right) + i\,sen\left(\frac{2k\pi}{6}\right)\right)$

$k=0,1,2,3,4,5 \rightarrow z^{1/6} = \sqrt{2}\,(\cos(0) + i\,sen(0)) = \sqrt{2}$ //

$= \sqrt{2}\left(\cos\frac{\pi}{3} + i\,sen\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}(1+i\sqrt{3})$ //

$= \sqrt{2}\left(\cos\frac{2\pi}{3} + i\,sen\frac{2\pi}{3}\right) = \frac{\sqrt{2}}{2}(-1+i\sqrt{3})$ //

$= \sqrt{2}(\cos\pi + i\,sen\,\pi) = -\sqrt{2}$ //

$= \sqrt{2}\left(\cos\frac{4\pi}{3} + i\,sen\frac{4\pi}{3}\right) = \frac{-\sqrt{2}}{2}(1+i\sqrt{3})$ //

$= \sqrt{2}\left(\cos\frac{5\pi}{6} + i\,sen\frac{5\pi}{6}\right) = \frac{\sqrt{2}}{2}(-\sqrt{3}+i)$ //

e. $\sqrt[4]{-8-8\sqrt{3}i} \rightarrow z = -8-8\sqrt{3}i \rightarrow z^{1/4} = (-8-8\sqrt{3}i)^{1/4}$ $\quad \tan\theta = 8\sqrt{3}/8 \rightarrow \theta = -\frac{2\pi}{3}$

$z^{1/4} = 16^{1/4}\left(\cos\left(\frac{-2\pi/3+2k\pi}{4}\right) + i\,sen\left(\frac{-2\pi/3+2k\pi}{4}\right)\right)$

$k=0,1,2,3 \rightarrow z^{1/4} = 2\left(\cos\left(\frac{-\pi}{6}\right) + i\,sen\left(\frac{-\pi}{6}\right)\right) = \sqrt{3}-i$ //

$= 2\left(\cos\frac{\pi}{3} + i\,sen\frac{\pi}{3}\right) = 1+i\sqrt{3}$ //

$= 2\left(\cos\frac{5\pi}{6} + i\,sen\frac{5\pi}{6}\right) = -\sqrt{3}+i$ //

$= 2\left(\cos\frac{4\pi}{3} + i\,sen\frac{4\pi}{3}\right) = -1-i\sqrt{3}$ //

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#6. a. $z = -ie$ $\theta = -\pi/2$ $n = 0$

$\text{Log } z = \text{Ln}(e) + i(-\pi/2) = 1 - i\frac{\pi}{2}$ //

b. $z = 1 - i$ $\tan\theta = -1/1 \rightarrow \theta = -\pi/4$ $n = 0$

$\text{Log } z = \text{Ln}(\sqrt{2}) + i(-\pi/4) = \frac{1}{4}(2\text{Ln}2 - i\pi)$ //

c. $z = e$ $\theta = 0$ $n = n$

$\text{Log } z = \text{Ln}(e) + i(0 + 2n\pi) = 1 + i\,2n\pi$ //

d. $z = i$ $\theta = \pi/2$ $n = n$

$\text{Log } z = \text{Ln}(1) + i(\pi/2 + 2n\pi) = i(\pi/2 + 2n\pi)$ //

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