Tarea 15-Feb.

Función de costo

$$\min_{\theta} \; \mathop{\mathbb{E}}_{x} \left\{ \| x_n - \hat{x}_n \|_2^2 \right\}$$

$$z_n = x_n \theta$$
$$\hat{x}_n = z_n \theta^T$$

$$\| x_n - \hat{x}_n \|_2^2 = \langle x_n - \hat{x}_n, x_n - \hat{x}_n \rangle$$

$$= \langle x_n - z_n \theta^T, x_n - z_n \theta^T \rangle$$

$$= x_n x_n^T - 2 x_n (z_n \theta^T) + z_n \theta^T (z_n \theta^T)^T$$

$$= x_n x_n^T - 2 x_n \theta z_n^T + z_n \theta^T \theta z_n^T$$

$$= x_n x_n^T - 2 x_n \theta \theta^T x_n^T + x_n \theta \underbrace{\theta^T \theta}_{1} \theta^T x_n^T$$

$$= x_n x_n^T - 2 x_n \theta \theta^T x_n^T + x_n \theta \theta^T x_n^T$$

$$= x_n x_n^T - x_n \theta \theta^T x_n^T$$

Así,

$$\min_{\theta} \; \mathop{\mathbb{E}}_{x} \left\{ x_n x_n^T - x_n \theta \theta^T x_n^T \right\}$$

$$= \min_{\theta} \; - \mathop{\mathbb{E}}_{x} \left\{ x_n \theta \theta^T x_n^T \right\}$$

Haciendo $z_n = x_n \theta$

$$= \min_{\theta} \; - \mathop{\mathbb{E}}_{x} \left\{ z_n z_n^T \right\}$$

$$= \min_{\theta} \; - \mathop{\mathbb{E}}_{x} \left\{ z_n^T z_n \right\}$$

$$= \min_{\theta} \; - \mathbb{E}_x \left\{ \theta^T x_n^T x_n \theta \right\}$$

$$= \min_{\theta} \; - \theta^T \mathbb{E}_x \left\{ x_n^T x_n \right\} \theta$$

$$= \min_{\theta} \; - \theta^T \Sigma_x \theta$$

$$= \boxed{\max_{\theta} \; \theta^T \Sigma_x \theta \quad \text{s.t} \quad \theta^T \theta = 1}$$