Demostraciones de probabilidad

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- $var\{x\} = \mathbb{E}\{x^2\} - \mathbb{E}^2\{x\}$

$$\begin{aligned} var\{x\} &= \mathbb{E}\{x^2\} - \mathbb{E}^2\{x\} \\ &= \mathbb{E}\{(x-\mu_x)^2\} \\ &= \mathbb{E}\{x^2 - 2\mu_x x + \mu_x^2\} \\ &= \mathbb{E}\{x^2\} - 2\mu_x\mathbb{E}\{x\} + \mathbb{E}\{\mu_x^2\} \\ &= \mathbb{E}\{x^2\} - 2\mathbb{E}\{x\}\mathbb{E}\{x\} + \mu_x^2 \\ &= \mathbb{E}\{x^2\} - 2\mathbb{E}^2\{x\} + \mathbb{E}^2\{x\} \\ &= \mathbb{E}\{x^2\} - \mathbb{E}^2\{x\} \end{aligned}$$

- $cov\{x,y\} = \mathbb{E}_{x,y}\{xy\} - \mathbb{E}\{x\}\mathbb{E}\{y\}$

$$\begin{aligned} cov\{x,y\} &= \mathbb{E}_{x,y}\{(x-\mu_x)(y-\mu_y)\} \\ &= \mathbb{E}_{x,y}\{xy - \mu_x y - x\mu_y + \mu_x\mu_y\} \\ &= \mathbb{E}_{x,y}\{xy\} - \mu_x\mathbb{E}\{y\} - \mathbb{E}\{x\}\mu_y + \mu_x\mu_y \\ &= \mathbb{E}_{x,y}\{xy\} - \mathbb{E}\{x\}\mathbb{E}\{y\} - \cancel{\mathbb{E}\{x\}\mathbb{E}\{y\}} + \cancel{\mathbb{E}\{x\}\mathbb{E}\{y\}} \\ &= \mathbb{E}_{x,y}\{xy\} - \mathbb{E}\{x\}\mathbb{E}\{y\} \end{aligned}$$

- $cov\{\mathbf{x},\mathbf{y}\} = \mathbb{E}_{\mathbf{x},\mathbf{y}}\{\mathbf{x}\mathbf{y}^T\} - \mathbb{E}\{\mathbf{x}\}\mathbb{E}\{\mathbf{y}^T\}$

$$\begin{aligned} cov\{\mathbf{x},\mathbf{y}\} &= \mathbb{E}_{\mathbf{x},\mathbf{y}}\{(\mathbf{x}-\mu_{\mathbf{x}})(\mathbf{y}-\mu_{\mathbf{y}})^T\} \\ &= \mathbb{E}_{\mathbf{x},\mathbf{y}^T}\{(\mathbf{x}-\mu_{\mathbf{x}})(\mathbf{y}^T-\mu_{\mathbf{y}}^T)\} \\ &= \mathbb{E}_{\mathbf{x},\mathbf{y}}\{\mathbf{x}\mathbf{y}^T - \mu_{\mathbf{x}}\mathbf{y}^T - \mathbf{x}\mu_{\mathbf{y}}^T + \mu_{\mathbf{x}}\mu_{\mathbf{y}}^T\} \\ &= \mathbb{E}_{\mathbf{x},\mathbf{y}}\{\mathbf{x}\mathbf{y}^T\} - \mu_{\mathbf{x}}\mathbb{E}\{\mathbf{y}^T\} - \mathbb{E}\{\mathbf{x}\}\mu_{\mathbf{y}}^T + \mu_{\mathbf{x}}\mu_{\mathbf{y}}^T \\ &= \mathbb{E}_{\mathbf{x},\mathbf{y}}\{\mathbf{x}\mathbf{y}^T\} - \mathbb{E}\{\mathbf{x}\}\mathbb{E}\{\mathbf{y}^T\} - \cancel{\mathbb{E}\{\mathbf{x}\}\mathbb{E}\{\mathbf{y}^T\}} + \cancel{\mathbb{E}\{\mathbf{x}\}\mathbb{E}\{\mathbf{y}^T\}} \\ &= \mathbb{E}_{\mathbf{x},\mathbf{y}}\{\mathbf{x}\mathbf{y}^T\} - \mathbb{E}\{\mathbf{x}\}\mathbb{E}\{\mathbf{y}^T\} \end{aligned}$$