PROGRAMMING IN HASKELL

Chapter 8 - Declaring Types and Classes

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Type Declarations

In Haskell, a new name for an existing type can be defined using a type declaration.

```
type String = [Char]
```

String is a synonym for the type [Char].

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Type declarations can be used to make other types easier to read. For example, given

```
type Pos = (Int,Int)
```

we can define:

```
origin :: Pos
origin = (0,0)

left :: Pos → Pos
left (x,y) = (x-1,y)
```

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Like function definitions, type declarations can also have parameters. For example, given

```
type Pair a = (a,a)
```

we can define:

```
mult :: Pair Int → Int
mult (m,n) = m*n

copy :: a → Pair a
copy x = (x,x)
```

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Type declarations can be nested:

```
type Pos = (Int,Int)

type Trans = Pos → Pos
```

✓

However, they cannot be recursive:

```
type Tree = (Int,[Tree])
```

✗

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Data Declarations

A completely new type can be defined by specifying its values using a data declaration.

```
data Bool = False | True
```

Bool is a new type, with two new values False and True.

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Note:

- The two values False and True are called the constructors for the type Bool.
- Type and constructor names must always begin with an upper-case letter.
- Data declarations are similar to context free grammars. The former specifies the values of a type, the latter the sentences of a language.

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Values of new types can be used in the same ways as those of built in types. For example, given

```
data Answer = Yes | No | Unknown
```

we can define:

```
answers :: [Answer]
answers = [Yes,No,Unknown]

flip :: Answer → Answer
flip Yes     = No
flip No      = Yes
flip Unknown = Unknown
```

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The constructors in a data declaration can also have parameters. For example, given

```
data Shape = Circle Float
           | Rect Float Float
```

we can define:

```
square :: Float → Shape
square n = Rect n n

area :: Shape → Float
area (Circle r) = pi * r^2
area (Rect x y) = x * y
```

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Note:

- Shape has values of the form Circle r where r is a float, and Rect x y where x and y are floats.
- Circle and Rect can be viewed as functions that construct values of type Shape:

```
Circle :: Float → Shape

Rect :: Float → Float → Shape
```

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Not surprisingly, data declarations themselves can also have parameters. For example, given

```
data Maybe a = Nothing | Just a
```

we can define:

```
safediv :: Int → Int → Maybe Int
safediv _ 0 = Nothing
safediv m n = Just (m `div` n)

safehead :: [a] → Maybe a
safehead [] = Nothing
safehead xs = Just (head xs)
```

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Recursive Types

In Haskell, new types can be declared in terms of themselves. That is, types can be recursive.

```
data Nat = Zero | Succ Nat
```

Nat is a new type, with constructors Zero :: Nat and Succ :: Nat → Nat.

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Note:

- A value of type Nat is either Zero, or of the form Succ n where n :: Nat. That is, Nat contains the following infinite sequence of values:

```
Zero

Succ Zero

Succ (Succ Zero)

⋮
```


- We can think of values of type Nat as <u>natural numbers</u>, where Zero represents 0, and Succ represents the successor function 1+.

- For example, the value

```
Succ (Succ (Succ Zero))
```

represents the natural number

```
1 + (1 + (1 + 0))  =  3
```


Using recursion, it is easy to define functions that convert between values of type Nat and Int:

```
nat2int :: Nat → Int
nat2int Zero     = 0
nat2int (Succ n) = 1 + nat2int n

int2nat :: Int → Nat
int2nat 0 = Zero
int2nat n = Succ (int2nat (n-1))
```


Two naturals can be added by converting them to integers, adding, and then converting back:

```
add :: Nat → Nat → Nat
add m n = int2nat (nat2int m + nat2int n)
```

However, using recursion the function add can be defined without the need for conversions:

```
add Zero     n = n
add (Succ m) n = Succ (add m n)
```


For example:

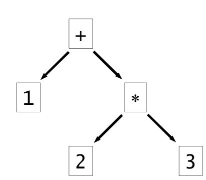
```
  add (Succ (Succ Zero)) (Succ Zero)
=
  Succ (add (Succ Zero) (Succ Zero))
=
  Succ (Succ (add Zero (Succ Zero)))
=
  Succ (Succ (Succ Zero))
```

Note:

- The recursive definition for add corresponds to the laws 0+n = n and (1+m)+n = 1+(m+n).

Arithmetic Expressions

Consider a simple form of <u>expressions</u> built up from integers using addition and multiplication.



Using recursion, a suitable new type to represent such expressions can be declared by:

```
data Expr = Val Int
          | Add Expr Expr
          | Mul Expr Expr
```

For example, the expression on the previous slide would be represented as follows:

```
Add (Val 1) (Mul (Val 2) (Val 3))
```


Using recursion, it is now easy to define functions that process expressions. For example:

```
size :: Expr → Int
size (Val n)   = 1
size (Add x y) = size x + size y
size (Mul x y) = size x + size y

eval :: Expr → Int
eval (Val n)   = n
eval (Add x y) = eval x + eval y
eval (Mul x y) = eval x * eval y
```


Note:

- The three constructors have types:

```
Val :: Int → Expr
Add :: Expr → Expr → Expr
Mul :: Expr → Expr → Expr
```

- Many functions on expressions can be defined by replacing the constructors by other functions using a suitable fold function. For example:

```
eval = folde id (+) (*)
```


Exercises

(1) Using recursion and the function add, define a function that multiplies two natural numbers.

(2) Define a suitable function folde for expressions and give a few examples of its use.

(3) Define a type Tree a of binary trees built from Leaf values of type a using a Node constructor that takes two binary trees as parameters.