* The simulation results from frequentist methods shows the fixed cubic spline with random intercept has very similar performance as the GAM. The, the Bayesian stan model, I used fixed spline with random intercept first since site-specific spline can be very challenging due to large number of parameters to be estimated.

**Simulation results (1) 200 iteration:**

**Frequentist GAM**

bias rmse true\_value.se est.se coverage

0.018 0.575 0.576 0.234 59.500

**Bayesian GAM using posterior median**

**brm(y ~ A + s(k, site, bs = "fs", k = 5), data = dd, family = gaussian(), #cores = 4,**

**iter = 2500, warmup = 500, refresh = 0,**

**control = list(adapt\_delta = 0.9))**

bias rmse coverage

0.018 0.574 60.000

**Bayesian GAM using posterior mean**

bias rmse coverage

0.018 0.574 60.000

**Simulation results (2) 200 iteration:**

**Frequentist GAM**

bias rmse true\_value.se est.se coverage

0.096 0.573 0.566 0.234 60.000

**Bayesian Spine using posterior median**

bias rmse est.se coverage

0.041 0.536 0.114 29.000

**Bayesian Spine using posterior mean**

bias rmse est.se coverage

0.042 0.536 0.114 29.000