Matrix Factorization and Movie Recommendations

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@luis_likes_math



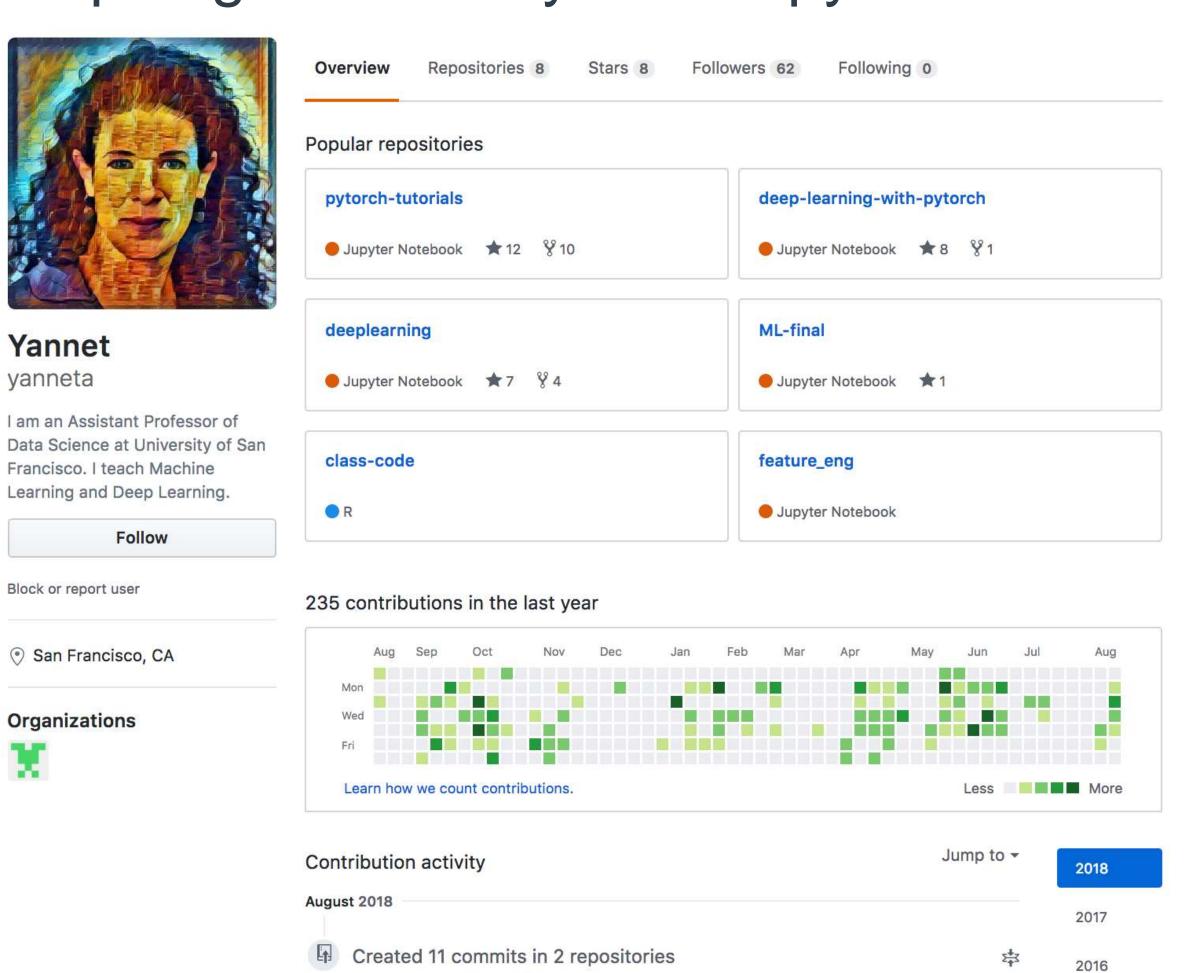
Yannet Interian @interian

Code



https://github.com/yanneta/pytorch-tutorials/blob/master/collaborative-filtering-nn.ipynb

2015



yanneta/pytorch-tutorials 10 commits

yanneta/dl-course 1 commit

Collaborative Filtering with Neural Networks

In this notebook we will write a matrix factorization model in pytorch to solve a recommendation problem. Then we will write a more general neural model for the same problem.

The MovieLens dataset (ml-latest-small) describes 5-star rating and free-text tagging activity from MovieLens, a movie recommendation service. It contains 100004 ratings and 1296 tag applications across 9125 movies. https://grouplens.org/datasets/movielens/. To get the data:

wget http://files.grouplens.org/datasets/movielens/ml-latest-small.zip

MovieLens dataset

```
[1]: from pathlib import Path
   import pandas as pd
   import numpy as np

[2]: PATH = Path("/Users/yinterian/teaching/deeplearning/data/ml-latest-small/")
   list(PATH.iterdir())

:[2]: [PosixPath('/Users/yinterian/teaching/deeplearning/data/ml-latest-small/links.csv'),
        PosixPath('/Users/yinterian/teaching/deeplearning/data/ml-latest-small/movies.csv'),
        PosixPath('/Users/yinterian/teaching/deeplearning/data/ml-latest-small/ratings.csv'),
        PosixPath('/Users/yinterian/teaching/deeplearning/data/ml-latest-small/README.txt'),
        PosixPath('/Users/yinterian/teaching/deeplearning/data/ml-latest-small/tags.csv'),
        PosixPath('/Users/yinterian/teaching/deeplearning/data/ml-latest-small/tiny_training2.csv'),
        PosixPath('/Users/yinterian/teaching/deeplearning/data/ml-latest-small/tiny_training2.csv')]

[3]: [1] head $PATH/ratings.csv
```





Binge-worthy TV Shows













Comedies











TV Action & Adventure













New Releases













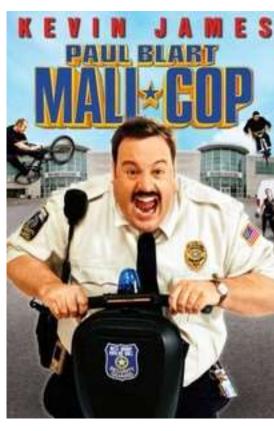
Netflix Universe



Ana



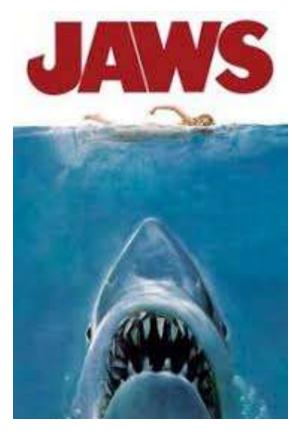
Betty



Movie 1



Movie 2



Movie 3



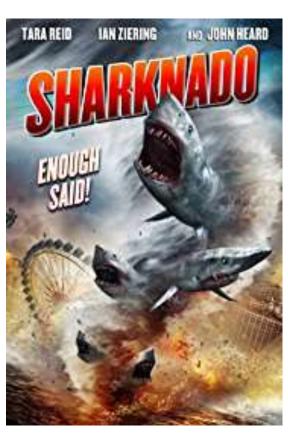
Carlos



Dana



Movie 4



Movie 5

Netflix ratings





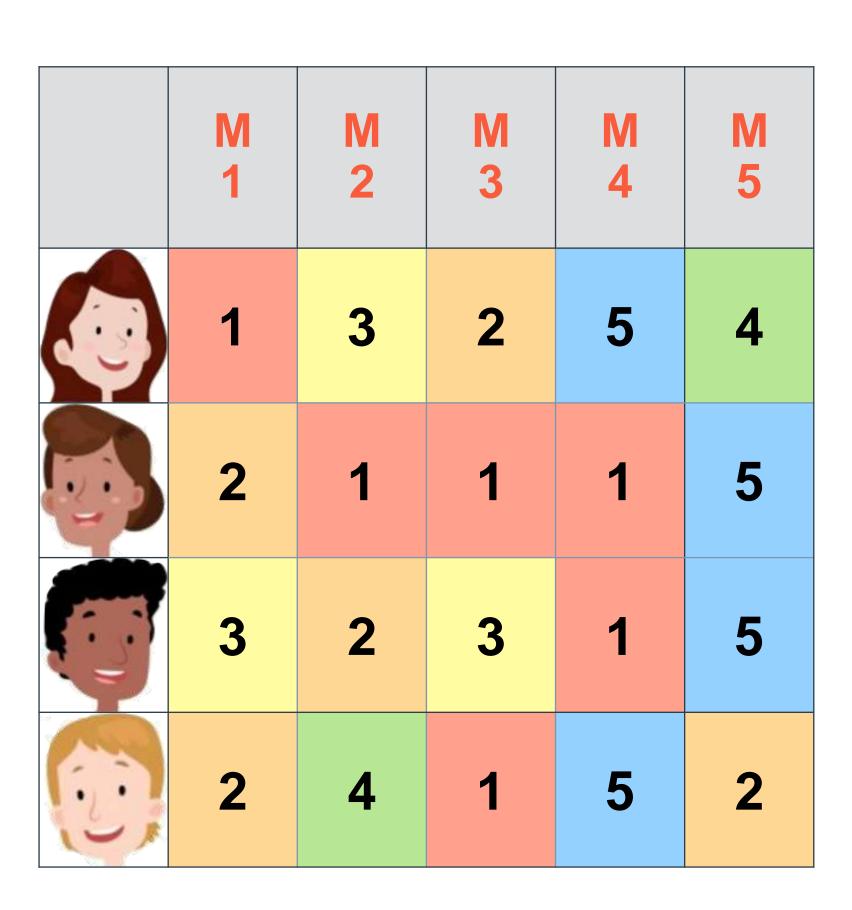
Movie 5

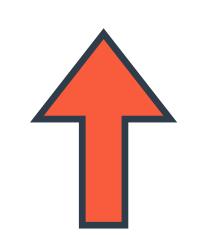
| M1 | M2 | M3 | M4 | M5 |
|----|-----------|----|----|----|
| 1 | 3 | 2 | 5 | 4 |
| 2 | 1 | 1 | 1 | 5 |
| 3 | 2 | 3 | 1 | 5 |
| 2 | 4 | 1 | 5 | 2 |

How do humans behave?

| M 1 | M 2 | M 3 | M 4 | M 5 |
|--------|------------|------------|--------|------------|
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |

| M 1 | M 2 | M 3 | M 4 | M 5 |
|--------|--------|--------|--------|------------|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |





| M1 | M2 | M3 | M4 | M5 |
|----|-----------|----|----|----|
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |

$$A = B = C = D$$

$$M1 = M2 = M3 = M4 = M5$$

How do humans behave?

| M 1 | M 2 | M 3 | M 4 | M 5 |
|--------|------------|------------|--------|------------|
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |

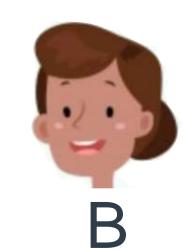
| M 1 | M 2 | M 3 | M 4 | M 5 |
|--------|--------|--------|--------|------------|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

| M 1 | M 2 | M 3 | M 4 | M 5 |
|--------|------------|------------|--------|------------|
| 1 | 3 | 2 | 5 | 4 |
| 2 | 1 | 1 | 1 | 5 |
| 3 | 2 | 3 | 1 | 5 |
| 2 | 4 | 1 | 5 | 2 |



| M1 | M2 | M3 | M4 | M5 |
|----|-----------|----|----|----|
| 1 | 3 | 2 | 5 | 4 |
| 2 | 1 | 1 | 1 | 5 |
| 3 | 2 | 3 | 1 | 5 |
| 2 | 4 | 1 | 5 | 2 |









M1

M2

M3

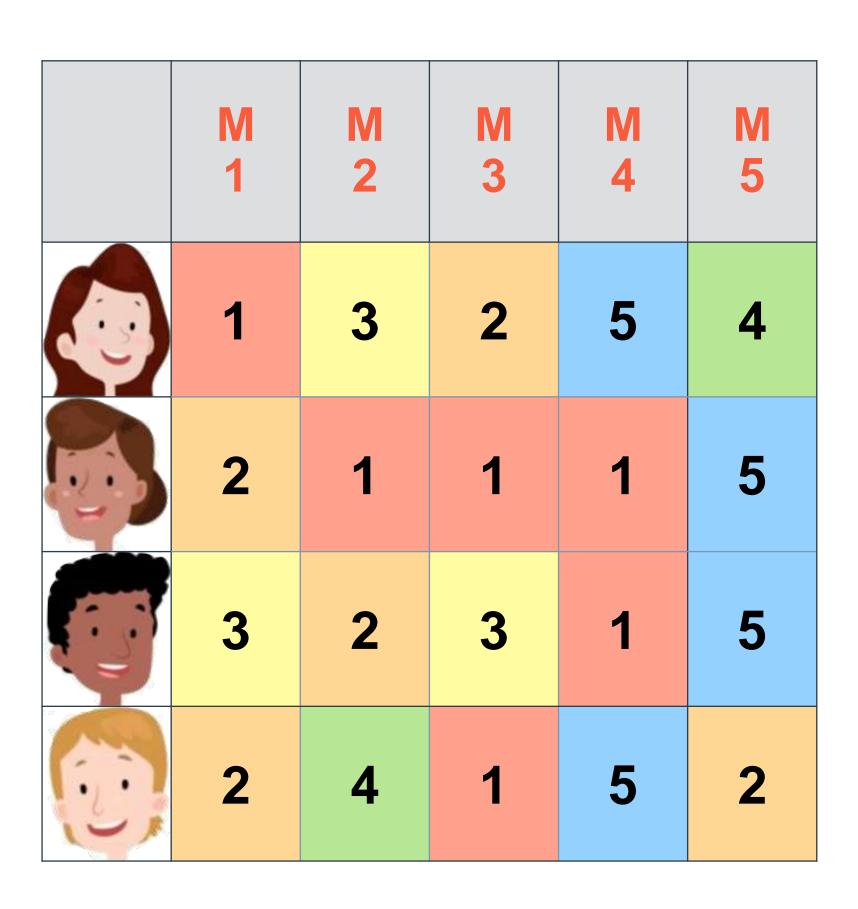
M4

M5

How do humans behave?

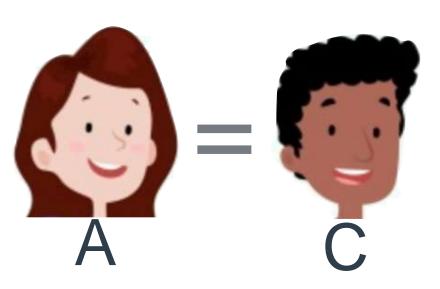
| M 1 | M 2 | M 3 | M 4 | M 5 |
|--------|------------|------------|--------|------------|
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |

| M 1 | M 2 | M 3 | M 4 | M 5 |
|--------|--------|--------|--------|------------|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |





| M1 | M2 | M3 | M4 | M5 | |
|----|----|----|----|----|--|
| 3 | 1 | 1 | 3 | 1 | |
| | | | | | |
| 3 | 1 | 1 | 3 | 1 | |
| | | | | | |

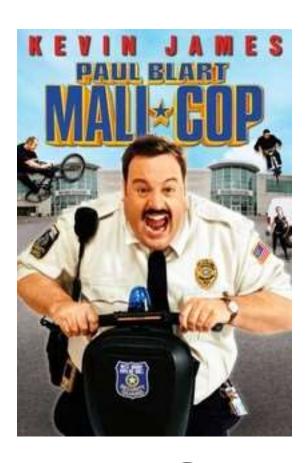


| M1 | M2 | M3 | M4 | M5 |
|----|----|----|----|----|
| 3 | | | 3 | |
| 1 | | | 1 | |
| 3 | | | 3 | |
| 4 | | | 4 | |

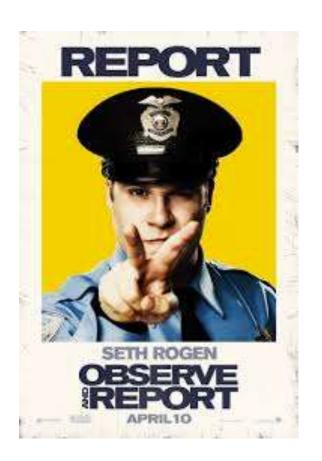




M1 = M4



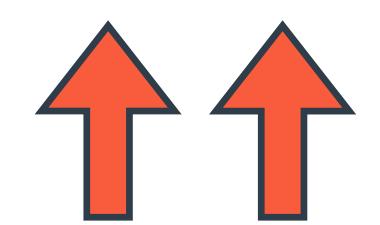
Mall Cop



Observe and Report

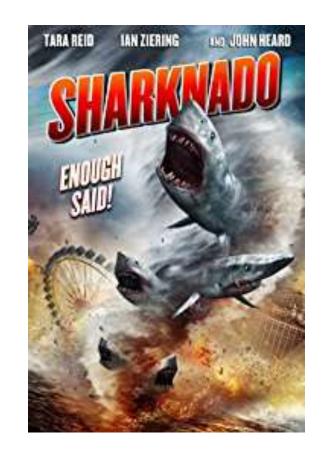
| M1 | M2 | M3 | M4 | M5 |
|----|----|----|-----------|----|
| | | | | |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

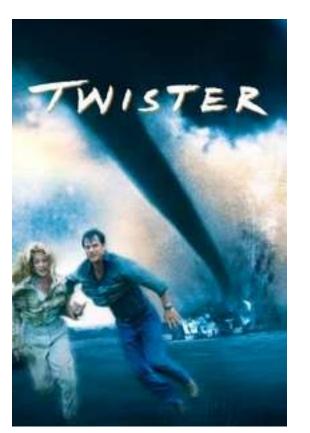
| M1 | M2 | M3 | M4 | M5 |
|----|----|----|----|----|
| | 1 | 1 | | 1 |
| | 2 | 4 | | 3 |
| | 1 | 1 | | 1 |
| | 3 | 5 | | 4 |

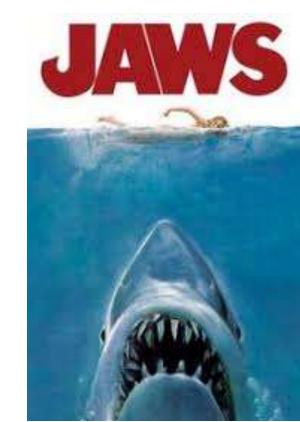




M5 = Average(M2, M3)



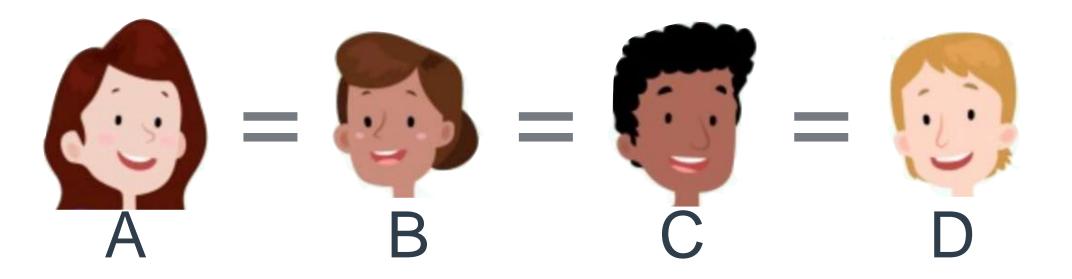




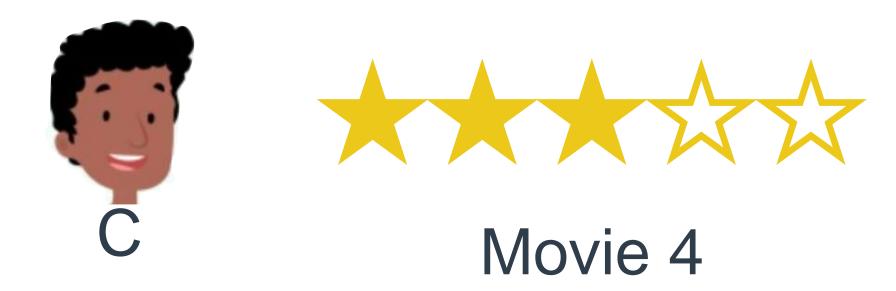
Guessing ratings

Recommender Systems

| M1 | M2 | M3 | M4 | M5 |
|----|-----------|----|-----------|----|
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | 3 | 3 |
| 3 | 3 | 3 | ? | 3 |
| 3 | 3 | 3 | 3 | 3 |



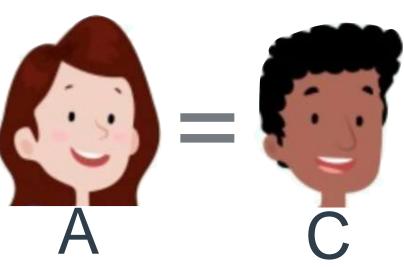
$$M1 = M2 = M3 = M4 = M5$$



Recommender Systems

| M1 | M2 | M3 | M4 | M5 | |
|----|-----------|----|----|----|---|
| 3 | 1 | 1 | 3 | 1 | • |
| 1 | 2 | 4 | 1 | 3 | |
| 3 | 1 | 1 | 3 | | • |
| 4 | 3 | 5 | 4 | 4 | |









Movie 5

Question: How do we figure out all these depencencies?

Answer: Matrix Factorization

Factorization

 $6 \times 4 = 24$

this x that =

| M1 | M2 | M3 | M4 | M5 |
|----|----|----|----|----|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

Features



Has a Sad Dog



Comedy



Sexy Canadian Ryan

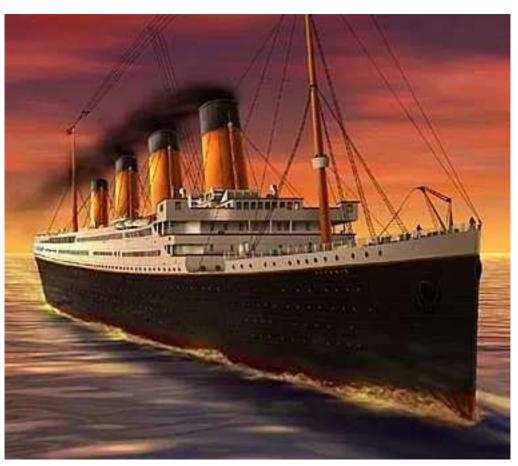




Big Boat



Meryl Streep





Action

Scary

Features





Dot Product



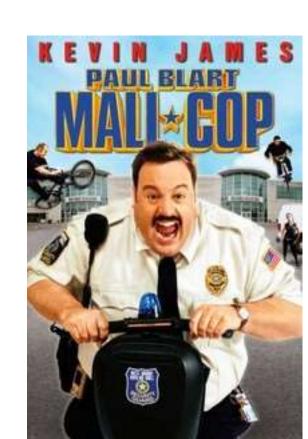












3



Dot Product







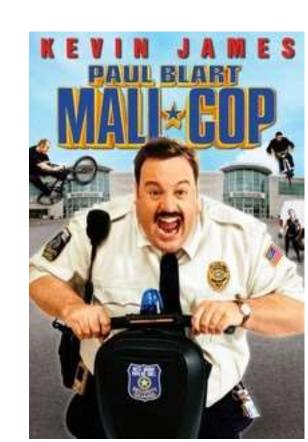


Betty









3

Dot Product











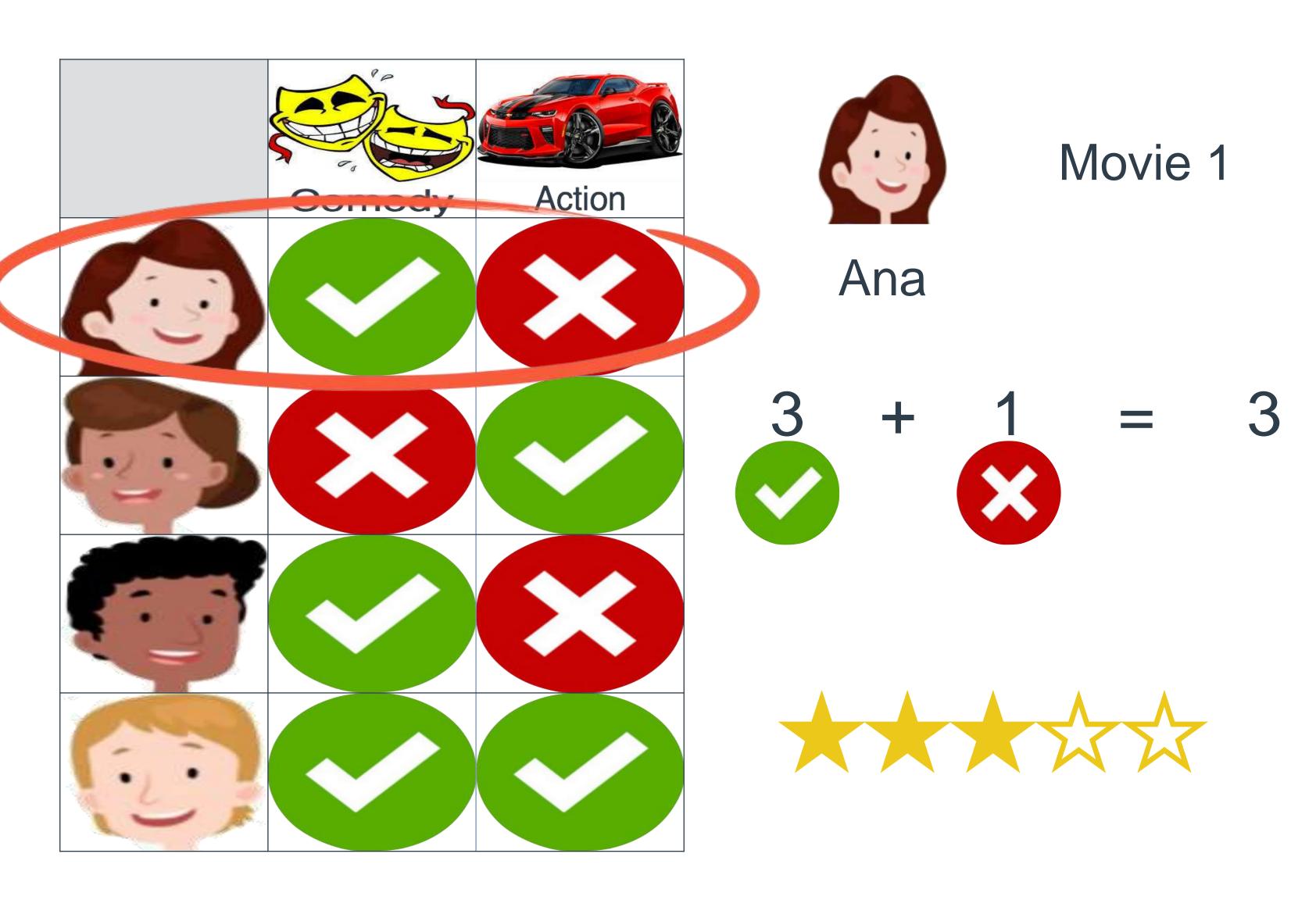






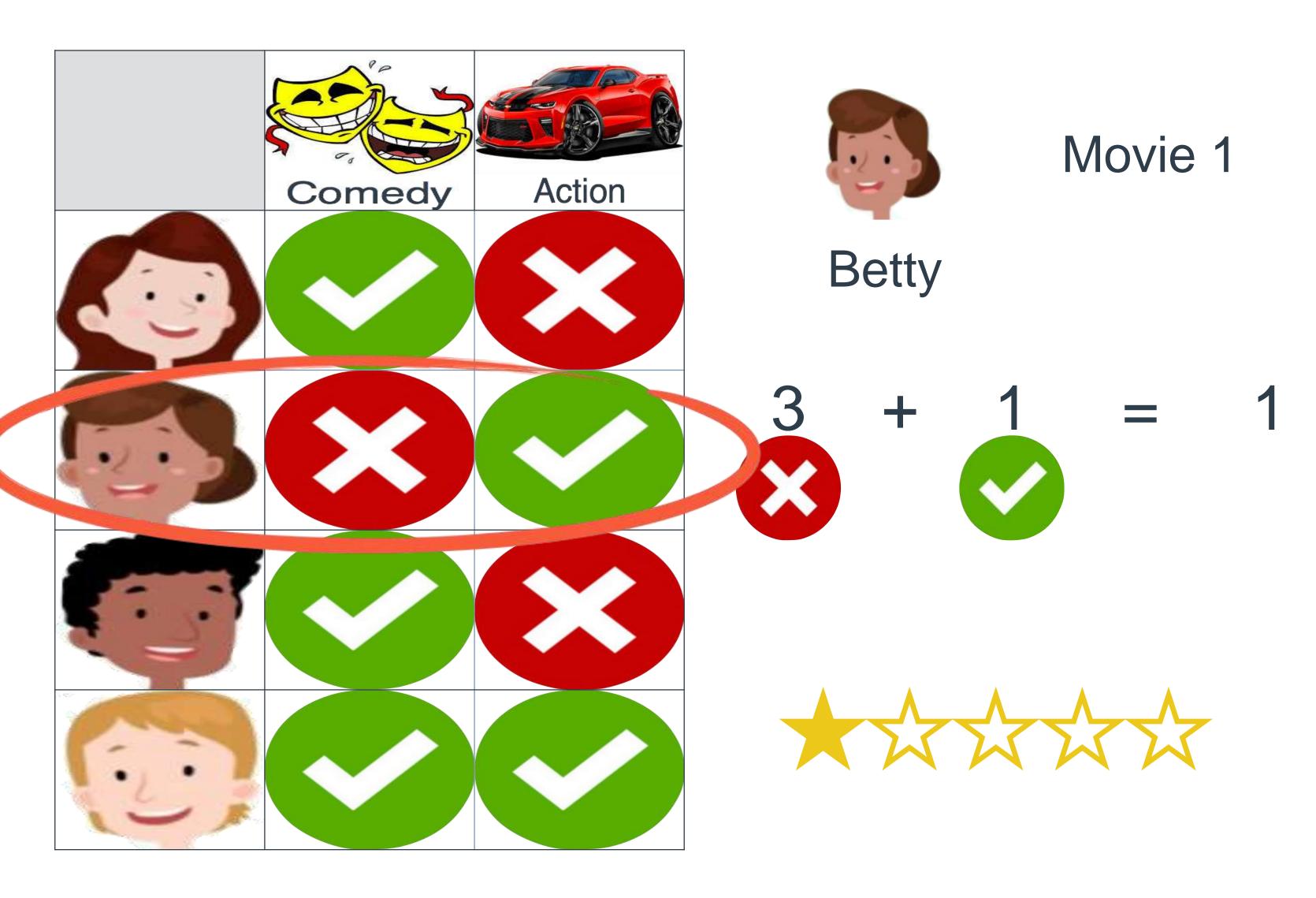
Categories

| | | Action |
|----|---|--------|
| M1 | 3 | 1 |
| M2 | 1 | 2 |
| M3 | 1 | 4 |
| M4 | 3 | 1 |
| M5 | 1 | 3 |



Categories

| | | Action |
|----|---|--------|
| M1 | 3 | 1 |
| M2 | 1 | 2 |
| M3 | 1 | 4 |
| M4 | 3 | 1 |
| M5 | 1 | 3 |



Categories

| | Comedy | Action | Com | edy Action | | Movie 5 |
|----|--------|--------|-----|---------------|------|---------|
| M1 | 3 | 1 | | Action Action | Dana | |
| M2 | 1 | 2 | | | 1 + | 3 = 4 |
| M3 | 1 | 4 | | | | |
| M4 | 3 | 1 | | | | |
| M5 | 1 | 3 | | | | |

| | Comedy | Action |
|----|--------|--------|
| M1 | 3 | 1 |
| M2 | 1 | 2 |
| M3 | 1 | 4 |
| M4 | 3 | 1 |
| M5 | 1 | 3 |

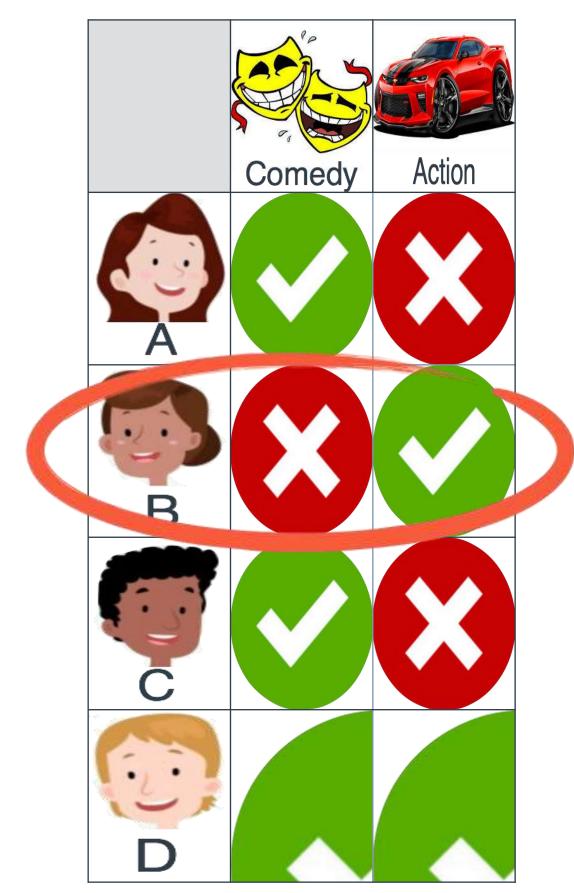
| Comedy | Action |
|--------|--------|
| | |
| | |
| | |
| | |

| M1 | M2 | M3 | M4 | M5 |
|----|-----------|----|-----------|----|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

Factorization

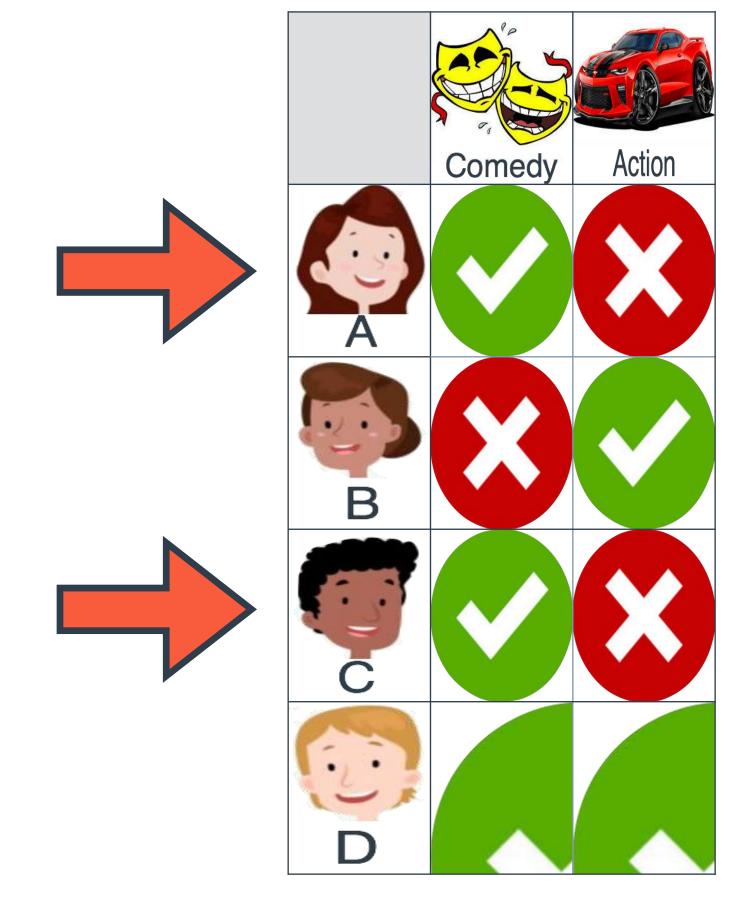
 $6 \times 4 = 24$



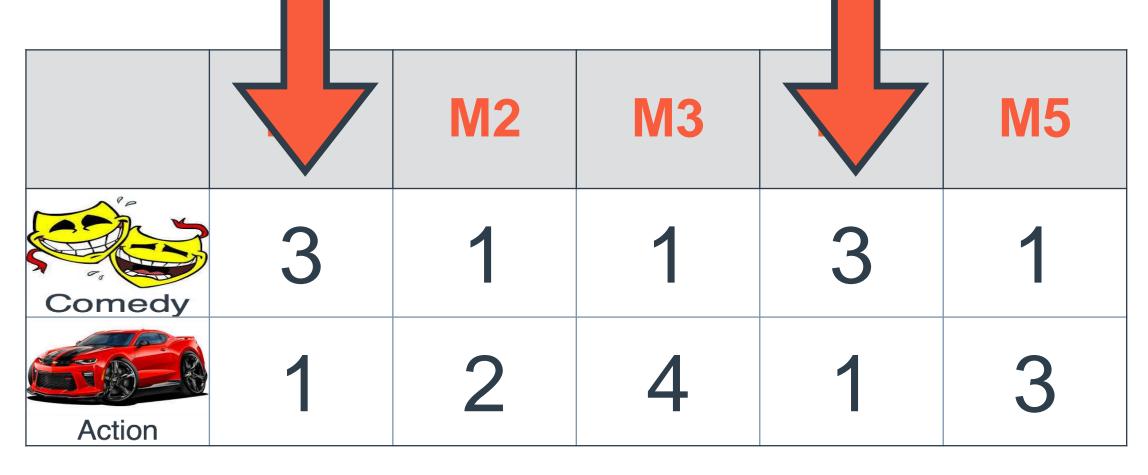


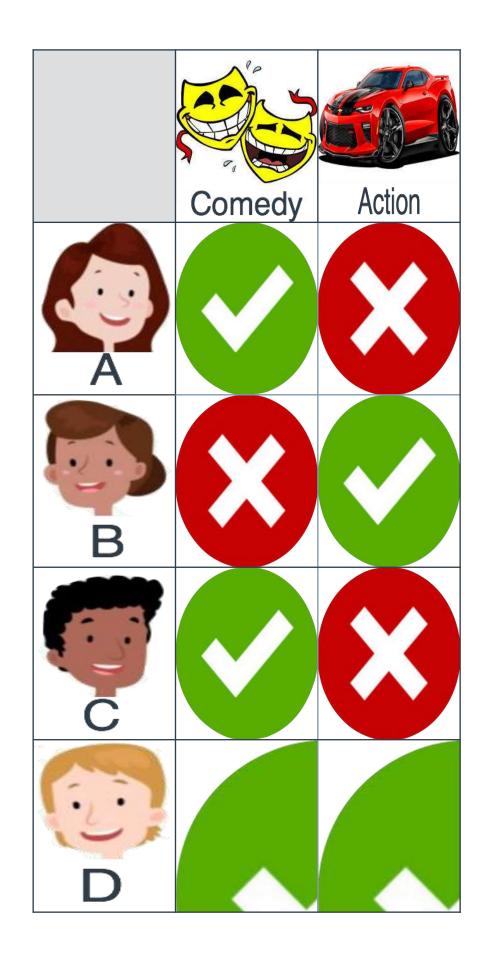
| M1 | M2 | M3 | M4 | M5 | | |
|----|-----------|----|-----------|----|--|--|
| 3 | 1 | 1 | 3 | 1 | | |
| 1 | 2 | 4 | 1 | 3 | | |
| 3 | 1 | 1 | 3 | 1 | | |
| 4 | 3 | 5 | 4 | 4 | | |

| | M1 | M2 | M3 | M4 | M5 |
|--------|----|----|----|----|----|
| Comedy | 3 | 1 | 1 | 3 | 1 |
| Action | 1 | 2 | 4 | 1 | 3 |



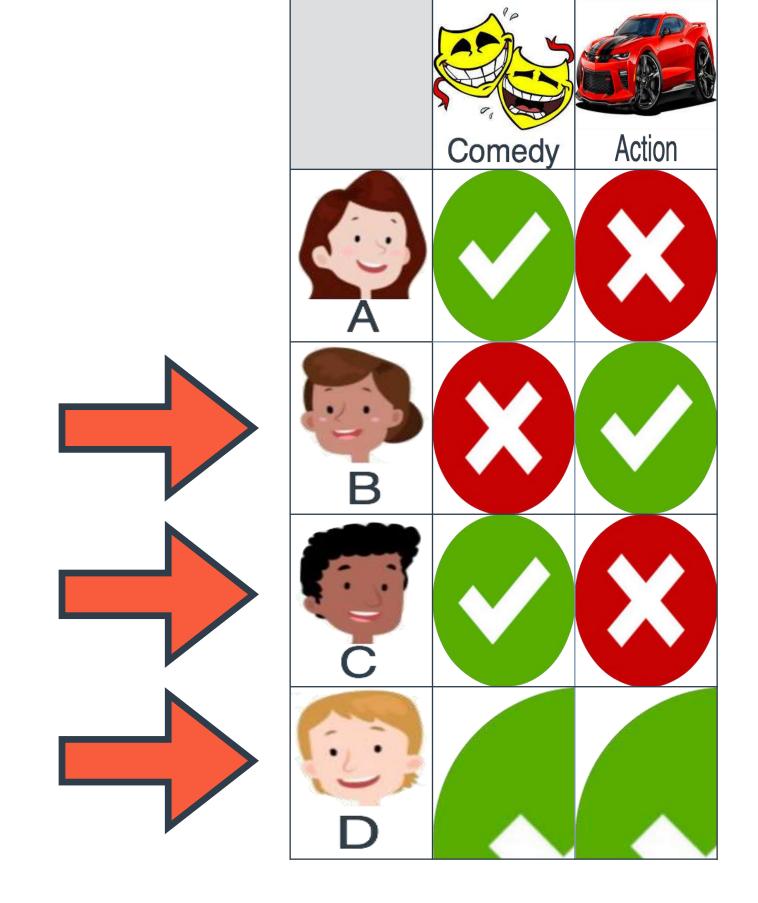
| M1 | M2 | M3 | M4 | M5 |
|----|-----------|----|----|----|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |



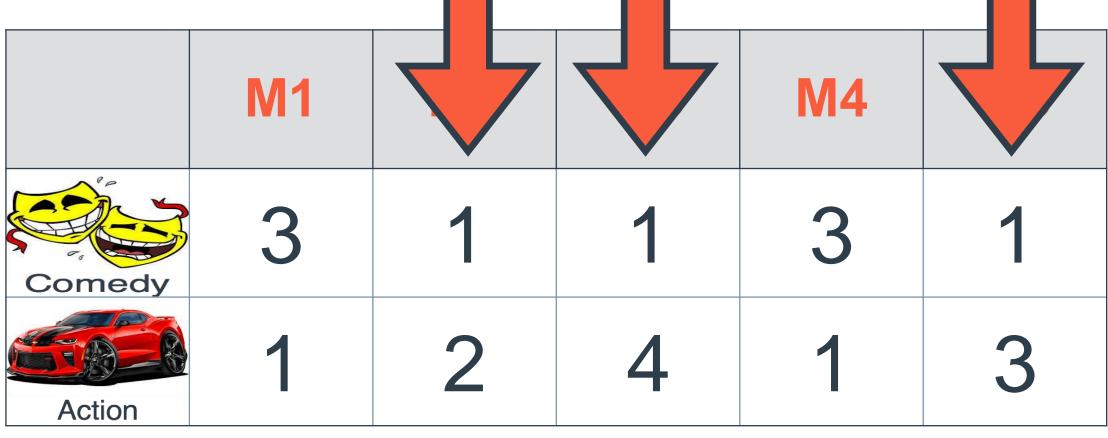


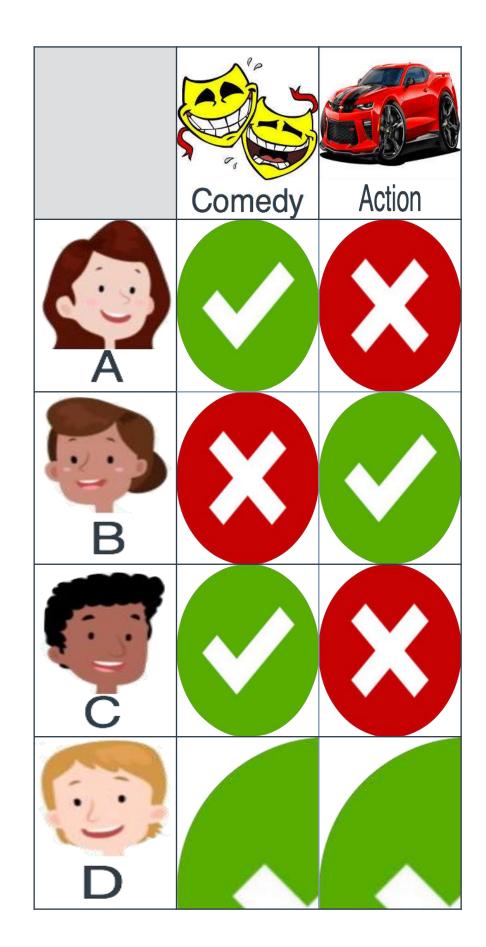
| M1 | M2 | M3 | M4 | M5 |
|----|-----------|----|----|----|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

| | M1 | M2 | M3 | M4 | M5 |
|--------|----|----|----|----|----|
| Comedy | 3 | 1 | 1 | 3 | 1 |
| Action | 1 | 2 | 4 | 1 | 3 |



| M1 | M2 | M3 | M4 | M5 |
|----|-----------|----|----|----|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

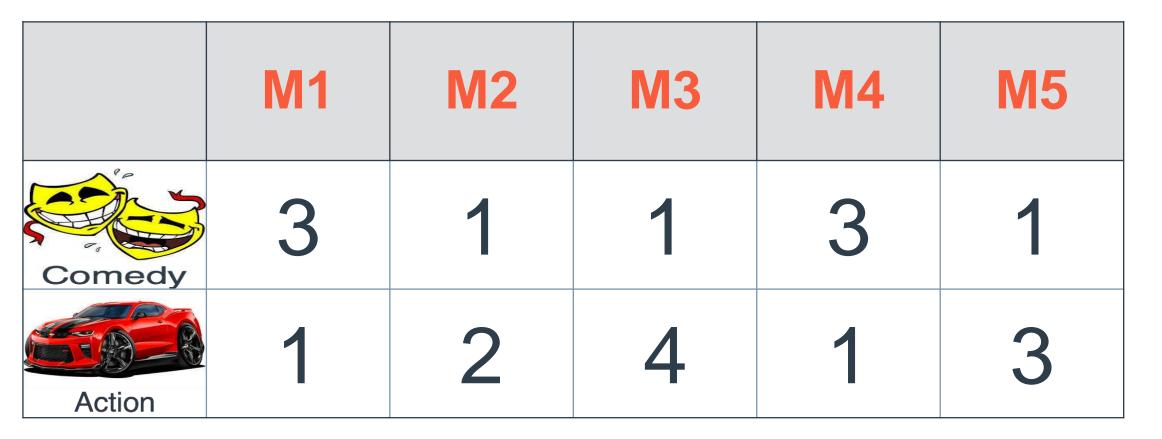


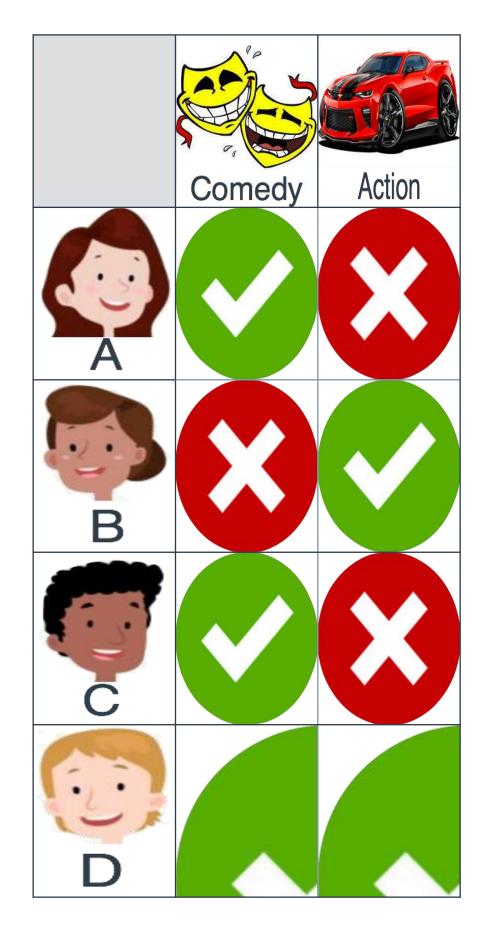


| M1 | M2 | M3 | M4 | M5 |
|----|-----------|----|-----------|----|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

Storage

Matrix Factorization





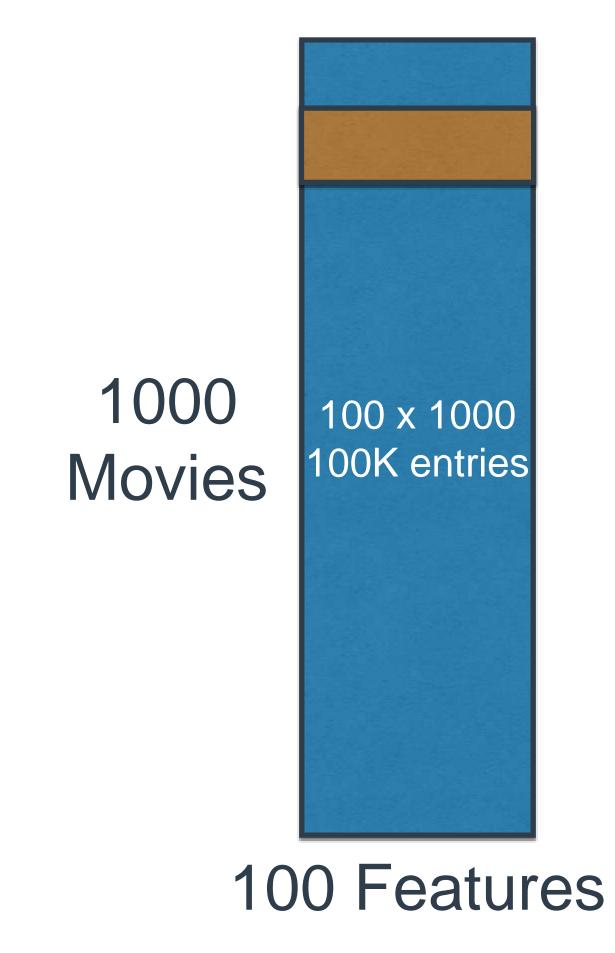
| M1 | M2 | M3 | M4 | M5 |
|----|-----------|----|----|----|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

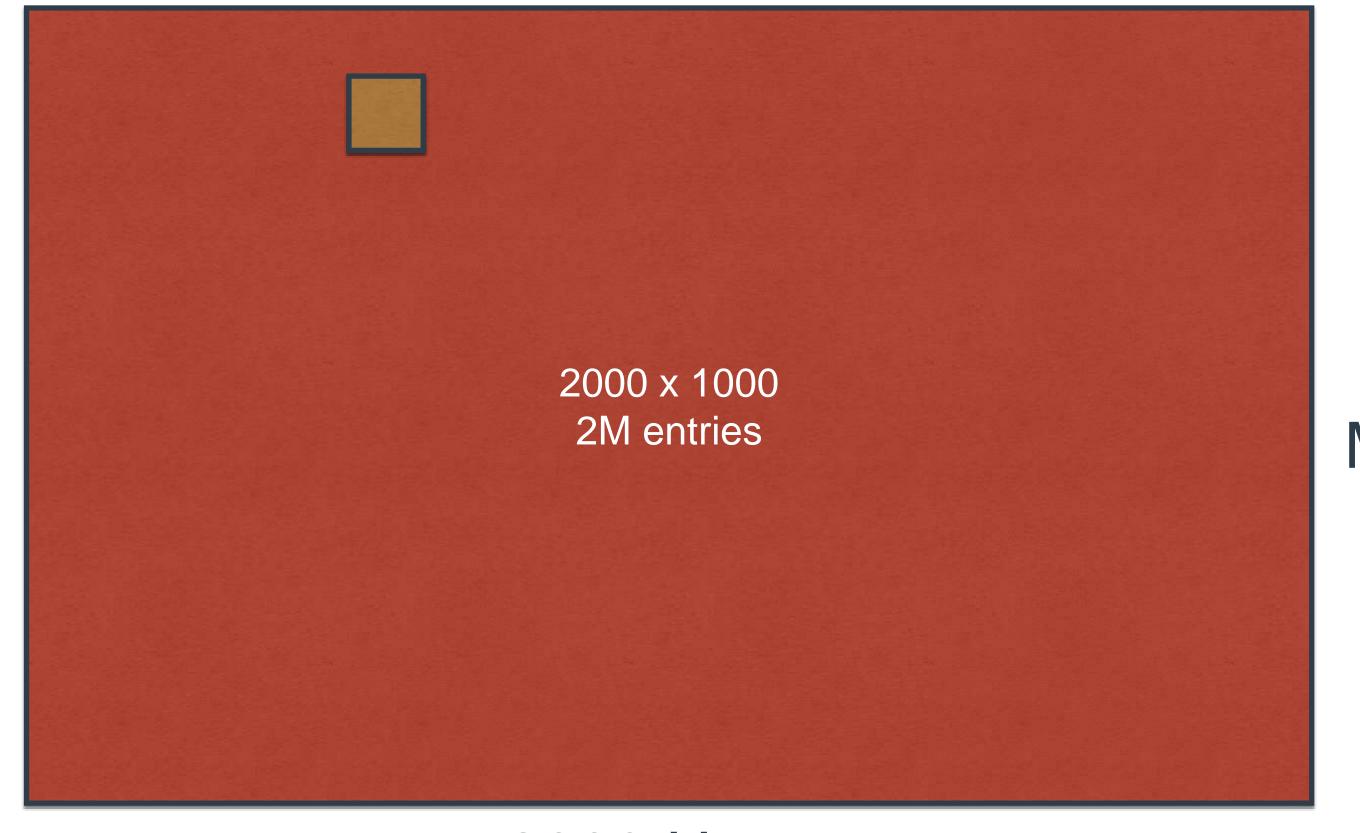
2000 Users





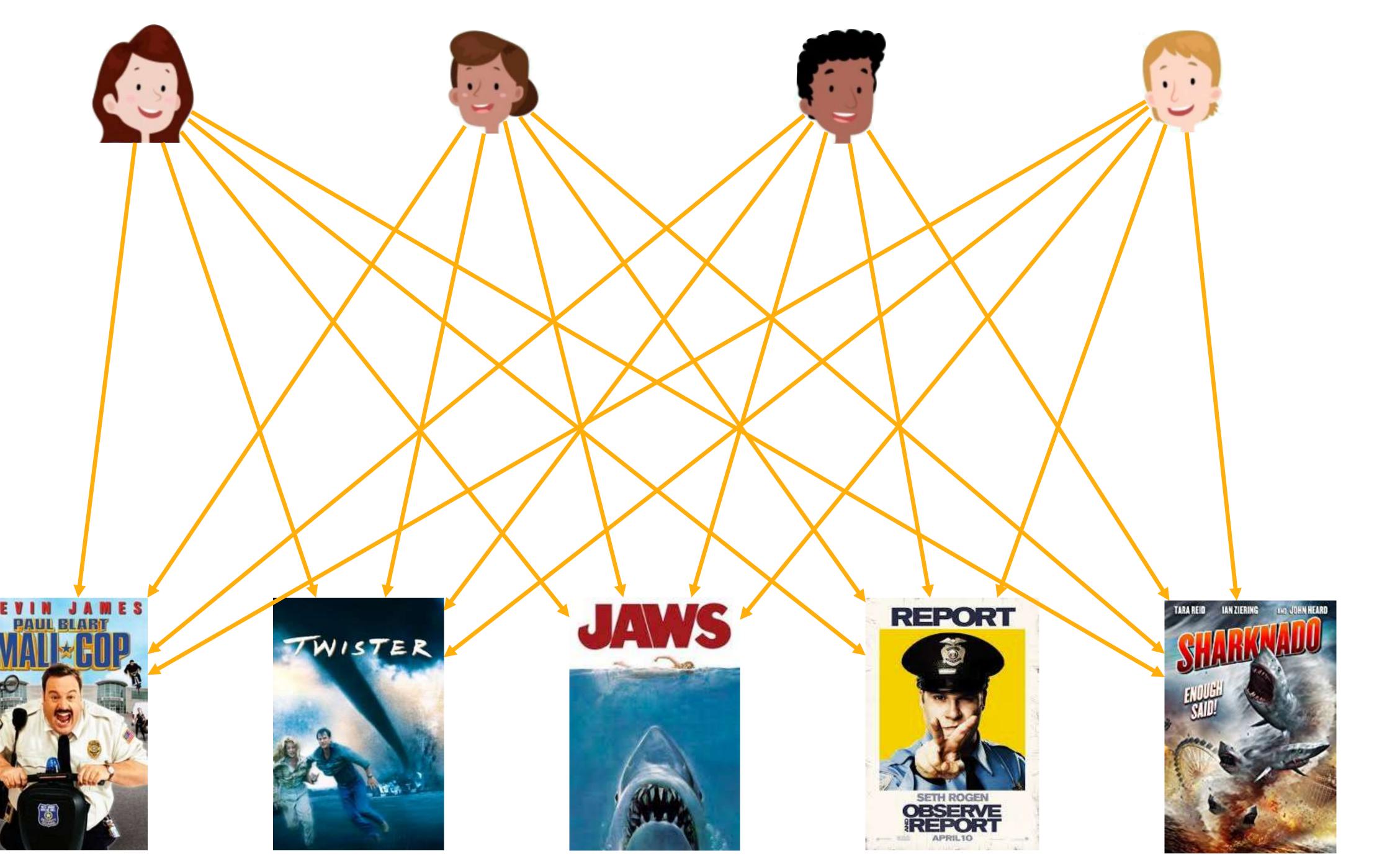
100 Features

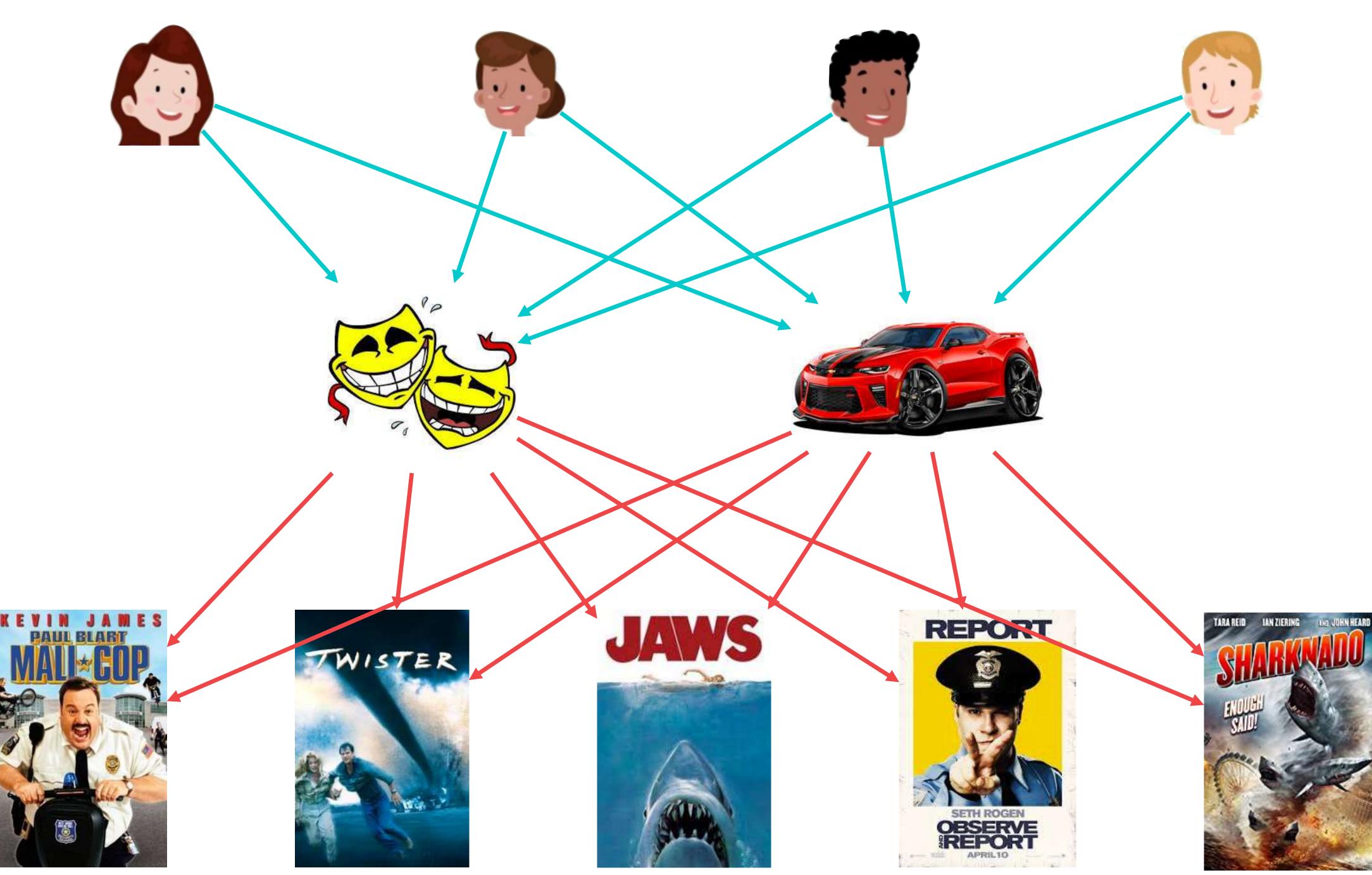




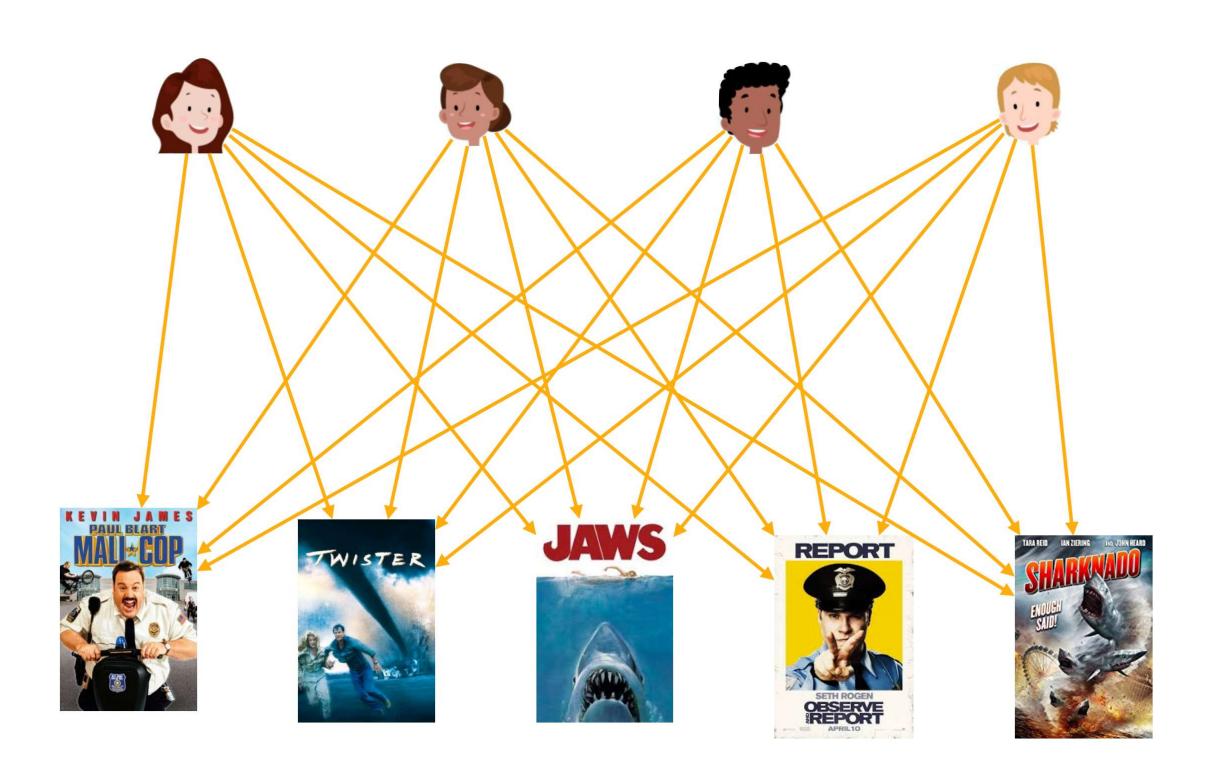
1000 Movies

2000 Users

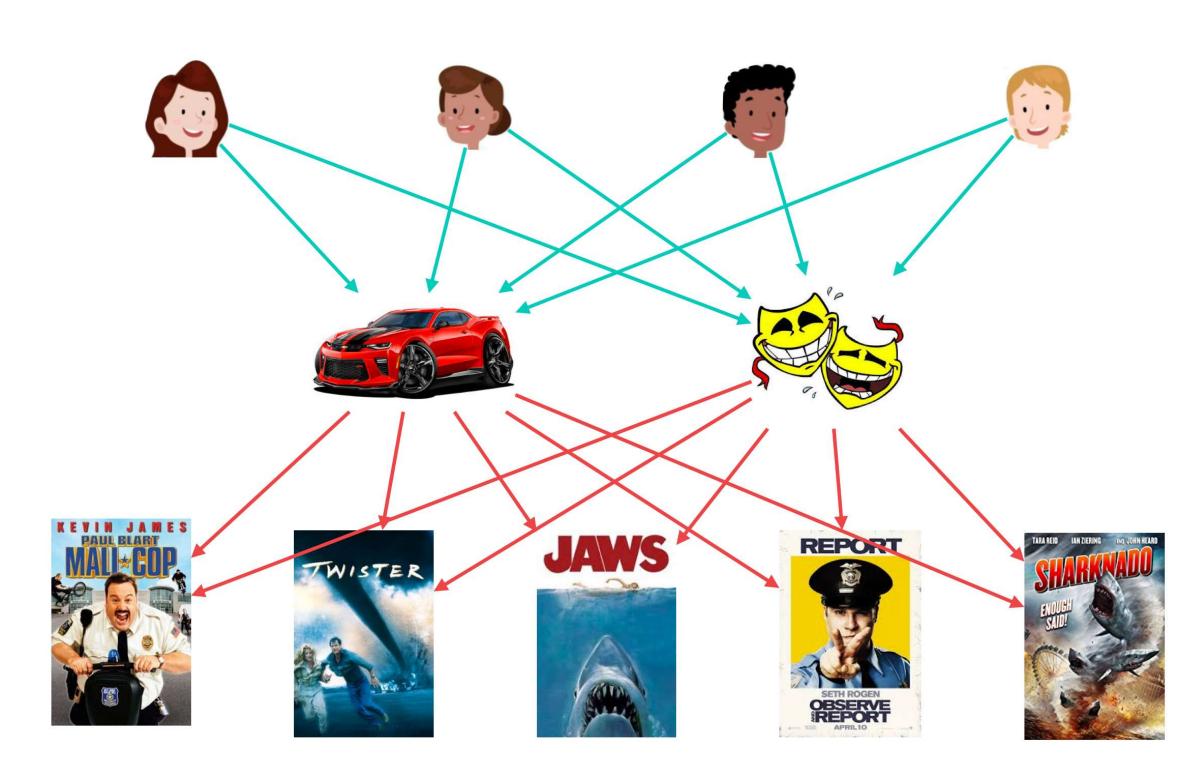




Quiz: How many parameters (arrows)?

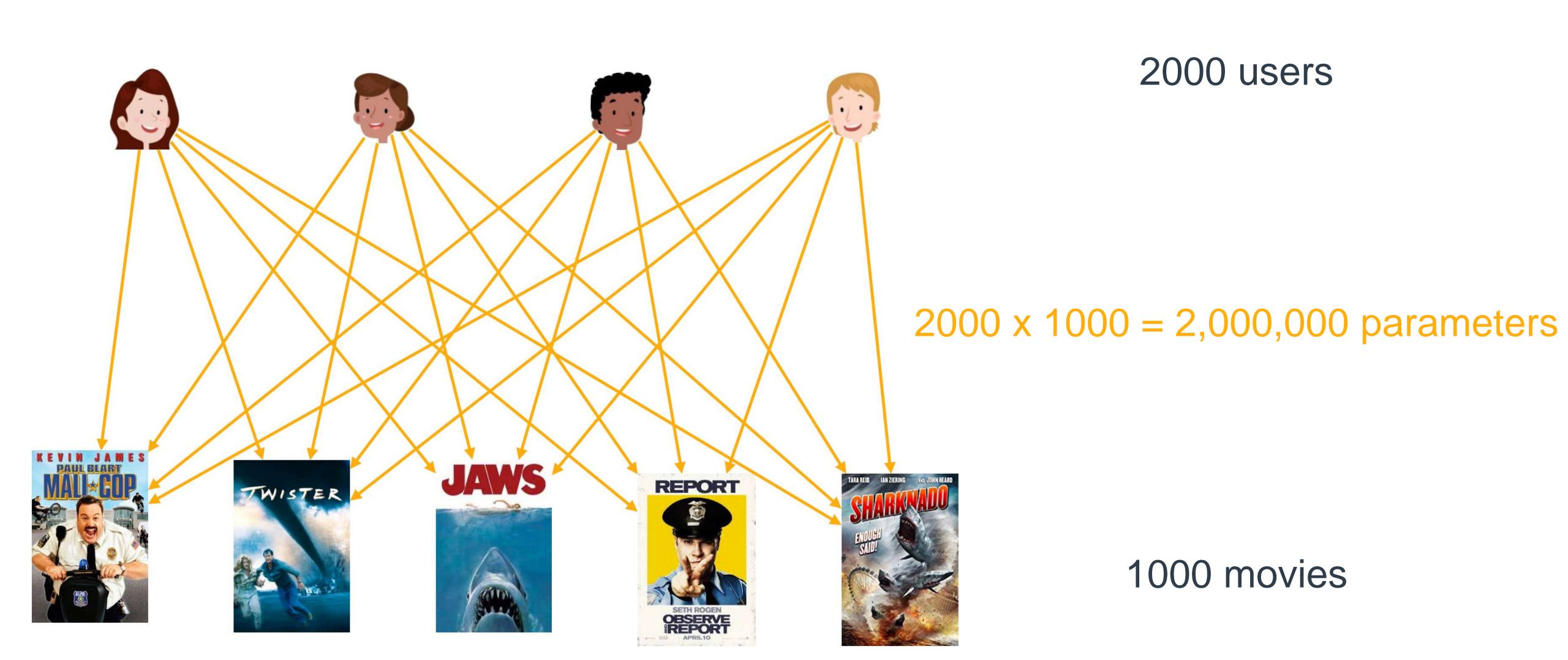


20 parameters

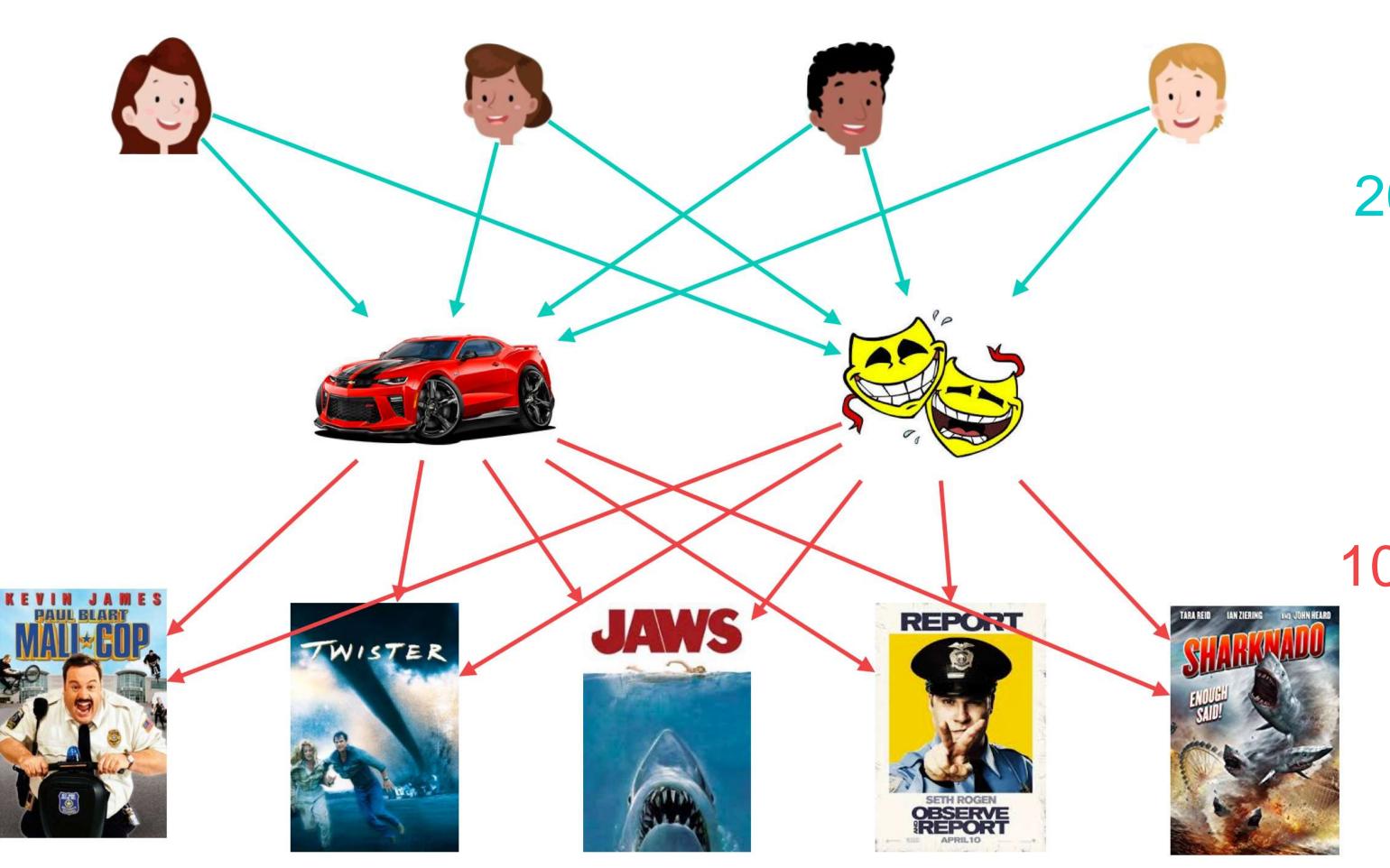


18 parameters

Quiz: How many parameters?



Quiz: How many parameters?



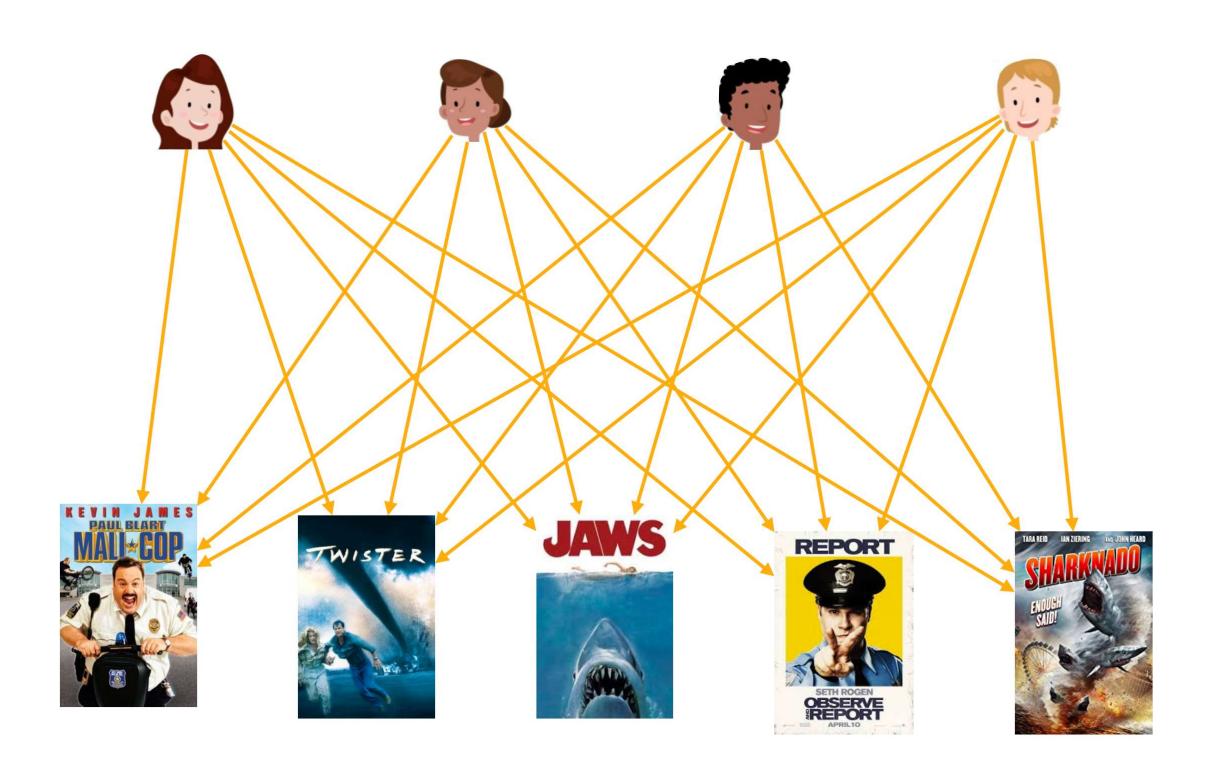
2000 users

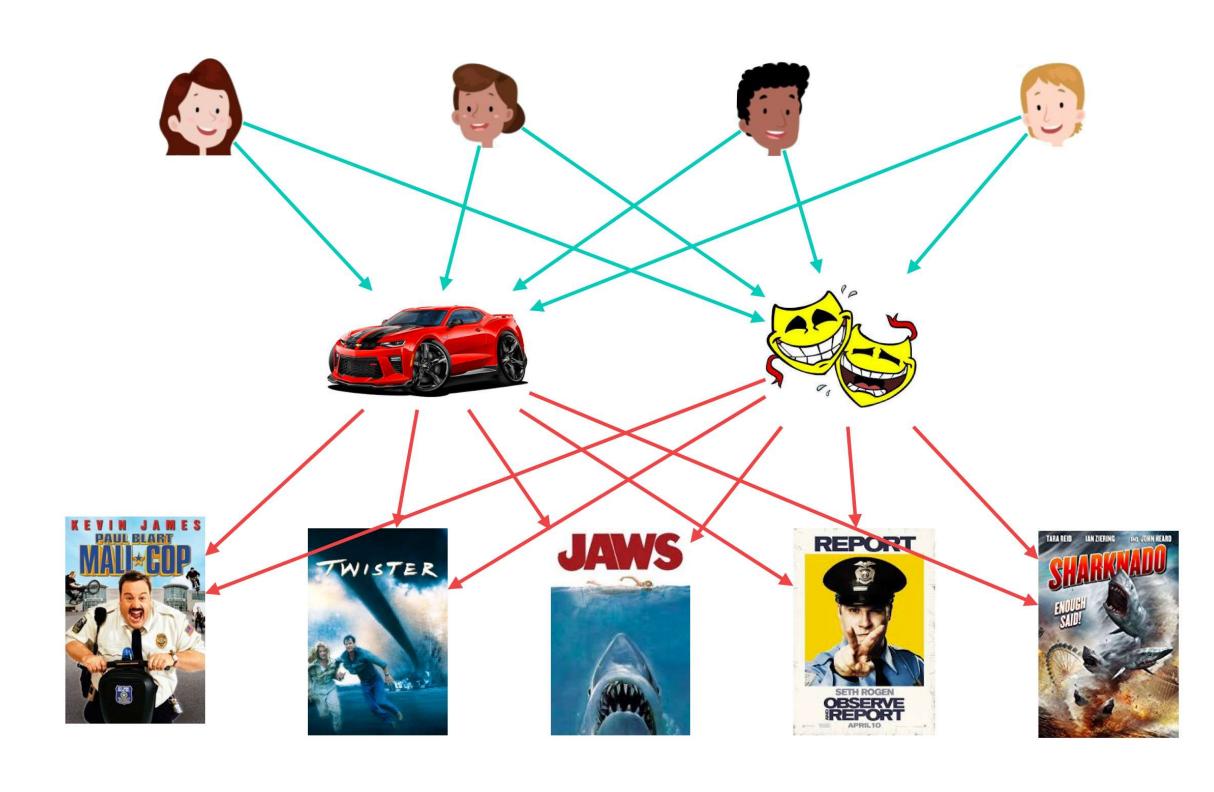
2000x100 = 200,000 parameters 300,000 parameters 100 features

 $100 \times 1000 = 100,000 \text{ parameters}$

1000 movies

Storage?

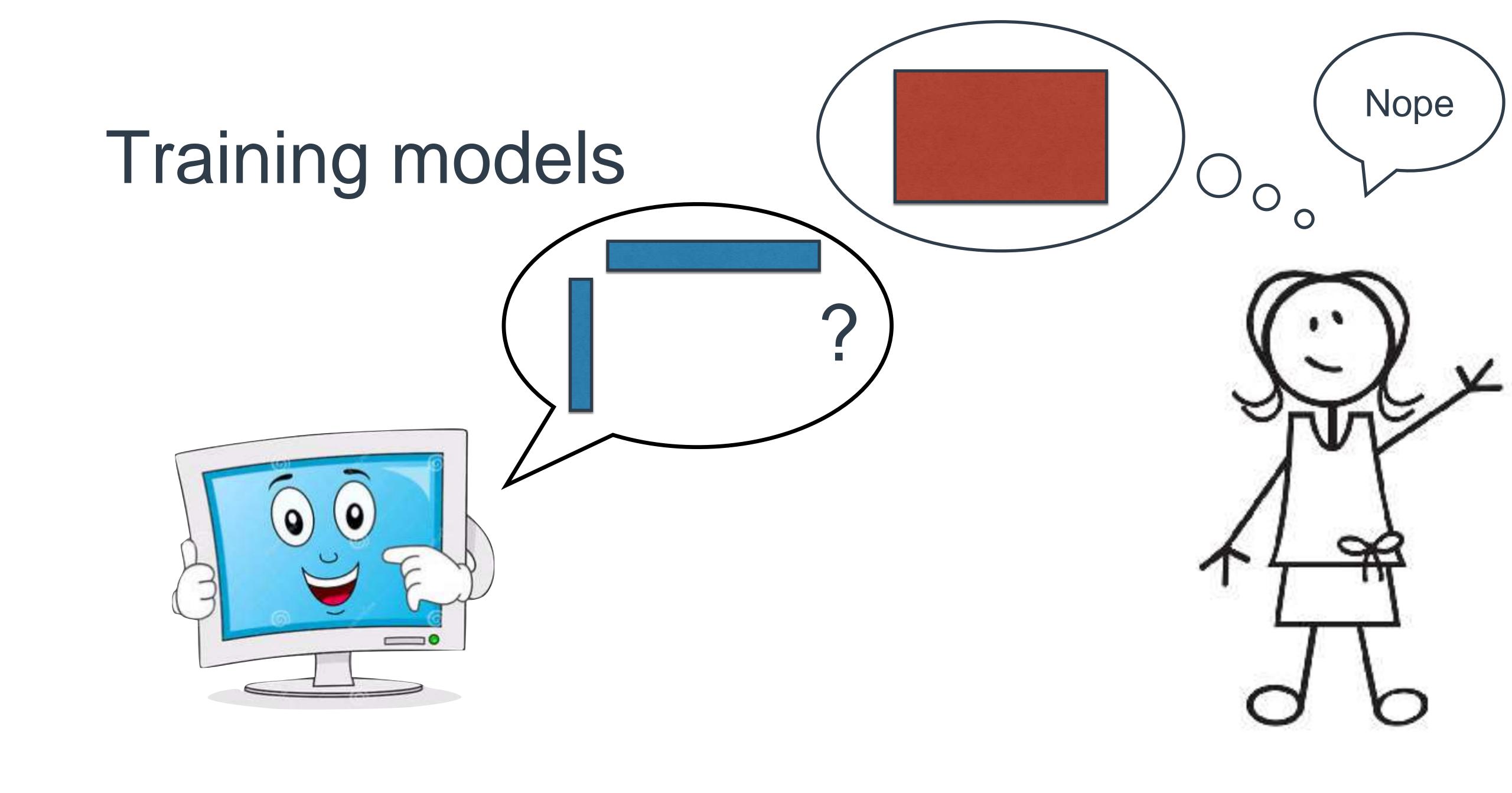


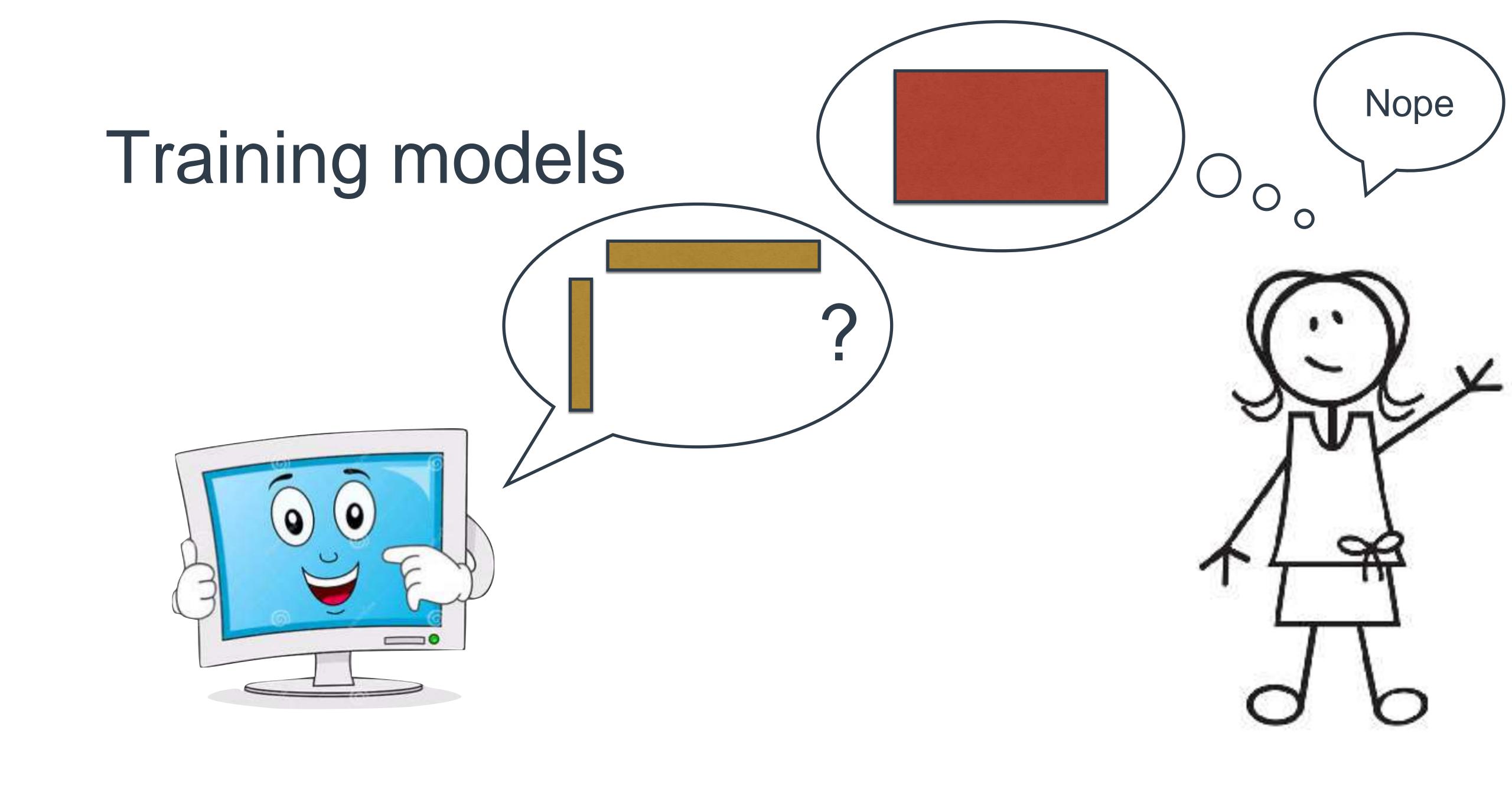


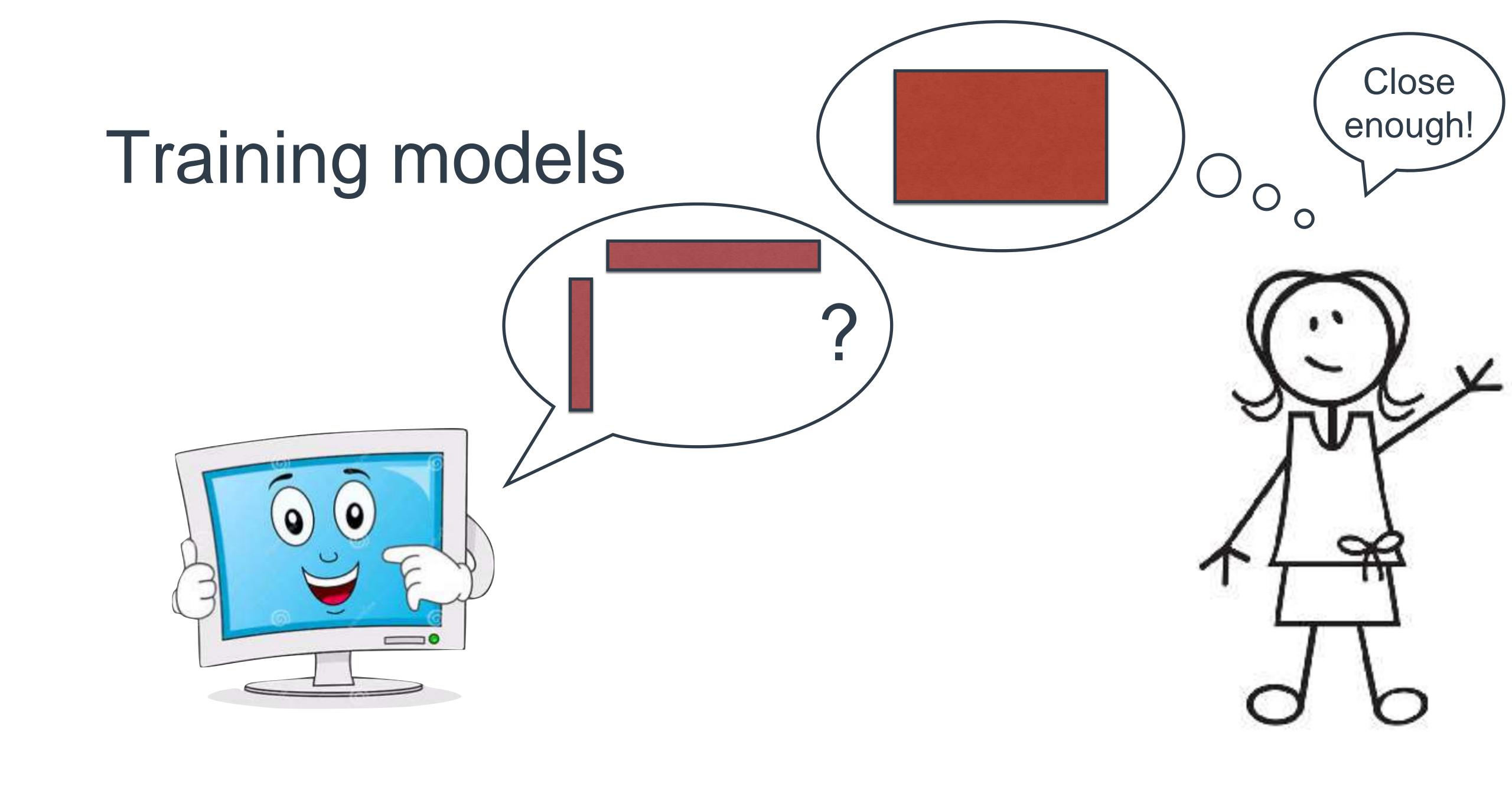
2M parameters

300K parameters

How to find the right factorization?







Gradient Descent

Matrix Factorization

| | M1 | M2 | M3 | M4 | M5 |
|--------|----|----|----|----|----|
| Comedy | 3 | 1 | 1 | 3 | 1 |
| Action | 1 | 2 | 4 | 1 | 3 |



1



| | Comedy | Action |
|---|--------|--------|
| A | 1 | 0 |
| B | 0 | 1 |
| O | 1 | 0 |
| D | 1 | 1 |

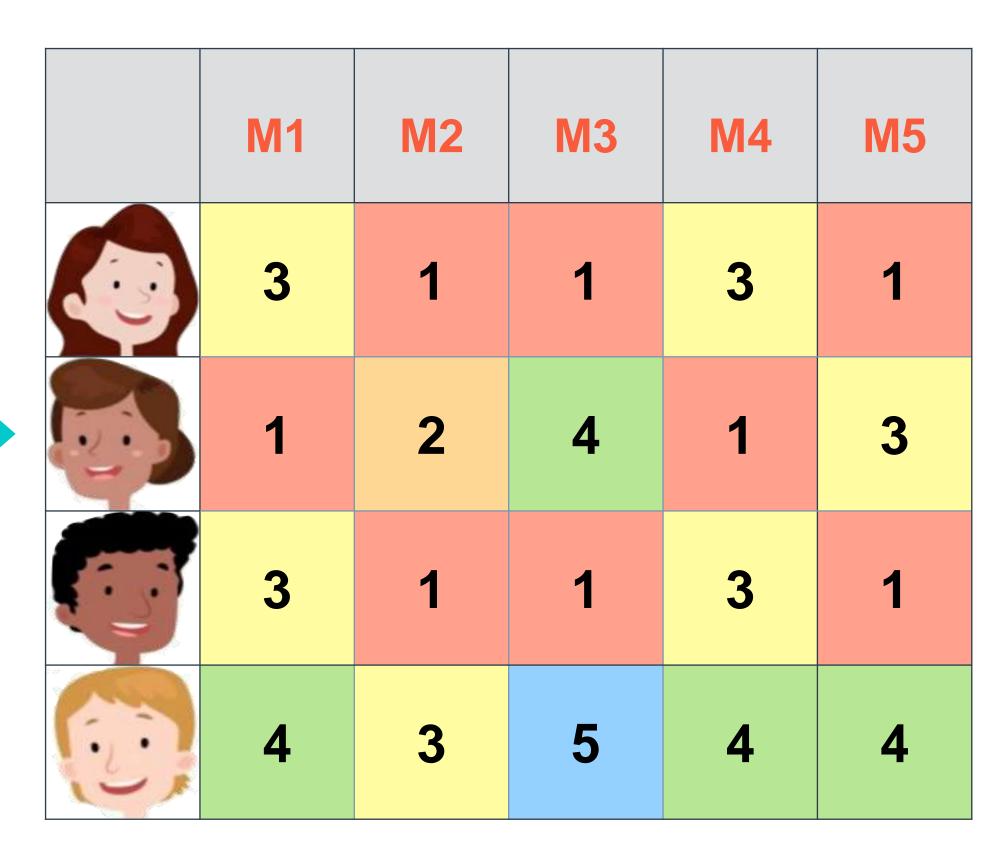
| M1 | M2 | M3 | M4 | M5 |
|----|----|----|-----------|----|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

| | IVI | M2 | M3 | M4 | M5 |
|----|-----|-----|-----|-----|-----|
| F1 | 1.2 | 3.1 | 0.3 | 2.5 | 0.2 |
| F2 | 2.4 | 1.5 | 4.4 | 0.4 | 1.1 |

| 1.2 | \times 0.2 | + 2 | .4 x | 0.5 | = 1 | .44 |
|-----|--------------|-----|------|-----|-----|-----|
|-----|--------------|-----|------|-----|-----|-----|

| | F1 | F2 |
|---|-----|-----|
| | 0.2 | 0.5 |
| B | 0.3 | 0.4 |
| C | 0.7 | 8.0 |
| D | 0.4 | 0.5 |

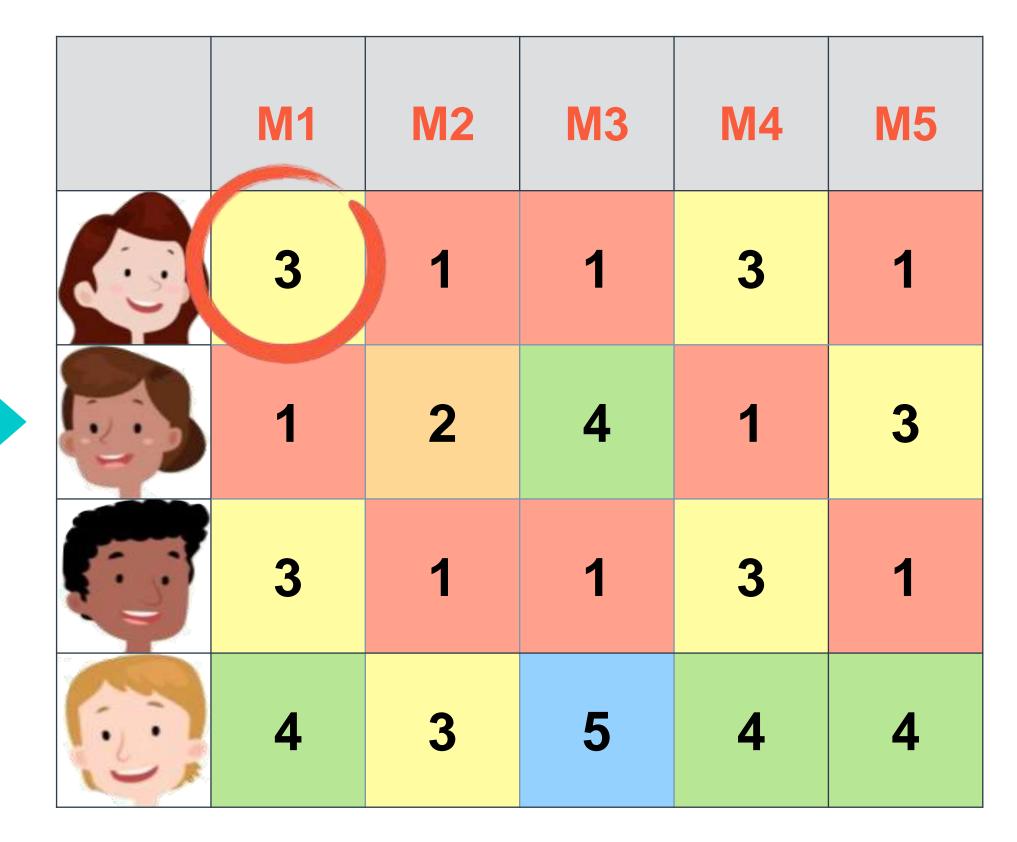
| M1 | M2 | M3 | M4 | M5 |
|------|-----------|------|-----------|------|
| 1.44 | 1.37 | 2.26 | 0.7 | 0.59 |
| 1.32 | 1.53 | 1.85 | 0.91 | 0.5 |
| 2.76 | 3.37 | 3.73 | 2.07 | 1.02 |
| 1.68 | 1.99 | 2.32 | 1.2 | 0.63 |



| | M1 | M2 | M3 | M4 | M5 |
|----|-----|-----------|-----|-----|-----|
| F1 | 1.2 | 3.1 | 0.3 | 2.5 | 0.2 |
| F2 | 2.4 | 1.5 | 4.4 | 0.4 | 1.1 |

| | F1 | F2 |
|---|-----|-----|
| A | 0.2 | 0.5 |
| B | 0.3 | 0.4 |
| C | 0.7 | 8.0 |
| D | 0.4 | 0.5 |

| M1 | M2 | M3 | M4 | M5 |
|------|------|------|-----------|------|
| 1.44 | 1.37 | 2.26 | 0.7 | 0.59 |
| 1.32 | 1.53 | 1.85 | 0.91 | 0.5 |
| 2.76 | 3.37 | 3.73 | 2.07 | 1.02 |
| 1.68 | 1.99 | 2.32 | 1.2 | 0.63 |

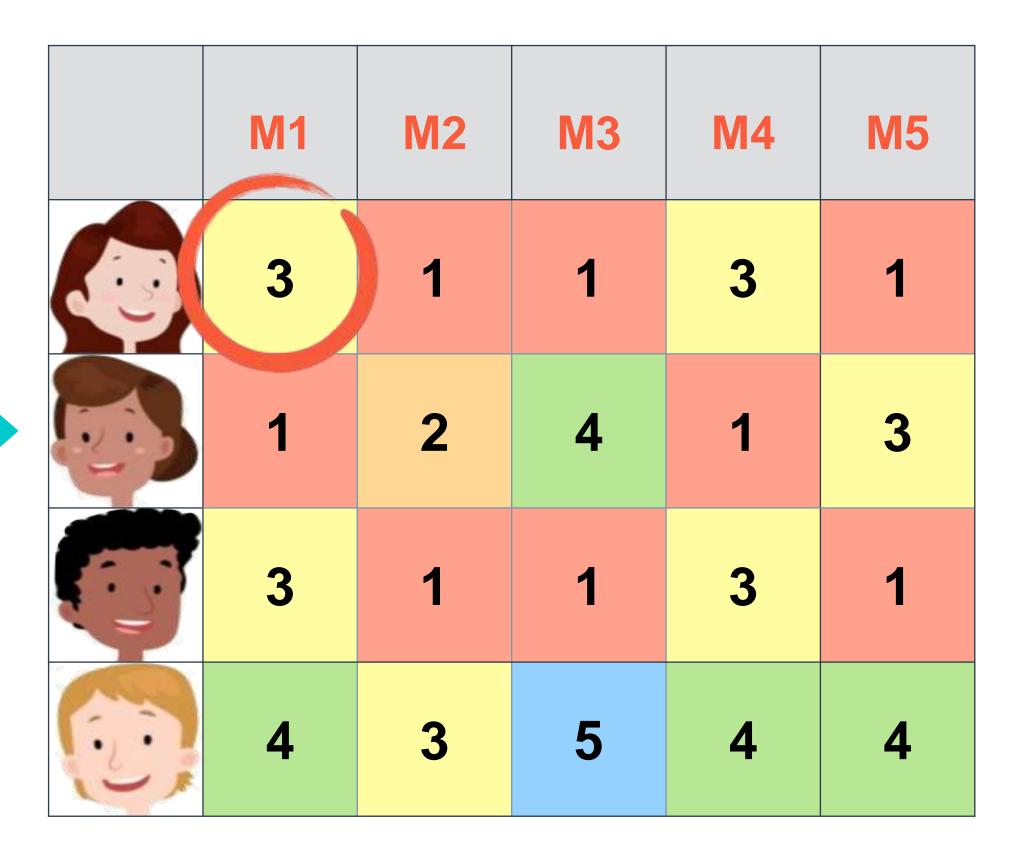


| | 1 | M2 | M3 | M4 | M5 |
|----|----------|-----|-----|-----|-----|
| F1 | 112 | 3.1 | 0.3 | 2.5 | 0.2 |
| F2 | 2.4 | 1.5 | 4.4 | 0.4 | 1.1 |

| $1.2 \times 0.2 + 2.4 \times 0.5 = 1.44$ | 1.2 | x C |)_2 | + 2 | 4 x | 0.5 | = 1 | _44 |
|--|-----|-----|-----|-----|-----|-----|-----|-----|
|--|-----|-----|-----|-----|-----|-----|-----|-----|

| | F1 | F2 |
|---|-----|-----|
| A | 0.2 | 0.5 |
| B | 0.3 | 0.4 |
| O | 0.7 | 8.0 |
| D | 0.4 | 0.5 |

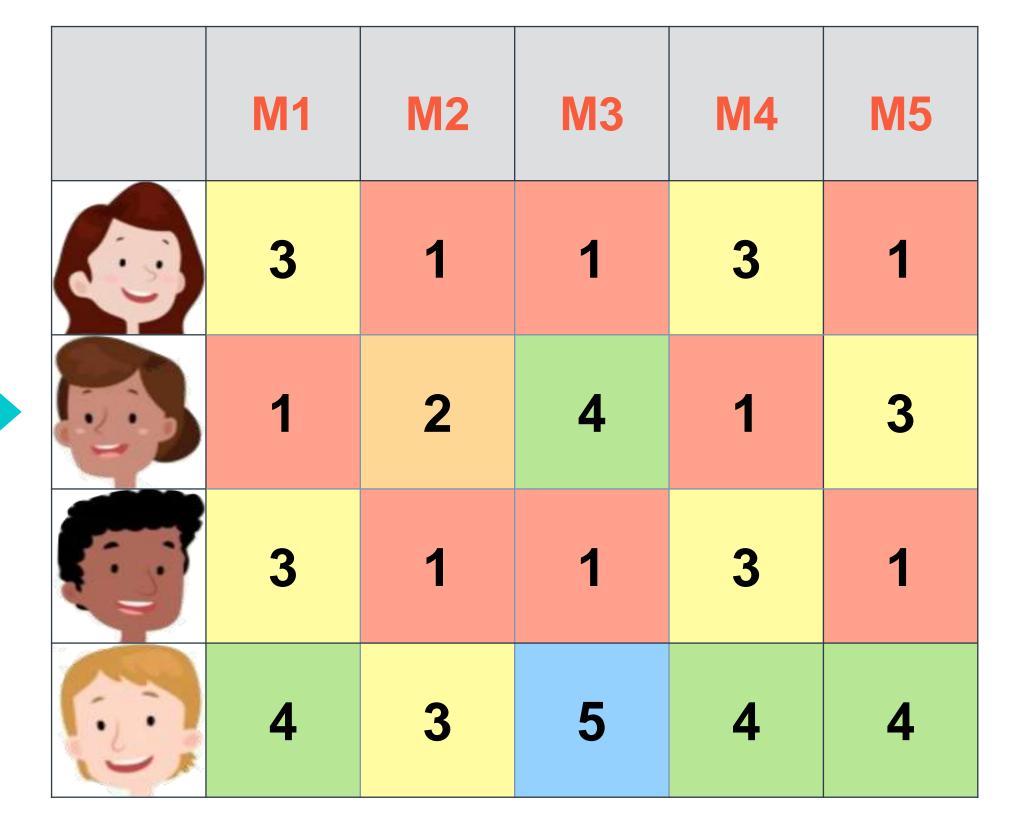
| M1 | M2 | M3 | M4 | M5 | |
|------|-----------|----|----|----|--|
| 1.44 | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |



| | M | M2 | M3 | M4 | M5 |
|----|----------|-----|-----|-----|-----|
| F1 | 1 | 3.1 | 0.3 | 2.5 | 0.2 |
| F2 | 2.5 | 1.5 | 4.4 | 0.4 | 1.1 |

| | F1 | F2 |
|---|-----|-----|
| A | 0.3 | 0.6 |
| B | 0.3 | 0.4 |
| C | 0.7 | 0.8 |
| D | 0.4 | 0.5 |

| M1 | M2 | M3 | M4 | M5 |
|----|----|----|----|----|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

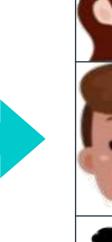


| | M | M2 | M3 | M4 | M5 |
|----|----------|-----|-----|-----|-----|
| F1 | 1 | 3.1 | 0.3 | 2.5 | 0.2 |
| F2 | 2.5 | 1.5 | 4.4 | 0.4 | 1.1 |

$$1.4 \times 0.3 + 2.5 \times 0.6 = 1.92$$

| | F1 | F2 |
|---|-----|-----|
| A | 0.3 | 0.6 |
| B | 0.3 | 0.4 |
| C | 0.7 | 0.8 |
| D | 0.4 | 0.5 |

| M1 | M2 | M3 | M4 | M5 | |
|------|----|----|----|----|--|
| 1.92 | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |



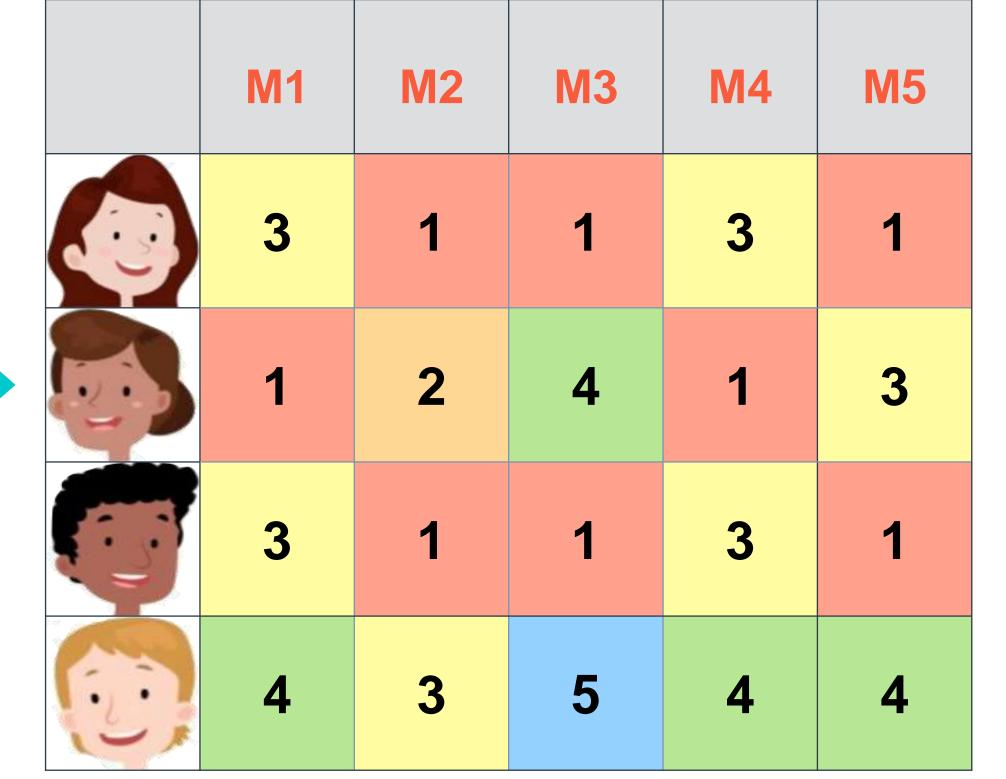
| M1 | M2 | M3 | M4 | M5 |
|----|-----------|-----------|-----------|----|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

| | M | M2 | M3 | M4 | M5 |
|----|----------|-----|-----|-----|-----|
| F1 | 1 | 3.1 | 0.3 | 2.5 | 0.2 |
| F2 | 2.5 | 1.5 | 4.4 | 0.4 | 1.1 |

| $1.4 \times 0.3 + 2.5 \times 0.6 = 1.92$ | 1 | .4 × | 0.3 | + 2 | 2.5 | x (| 0.6 | = | 1 | .9 |
|--|---|------|-----|-----|-----|-----|-----|---|---|----|
|--|---|------|-----|-----|-----|-----|-----|---|---|----|

| | F1 | F2 |
|---|-----|-----|
| | 0.3 | 0.6 |
| B | 0.3 | 0.4 |
| C | 0.7 | 0.8 |
| D | 0.4 | 0.5 |

| M1 | M2 | M3 | M4 | M5 |
|------|-----------|----|----|----|
| 1.92 | 1.83 | | | |
| | | | | |
| | | | | |
| | | | | |

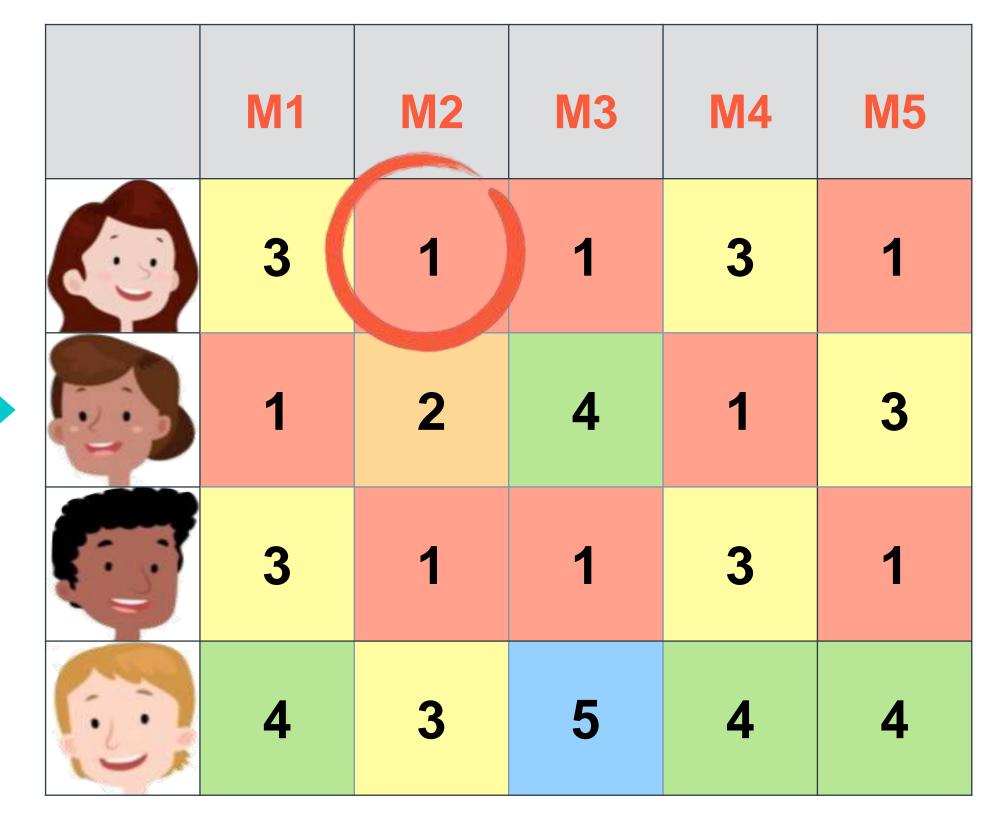


| | M1 | M2 | M3 | M4 | M5 |
|----|-----|-----|-----|-----|-----|
| F1 | 1.4 | 311 | 0.3 | 2.5 | 0.2 |
| F2 | 2.5 | 15 | 4.4 | 0.4 | 1.1 |

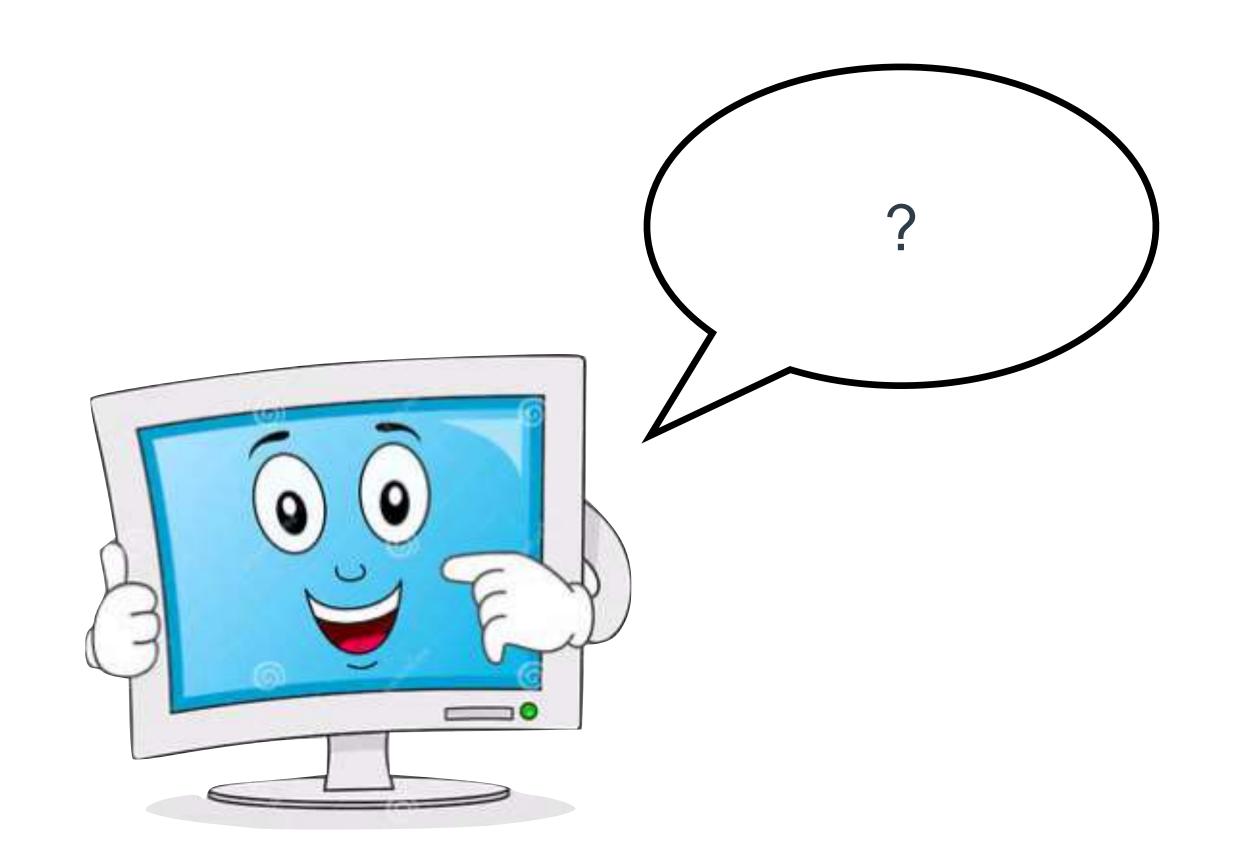
| $3.1 \times 0.3 + 1.5 \times$ | U.D | | .83 |
|-------------------------------|-----|--|-----|
|-------------------------------|-----|--|-----|

| | F1 | F2 |
|---|-----|-----|
| A | 0.3 | 0.6 |
| B | 0.3 | 0.4 |
| C | 0.7 | 0.8 |
| D | 0.4 | 0.5 |

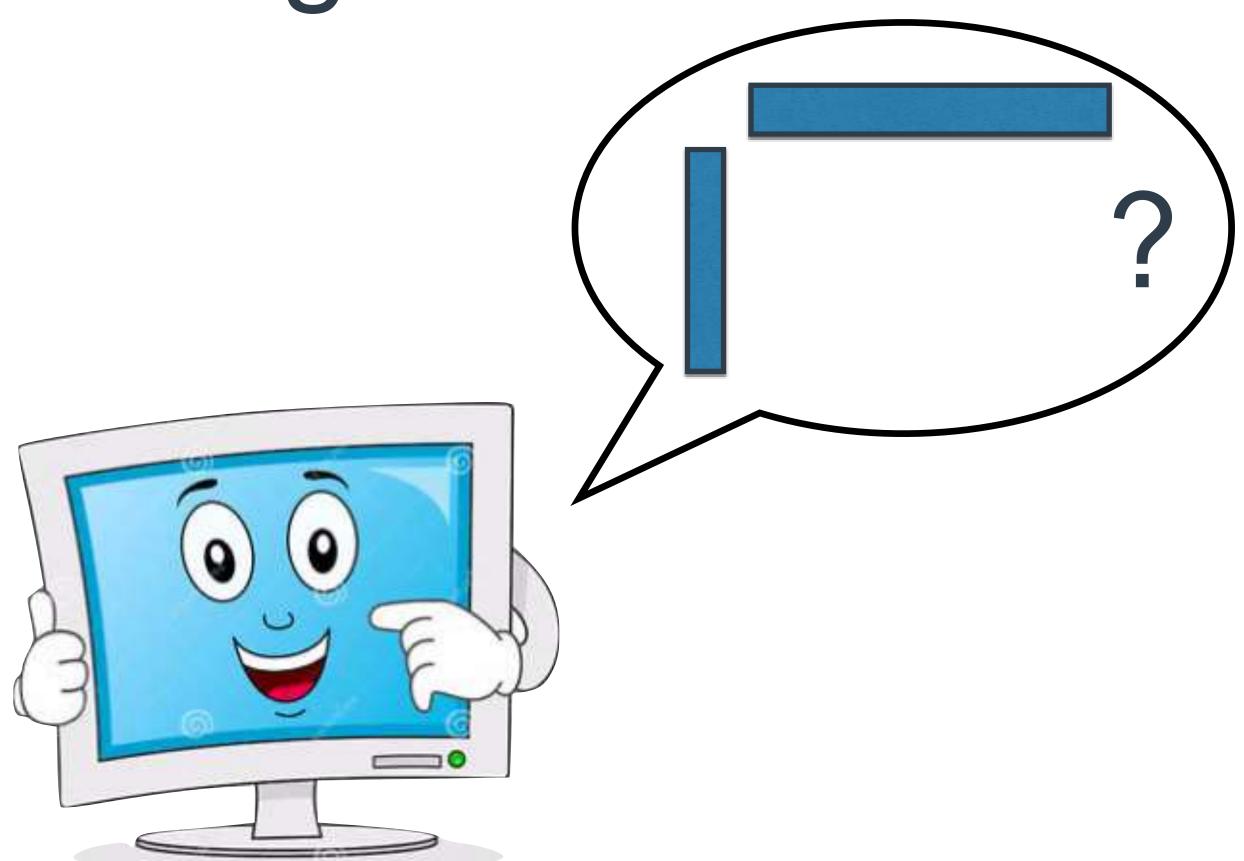
| M1 | M2 | M3 | M4 | M5 | |
|------|------|----|-----------|----|---|
| 1.92 | 1.83 | | | | |
| | | | | | • |
| | | | | | |
| | | | | | |



Error function

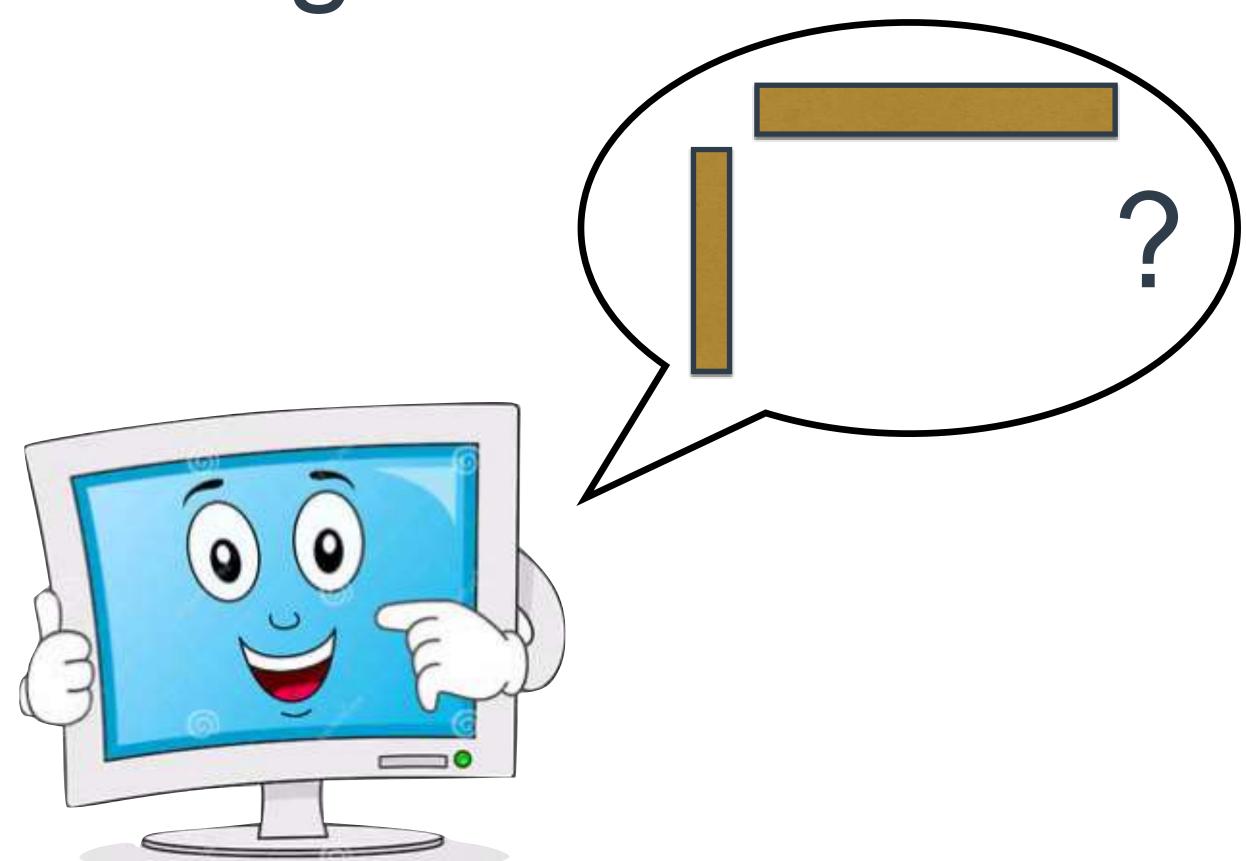






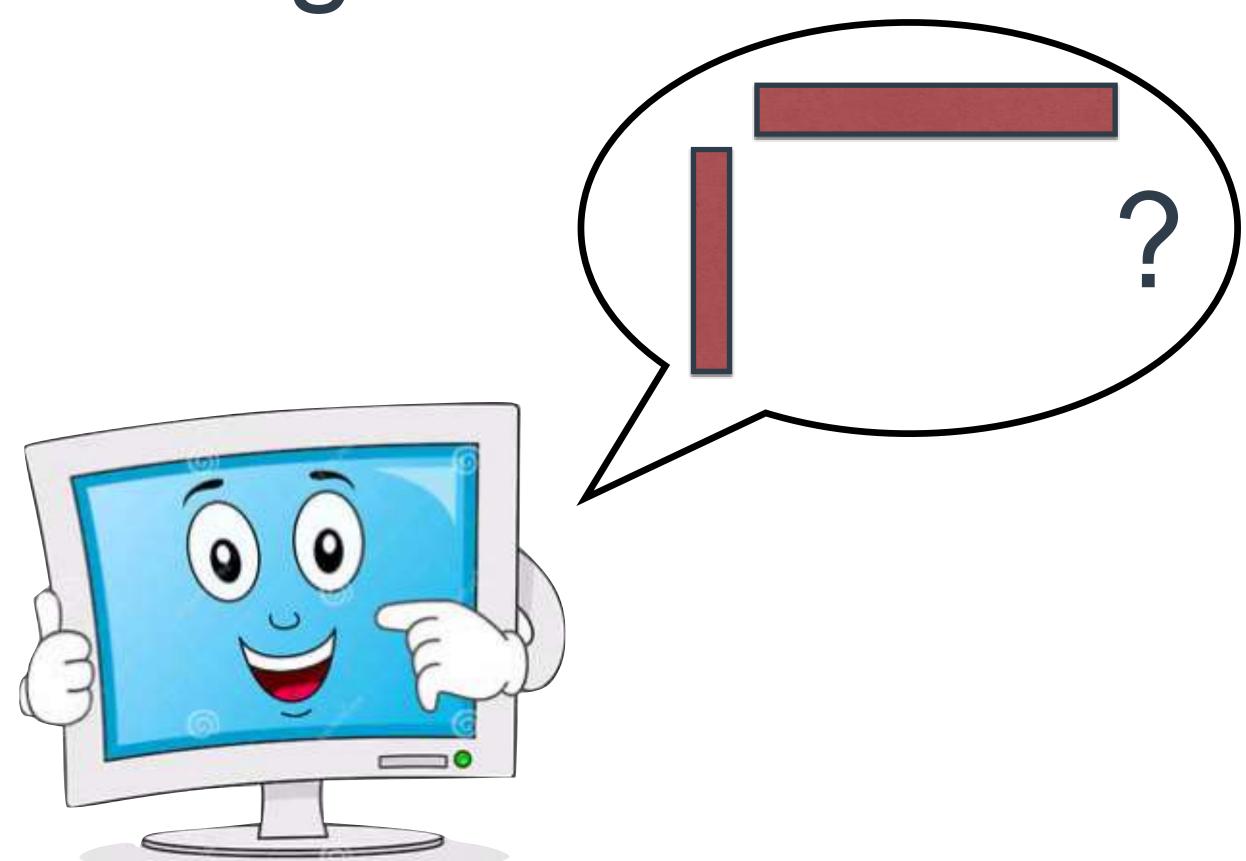
You were wrong by 10.3





You were wrong by 5.32

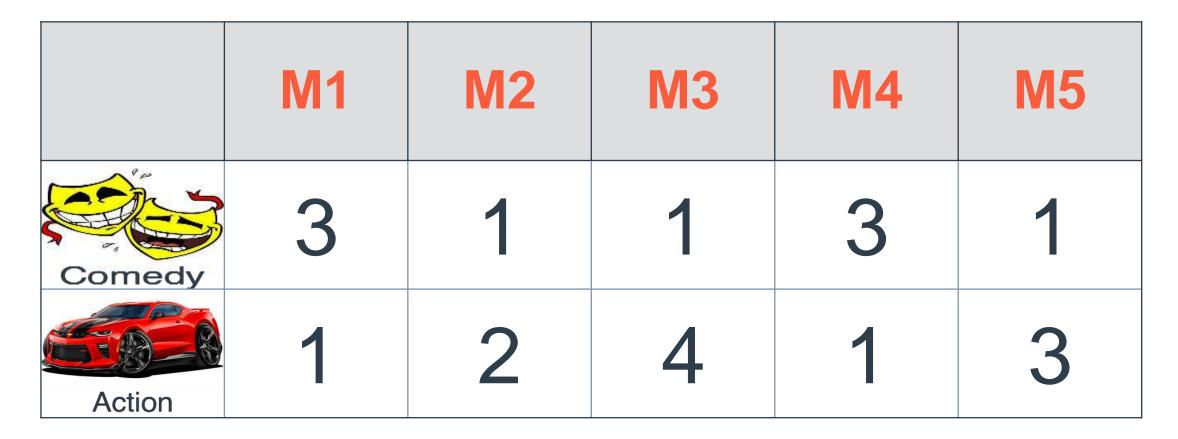




You were wrong by 1.23



Matrix Factorization



| | Comedy | Action |
|---|--------|--------|
| A | 1 | 0 |
| B | 0 | 1 |
| C | 1 | 0 |
| D | 1 | 1 |

| M1 | M2 | M3 | M4 | M5 |
|----|-----------|----|----|----|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

Error Function

| | M1 | M2 | M3 | M4 | M5 |
|----|-----|-----------|-----|-----|-----|
| F1 | 1.2 | 3.1 | 0.3 | 2.5 | 0.2 |
| F2 | 2.4 | 1.5 | 4.4 | 0.4 | 1.1 |

| | F1 | F2 |
|---|-----|-----|
| A | 0.2 | 0.5 |
| B | 0.3 | 0.4 |
| C | 0.7 | 0.8 |
| D | 0.4 | 0.5 |

| M1 | M2 | M3 | M4 | M5 | |
|------|-----------|----|----|----|--|
| 1.44 | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

Error = $(3 - 1.44)^2$

| M1 | M2 | M3 | M4 | M5 |
|----|----|----|----|----|
| 3 | 1 | 1 | 3 | 1 |
| 1 | 2 | 4 | 1 | 3 |
| 3 | 1 | 1 | 3 | 1 |
| 4 | 3 | 5 | 4 | 4 |

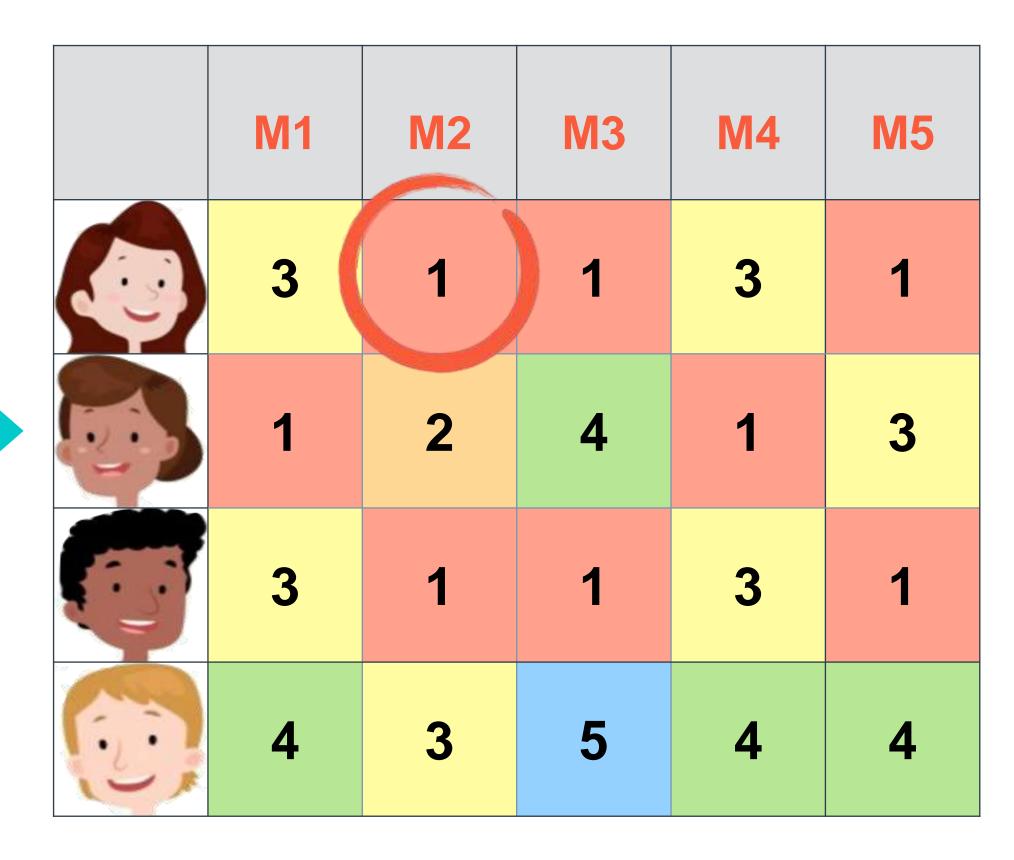
Error Function

| | M1 | 12 | M3 | M4 | M5 |
|----|-----|------|-----|-----|-----|
| F1 | 1.2 | 1-1- | 0.3 | 2.5 | 0.2 |
| F2 | 2.4 | 15 | 4.4 | 0.4 | 1.1 |

| | Error = $(3 - 1.44)^2$ |
|------------|------------------------|
| Derivative | $+(1-1.37)^2$ |
| | + |

| | F1 | F2 |
|---|-----|-----|
| A | 0.2 | 0.5 |
| B | 0.3 | 0.4 |
| C | 0.7 | 0.8 |
| D | 0.4 | 0.5 |

| M1 | M2 | M3 | M4 | M5 | |
|------|------|----|-----------|----|--|
| 1.44 | 1.37 | | | | |
| | | | | | |
| | | | | | |
| | | | | | |



Summary: How to use this?

| | M1 | M2 | M3 | M4 | M5 |
|----|----|----|----|----|----|
| F1 | 3 | 1 | 1 | 3 | 1 |
| F2 | 1 | 2 | 4 | 1 | 3 |

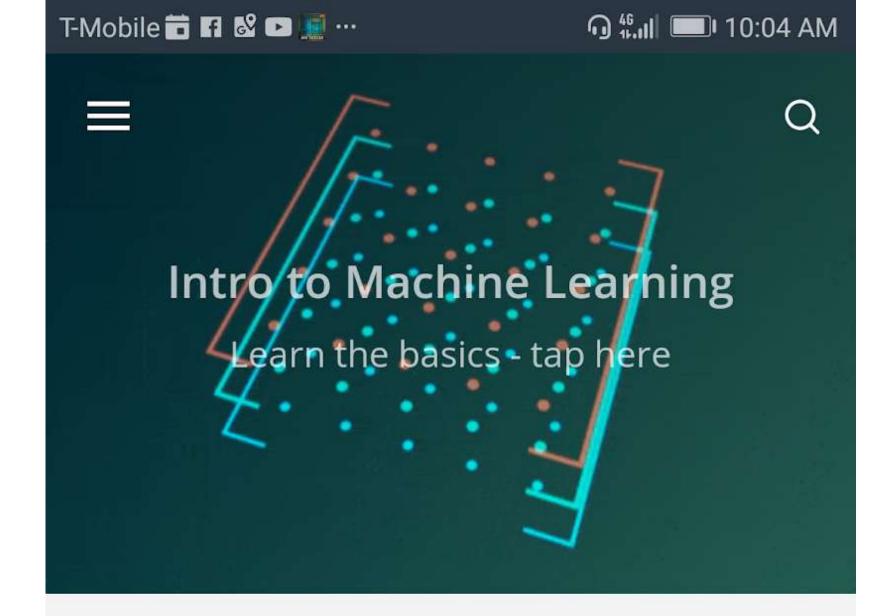
| | F1 | F2 |
|---|----|----|
| A | 1 | 0 |
| B | 0 | 1 |
| C | 1 | 0 |
| D | 1 | 1 |

| | M1 | M2 | M3 | M4 | M5 |
|---|----|----|----|----|----|
| | 3 | 1 | 1 | 3 | 1 |
| B | 1 | 2 | 4 | 1 | 3 |
| C | 3 | 1 | 1 | 3 | 1 |
| | 4 | 3 | 5 | 4 | 4 |

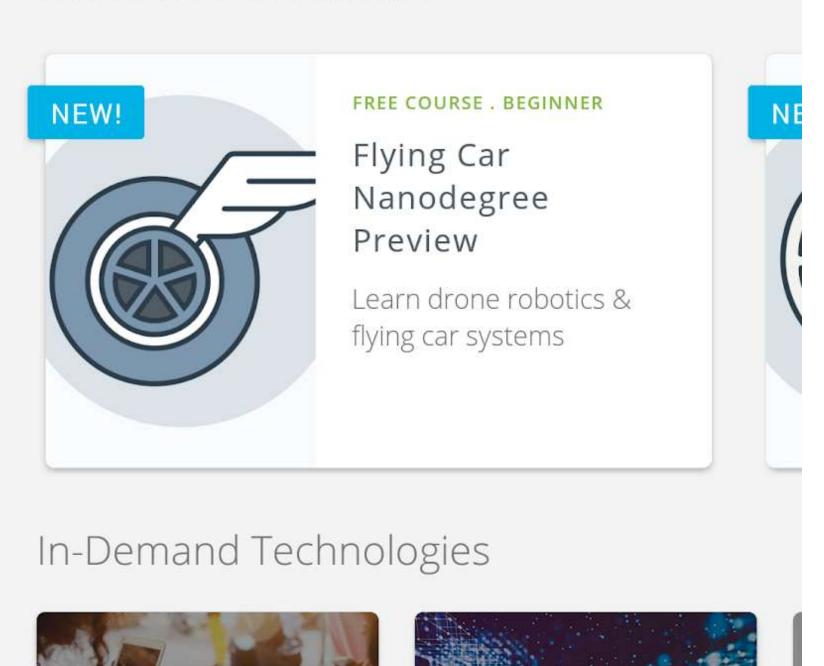


M3





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Thank you!

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