Telecom Systems (Week 3)
Class exercises

Queen Mary University of London

Email: yue.gao@qmul.ac.uk **web:** http://www.eecs.qmul.ac.uk/~yueg/

Class exercise – 1

- A multilevel digital communication system is to operate at a data rate of 1500kb/s. If a 4-bit words are encoded into each level for transmission over the channel. What is the minimum required bandwidth for each channel?

minimum required bandwidth → Lower bound

Solution to CE-1:

$$D = \frac{R}{l} = \frac{1500}{4} = 375 \text{ kbaud}$$

$$B \geq \frac{N}{2T} = \frac{1}{2}D = \frac{1}{2} * 375 = 187.5 \text{ kHz} \rightarrow \text{minimum bandwidth}$$

Class exercise - 2

- A multilevel digital communication system sends of 16 possible levels over the channel every 0.8ms?
  - a) what is the number of bits corresponding to each level?
  - b)What is the baud rate?
  - c) What is the bit rare?

Solution to CE-2:

(a) $L = 2^l = 16 \rightarrow l = 4\ bits\ /\ level$

(b) $D = \frac{N}{T_0} = \frac{1}{T_s} = \frac{1}{0.8*10^{-3}s} = 1.25\ kbaud$

(c) $\mathrm{R} = lD = 4 * 1250 = 5\ kbits\ /\ s$

Class exercise - 3

A binary waveform of 9600 bits/s is converted into an 8-level waveform that is passing through a channel with a raised cosine roll-off Nyquist filter. The channel has a conditional (equalised) phase response out to 2.4KHz:

(a) What is the baud rate?
(b) What is the roll-off factor?

Solutions to CE-3

(a) $L = 8 = 2^l \rightarrow\ l = 3\ bits\ /\ level$

$$D = \frac{R}{l} = \frac{9600\ bits\ /\ s}{3bits\ /\ symbol} = 3.2\ k\ symbols\ /\ s$$

(b) $D = \frac{2B}{1+r} = \frac{2*2.4\ k}{1+r} = 3.2\ k,\ r = 0.5$

Class exercise - 4

An analog signal is to be converted into a PCM signal that is a binary polar NRZ line code. The signal is transmitted over a channel that is absolutely band limited to 4kHz. Assume that the PCM quantizer has 16 steps, and the overall equivalent system transfer function is of the raised cosine roll-off type with r=0.5. (Note: the binary signalling is used)

(a) Find the maximum PCM bit rate that can be supported by this system without introducing ISI.
(b) Find the bandwidth that can be permitted for the analog signal.

Solutions to CE-4

$M = 16 = 2^n \rightarrow \; n = 4$

(a) Binary PCM: $\rightarrow l = 1, R = nf_s = 4f_s = D$

$$D = \frac{2B}{1+r} = \frac{2 * 4\,k}{1\; + 0.5} = 5.33\; kbits\;/\;s$$

(b)$f_s = \frac{D}{4} = \frac{5.33}{4} = 1.33\; kHz,$

$$B_{analog\_Max} = \frac{f_s}{2} = \frac{1.33\; kHz}{2} = 677\; Hz$$

Class exercise - 5

- Consider a random data pattern consisting of binary 1's and 0's where the probability of obtaining either a binary 1 or binary 0 is 50%. Calculate the PSD for the unipolar NRZ signalling as a function of T_b (the time needed to send 1 bit of data).

Solutions to CE-5 (5-1)

- In general, the PSD is given by equation (3-6a) as,

$$P(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k) e^{j2\pi k f T_s}$$

- The autocorrelation is calculated by (3-7) $\mathrm{R}_{unipolar}(k) = \begin{cases} \frac{1}{2}A^2 & k = 0 \\ \frac{1}{4}A^2 & k \neq 0 \end{cases}$

- So

$$P(f) = \frac{|F(f)|^2}{T_b} \sum_{k=-\infty}^{\infty} R(k) e^{j2\pi k f T_b}$$

$$= \frac{|F(f)|^2}{T_b} \left[R(0) + \sum_{k=-\infty}^{-1} R(k) e^{j2\pi k f T_b} + \sum_{k=1}^{\infty} R(k) e^{j2\pi k f T_b} \right]$$

$$= \frac{|F(f)|^2}{T_b} \left[R(0) + \sum_{k=1}^{\infty} R(k) e^{-j2\pi k f T_b} + \sum_{k=1}^{\infty} R(k) e^{j2\pi k f T_b} \right]$$

Solutions to CE-5 (5-2)

$$=\frac{|F(f)|^2}{T_b}\left[R(0)+\sum_{k=1}^{\infty}R(k)(e^{-j2\pi kfT_b}+e^{j2\pi kfT_b})\right]$$

$$=\frac{|F(f)|^2}{T_b}\left[R(0)+2\sum_{k=1}^{\infty}R(k)\cos(2\pi kfT_b)\right]$$

$$=\frac{|F(f)|^2}{T_b}\left[\frac{A^2}{2}+2*\frac{A^2}{4}\sum_{k=1}^{\infty}\cos(2\pi kfT_b)\right]$$

$$=\frac{A^2}{4}\frac{|F(f)|^2}{T_b}\left[2+2\sum_{k=1}^{\infty}\cos(2\pi kfT_b)\right]$$

With a weight of ½ at R(0)

$$=\frac{A^2}{4}\frac{|F(f)|^2}{T_b}\left[1+\sum_{k=-\infty}^{\infty}e^{j2\pi kfT_b}\right]$$

Solutions to CE-5 (5-3)

$$P_{unipolar\_NRZ}(f) = \frac{A^2}{4}\frac{|F(f)|^2}{T_b}\left[1 + \sum_{k=-\infty}^{\infty} \delta(f - \frac{n}{T_b})\right]$$

$$f(t) = Rect\left(\frac{t}{T_b}\right) \leftrightarrow F(f) = T_b \frac{\sin \pi f(T_b)}{\pi f T_b}$$

$$P_{unipolar\_NRZ}(f) = \frac{A^2 T_b}{4}\left(\frac{sin\pi f T_b}{\pi f T_b}\right)^2 \left[1 + \frac{1}{T_b}\delta(f)\right]$$