Tutorial: Linear Algebra

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Question 1

Consider the pair of vectors:

$$\psi_0 = (1,1) \qquad \psi_1 = \sqrt{\tfrac{1}{2}}(1,-1)$$

By calculating relevant inner products and norms, identify whether or not these vectors form an orthogonal or an orthonormal set.
Sketch these two vectors on a diagram to confirm your answer

Question 2

Two sets of functions are given by:

$$\{\Psi_1\}=[(2,0),(a,2)]$$

$$\{\Psi_2\}=[(a,-1/8),(0,b)]$$

State the condition required for these two sets to be a Dual Basis and determine the corresponding values of a and b.

Question 3

The matrix A is:

$$A=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$$

- Find an orthogonal basis of the NULL space of A.
- Find the RANK of A.
- Find an orthonormal basis of the ROW space of A.

Question 4

Vectors v_1 and v_2 span the space V.

$$v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Do v_1 and v_2 form an orthonormal basis for V?
If they do not, then find an orthonormal basis for V.

Question 5

$$v_1 = \frac{1}{3\sqrt{2}}\begin{bmatrix}1\\1\\-4\end{bmatrix},\ v_2 = \frac{1}{3}\begin{bmatrix}2\\2\\1\end{bmatrix},\ v_3 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\end{bmatrix}$$

Show that these three vectors are an orthonormal basis for R^3