EBU6018
Advanced Transform Methods

Tut: MRA/WTFB/Haar Transform

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Q1

- Multiresolution Analysis (MRA) is used to separate data into course and fine detail.
- Apply the transform defined by
- $$x_{n-1,i} = (x_{n,2i} + x_{n,2i+1})/2$$
- $$d_{n-1,i} = (x_{n,2i} - x_{n,2i+1})/2$$
- to the sequence
- $$[x_{n,i}] = [6, 8, 3, 11, 9, 5, 7, 2]$$
- Where $i = 0…..7$, is the index position in the sequence, and
- n is the level. The next level is n-1.
- At each level, calculate the sequencies for $x_{n-1,i}$ and $d_{n-1,i}$
- Continue till no further levels are possible.
- i) State the significance of the first element in the final level.
- ii) Has any information been lost in the process?
- iii) Comment on how this process could be used to compress the data.

Q1 Answer

- n=3   [6, 8, 3, 11, 9, 5, 7, 2]
- n=2   [7, 7, 7, 4.5, -1, -4, 2, 2.5]
- n=1   [7, 5.75, 0, 1.25, -1, -4, 2, 2.5]
- n=0   [6.375, 0.625, 0, 1.25, -1, -4, 2, 2.5]
- i) The first element is the average of all the elements in the original sequence
- ii) No information has been lost
- iii) Because most of the values in the final level are small, potentially fewer bits would be required to store it. Where there are zeroes, they do not need to be stored, although the positions of the other values would need to be stored. Small values could be replaced by zeroes without significant loss of detail, this can be done by applying a threshold value, the bigger the threshold the greater the loss of detail.

Q2

- Multiresolution Analysis (MRA) is used to separate data into course and fine detail.

-         Apply the transform defined by

-                 $x_{n-1},i = (x_{n,2i} + x_{n,2i+1})/2$

-                 $d_{n-1},i = (x_{n,2i} - x_{n,2i+1})/2$

- to the sequence

-                 $[x_{n,i}] = [6, 7, 3, 4, 8, 2, 1, 9]$

- Where i = 0…..7, is the index position in the sequence, and

-                 n is the level. The next level is n-1.

-         At each level, calculate the sequencies for x_{n-1},i and d_{n-1},i

-         Continue till no further levels are possible.

-       i) State the significance of the first element in the final level.

-       ii) Has any information been lost in the process?

-       iii) Comment on how this process could be used to compress the data.

Q3

- Multiresolution Analysis (MRA) is used to separate data into course and fine detail.
- Apply the transform defined by
- $$x_{n-1},i = (x_{n,2i}+x_{n,2i+1})/2$$
- $$d_{n-1},i = (x_{n,2i}-x_{n,2i+1})/2$$
- to the sequence
- $$[x_{n,i}] = [12, 8, 13, 10, 6, 7, 11, 14]$$
- Where i = 0…..7, is the index position in the sequence, and
- n is the level. The next level is n-1.
- At each level, concatenate the sequencies for x_{n-1},i and d_{n-1},i
- Continue till no further levels are possible.
- i) State the significance of the first element in the final level.
- ii) Has any information been lost in the process?
- iii) Comment on how this process could be used to compress the data.

Q4

Suppose we have a Haar wavelet transform analysis filterbank which uses a low-pass filter

$$h_0[0] = h_0[1] = \tfrac{1}{2}$$

and a high-pass filter $\quad h_1[0] = \tfrac{1}{2} \quad h_1[1] = -\tfrac{1}{2}$

.Use the recursive equations:

$$c_{m-1,n} = \sqrt{2} \cdot \frac{1}{2}(c_{m,2n} + c_{m,2n+1})$$

$$= \frac{1}{\sqrt{2}}(c_{m,2n} + c_{m,2n+1})$$

$$d_{m-1,n} = \frac{1}{\sqrt{2}}(c_{m,2n} - c_{m,2n+1})$$

to calculate the Haar wavelet transform for a sampled signal s[n] = [3 2 5 -2] after 1 and 2 stages of the transform filterbank.

Q5

Suppose we have a Haar wavelet transform analysis filterbank which uses a low-pass filter

$$h_0[0] = h_0[1] = \frac{1}{2}$$

and a high-pass filter $h_1[0] = \frac{1}{2} \quad h_1[1] = -\frac{1}{2}$

. Using the recursive equations, calculate the Haar wavelet transform for a sampled signal

,

$$s[n] = [1,\ 2,0,-2]$$

after 1 and 2 stages of the transform filterbank.

. Calculate the inverse wavelet transform of your result using the resynthesis filterbank, to confirm that the result of the inverse transform is the original

Q6

For the sequences of Q4 and Q5, perform the Haar Transform directly using the Haar Matrix.