SOLUTIONS

Module:	Telecom Systems		
Module Code	EBU5302	Paper	A
Time allowed	2hrs	Filename	Solutions_201920_EBU5302_A
Rubric	ANSWER ALL FOUR QUESTIONS		
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Question 1:

a) Signals that travel through transmission media will always be corrupted by attenuation, distortion and noise. Explain the concept of attenuation, distortion, and noise. You may want to use figures to support your explanations (e.g., with using transmitting signals and received signals).

[6 marks]

b) This question concerns A/D conversion. A continuous-time signal $x(t)$ is sampled, producing the signal $x[n]$, and then quantised, resulting in the signal $x_Q[n]$. The following diagram (**Figure 1**) illustrates this process:

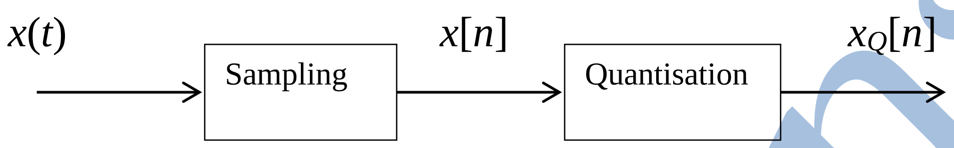


Figure 1

Assuming that:

- The bandwidth of $x(t)$ is W = 10 Hz.
- The sampling frequency is five times the Nyquist rate.
- The amplitude of $x[n]$ is distributed uniformly in the interval [-1, 1].
- A 4-bit uniform quantiser is used.

Determine:

i) The quantisation regions. Assign a binary code to each region. [4 marks]
ii) The power of the quantisation error. [3 marks]
iii) The resulting bit rate after encoding $x_Q[n]$ with your 4-bit coding strategy. [2 marks]

[9 marks]

c) A source emits 5 symbols (A, B, C, D, E) with probabilities as shown in **Figure 2**.

A	*B*	*C*	*D*	**E**
0.4	0.2	0.2	0.1	0.1

Figure 2

i) Calculate the entropy of the source

[2 marks]

ii) Compare the answer in (i) with the number of bits/symbol that would have to be used if the symbols were to be encoded without any consideration of the probability.

[2 marks]

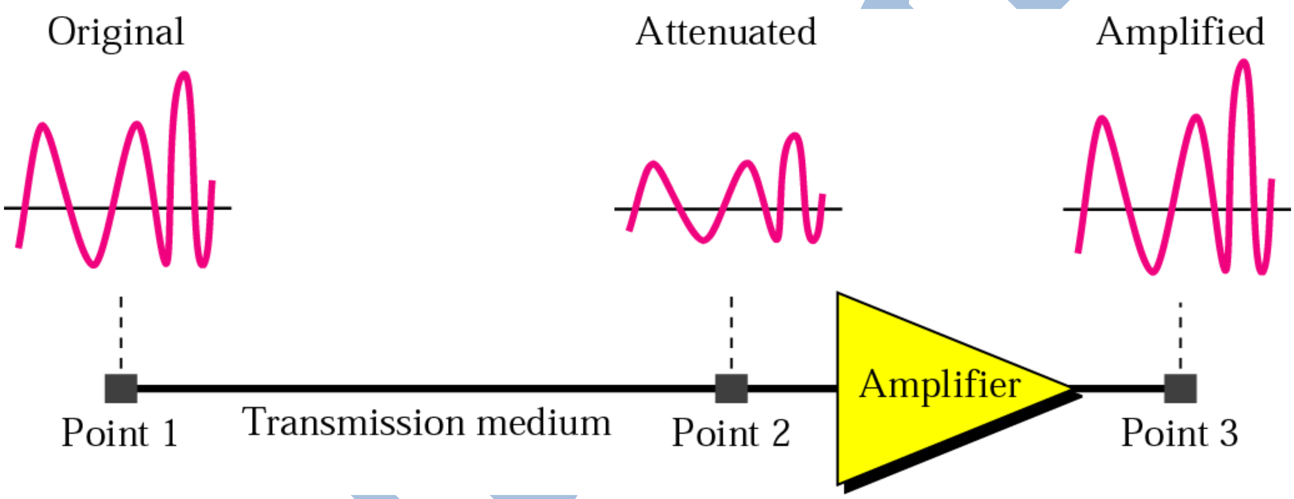
iii) Code this data with a Huffman variable-length code representing each symbol and determine the average number of bits/symbol that could be used to transmit this source.

[6 marks]

Answers:

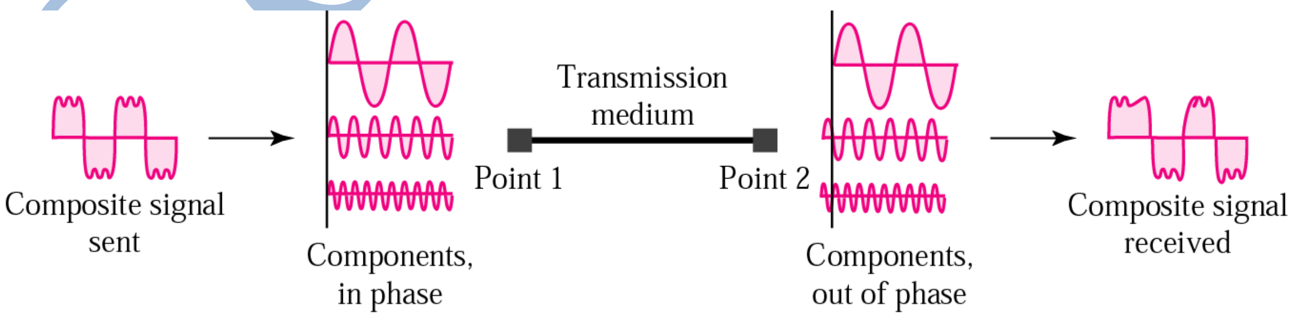
a)

Attenuation means the loss of energy. **[1 mark]**



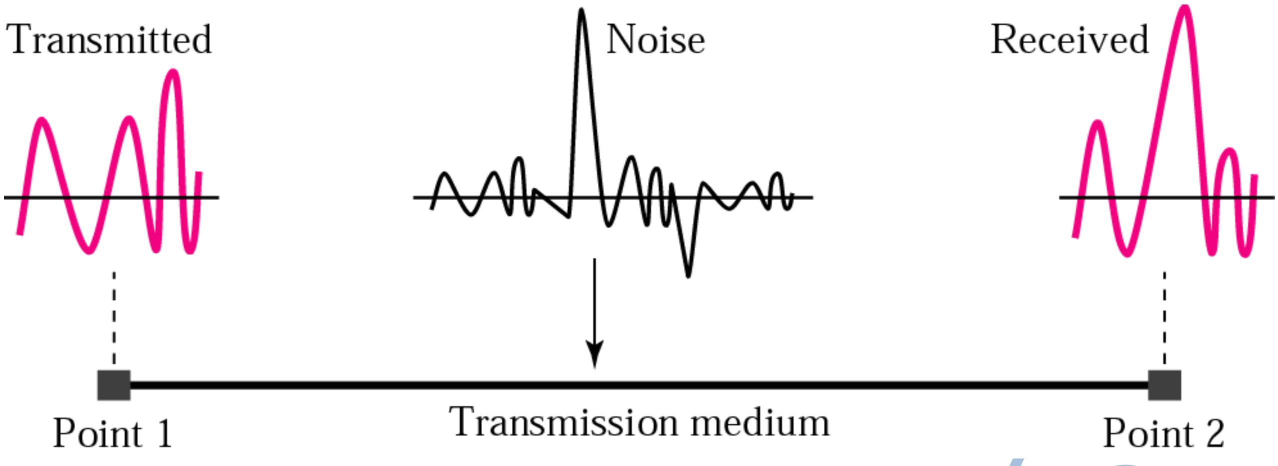
[1 mark]

Distortion: signal changes in its form or shape. **[1 mark]**



[1 mark]

Noise is the main source of a signal being corrupted. **[1 mark]**



[1 mark]

b) Since a 4-bit uniform quantiser is used, the amplitude range is divided into 16 equally sized regions **[1 mark]**
The signals amplitude range is 1-(-1)=2. **[1 mark]**
Hence, the size of the quantisation regions is q=2/16=1/8=0.125 and they are [-1,-7/8, [-7/8, -6/8], [-6/8, -5/8], [-5/8, -4/8], [-4/8, -3/8], [-3/8, -2/8], [-2/8, -1/8], [-1/8, 0], [0,1/8], [1/8, 2/8], [2/8, 3/8], [3/8, 4/8], [4/8, 5/8], [5/8, 6/8], [6/8, 7/8] and [7/8, 1]. **[1 mark]**
We can assign the following binary code to each region 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111. **[1 mark]**

Since the signal's amplitude is distributed uniformly, so is the quantisation error. **[1 mark]**
Under these circumstances, the power of the quantisation error is Pe=q^2/12. **[1 mark]**
Hence, substituting q with 1/8 we obtain Pe=$(1/8)^2$/12. **[1 mark]**

Since the Nyquist rate is twice the bandwidth of $x(t)$, the sampling frequency is 5x2x10Hz = 100 Hz. **[1 mark]**
Since each sample is encoded by 4 bits, the resulting bit rate is 4x100=400 bps. **[1 mark]**

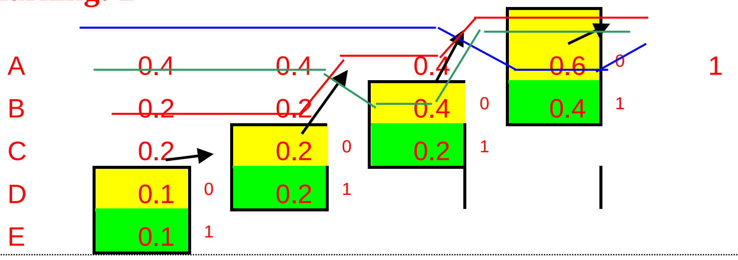
c)

i) $\boldsymbol{H} = \sum \boldsymbol{p_i} \log_2(1/\ \boldsymbol{p_i})$ or $H = \sum p_i \dfrac{\log_{10}(1/\,p_i)}{\log_{10} 2} = \dfrac{1}{\log_{10} 2} \sum p_i \log_{10}(1/\,p_i)$ bits/symbol

	pi	pi*log2(1/pi)
A	0.4	0.528771238
B	0.2	0.464385619
C	0.2	0.464385619
D	0.1	0.332192809
E	0.1	0.332192809
	1	2.121928095

=2.12 bits/symbol
[2 marks]
ii) No source coding – 5 symbols so 3 bits would be required. **[2 marks]**

Marking: 2



	Reverse →	Real code ←	Ni	Pi	ni*pi
A	1	1	1	0.4	0.4
B	10	01	2	0.2	0.4
C	000	000	3	0.2	0.6
D	0100	0010	4	0.1	0.4
E	1100	0011	4	0.1	0.4
			average no of bits/symbol		**2.2**

Note – the exact format may be different of some of the ordering is changed.
[6 marks]
Marking: 2 for principle of Huffman, 2 for working, 2 for sum(Ni*pi)

Question 2

A digital information source produces binary sequences at a rate of 2 kbps. The probability of producing the value 0 is $p_0 = 0.2$. A Hamming code with the following parity check matrix **H** is employed to protect information against errors:

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The resulting binary sequences are transmitted through an AWGN digital channel with a bandwidth W = 20 kHz and a SNR = 20 dB.

a) Determine the information rate of the digital information source and the channel capacity. According to the channel-coding theorem, can the information produced by the source be transmitted through the digital channel without errors? **[6 marks]**
b) Based on the parity check matrix **H**, determine the length of the input information sequences and the length of the code words. Calculate the code rate of this Hamming code and the resulting transmission rate. **[3 marks]**
c) How can the code words of this Hamming code be obtained? Find out the code words corresponding to the information sequences 0100 and 1000. **[6 marks]**
d) How can the minimum distance be obtained? Determine the number of errors can be detected and corrected in this Hamming code. **[5 marks]**
e) Decode the following received sequence **r** = 1010101. **[5 marks]**

Answer:

a) The entropy of a binary information source that produces both values with the same probability is $H_B = -\,0.2 \log_2 0.2 - 0.8 \log_2 0.8 = 0.72$ information-bit/symbol-bit. [1 mark]

Hence the information rate is $R_I = 0.72 \times 2$kbps $= 1.44$ kbps. [1 mark]

The capacity of an AWGN channel is defined by Shannon's formula, $C = W \log_2 (1+\mathrm{SNR})$. [1 mark]

By converting the SNR from dB to a ratio, we obtain SNR = 100. [1 mark]

The capacity of the AWGN channel is consequently $C = 20\text{x}10^3 \log_2 101$ Hz ≈ 133.16 kHz. [1 mark]

Since $R_I < C$, information theory predicts that information can be transmitted at a rate R_I with a probability of errors arbitrarily small. [1 marks]

b) The dimensions of the parity check matrix are m x n, where n is the length of a code word, $m = n - k$, and k is the length of information sequences. [1 mark]

This is then a (7,4) Hamming code and its code rate is $R_C = k/n = 4/7$. [1 mark]

The resulting transmission rate can be obtained as $R_B = 2\text{kbps} * 1/R_c = 2000 \times 7/4 = 3.5$ kbps. [1 mark]

c) Based on the parity check matrix **H**, we first obtain the matrix **P**:

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \text{[1 mark]}$$

The generator matrix **G** will then be

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \text{[2 marks]}$$

Code words **c** can be obtained by multiplying each 4-bit information sequence **x** by the generator matrix **G**, **c** = **xG**. [1 marks]

By using this expression, the code words corresponding to the sequences 0100 and 1000 are, respectively, 0100011 and 1000110. [2 marks]

d) Hamming codes belong to the family of linear block codes. Hence, the minimum distance can be obtained as the minimum weight, where the weight of a code word is defined as the number of bits of value 1 in each sequence. [2 marks]

Since the minimum distance is 3, up to 2 errors will be detected and 1 error will be corrected. [3 marks]

e) In order to decode the received sequence, we first compute its syndrome $\mathbf{s} = \mathbf{cH}^t + \mathbf{yH}^t = \mathbf{yH}^t$ [1 mark]
The syndrome sequence corresponding to 1010101 is $\mathbf{s}$ = 110. [2 mark]

The error sequence corresponding to this syndrome is $\mathbf{e}$ = 1000000. [1 marks]

Hence, the transmitted code word is $\mathbf{c}$ = 1010101 + 1000000 = 0010101. [1 mark]

Question 3

(a) For the binary value of 1011001, draw the line coding signalling format of
   i) Unipolar RZ
   ii) Bipolar RZ
   iii) Manchester NRZ **[6 marks]**

b) What important information eye patterns can provide about the characteristics of a signal? Briefly explain them.
[4 marks]

c) Calculate the autocorrelation function R(k) of the unipolar NRZ signalling assuming bit to bit data independence and equally likely occurrence of two voltage levels.
[7 marks]

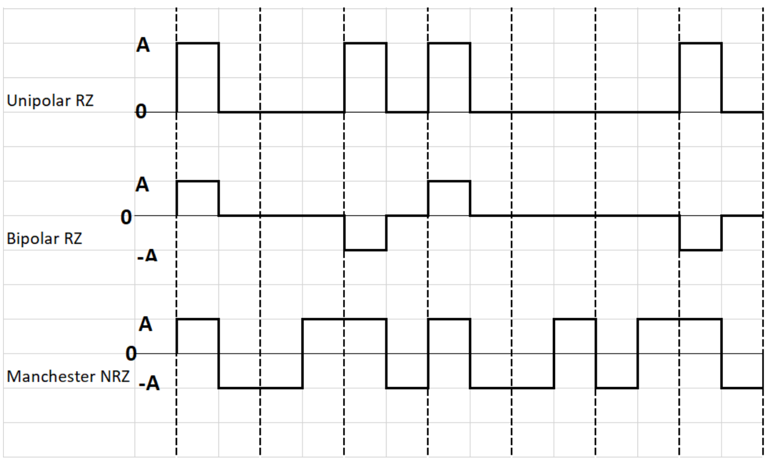
d) A binary waveform of 12 kbits/s is converted into a 16-level waveform that is passing through a channel with a raised cosine roll-off Nyquist filter. The channel has a conditional (equalised) phase response out to 2.25 KHz:

   i) What is the baud rate?
   ii) What is the roll-off factor?

[5 marks]

e) What types of synchronisation are usually used in digital communications?
[3 marks]

Answer:
a)



[2 marks for each correct line coding signal] b)

The **timing error** allowed on the sampler at the receiver is given by the width inside the eye, called the eye opening. Of course, the preferred time for sampling is at the point where the vertical opening of the eye is largest. [2 marks]

The **sensitivity to timing error** is given by the slope of the open eye (evaluated at, or near, the zero-crossing point). [1 marks]

The ***noise margin*** of the system is given by the height of the eye opening. [1 mark]

c) For unipolar signalling, the possible levels for the a's are +A and 0 V. [0.5 marks] Assuming that these values are equally likely to occur and that data are independent. Now, we need to evaluate R(k) as defined by:

$$\mathrm{R}(k) = \sum_{i=1}^{l} (a_n a_{n+k})_i P_i$$ [0.5 marks]

For k=0, the possible products of $a_n a_n$ are $A \times A = A^2$ and $0 \times 0 = 0$, [1 mark] and consequently, $I = 2$. For random data, the probability of having A^2 is 50%, and the probability of having 0 is 50% [1 mark],

so that

$$\mathrm{R}(0) = \sum_{i=1}^{2} (a_n a_n)_i P_i = A^2 \cdot \frac{1}{2} + 0 \cdot \frac{1}{2} = \frac{1}{2} A^2$$ [1 mark]

For $k \neq 0$, there are I=4 possibilities for the product values:

$A \times A,\ A \times 0,\ and\ 0 \times A,\ 0 \times 0$. They all occur with a probability of ¼. [1 mark]

Thus,

$$\mathrm{R}(k) = \sum_{i=1}^{4} (a_n a_{n+k})_i P_i = A^2 \cdot \frac{1}{4} + 0 \cdot \frac{1}{4} + 0 \cdot \frac{1}{4} + 0 \cdot \frac{1}{4} = \frac{1}{4} A^2$$ [1 mark]

Overall,

$$\mathrm{R}_{unipolar}(k) = \begin{cases} \frac{1}{2} A^2 & k = 0 \\ \frac{1}{4} A^2 & k \neq 0 \end{cases}$$ [1 mark]

(d)

$L = 16 = 2^l$ [1 mark]

→ $l = \log_2 16 = 4\ bits\ /\ level$ [1 mark]

$D = \frac{R}{l} = \frac{12 \times 10^3\ bits\ /\ s}{4\ bits\ /\ symbol} = 3 \times 10^3\ symbols/s$ [1 marks]

$D = \frac{2B}{1+r} = \frac{2 \times 2.4 \times 10^3}{1+r} = 3 \times 10^3$, [1 mark]

$1 + r = \frac{2 \times 2.25 \times 10^3}{3 \times 10^3} = 1.5$ [0.5 marks]

$r = 0.5$ [0.5 mark]

(e)

Digital communications usually need at least three types of synchronisation signals: [1 mark for each of the followings, 3 in total]

- Bit sync, to distinguish one bit interval form another;

- Frame sync, to distinguish groups of data;
- Carrier sync, for bandpass signalling with coherent detection.

Question 4:

a) Explain the characteristics of amplitude-shift keying (ASK), frequency-shift keying (FSK) and phase-shift keying (PSK). Assuming that the digital data to be transmitted is 00110100010, please draw the analog waveforms of modulating for digital data above for ASK, binary FSK and binary PSK.

[6 marks]

b) OFDM is widely used in current 4G communication systems.

i) What is OFDM short for?

[1 marks]

ii) Please explain the key advantages of OFDM.

[2 marks]

iii) Illustrate the purpose of Cyclic Prefix (Guard Interval) for OFDM symbols in time domain by showing the direct and delayed paths. You may want to use diagrams to explain your answer.

[6 marks]

iv) What are the key differences of single carrier systems, multi-carrier systems and OFDM systems? You may want to use diagrams to explain your answer.

[3 marks]

c) In cellular communication systems, cells sufficiently distant from each other can use the same channel (frequency).

i) Please use mathematical approaches with possible geometry diagrams to show the relationships between cluster radius (Ru) and cell radius (r). (Assuming *a* is distance between *i* cells, *b* is distance between *j* cells, you final expressions of Ru can be expressed by i, j, r). You may want to use cluster size N=7 as an example to show your derivations.

[5 marks]

ii) What is the effect of cluster size for frequency reuse in the cellular communication systems?

[2 marks]

Answer:

– Amplitude-shift keying (ASK)

Amplitude difference of carrier frequency

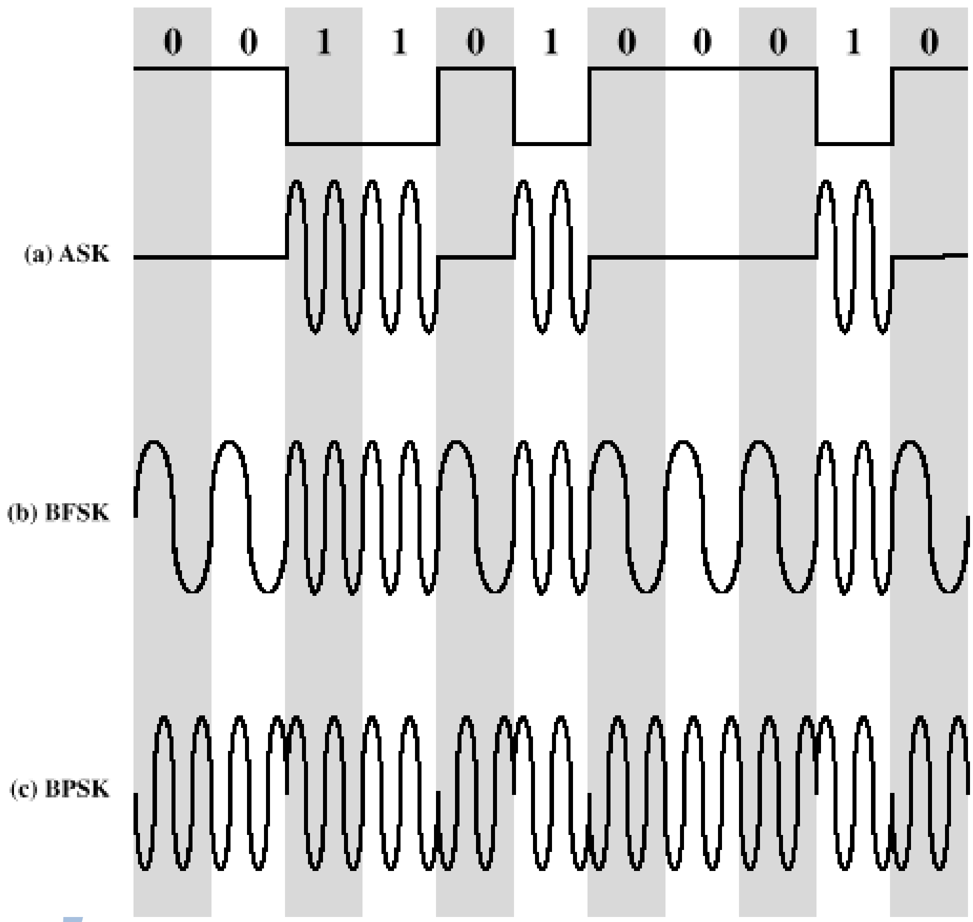
– Frequency-shift keying (FSK)

Frequency difference near carrier frequency

– Phase-shift keying (PSK)

Phase of carrier signal shifted

[3 marks] One mark for each item



[3 marks] One mark for each item

b)

i) orthogonal frequency multiple access

[1 mark]

ii) Higher spectral efficiency in real-life time dispersive channels

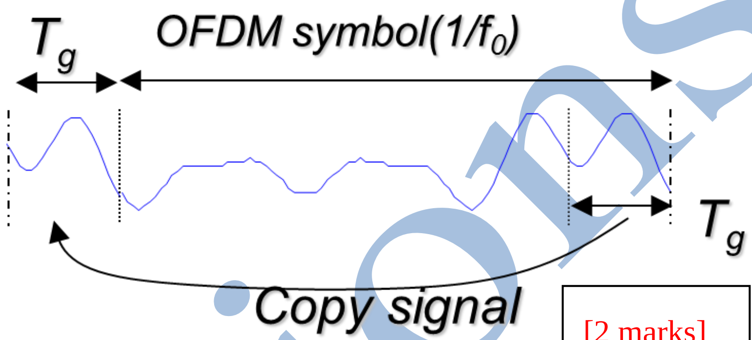
More robust – less multi-path interference

Easy to integrate MIMO technologies

Simpler receiver to cope with real-life time dispersive channels □ lower cost

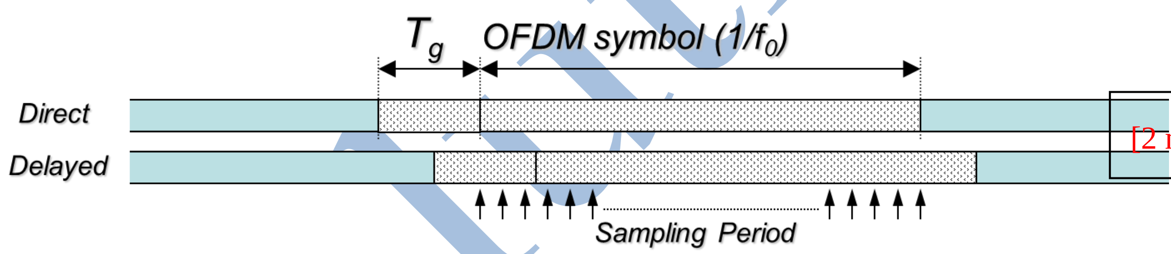
[2 marks](Answer each two can obtain the two marks)

iii) Cyclic Prefix (Guard Interval) for OFDM symbols



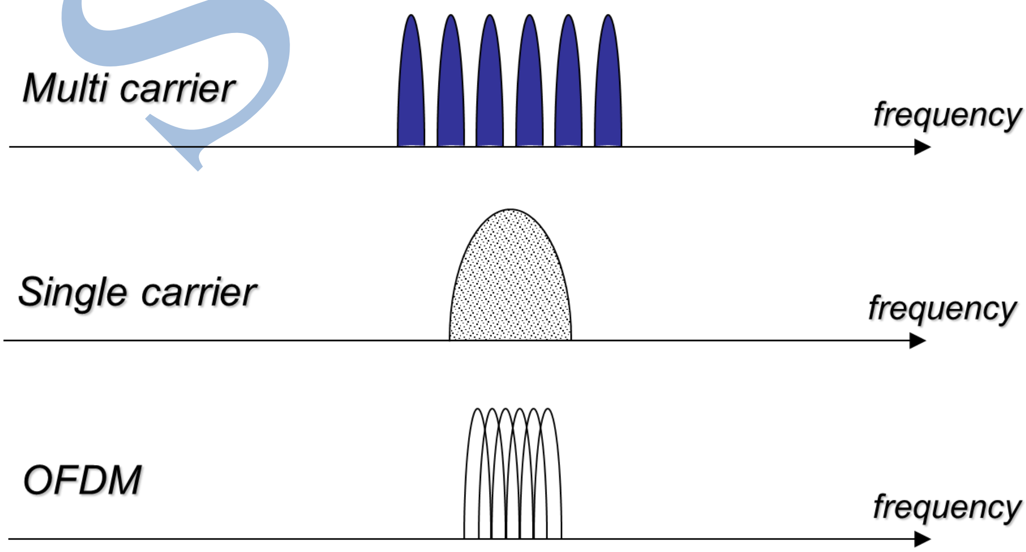
[2 marks]

By adding the Gurard Interval Period, Inter Symbol Interference (ISI) can be avoided. [2 marks]



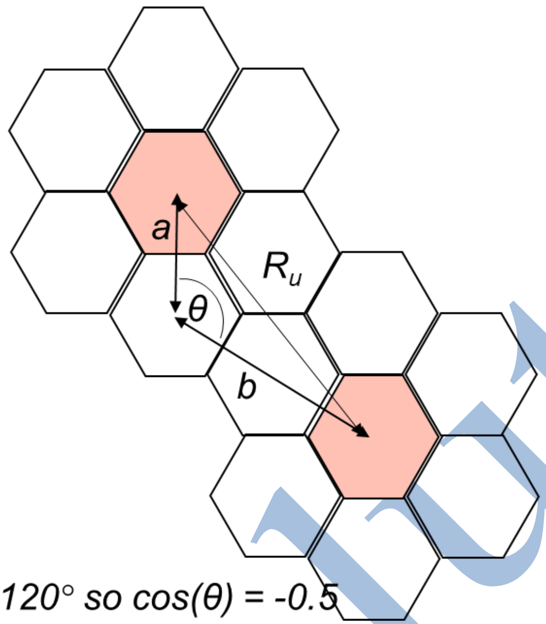
[2 marks]

iv) **One mark for each one. [3 marks]**



c)

i) **[2 marks] for figure, [3 marks for derivation]**



$\theta = 120°$ so $\cos(\theta) = -0.5$

In general

a is distance between i cells

$$a = ir\sqrt{3}$$

b is distance between j cells

$$b = jr\sqrt{3}$$

$$R_u^2 = i^2r^2 3 + j^2r^2 3 + 2\times 0.5 ijr^2 3$$

$$R_u = \left(\sqrt{i^2 + j^2 + ij}\right)\left(r\sqrt{3}\right)$$

ii) Bigger cluster will result in

less interference from next cell using the same frequency

lower capacity – bandwidth available in cell is F_A/N

(F_A is frequency spectrum allocated)

[2 marks]