# § 2.5 Electric Potential

- **Why do we introduce such a parameter?**
- Static E-field is a conservative field.
- This field has divergence but not curl. $\nabla \times \vec{E} = 0$
- Recall related *Identical Equation for a conservative field*
  - The curl of a scalar's gradient is always zero.

$$\nabla \times \nabla U \equiv 0$$

  - It's reversible. If the curl of a vector field is 0, the vector must be a scalar's gradient.

$$\nabla \times \vec{F} \equiv 0 \quad \Rightarrow \quad \vec{F} = \nabla U$$

$$\because \nabla \times \vec{E} = 0 \longrightarrow \vec{E} = -\nabla \psi$$

1. Static E-field is a conservative field

$$\because \nabla \times \vec{E} = 0 \qquad \therefore \exists U, \quad \vec{E} = \nabla U$$

2. Thus we define a scalar function: $\psi$

$$\vec{E} = \underline{-\nabla}\,\psi$$

Negative gradient of this scalar is E-intensity.

## 3. In Cartesian Coordinates

$$\vec{E} = -\nabla \psi \qquad \nabla \psi = \left\{ \frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z} \right\}$$

$$\vec{E} = -\left( \vec{a}_x \frac{\partial \psi}{\partial x} + \vec{a}_y \frac{\partial \psi}{\partial y} + \vec{a}_z \frac{\partial \psi}{\partial z} \right)$$

1. 电位场的梯度是电场强度
2. 电力线的方向是电位的最大变化率方向
3. 描述的是最大电位的减小率（负号）
4. 变化率的大小是电场强度

**How about gradient in other coordinate?**

# Gradient in different coordinates

**Cartesian Coordinates**

$$\nabla = \vec{a}_x \frac{\partial}{\partial x} + \vec{a}_y \frac{\partial}{\partial y} + \vec{a}_z \frac{\partial}{\partial z}$$

**Cylindrical Coordinates**

$$\nabla = \vec{a}_r \frac{\partial}{\partial r} + \vec{a}_\varphi \frac{1}{r} \frac{\partial}{\partial \varphi} + \vec{a}_z \frac{\partial}{\partial z}$$

**Spherical Coordinates**

$$\nabla = \vec{a}_R \frac{\partial}{\partial R} + \vec{a}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} + \vec{a}_\varphi \frac{1}{R \cdot \sin\theta} \frac{\partial}{\partial \varphi}$$

# 4. Potential Difference Between Spot A & B

$$\psi_B - \psi_A = \int_B^A \vec{E} \bullet d\vec{l} \qquad\qquad \vec{E} = -\nabla \psi$$

**Physical Meaning:**

$$\psi_B - \psi_A = \int_B^A \underline{\underline{(1\vec{E})}} \bullet d\vec{l}$$

*Work by electrostatic force when moving unit charge from B to A.*

This work is related only to the starting- & ending spots, regardless of the path.

Similar to that by gravity.

## 5. Reference Potential

$$\psi_B - \psi_A = \int_B^A \vec{E} \bullet d\vec{l} \qquad \text{Let} \quad \psi_A = 0$$

$$\boxed{\therefore \psi_B = \int_B^{\text{Ref. Spot}} \vec{E} \bullet d\vec{l}}$$

$$\psi_\infty = 0$$

Usually, we assume potential at infinity as 0.

## 6. Potential in a Field of a Point Charge

$$\psi_P = \int_P^\infty \vec{E} \bullet d\vec{l} = \int_P^\infty \frac{q}{4\pi\varepsilon_0 R^2} dR = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{R}$$

$$\boxed{\psi_P = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{R}}$$

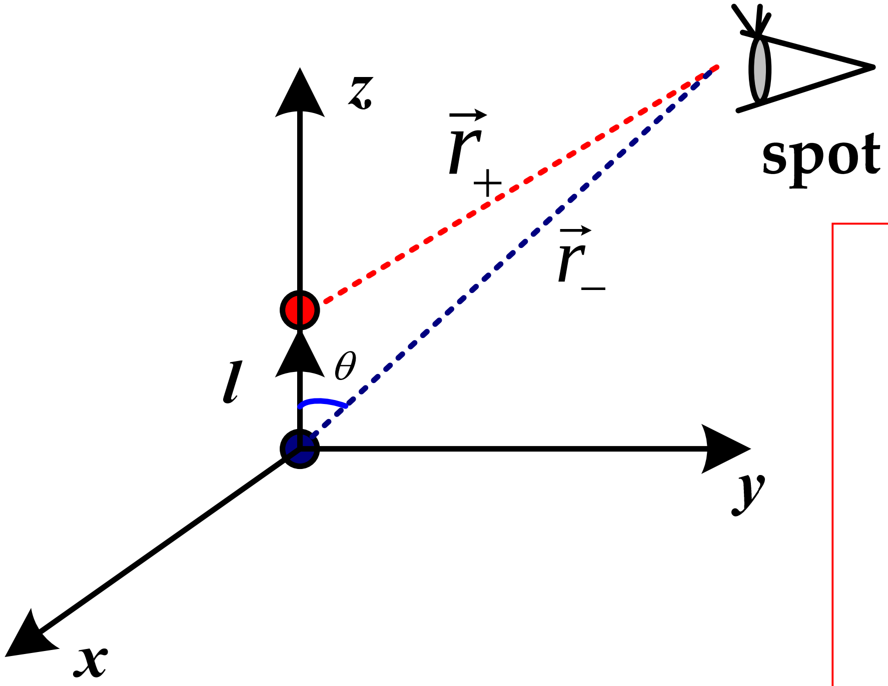
# Potential in Complicate Distribution of Charges

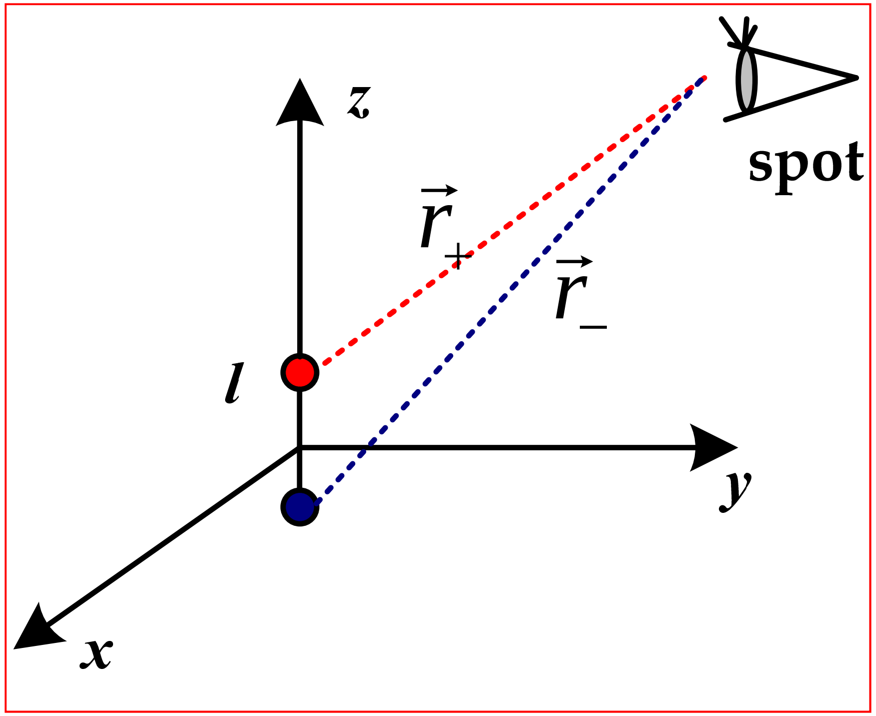
$$\boxed{\psi = \int \frac{dq}{4\pi\varepsilon R}}$$

- *dq* should be obtained according to the distribution of charges.
  - ***Bulk*** charges, ***surface*** charges, ***line*** charges——integral
  - Scattering charges——sum
- If in space, use $\varepsilon_0$ in above equation.
- If E-Intensity is known, just apply

$$\psi_B = \int_B^\infty \vec{E} \bullet d\vec{l}$$

# Example 1. Potential in Field of Electric Dipole

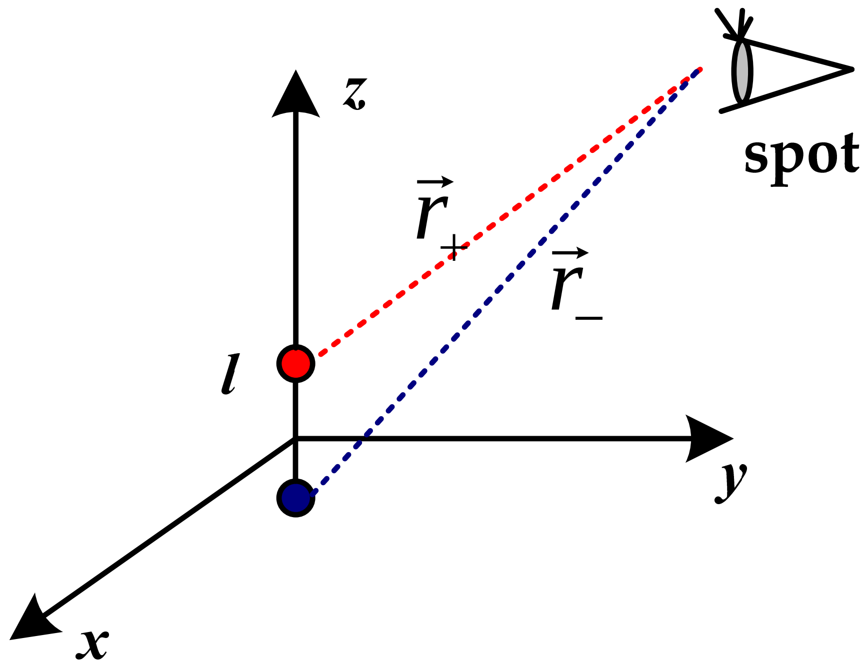


Scattering charges——sum

$$\psi = \frac{q}{4\pi\varepsilon_0}\cdot\left(\frac{1}{r_+}+\frac{-1}{r_-}\right)$$

In spherical coordinates



# Summary of Potential Calculation

$$\boxed{\therefore \psi_B = \int_B^{\text{Ref. Spot}} \vec{E} \bullet d\vec{l}}$$

$$\boxed{\psi_B = \int_B^{\infty} \vec{E} \bullet d\vec{l}}$$

$$\boxed{\psi_P = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{R}}$$

$$\boxed{\psi = \int \frac{dq}{4\pi\varepsilon R}}$$

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**——Now, let's go on.**