EBU6018
Advanced Transform Methods

Tutorial - DCT

Andy Watson

1-Dimensional DCT

$$DCT[k] = c(k)\sum_{n=0}^{N-1} s[n]\cos\frac{\pi(2n+1)k}{2N} \qquad k = 0,1,2...N-1$$

$$DCT[k] = \langle s, \psi_k \rangle$$

$$c(k) = \begin{cases} \sqrt{1/N} & k = 0 \\ \sqrt{2/N} & k \neq 0 \end{cases}$$

c(k) is the normalisation factor.

- Orthonormal

$$\langle \psi_m, \psi_n \rangle = \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

The Basis Functions Ψ_k are the cosine terms in the definition. They are calculated for each value of k, with n = 0….N-1

DCT Question 1

If N = 4 (input is a 4-point sequence)

For each value of k = 0….N-1, insert n = 0….N-1:

$$\psi_0 = (1,1,1,1)/2$$

$$\psi_1 = \sqrt{1/2}(\cos(\pi/8), \cos(3\pi/8), \cos(5\pi/8), \cos(7\pi/8))$$

$$\psi_2 = \sqrt{1/2}(\cos(\pi/4), \cos(3\pi/4), \cos(5\pi/4), \cos(7\pi/4))$$

$$\psi_3 = \sqrt{1/2}(\cos(3\pi/8), \cos(9\pi/8), \cos(15\pi/8), \cos(5\pi/8))$$

$$DCT[0] = \frac{1}{\sqrt{N}} \sum_{n=0}^{3} s[n] \qquad DCT[1] = \sqrt{\frac{2}{N}} \sum_{n=0}^{3} s[n] \cos\frac{\pi(2n+1)}{8}$$

$$DCT[2] = \sqrt{\frac{2}{N}} \sum_{n=0}^{3} s[n] \cos\frac{\pi(2n+1)}{4} \qquad DCT[3] = \sqrt{\frac{2}{N}} \sum_{n=0}^{3} s[n] \cos\frac{\pi(2n+1)3}{8}$$

Question

These 4 Basis Functions can be written in Matrix format.

- Calculate the elements of the 4x4 Basis Function Matrix.
- Then determine the output sequence if the input sequence is s[n] = [2, 3, 1, 4]

4x4 DCT Basis Matrix

$$\Psi = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

4x4 DCT Basis Matrix

$$\Psi = \begin{bmatrix} 0.50 & 0.50 & 0.50 & 0.50 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.50 & -0.50 & -0.50 & 0.50 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{bmatrix}$$

4x4 DCT Transform

$$\text{DCT}=\begin{bmatrix} 0.50 & 0.50 & 0.50 & 0.50 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.50 & -0.50 & -0.50 & 0.50 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \\ 4 \end{bmatrix}$$

4x4 DCT Transform

$$\text{DCT} = \begin{bmatrix} 5.00 \\ -2.10 \\ 1.00 \\ -1.84 \end{bmatrix}$$

DCT Question 2

The1D Discrete Cosine Transform (DCT) is defined:

$$DCT[k] = c(k)\sum_{n=0}^{N-1} s[n]\cos\frac{\pi(2n+1)k}{2N}$$

$$c(k) = \begin{cases} \sqrt{1/N} & k = 0 \\ \sqrt{2/N} & k \neq 0 \end{cases}$$

(i) Derive the basis functions for a 3-point DCT, for k=n=0, 1, 2.
(ii) Calculate, to 3 decimal places, the DCT of the input sequence: s(n)=[7, -2, 5].

Solution

$k = 0, \quad \Psi_0 = \sqrt{\frac{1}{3}}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

$k = 1, \quad \Psi_1 = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos\frac{\pi}{6} & \cos\frac{3\pi}{6} & \cos\frac{5\pi}{6} \end{bmatrix}$

$k = 2, \quad \Psi_2 = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos\frac{2\pi}{6} & \cos\frac{6\pi}{6} & \cos\frac{10\pi}{6} \end{bmatrix}$

Solution

$$\Psi = \begin{bmatrix} 0.577x1 & 0.577x1 & 0.577x1 \\ 0.816x0.866 & 0.816x0 & 0.816x(-0.866) \\ 0.816x0.5 & 0.816x(-1) & 0.816x0.5 \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0.577 & 0.577 & 0.577 \\ 0.707 & 0 & -0.707 \\ 0.408 & -0.816 & 0.408 \end{bmatrix}$$

Solution

$$\text{DCT} = \begin{bmatrix} 0.577 & 0.577 & 0.577 \\ 0.707 & 0 & -0.707 \\ 0.408 & -0.816 & 0.408 \end{bmatrix} \begin{bmatrix} 7 \\ -2 \\ 5 \end{bmatrix}$$

Solution

$$\text{DCT} = \begin{bmatrix} 5.77 \\ 1.414 \\ 6.528 \end{bmatrix}$$