

# SOLUTIONS

Module:	Telecom Systems		
Module Code	EBU5302	Paper	B
Time allowed	2hrs	Filename	Solutions_201920_EBU5302_B
Rubric	ANSWER ALL FOUR QUESTIONS		
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Solutions

**Question 1**

- a) Explain the methods of circuit switching and packet switching, identifying the main advantage and the main disadvantage of each method.

**[4 marks]**

- b) A music signal  $x(t)$  has a bandwidth of 10 kHz. Signal  $x(t)$  is sampled at a rate 30% higher than the Nyquist rate to provide a guard band.  $x(t)$  is quantised by a uniform quantiser  $Q$ . Symbol A to H represent the amplitudes produced by the quantiser. The probability pm of each symbol is shown in the following table:

Table 1

Symbol	A	B	C	D	E	F	G	H
P(m)	0.1	0.3	0.08	0.25	0.03	0.05	0.18	0.01

- i) What is the minimum sampling rate for  $x(t)$ ?

**[2 marks]**

- ii) Design a Huffman code for the information produced by  $Q$ . Please explain the principles of source coding with the designed code.

**[7 marks]**

- iii) What is average number of bits to be transmitted of the designed Huffman code?

**[2 marks]**

- iv) What is the information content of  $x(t)$ ? What is the source entropy of  $x(t)$ ?

**[4 marks]**

- v) What is the maximum entropy of  $x(t)$ ? What is the source efficiency of  $x(t)$ ?

**[4 marks]**

- vi) What is the code efficiency for  $x(t)$ ?

**[2 marks]****Answers:****a)**

In circuit switching, a dedicated path connecting source and destination is established. **[1 mark]**

In packet switching, data is split into packets that can follow different paths to reach their destination. **[1 mark]**

In circuit switching, resources are guaranteed but they must be wasted. **[1 mark]**

In packet switching, resources are not guaranteed but they can be shared. [1 mark]

b)

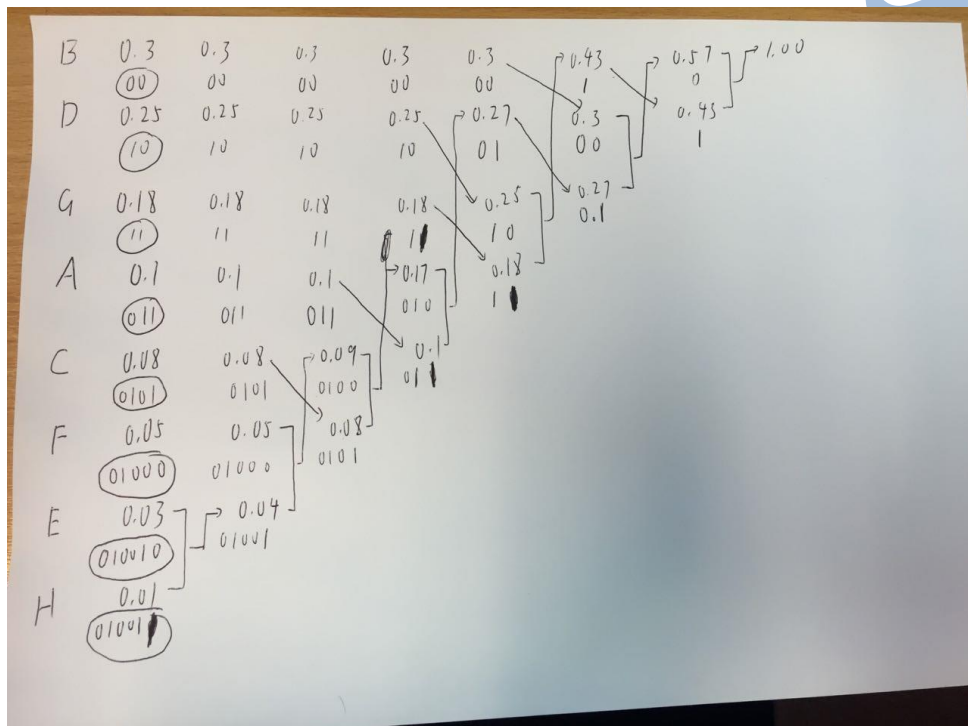
- i) The Nyquist sampling rate for  $x(t)$  is  $R_N = 2 * 10\text{kHz} = 20\text{ kHz}$  (samples per second).

[1 mark]

The actual sampling rate is  $R_s = 20\text{ kHz} * 1.3 = 26\text{ kHz}$ .

[1 mark]

ii)



[3 marks]

Symbol	A	B	C	D	E	F	G	H
Codeword	011	00	0101	10	010010	01000	11	010011

[2 marks]

- ◆ The event with high probability uses short code

[1 mark]

- ◆ The event with low probability uses long code

[1 mark]

iii) Average code length  $= \sum p_i n_i$  [1 mark]  
 $= 0.1*3 + 0.3*2 + 0.08*4 + 0.25*2 + 0.03*6 + 0.05*5 + 0.18*2 + 0.01*6$   
 $= 2.57$  [1 mark]

iv) The information content is:

$$I = \sum \log_2 (1/p_i)$$
 [1 mark]
$$= 1/\lg 2 * (\lg 1 + \lg 1/0.3 + \lg 1/0.08 + \lg 1/0.25 + \lg 1/0.05 + \lg 1/0.18 + \lg 1/0.01)$$

$$= 3.3 * (0.52 + 1.10 + 0.60 + 1.30 + 0.74 + 2)$$

$$= 29.20$$
 [1 mark]

The source entropy is

$$H = \sum_i p_i \log_2 (1/p_i)$$
 [1 mark]
$$= 1/\lg 2 * (0.1 * \lg 1 + 0.3 * \lg 1/0.3 + 0.08 * \lg 1/0.08 + 0.25 * \lg 1/0.25 + 0.05 * \lg 1/0.05 + 0.18 * \lg 1/0.18 + 0.01 * \lg 1/0.01)$$

$$= 3.3 * (0.3 * 0.52 + 0.08 * 1.10 + 0.25 * 0.60 + 0.05 * 1.30 + 0.18 * 0.74 + 0.01 * 2)$$

$$= 2.52$$
 [1 mark]

v) The maximum entropy is  $H_{\max} = \log_2(N) = \log_2(8)$  [1 mark]  
 $= 3$  [1 mark]  
 The source efficiency is  $\eta_{\text{source}} = \frac{H}{H_{\max}}$  [1 mark]  
 $= 84.15\%$  [1 mark]

vi) The code efficiency is

$$\eta_{\text{code}} = \frac{H}{L}$$
 [1 mark]
$$= 98.2\%$$
 [1 mark]

**Question 2**

- a) Consider an AWGN channel with bandwidth 50MHz, received signal power 10mW, and noise PSD (power spectral density)  $N_0/2$ , where  $N_0 = 2 \times 10^{-9}$  W/Hz. How much does capacity increase by doubling the received power? How much does capacity increase by doubling the channel bandwidth? [6 marks]

- b) Consider a (7,4) code with generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

- i) Find all the codewords of the code. [8 marks]  
 ii) What is the minimum distance of the code? [1 mark]  
 iii) Find the parity check matrix of the code. [4 marks]  
 iv) Find the syndrome for the received vector  $R = [1101011]$ . [6 marks]

**Answer**

- a) The received  $SNR = P / N_0 B = 0.01 / 50 \times 10^6 \times 2 \times 10^{-9} = 0.1 = -10$  dB [1 mark] and  $C = B \log_2 (1 + SNR) = 6.87$  Mbps [1 mark]  
 If double received signal power,  $SNR_1 = 0.02 / 50 \times 10^6 \times 2 \times 10^{-9} = 0.2 = -7$  dB [1 mark] and  $C = B \log_2 (1 + SNR) = 13.14$  Mbps [1 mark]  
 If double channel bandwidth,  $SNR_1 = 0.01 / 100 \times 10^6 \times 2 \times 10^{-9} = 0.05 = -13$  dB [1 mark] and  $C = B \log_2 (1 + SNR) = 7.03$  Mbps [1 mark]

b)

- i. [8 marks, each row counts 0.5]

Message	Codeword	Weight
0000	0000000	0
0001	0001001	2
0010	0010010	2
0011	0011011	4
0100	0100101	3
0101	0101100	3
0110	0110111	5
0111	0111110	5
1000	1000110	3
1001	1001111	5
1010	1010100	3
1011	1011101	5
1100	1100011	4
1101	1101010	4
1110	1110001	4
1111	1111000	4

ii.  $d_{min}$ =minimum weight=2 [1 mark]

iii.  $G = [I_k | P]$   $H = [P^T | I_{n-k}]$  [1 mark]

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad [3 \text{ marks}]$$

iv. In order to decode a sequence, we first need to obtain the syndrome  $s = rH^T = eH^T$ . [1 mark].  
In the sequence 1101011 the syndrome is  $s=001$ . [2 marks]

The corresponding error sequence is  $e = 0000001$ . [1 mark]

Hence, the transmitted code word is  $c = r + e = 1101011 + 0000001 = 1101010$ . [1 mark]

The information sequence can be identified in the first 4 bits:  $x = 1101$ . [1 mark]

### Question 3

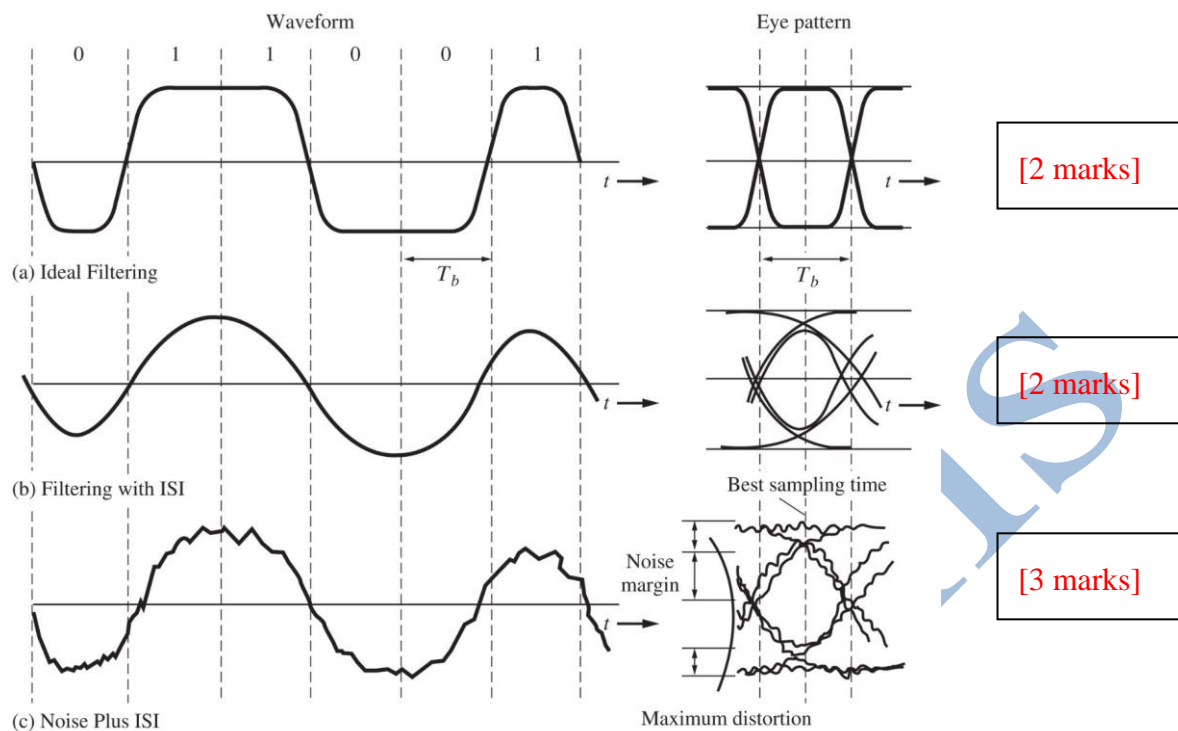
- Using diagrams, illustrate ideal i) filtering, ii) filtering with ISI and iii) noise plus ISI polar NRZ waveforms and corresponding eye patterns. [7 marks]
- What important information eye patterns can provide about the characteristics of a signal? Briefly explain them. [4 marks]
- Calculate the autocorrelation function  $R(k)$  of the polar NRZ signalling assuming bit to bit data independence and equally likely occurrence of two voltage levels [7 marks]
- A random binary data sequence 110100101... is transmitted by using a Manchester (split-phase) line code with the pulse  $p(t)$  shown below. Sketch the wave form  $y(t)$ .

$$p(t) = \text{rect}\left(\frac{t + T_b/4}{T_b/2}\right) - \text{rect}\left(\frac{t - T_b/4}{T_b/2}\right)$$

- List two methods to improve the inter-symbol interference (ISI). [5 marks]  
[2 marks]

Answer:

a)



- b) The **timing error** allowed on the sampler at the receiver is given by the width inside the eye, called the eye opening. Of course, the preferred time for sampling is at the point where the vertical opening of the eye is largest. [2 marks]

The **sensitivity to timing error** is given by the slope of the open eye (evaluated at, or near, the zero-crossing point). [1 marks]

The **noise margin** of the system is given by the height of the eye opening. [1 mark]

c)

For polar NRZ signalling, the possible levels for a's are +A and -A V [1 mark].

For equally likely occurrences of +A and -A, and assuming that the data are independent from bit to bit, we get:

$$R(k) = \sum_{i=1}^l (a_n a_{n+k})_i P_i \quad [1 \text{ mark}]$$

$$R(0) = \sum_{i=1}^2 (a_n a_n)_i P_i = A^2 \cdot \frac{1}{2} + (-A)^2 \cdot \frac{1}{2} = A^2 \quad [2 \text{ marks}]$$

For  $k \neq 0$ ,

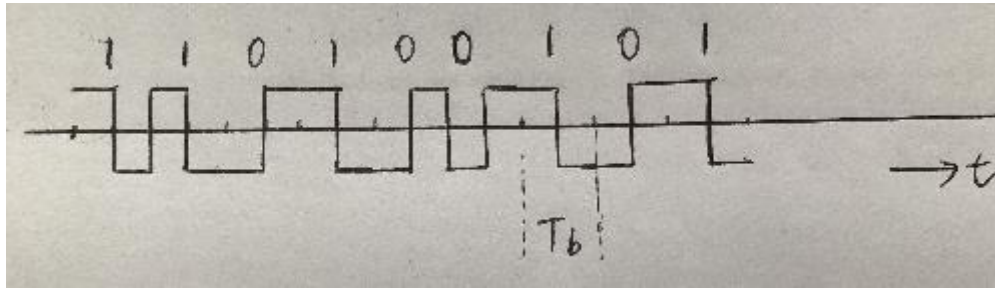
$$R(k) = \sum_{i=1}^4 (a_n a_{n+k})_i P_i = A^2 \cdot \frac{1}{4} + (-A)(+A) \cdot \frac{1}{4} + (+A)(-A) \cdot \frac{1}{4} + (-A)^2 \cdot \frac{1}{4} = 0$$

[2 marks]

Thus,

$$R_{\text{polar}}(k) = \begin{cases} A^2 & k = 0 \\ 0 & k \neq 0 \end{cases} \quad [1 \text{ mark}]$$

- d) The wave form y(t) is as below:



[5 marks, 0.5 mark each bit, 1 mark for  $T_b$ ]

e)

- ISI can be reduced by increasing the channel bandwidth. [1 mark]
- Reshape the pulse from the transmitter to minimise the ISI. [1 mark]

#### Question 4:

- a) What are FDMA, TDMA, CDMA and OFDMA? You may want to use diagrams to illustrate your answers. What are the key features and applications for each of them?

[12 marks]

- b) Draw a block diagram to show how an OFDM demodulator works.

[9 marks]

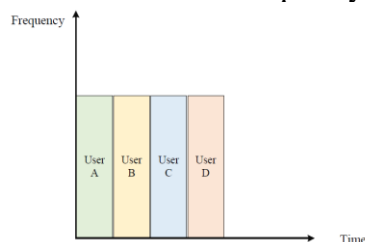
- c) Determine the isotropic free space loss at 4GHz for the shortest path for a point-to-point system. The shortest path from the transmitter to the receiver is 35853 km.

[4 marks]

**Answer:**

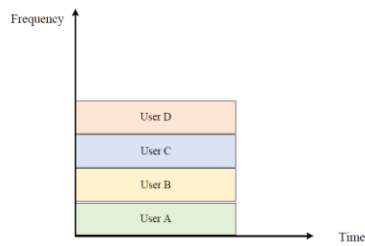
a)

- ◆ FDMA: different frequency bands are assigned to different users.

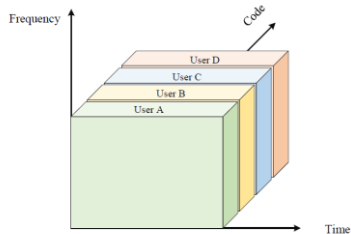


- ◆ TDMA: different time slots are assigned to different users.





- ◆
- ◆ CDMA: different codes are assigned to different users.



[0.5 mark for each description and 0.5 mark for diagram]

- ◆ **OFDMA =Orthogonal Frequency Division Multiple Access**
- ◆ OFDMA is a multi-user version of the popular OFDM digital modulation scheme. Multiple access is achieved in OFDMA by assigning subsets of subcarriers to individual users.

[1 mark]

Application:

FDMA: All 1G systems use FDMA.

TDMA: Most 2G systems use TDMA

CDMA: Some 2G and most 3G systems

OFDMA: 4G systems and 5G systems

[1 mark for each]

TDMA: Single carrier frequency with multiple users.

Non-continuous transmission.

Each user occupies a cyclically repeating time slot.

[stating any of them can obtain 1 mark]

FDMA: Assign each user to a particular channel.

Transmit signals simultaneously and continuously

[stating any of them can obtain 1 mark]

CDMA: All users use same time and frequency.

Narrowband signals multiplied by wideband spreading codes.

[stating any of them can obtain 1 mark]

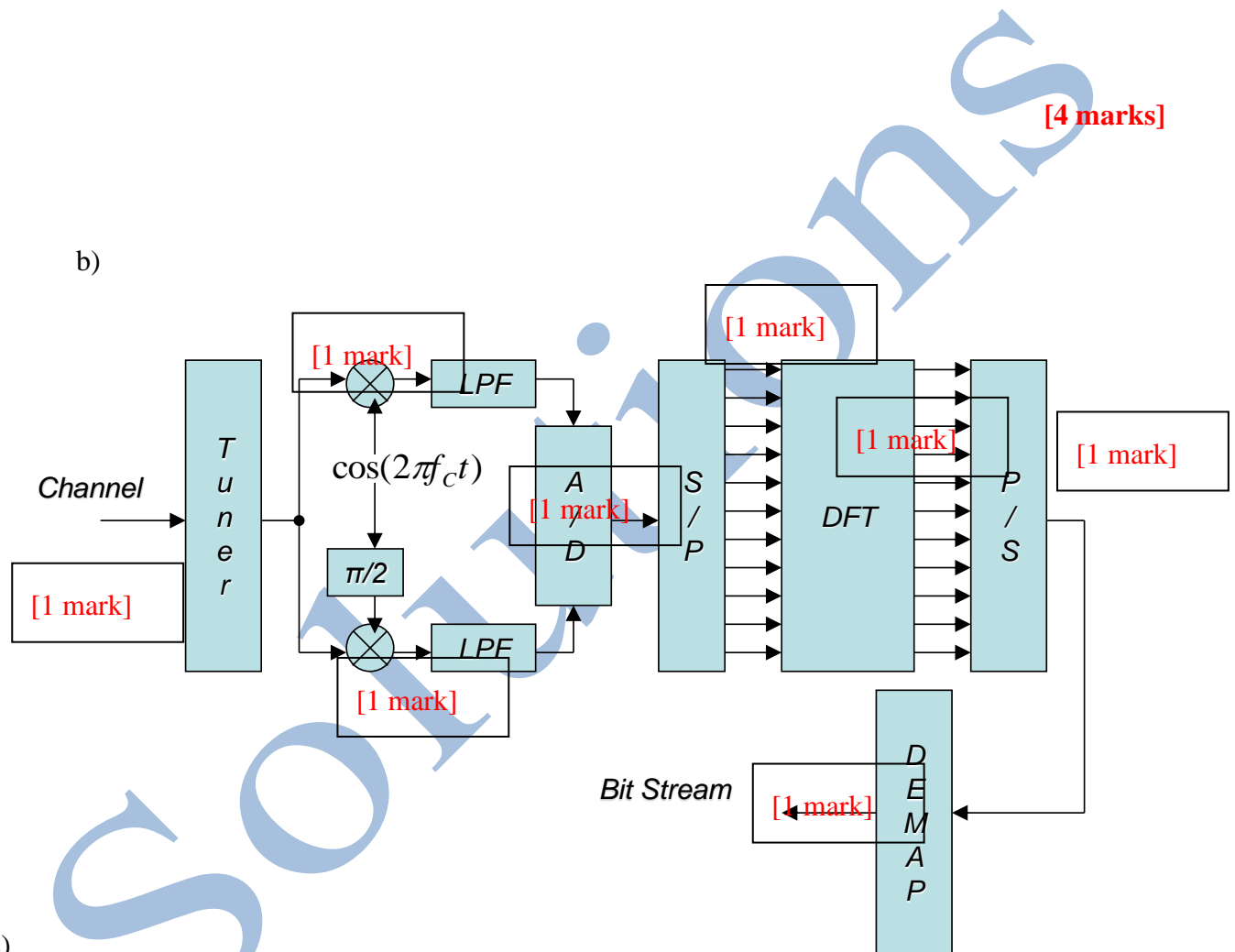
OFDMA: Higher spectral efficiency in real-life time dispersive channels

More robust – less multi-path interference

Easy to integrate MIMO technologies

Simpler receiver to cope with real-life time dispersive channels ☐ lower cost

[stating any of them can obtain 1 mark]



c)

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{4 \times 10^9} = 0.075m \quad [1 \text{ mark}]$$

$$L_{dB} = -20 \log \frac{P_t}{P_r} = 20 \log \left( \frac{4\pi d}{\lambda} \right) = -20 \log(\lambda) + 20 \log(d) + 21.98dB \quad [2 \text{ marks}]$$

[1 mark]

$$L_{dB} = -20 \log(0.075) + 20 \log(35.853 \times 10^6) + 21.98dB = 195.6dB$$

Solutions