SOLUTIONS

Module:	Telecoms Systems		
Module Code	EBU5302	Paper	A
Time allowed	2hrs	Filename	Solutions_1819_EBU5302_A
Rubric	ANSWER ALL FOUR QUESTIONS		
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Question 1

Let x(t) be a band-limited signal with bandwidth W = 30kHz. Signal *x(t)* is sampled at a rate 50% higher than the Nyquist rate to provide a guard band. *x(t)* is quantised by a uniform quantiser *Beta*. Symbol A to H represent the amplitudes produced by the quantiser. The probability pm of each symbol is shown in the following table. Here *P(m)* is defined as the probability of each symbol *m*.

Table 1

Symbol	A	B	C	D	E	F	G	H
P(m)	0.1	0.3	0.08	0.25	0.03	0.05	0.18	0.01

a) What is the minimum sampling rate that avoids aliasing for x(t)? **[2 marks]**

b) Design a Huffman code for the information produced by *Beta*. Please explain the principles of source coding with the designed code.

[8 marks]

c) What is average number of bits to be transmitted of the designed Huffman code? **[2 marks]**

d) What is the information content of the symbols produced by *Beta*? What is the source entropy of the sequence of symbols?

[4 marks]

e) What is the maximum entropy of the discrete source that produces 8 symbols? What is the source efficiency of the sequence produced by *Beta*? What is the code efficiency for the designed Huffman coding scheme?

[3 marks]

f) Explain the Sampling Theorem with appropriate diagrams (Please explain both from time domain and frequency domain). Based on the frequency domain diagram, please use diagrams to explain why it is easier to design a simple filter with the aid of applying oversampling.

[6 marks]

Answers:

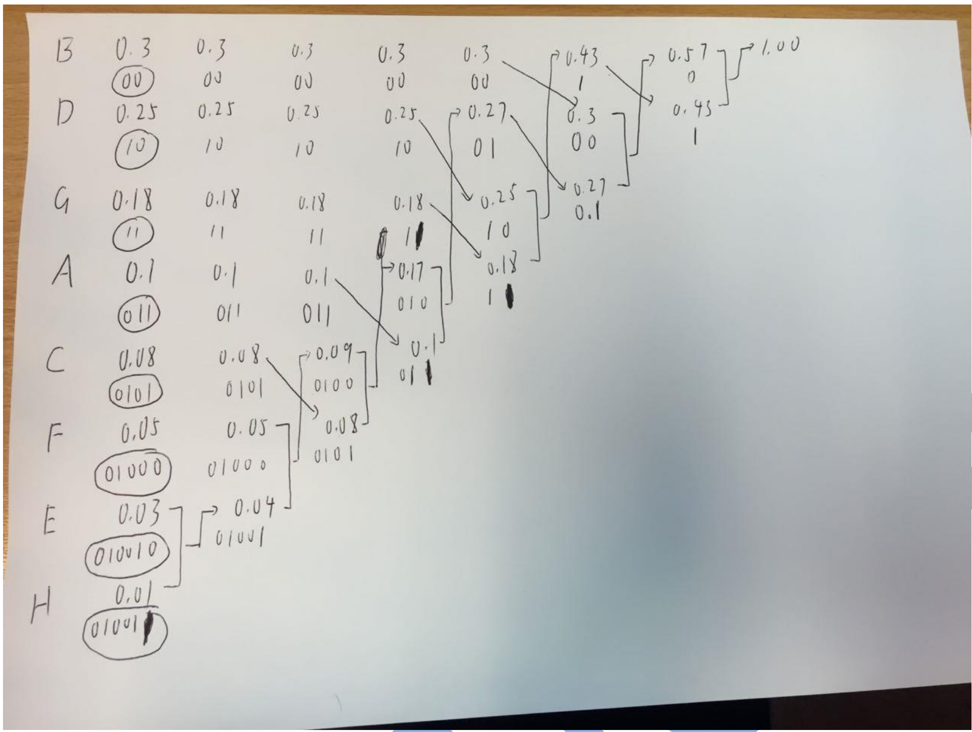
a) The Nyquist sampling rate for $x(t)$ is R_N = 2* 30kHz = 60 kHz (samples per second).

[1 mark]

The actual sampling rate is Rs = 60 kHz*1.5=90 kHz.

[1 mark]

b)



Show the Huffman coding procedure 2 marks. Each correctly combining and moving arrow 0.5 mark, up to 2 marks.

[4 marks]

Symbol	A	B	C	D	E	F	G	H
Codeword	011	00	0101	10	010010	01000	11	010011

[2 marks]

- The event with high probability uses short code

[1 mark]

- The event with low probability uses long code

[1 mark]

c) Average code length $=\sum p_i n_i$ [1 mark]
=0.1*3+0.3*2+0.08*4+0.25*2+0.03*6+0.05*5+0.18*2+0.01*6

=2.57 [1 mark]

d) The information content is:

$I = \sum \log_2 (1/p_i)$ [1 mark]

=1/lg2*(lg1+lg1/0.3+lg1/0.08+lg1/0.25+lg1/0.05+lg1/0.18+lg1/0.01)

=3.3*(0.52+1.10+0.60+1.30+0.74+2)

=29.20 [1 mark]

The source entropy is

$H = \sum_i p_i \, log_2(1/p_i)$ [1 mark]

=1/lg2*(0.1*lg1+0.3*lg1/0.3+0.08*lg1/0.08+0.25*lg1/0.25+0.05*lg1/0.05+0.18*lg1/0.18+0.01*lg1/0.01)

=3.3*(0.3*0.52+0.08*1.10+0.25*0.60+0.05*1.30+0.18*0.74+0.01*2)

=2.52

[1 mark]

e) The maximum entropy is Hmax=log_2(N)= log_2(8) =3 [1 mark]

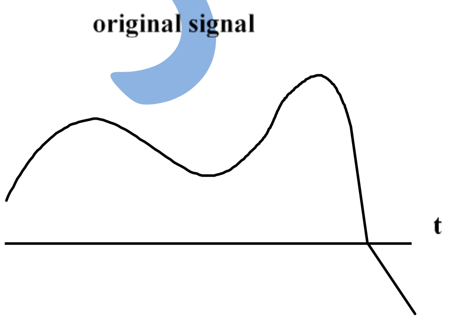
The source efficiency is $\eta_{source} = \frac{H}{Hmax}$
=84.15% [1 mark]

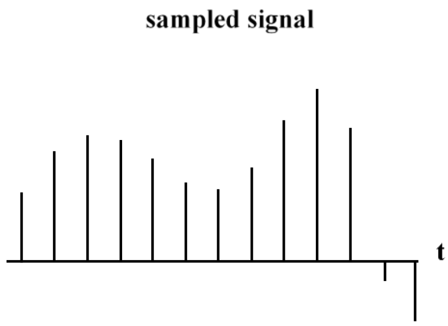
The code efficiency is

$\boldsymbol{\eta_{code}} = \frac{\boldsymbol{H}}{\boldsymbol{L}}$

=98.2% [1 mark]

f) Time domain





[1 mark]

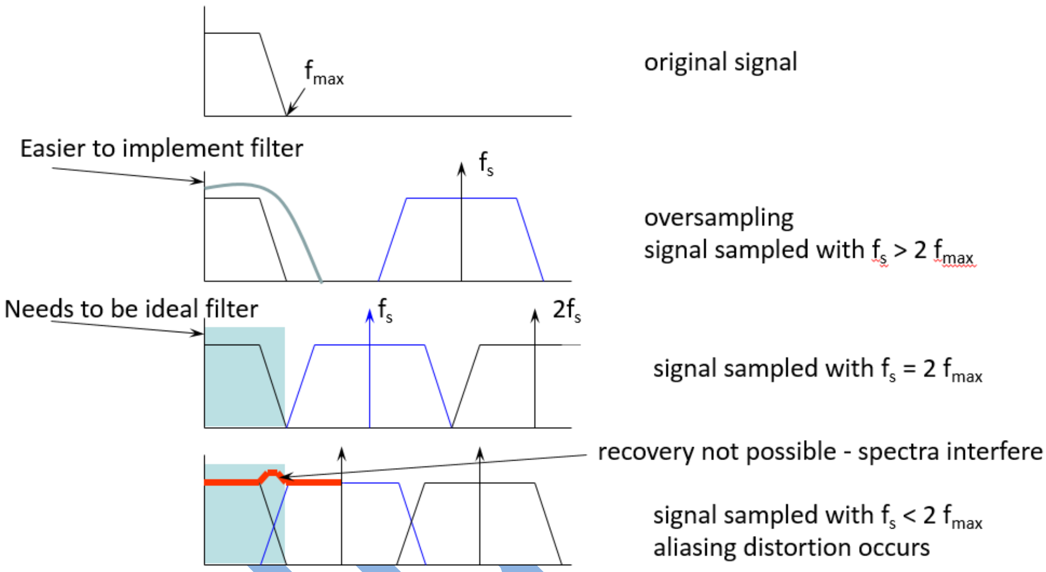
Given f_{max} is the highest frequency present in the original signal and f_s is the sampling frequency.
Sampling theorem states: To prevent aliasing and hence to allow the original signal to be recovered the sampling frequency (f_s) must be given by: $f_s \geq 2 \, f_{max}$

[1 mark]

Explanation: refer to the frequency domain. State that the frequency replicas repeat around multiples of the sampling frequency.

[1 mark]

Frequency domain and easy to implement filter



[3 marks]

Question 2

a) Two binary random variables X and Y are distributed according to the joint PMF (Probability Mass Function) given by

$$P\ (X=0,\ Y=1) = \frac{1}{4};$$
$$P\ (X=1,\ Y=1) = \frac{1}{2};$$
$$P\ (X=0,\ Y=0) = \frac{1}{4}.$$

Determine $H(X, Y), H(X), H(Y), H(X|Y), H(Y|X)$ *and* $I(X;Y)$.

[11 marks]

Answer

$P\ (X=1,\ Y=0) = 0$ [1 mark]

$H(X,Y)$=$-\sum_{x,y} p(x,y)\log p(x,y)$= $\frac{1}{4}$*log4+$\frac{1}{4}$*log4+$\frac{1}{2}$*log2 = $\frac{3}{2}$ [1 mark]

$P(X=0)=P(X=0,Y=0)+P(X=0,Y=1)= \frac{1}{4}+\frac{1}{4}=\frac{1}{2}$ [1 mark]

$P(X=1)=1-P(X=0)= \frac{1}{2}$ [1 mark]

$H(X)$= $-\sum_x p(x)\log p(x)$ = $\frac{1}{2}$*log2+$\frac{1}{2}$*log2=1 [1 mark]

$P(Y=0)=P(X=0,Y=0)+P(X=1,Y=0)= \frac{1}{4}+0=\frac{1}{4}$ [1 mark]

$P(Y=1)=1-P(Y=0)= \frac{3}{4}$ [1 mark]

$H(Y)$= $-\sum_y p(y)\log p(y)$ = $\frac{1}{2}$*log2+$\frac{3}{4}$*log$\frac{4}{3}$=0.81 [1 mark]

$H(Y|X)=H(X,Y)-H(X) = 1.5-1=0.5$ [1 mark]

$H(X|Y)=H(X,Y)-H(Y) = 1.5-0.81=0.687$ [1 mark]

$I(X;Y)=H(X)-H(X/Y) = 1-0.687=0.313$ [1 mark]

[11 marks]

b) Given (5,2) code defined as below, What can you tell when a sequence r=10101 received?

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

[14 marks]

Answer

The minimum distance of these code words are d_{min}=2 [1 mark]

For error detection, it should meet the condition that $d_{min} \geq N_d + 1$, so 1 error can be detected [1 mark]

For error correction, it should meet the condition that $d_{min} \geq 2N_c + 1$, so NO error can be corrected [1 mark]

The parity check matrix of this code set is [2 mark]

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \text{ or } H^t = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Knowing the valid codewords, we can say that r is not a codeword [2 marks]. By multiplying by the transpose of the parity check matrix we obtain the syndrome sequence s = 001 [2 marks].

Since the syndrome is not zero [1 mark], with absolute certainty the received sequence contains errors. [2 marks] However, based on the syndrome sequence alone, it is impossible to say how many errors it contains or which bits are erroneous [2 marks].

[14 marks]

Question 3

1) Consider a full-width rectangular pulse shape with unit level value.

$$p(t) = Rect\,(\frac{t}{T_b})$$

a) Find PSDs for the polar, unipolar, and bipolar signalling

[17 marks]

b) Sketch roughly the PSDs and find their bandwidths. For each case, compare the bandwidth to the case where $p(t)$ is a half-width rectangular pulse.

[8 marks]

Answer

a) The general expression of PSD of a digital signal is

$$P(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k)e^{j2\pi kfT_s}$$

[1 mark]

And

$$\mathrm{R}(k) = \sum_{i=1}^{l} (a_n a_{n+k})_i\, P_i$$

[1 mark]

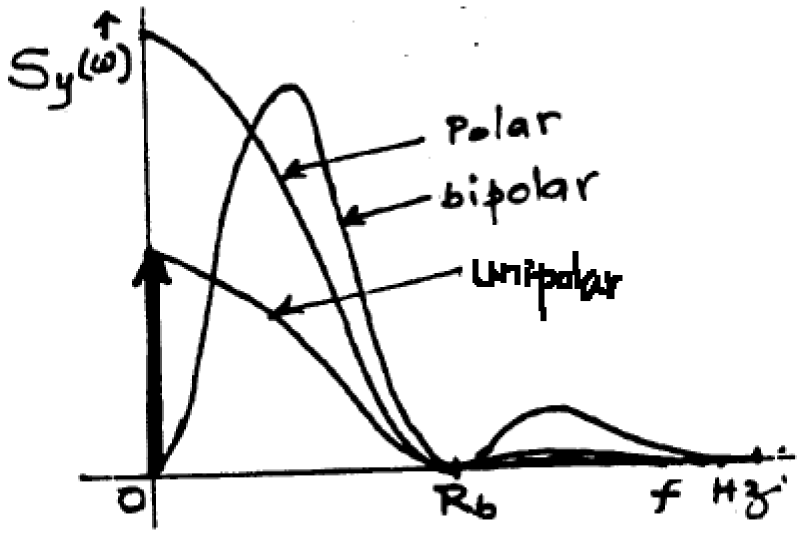
So we have the following values when $T_s = T_b$:

	Polar	Unipolar	Bipolar
R_0	1	1/2	1/2
R_1	0	1/4	-1/4
R_n $(n > 1)$	0	1/4	0
$P\ (f)$	$T_b sinc^2(\pi f T_b)$	$\frac{\|F(f)\|^2}{4T_b} + \frac{\|F(f)\|^2}{4T_b^2} \sum_{-\infty}^{+\infty} \delta(f - \frac{n}{T_b})$ =	$\frac{\|F(f)\|^2}{T_b} (\frac{1}{2} - \frac{1}{2} \cos 2\pi n f T_b)$ =

		$\frac{T_b sinc^2(\pi f T_b)}{4} + \frac{sinc^2(\pi f T_b)}{4}\sum_{-\infty}^{+\infty}\delta(f-\frac{n}{T_b})$ As for $sinc^2(\pi f T_b) = 0$ for $f = n/T_b$ when $n \neq 0$ and $sinc^2(\pi f T_b) = 1$ for n=0 So $\frac{T_b sinc^2(\pi f T_b)}{4} + \frac{1}{4}\sum_{-\infty}^{+\infty}\delta(f)$	$\frac{sinc^2(\pi f T_b)}{2T_b} sin^2(\pi f T_b)$

[15 marks, 1 mark each for row 1-3, 2 marks each for row 4]

b)



[4 marks, 1 mark each line in the figure, including DC]

From this spectra, we can see the bandwidth for the three cases are the same, which is R_b Hz.

[1 mark]

When half-width pulse is adopted, the bandwidth for the three cases are as below:

Bandwidth (Hz)	Polar	Unipolar	Bipolar
full-width rectangular pulse	R_b	R_b	R_b
half-width rectangular pulse	$2R_b$	$2R_b$	R_b

[3 marks, 1 for each in the 2nd row]

Question 4

a) A multilevel digital communication system is to operate at a data rate of 15 kbits/s. If 4-bit words are encoded into each level for transmission over the channel, what is minimum required bandwidth for the channel?

[4 marks]

Answer:

(a) $D = \frac{R}{\ell} = \frac{1{,}500k}{4} = 375 \text{ k baud}$ [2 mark]

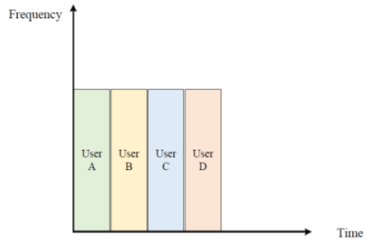
$B \geq \frac{1}{2}\frac{N}{T} = \frac{1}{2}D = \frac{1}{2}(375k) = \underline{187.5 \text{ kHz}}$ minimum BW [2 marks]

b) What are FDMA, TDMA and CDMA? You may want to use diagrams to illustrate your answers. What are the corresponding application scenarios for FDMA, TDMA, and CDMA (which generations of communications systems)? What are the key features for each of them?

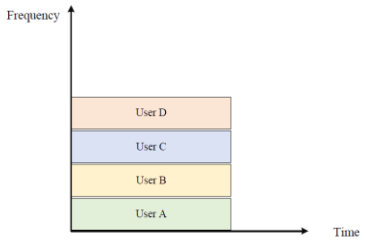
[12 marks]

Answer

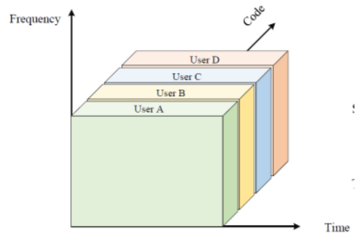
- FDMA: different frequency bands are assigned to different users.



- TDMA: different time slots are assigned to different users.



- CDMA: different codes are assigned to different users.



[1 mark for each description and 1 mark for diagram]

[6 marks]

Application:

FDMA: All 1G systems use FDMA.

TDMA: Most 2G systems use TDMA

CDMA: Some 2G and most 3G systems

[1 mark for each]

[3 marks]

TDMA: Single carrier frequency with multiple users.
Non-continuous transmission.
Each user occupies a cyclically repeating time slot.

[stating any of them can obtain 1 mark]

FDMA: Assign each user to a particular channel.
Transmit signals simultaneously and continuously

[stating any of them can obtain 1 mark]

CDMA: All users use same time and frequency.
Narrowband signals multiplied by wideband spreading codes.
[stating any of them can obtain 1 mark]

[3 marks]

c) Frequency reuse improves the SNR from co-channel interference but reduces the capacity in each cell; diversity also improves SNR by up to about 6dB. Using a **mathematical approach**, explain how adding diversity to 3-cell cluster can give the same overall SNR from co-channel interference as a 7-cell cluster, but gives more capacity in each cell.

[9 marks]

Answer:

Diversity gain is about 4-6dB. (**1 mark)**

Frequency re-use distance $R_u = r\sqrt{3N}$ (**1 mark)** so if $P \propto 1/r^4$ then received power from neighbouring cell of same frequency is $P = P_0/(r\sqrt{3N})^4$ **(2 marks)** but in each case there are 6 nearest co-channel neighbours **(2 marks, 1 for each N)** $P_7/P_3 = \frac{P_0/(r\sqrt{3\times 3})^4}{P_0/(r\sqrt{3\times 7})^4}$ **(1 mark)** so

$\frac{P_7}{P_3} = \left(\frac{3}{7}\right)^2$ or in dB = -7.3 dB (accept approx -6dB as correct) **(1 mark)**

If 6dB gain is achieved by adding the diversity the -7dB reduction is more or less cancelled out **(1 mark)**