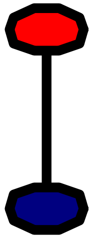
§ 5.3 Magnetic Dipole

Only a brief introduction and comparison

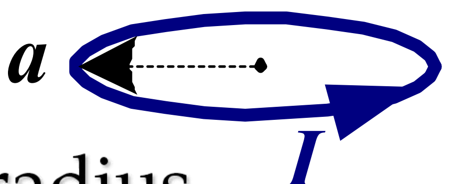
Electric Dipole

A pair of opposite charges very close to each other.

- Distance: l
- Point charges: $q_1=q$、 $q_2=-q$



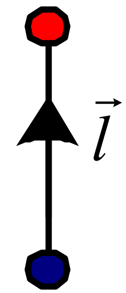
Magnetic Dipole



A circular current with a very small radius

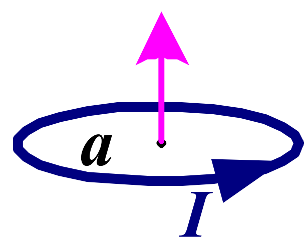
Electric Dipole Moment

$$\vec{p} = q\vec{l}$$

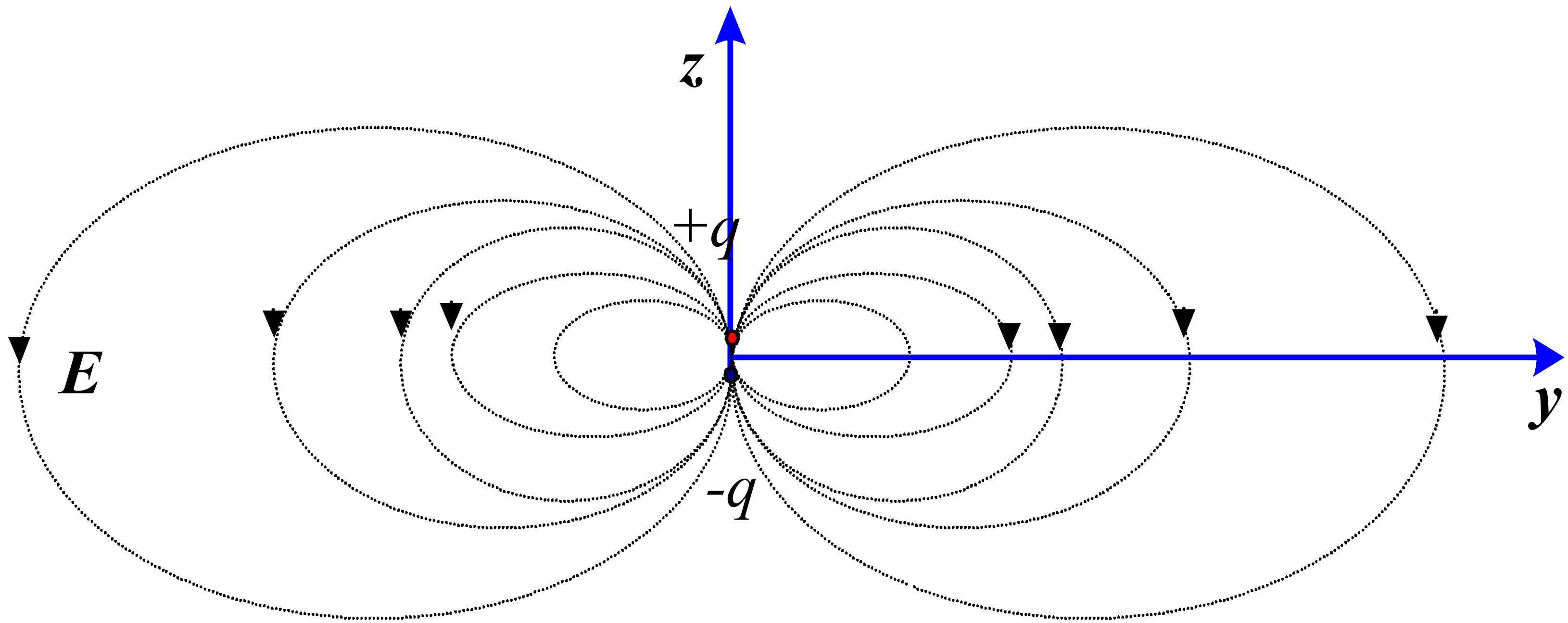


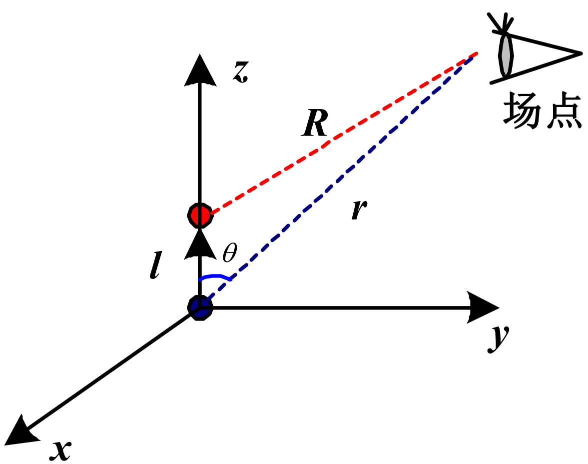
Magnetic Dipole Moment

$$\vec{p}_m = I\vec{S}$$



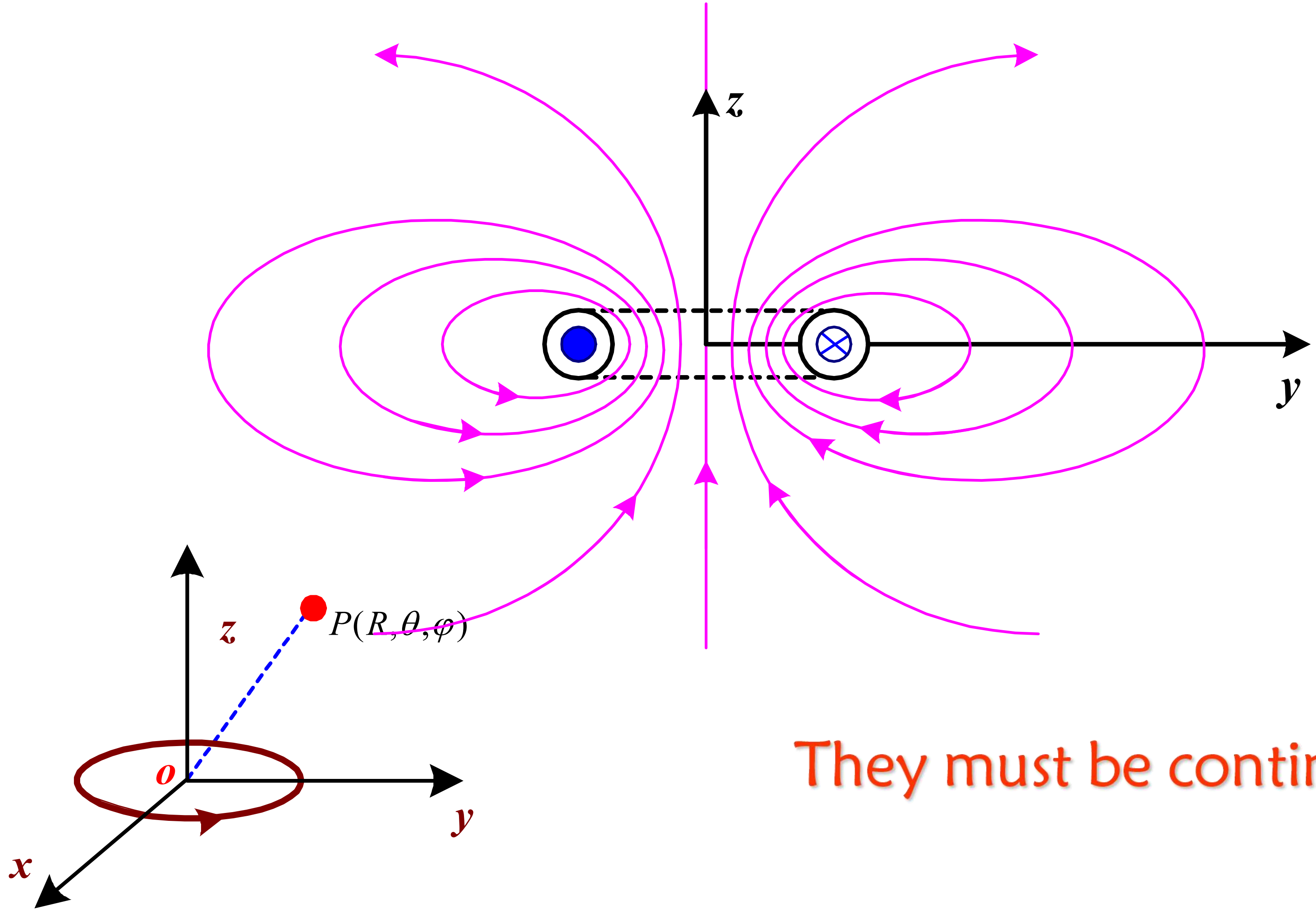
Lines of E-Flux for E-Dipole





Unnecessary to be continuous.

Lines of M-Flux for M-Dipole



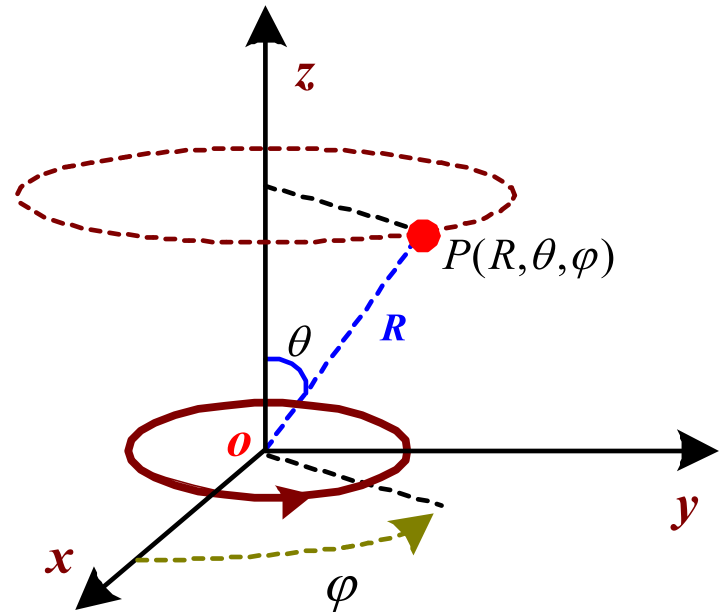
They must be continuous.

Key Parameters for M-Dipole

$$\vec{A} = \vec{a}_\varphi \left(\frac{\mu_0 I a^2 \cdot \sin\theta}{4R^2} \right)$$

$$\vec{A} = \frac{\mu_0 \vec{p}_m \times \vec{a}_R}{4\pi \cdot R^2} = -\frac{\mu_0}{4\pi} \vec{p}_m \times \nabla\left(\tfrac{1}{R}\right)$$

$$\vec{B} = \nabla \times \vec{A}$$



$$\vec{B} = \vec{a}_R \frac{\mu_0 P_m}{2\pi r^3} \cos\theta + \vec{a}_\theta \frac{\mu_0 P_m}{4\pi r^3} \sin\theta$$

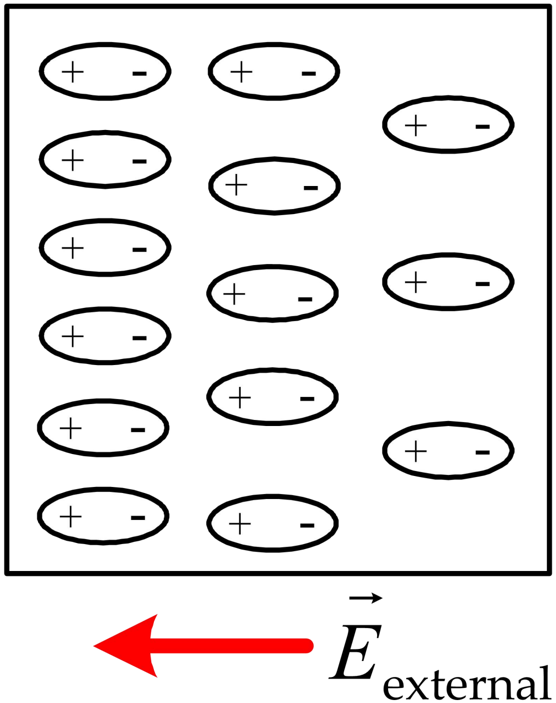
$$\vec{E} = \vec{a}_R \frac{P_e}{2\pi\varepsilon_0 r^3} \cos\theta + \vec{a}_\theta \frac{P_e}{4\pi\varepsilon_0 r^3} \sin\theta$$

§ 5.4 Material in M-Field

Magnetization

Recall that

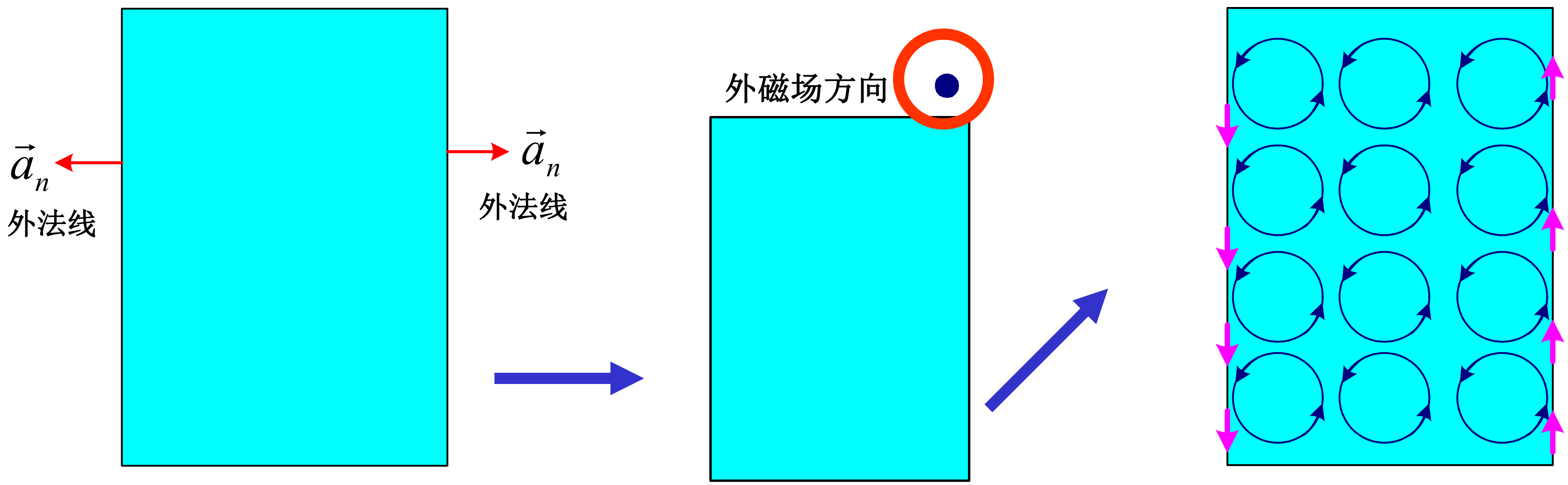
- Materials in E-field will be polarized. Subjected into an E-field, E-dipoles begin to queue orderly which induce bound charges on the surface.



Materials in M-field will be magnetized

- Molecule currents, or atom currents are actually M-dipoles.
- These M-dipoles oriented at random without external M-field.
- With external M-field, all M-dipoles point to the same direction, which is called magnetization.
  - Diamagnetic（反磁性体）: substance inside which the M-intensity is weaker than external M-intensity.
  - Paramagnetic（顺磁性的）: substance inside which the M-intensity is stronger than external M-intensity

- Due to magnetization, all M-dipoles queue orderly and thus yield a kind of surface current, called bound current, or magnetization current.



Magnetization Intensity 磁化强度(Optional)

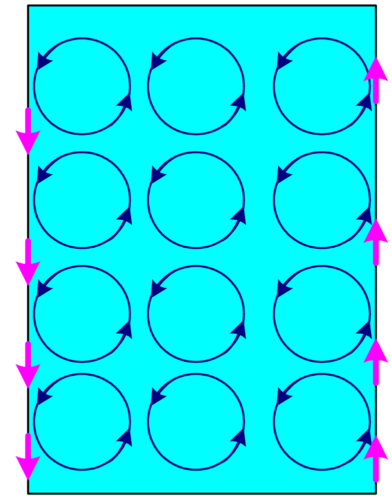
$$\vec{M} = \lim_{\Delta\tau \to 0} \frac{\sum \vec{p}_m}{\Delta\tau} \qquad (A/m)$$

The magnetic dipole moment per unit volume

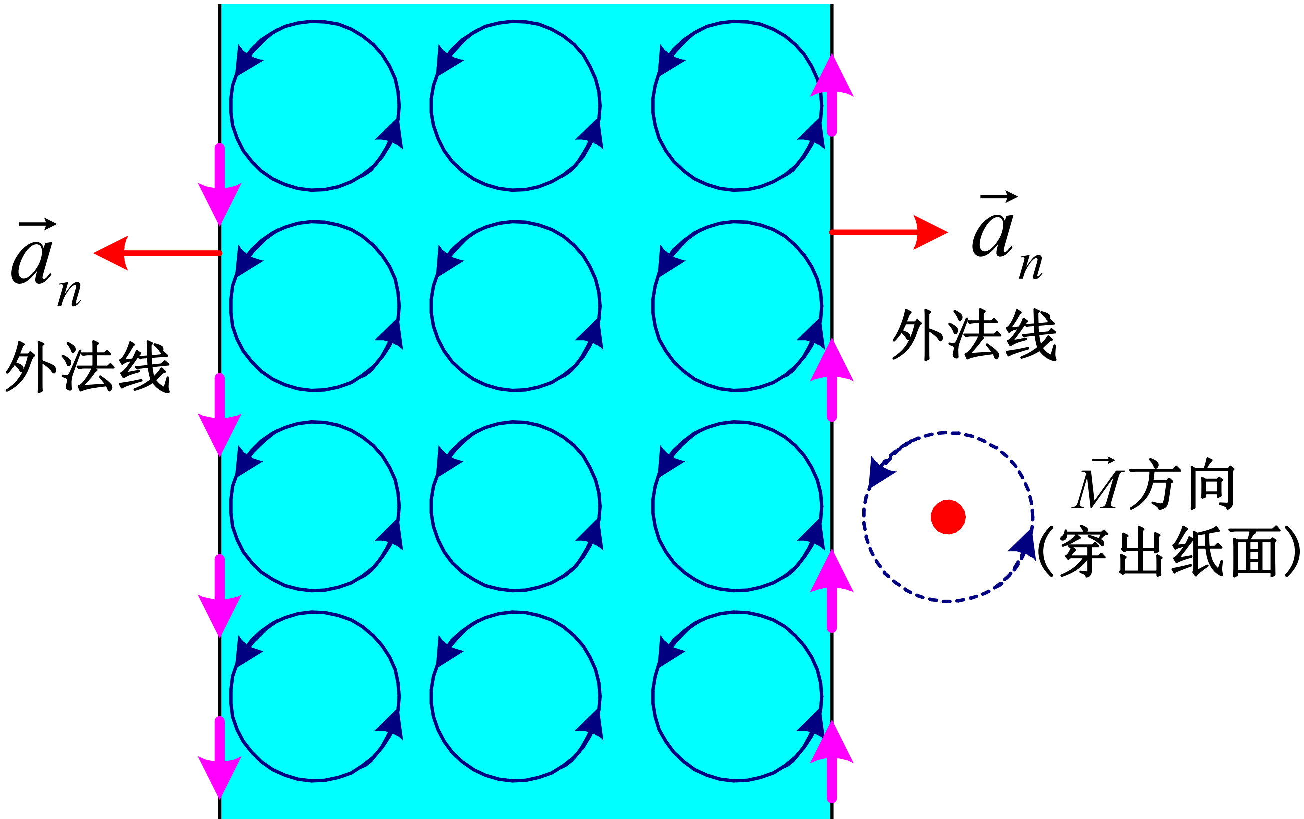
Recall that the polarization intensity is E-moment per unit volume

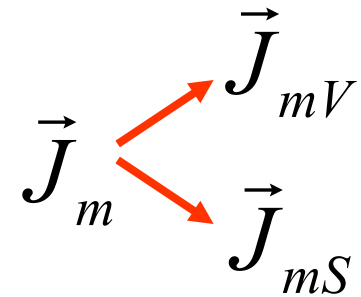
Density of Magnetization current

$$\begin{cases} \vec{J}_m = \nabla \times \vec{M} & (A/m^2) \\ \vec{J}_{ms} = \vec{M} \times \vec{a}_n & (A/m) \end{cases}$$



If a homogeneous substance is magnetized uniformly, the net current inside must be 0.





Adjacent M-dipoles will counteract each other

$$\begin{cases} \vec{J}_{mV} = \nabla \times \vec{M} = 0 \\ \vec{J}_{mS} = \vec{M} \times \vec{a}_n = ? \end{cases}$$

M-Intensity & Relative Permeability(磁导率)

Question:

External M-field + Magnetized Substance → New M-field
How to describe new M-field inside the magnetized substance?

Solution:

Recall that for magnetostatics in free space we have
$$\begin{cases} \nabla \bullet \vec{B} = 0 \\ \dfrac{1}{\mu_0} \nabla \times \vec{B} = \vec{J} \end{cases}$$

For new M-field inside the magnetized substance

$$\frac{1}{\mu_0} \nabla \times \vec{B} = \vec{J} + \vec{J}_M$$

Corresponding to free current

Corresponding to magnetization current

$$\frac{1}{\mu_0}\nabla\times\vec{B}=\vec{J}+\vec{J}_M=\vec{J}+\nabla\times\vec{M} \qquad \therefore \nabla\times\left(\frac{\vec{B}}{\mu_0}-\vec{M}\right)=\vec{J}$$

Magnetic Field Intensity

$$\vec{H}=\frac{\vec{B}}{\mu_0}-\vec{M} \qquad (A/m)$$

$$\therefore \nabla\times\vec{H}=\vec{J} \ \text{(volume density of free current)}$$

In comparison with electrostatics:

$$\oint_S \vec{E}\bullet d\vec{S}=\frac{\sum q_{fc}+\sum q_{pc}}{\varepsilon_0} \qquad \nabla\bullet(\varepsilon_0\vec{E}+\vec{P})=\rho_{fc}$$

$$\nabla\bullet\vec{D}=\rho_{fc}$$

$\nabla \times \vec{H} = \vec{J}$ (volume density of free current)

so
$$\int_S \left(\nabla \times \vec{H}\right) \bullet d\vec{S} = \int_S \left(\vec{J}\right) \bullet d\vec{S}$$

Applying Stokes's Law, we have

$$\oint_C \vec{H} \bullet d\vec{l} = I$$

What is the Unit of "Magnetic Field Intensity" ?

$$\vec{H}: \quad (A/m)$$

How about "Electric Field Intensity" ?

$$\boxed{\vec{E}: \quad (V/m)}$$

In Linear & Isotropic Materials

$$\boxed{\vec{M} = \chi_m \vec{H}}$$

χ_m : magnetic susceptibility （磁化率 无量纲）

$$\because \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\boxed{\vec{B} = \ldots = \mu_0 (1 + \chi_m)\vec{H} = \mu_0 \mu_r \vec{H} = \mu \vec{H}}$$

$\vec{B}: \quad (Wb/m^2)$ $\qquad \vec{H}: \quad (A/m)$

μ_r : relative permeability（相对磁导率）

μ : absolute permeability （绝对磁导率）

$$\boxed{\mu_r = 1 + \chi_m = \mu / \mu_0}$$

In comparison with electrostatics:

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} \qquad \vec{D} = \varepsilon_0 (1 + \chi_e)\vec{E} = \varepsilon \vec{E}$$

A Discussion on Relative Permeability μ_r

1. *diamagnetic*抗磁性材料 $\mu_r \leq 1$ $\chi_m \approx -0$

Copper, lead, gold, silver, etc..

2. *paramagnetic*顺磁性材料 $\mu_r \geq 1$ $\chi_m \approx +0$

Aluminum, tungsten (钨), etc..

3. *ferromagnetic*铁磁性材料 $\mu_r >> 1$ $\chi_m >> 0$

Cobalt (钴), iron, etc..

Summary on Material Parameters

真空中磁导率(*Permeability*)：

$$\mu_0 = 4\pi \cdot 10^{-7} (H/m)$$

$$\frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}} = c$$

真空中介电常数(*Dielectric Constant*)：

$$\varepsilon_0 = \frac{1}{4\pi \cdot 9 \times 10^9} = 8.85 \times 10^{-12} (F/m)$$

简单媒质——线性、均匀、各向同性

磁化率χ_m：无单位、常数

相对磁导率μ_r：无单位、常数