which is of the same form as the expression in Eq. (6–44) for **B** at a distant point due to a single magnetic dipole having a moment $I\pi b^2$. ∎

This problem can be solved just as easily by using the equivalent magnetization current density concept. (See Problem P.6–25.)

6–7 Magnetic Field Intensity and Relative Permeability

Because the application of an external magnetic field causes both an alignment of the internal dipole moments and an induced magnetic moment in a magnetic material, we expect that the resultant magnetic flux density in the presence of a magnetic material will be different from its value in free space. The macroscopic effect of magnetization can be studied by incorporating the equivalent volume current density, $\mathbf{J}_m$ in Eq. (6–62), into the basic curl equation, Eq. (6–7). We have

$$\frac{1}{\mu_0}\nabla \times \mathbf{B} = \mathbf{J} + \mathbf{J}_m = \mathbf{J} + \nabla \times \mathbf{M}$$

or

$$\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M}\right) = \mathbf{J}. \tag{6–74}$$

We now define a new fundamental field quantity, the ***magnetic field intensity*** **H**, such that

$$\boxed{\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \qquad (\text{A/m}).} \tag{6–75}$$

The use of the vector **H** enables us to write a curl equation relating the magnetic field and the distribution of free currents in any medium. There is no need to deal explicitly with the magnetization vector **M** or the equivalent volume current density $\mathbf{J}_m$. Combining Eqs. (6–74) and (6–75), we obtain the new equation

$$\boxed{\nabla \times \mathbf{H} = \mathbf{J} \qquad (\text{A/m}^2),} \tag{6–76}$$

where **J** (A/M^2) is the volume density of *free current.* Equations (6–6) and (6–76) are the two fundamental governing differential equations for magnetostatics. The permeability of the medium does not appear explicitly in these two equations.

The corresponding integral form of Eq. (6–76) is obtained by taking the scalar surface integral of both sides:

$$\int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{s} = \int_S \mathbf{J} \cdot d\mathbf{s} \tag{6–77}$$

or, according to Stokes's theorem,

$$\boxed{\oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = I \qquad \text{(A)},} \tag{6-78}$$

where C is the contour (closed path) bounding the surface S and I is the total free current passing through S. The relative directions of C and current flow I follow the right-hand rule. Equation (6–78) is another form of ***Ampère's circuital law***: It states that ***the circulation of the magnetic field intensity around any closed path is equal to the free current flowing through the surface bounded by the path.*** As we indicated in Section 6–2, Ampère's circuital law is most useful in determining the magnetic field caused by a current when cylindrical symmetry exists—that is, when there is a closed path around the current over which the magnetic field is constant.

When the magnetic properties of the medium are *linear* and *isotropic*, the magnetization is directly proportional to the magnetic field intensity:

$$\mathbf{M} = \chi_m \mathbf{H}, \tag{6-79}$$

where χ_m is a dimensionless quantity called ***magnetic susceptibility***. Substitution of Eq. (6–79) in Eq. (6–75) yields

$$\boxed{\begin{aligned} \mathbf{B} &= \mu_0(1 + \chi_m)\mathbf{H} \\ &= \mu_0\mu_r\mathbf{H} = \mu\mathbf{H} \qquad (\text{Wb/m}^2) \end{aligned}} \tag{6-80a}$$

or

$$\boxed{\mathbf{H} = \frac{1}{\mu}\mathbf{B} \qquad (\text{A/m}),} \tag{6-80b}$$

where

$$\boxed{\mu_r = 1 + \chi_m = \frac{\mu}{\mu_0}} \tag{6-81}$$

is another dimensionless quantity known as the ***relative permeability*** of the medium. The parameter $\mu = \mu_0\mu_r$ is the ***absolute permeability*** (or sometimes just ***permeability***) of the medium and is measured in H/m; χ_m, and therefore μ_r, can be a function of space coordinates. For a simple medium—linear, isotropic, and homogeneous—χ_m and μ_r are constants.

The permeability of most materials is very close to that of free space (μ_0). For ferromagnetic materials such as iron, nickel, and cobalt, μ_r could be very large (50–5000 and up to 10^6 or more for special alloys); the permeability depends not only on the magnitude of $\mathbf{H}$ but also on the previous history of the material. Section 6–9 contains some qualitative discussions of the macroscopic behavior of magnetic materials.

At this point we note a number of analogous relations between the quantities in electrostatics and those in magnetostatics as follows:

Electrostatics	Magnetostatics
$\mathbf{E}$	$\mathbf{B}$
$\mathbf{D}$	$\mathbf{H}$
ϵ	$\frac{1}{\mu}$
$\mathbf{P}$	$-\mathbf{M}$
ρ	$\mathbf{J}$
V	$\mathbf{A}$
$\cdot$	$\times$
$\times$	$\cdot$

With the above table, most of the equations relating the basic quantities in electrostatics can be converted into corresponding analogous ones in magnetostatics.

6–8 Magnetic Circuits

In electric-circuit problems we are required to find the voltages across and the currents in various branches and elements of an electric network that are excited by voltage and/or current sources. There is an analogous class of problems dealing with magnetic circuits. In a magnetic circuit we are generally concerned with the determination of the magnetic fluxes and magnetic field intensities in various parts of a circuit caused by windings carrying currents around ferromagnetic cores. Magnetic circuit problems arise in transformers, generators, motors, relays, magnetic recording devices, and so on.

Analysis of magnetic circuits is based on the two basic equations for magnetostatics, (6–6) and (6–76), which are repeated below for convenience:

$$\nabla \cdot \mathbf{B} = 0, \tag{6–82}$$

$$\nabla \times \mathbf{H} = \mathbf{J}. \tag{6–83}$$

We have seen in Eq. (6–78) that Eq. (6–83) converts to Ampère's circuital law. If the closed path C is chosen to enclose N turns of a winding carrying a current I that excites a magnetic circuit, we have

$$\oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = NI = \mathscr{V}_m. \tag{6–84}$$

The quantity $\mathscr{V}_m$ $(=NI)$ here plays a role that is analogous to electromotive force (emf) in an electric circuit and is therefore called a ***magnetomotive force*** (mmf). Its SI unit is ampere (A); but, because of Eq. (6–84), mmf is frequently measured in ampere-turns (A·t). An mmf is *not* a force measured in newtons.

EXAMPLE 6–10 Assume that N turns of wire are wound around a toroidal core of a ferromagnetic material with permeability μ. The core has a mean radius r_o, a circular cross section of radius a ($a \ll r_o$), and a narrow air gap of length ℓ_g, as shown in Fig. 6–13. A steady current I_o flows in the wire. Determine (a) the magnetic flux density, $\mathbf{B}_f$, in the ferromagnetic core; (b) the magnetic field intensity, $\mathbf{H}_f$, in the core; and (c) the magnetic field intensity, $\mathbf{H}_g$, in the air gap.

Solution

a) Applying Ampère's circuital law, Eq. (6–84), around the circular contour C in Fig. 6–13, which has a mean radius r_o, we have

$$\oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = NI_o. \tag{6–85}$$

If flux leakage is neglected, the same total flux will flow in both the ferromagnetic core and in the air gap. If the fringing effect of the flux in the air gap is also neglected, the magnetic flux density $\mathbf{B}$ in both the core and the air gap will also be the same. However, because of the different permeabilities, the magnetic field intensities in both parts will be different. We have

$$\mathbf{B}_f = \mathbf{B}_g = \mathbf{a}_\phi B_f, \tag{6–86}$$

where the subscripts f and g denote ferromagnetic and gap, respectively. In the ferromagnetic core,

$$\mathbf{H}_f = \mathbf{a}_\phi \frac{B_f}{\mu}; \tag{6–87}$$

and, in the air gap,

$$\mathbf{H}_g = \mathbf{a}_\phi \frac{B_f}{\mu_0}. \tag{6–88}$$

Substituting Eqs. (6–87) and (6–88) in Eq. (6–85), we obtain

$$\frac{B_f}{\mu}(2\pi r_o - \ell_g) + \frac{B_f}{\mu_0}\ell_g = NI_o$$

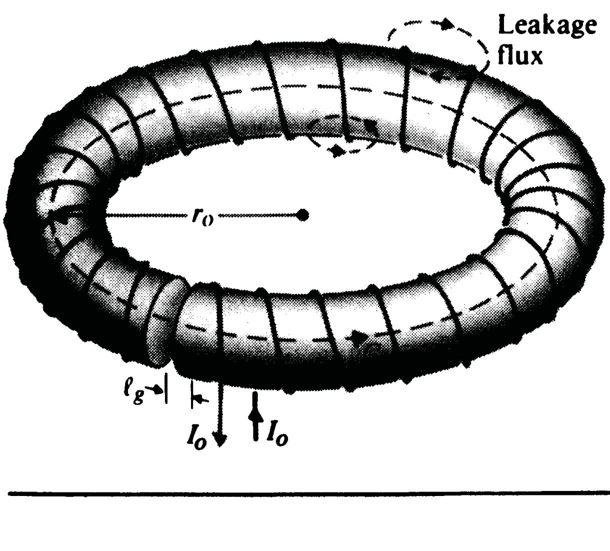


FIGURE 6–13
Coil on ferromagnetic toroid with air gap (Example 6–10).

and

$$\mathbf{B}_f = \mathbf{a}_\phi \frac{\mu_0 \mu N I_o}{\mu_0(2\pi r_o - \ell_g) + \mu \ell_g}. \tag{6–89}$$

b) From Eqs. (6–87) and (6–89) we get

$$\mathbf{H}_f = \mathbf{a}_\phi \frac{\mu_0 N I_o}{\mu_0(2\pi r_o - \ell_g) + \mu \ell_g}. \tag{6–90}$$

c) Similarly, from Eqs. (6–88) and (6–89) we have

$$\mathbf{H}_g = \mathbf{a}_\phi \frac{\mu N I_o}{\mu_0(2\pi r_o - \ell_g) + \mu \ell_g}. \tag{6–91}$$

Since $H_g/H_f = \mu/\mu_0$, the magnetic field intensity in the air gap is much stronger than that in the ferromagnetic core. ■

If the radius of the cross section of the core is much smaller than the mean radius of the toroid, the magnetic flux density **B** in the core is approximately constant, and the magnetic flux in the circuit is

$$\Phi \cong BS, \tag{6–92}$$

where S is the cross-sectional area of the core. Combination of Eqs. (6–92) and (6–89) yields

$$\Phi = \frac{N I_o}{(2\pi r_o - \ell_g)/\mu S + \ell_g/\mu_0 S}. \tag{6–93}$$

Equation (6–93) can be rewritten as

$$\Phi = \frac{\mathscr{V}_m}{\mathscr{R}_f + \mathscr{R}_g}, \tag{6–94}$$

with

$$\mathscr{R}_f = \frac{2\pi r_o - \ell_g}{\mu S} = \frac{\ell_f}{\mu S}, \tag{6–95}$$

where $\ell_f = 2\pi r_o - \ell_g$ is the length of the ferromagnetic core, and

$$\mathscr{R}_g = \frac{\ell_g}{\mu_0 S}. \tag{6–96}$$

Both $\mathscr{R}_f$ and $\mathscr{R}_g$ have the same form as the formula, Eq. (5–27), for the d-c resistance of a straight piece of homogeneous material with a uniform cross section S. Both are called ***reluctance***: $\mathscr{R}_f$, of the ferromagnetic core; and $\mathscr{R}_g$, of the air gap. The SI unit for reluctance is reciprocal henry (H^{-1}). The fact that Eqs. (6–95) and (6–96) are as they are, even though the core is not straight, is a consequence of assuming that **B** is approximately constant over the core cross section.

Equation (6–94) is analogous to the expression for the current I in an electric circuit, in which an ideal voltage source of emf $\mathscr{V}$ is connected in series with two

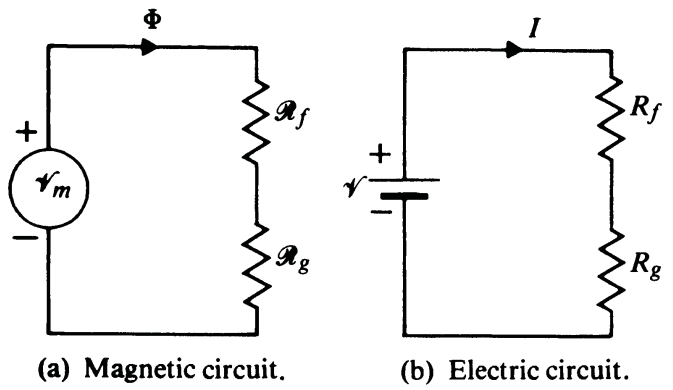


FIGURE 6–14
Equivalent magnetic circuit and analogous electric circuit for toroidal coil with air gap in Fig. 6–13.

resistances R_f and R_g:

$$I = \frac{\mathscr{V}}{R_f + R_g}. \tag{6-97}$$

The analogous magnetic and electric circuits are shown in Figs. 6–14(a) and 6–14(b), respectively. Magnetic circuits can, by analogy, be analyzed by the same techniques we have used in analyzing electric circuits. The analogous quantities are as follows:

Magnetic Circuits	Electric Circuits
mmf, $\mathscr{V}_m\ (=NI)$	emf, $\mathscr{V}$
magnetic flux, Φ	electric current, I
reluctance, $\mathscr{R}$	resistance, R
permeability, μ	conductivity, σ

In spite of this convenient likeness an exact analysis of magnetic circuits is inherently very difficult to achieve.

First, it is very difficult to account for leakage fluxes, fluxes that stray or leak from the main flux paths of a magnetic circuit. For the toroidal coil in Fig. 6–13, leakage flux paths encircle every turn of the winding; they partially transverse the space around the core, as illustrated, because the permeability of air is not zero. (There is little need for considering leakage currents outside the conducting paths of electric circuits that carry direct currents. The reason is that the conductivity of air is practically zero compared to that of a good conductor.)

A second difficulty is the fringing effect that causes the magnetic flux lines at the air gap to spread and bulge.† (The purpose of specifying the "narrow air gap" in Example 6–10 was to minimize this fringing effect.)

† To obtain a more accurate numerical result, it is customary to consider the effective area of the air gap as slightly larger than the cross-sectional area of the ferromagnetic core, with each of the lineal dimensions of the core cross section increased by the length of the air gap. If we were to make a correction like this in Eq. (6–86), $\mathbf{B}_g$ would become

$$B_g = \frac{a^2 B_f}{(a + \ell_g)^2} < B_f.$$

A third difficulty is that the permeability of ferromagnetic materials depends on the magnetic field intensity; that is, **B** and **H** have a nonlinear relationship. (They might not even be in the same direction). The problem of Example 6–10, which assumes a given μ before either $\mathbf{B}_c$ or $\mathbf{H}_c$ is known, is therefore not a realistic one.

In a practical problem the B–H curve of the ferromagnetic material, such as that shown later in Fig. 6–17, should be given. The ratio of B to H is obviously not a constant, and B_f can be known only when H_f is known. So how does one solve the problem? Two conditions must be satisfied. First, the sum of $H_g\ell_g$ and $H_f\ell_f$ must equal the total mmf NI_o:

$$H_g\ell_g + H_f\ell_f = NI_o. \tag{6–98}$$

Second, if we assume no leakage flux, the total flux Φ in the ferromagnetic core and in the air gap must be the same, or $B_f = B_g$:†

$$B_f = \mu_0 H_g. \tag{6–99}$$

Substitution of Eq. (6–99) in Eq. (6–98) yields an equation relating B_f and H_f in the core:

$$B_f + \mu_0 \frac{\ell_f}{\ell_g} H_f = \frac{\mu_0}{\ell_g} NI_o. \tag{6–100}$$

This is an equation for a straight line in the B–H plane with a negative slope $(-\mu_0\ell_f/\ell_g)$. The intersection of this line and the given B–H curve determines the operating point. Once the operating point has been found, μ and H_f and all other quantities can be obtained.

The similarity between Eqs. (6–94) and (6–97) can be extended to the writing of two basic equations for magnetic circuits that correspond to Kirchhoff's voltage and current laws for electric circuits. Similar to Kirchhoff's voltage law in Eq. (5–41), we may write, for any closed path in a magnetic circuit,

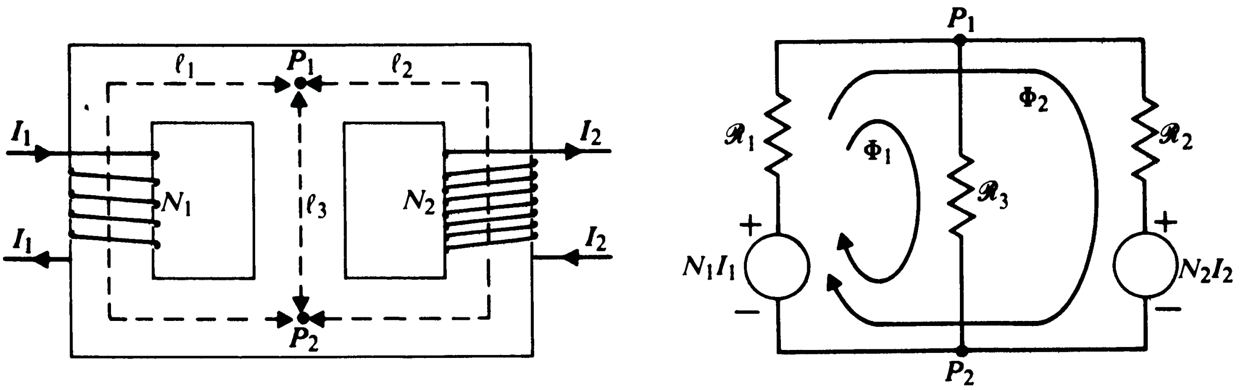
$$\boxed{\sum_j N_j I_j = \sum_k \mathscr{R}_k \Phi_k.} \tag{6–101}$$

Equation (6–101) states that ***around a closed path in a magnetic circuit the algebraic sum of ampere-turns is equal to the algebraic sum of the products of the reluctances and fluxes.***

Kirchhoff's current law for a junction in an electric circuit, Eq. (5–47), is a consequence of $\nabla \cdot \mathbf{J} = 0$. Similarly, the fundamental postulate $\nabla \cdot \mathbf{B} = 0$ in Eq. (6–82) leads to Eq. (6–9). Thus, we have

$$\boxed{\sum_j \Phi_j = 0,} \tag{6–102}$$

† This assumes an equal cross-sectional area for the core and the gap. If the core were to be constructed of insulated laminations of ferromagnetic material, the effective area for flux passage in the core would be smaller than the geometrical cross-sectional area, and B_c would be larger than B_g by a factor. This factor can be determined from the data on the insulated laminations.



(a) Magnetic core with current-carrying windings. (b) Magnetic circuit for loop analysis.

FIGURE 6–15
A magnetic circuit (Example 6–11).

which states that ***the algebraic sum of all the magnetic fluxes flowing out of a junction in a magnetic circuit is zero.*** Equations (6–101) and (6–102) form the bases for the loop and node analysis, respectively, of magnetic circuits.

EXAMPLE 6–11 Consider the magnetic circuit in Fig. 6–15(a). Steady currents I_1 and I_2 flow in windings of N_1 and N_2 turns, respectively, on the outside legs of the ferromagnetic core. The core has a cross-sectional area S_c and a permeability μ. Determine the magnetic flux in the center leg.

Solution The equivalent magnetic circuit for loop analysis is shown in Fig. 6–15(b). Two sources of mmf's, N_1I_1 and N_2I_2, are shown with proper polarities in series with reluctances $\mathscr{R}_1$ and $\mathscr{R}_2$, respectively. This is obviously a two-loop network. Since we are determining magnetic flux in the center leg P_1P_2, it is expedient to choose the two loops in such a way that only one loop flux (Φ_1) flows through the center leg. The reluctances are computed on the basis of average path lengths. These are, of course, approximations. We have

$$\mathscr{R}_1 = \frac{\ell_1}{\mu S_c}, \tag{6–103a}$$

$$\mathscr{R}_2 = \frac{\ell_2}{\mu S_c}, \tag{6–103b}$$

$$\mathscr{R}_3 = \frac{\ell_3}{\mu S_c}. \tag{6–103c}$$

The two loop equations are, from Eq. (6–101),

$$\textit{Loop } 1: \quad N_1I_1 = (\mathscr{R}_1 + \mathscr{R}_3)\Phi_1 + \mathscr{R}_1\Phi_2; \tag{6–104}$$

$$\textit{Loop } 2: \quad N_1I_1 - N_2I_2 = \mathscr{R}_1\Phi_1 + (\mathscr{R}_1 + \mathscr{R}_2)\Phi_2. \tag{6–105}$$

Solving these simultaneous equations, we obtain

$$\Phi_1 = \frac{\mathscr{R}_2 N_1 I_1 + \mathscr{R}_1 N_2 I_2}{\mathscr{R}_1\mathscr{R}_2 + \mathscr{R}_1\mathscr{R}_3 + \mathscr{R}_2\mathscr{R}_3}, \tag{6–106}$$

which is the desired answer. ■

Actually, since the magnetic fluxes and therefore the magnetic flux densities in the three legs are different, different permeabilities should be used in computing the reluctances in Eqs. (6–103a), (6–103b), and (6–103c). But the value of permeability, in turn, depends on the magnetic flux density. The only way to improve the accuracy of the solution, provided that the B–H curve of the core material is given, is to use a procedure of successive approximation. For instance, Φ_1, Φ_2, and Φ_3 (and therefore B_1, B_2, and B_3) are first solved with an assumed μ and reluctances computed from the three parts of Eq. (6–103). From B_1, B_2, and B_3 the corresponding μ_1, μ_2, and μ_3 can be found from the B–H curve. These will modify the reluctances. A second approximation for B_1, B_2, and B_3 is then obtained with the modified reluctances. From the new flux densities, new permeabilities and new reluctances are determined. This procedure is repeated until further iterations bring little change in the computed values.

We remark here that the currents in the windings in Fig. 6–15(a) are independent of time and that Example 6–11 is strictly a d-c magnetic circuit problem. If the currents vary with time, we must deal with the effects of electromagnetic induction, and we will have a transformer problem. Other fundamental laws are involved, which we shall discuss in Chapter 7.

6–9 Behavior of Magnetic Materials

In Eq. (6–79), Section 6–7, we described the macroscopic magnetic property of a linear, isotropic medium by defining the magnetic susceptibility χ_m, a dimensionless coefficient of proportionality between magnetization $\mathbf{M}$ and magnetic field intensity $\mathbf{H}$. The relative permeability μ_r is simply $1 + \chi_m$. Magnetic materials can be roughly classified into three main groups in accordance with their μ_r values. A material is said to be

Diamagnetic, if $\mu_r \lesssim 1$ (χ_m is a very small negative number).

Paramagnetic, if $\mu_r \gtrsim 1$ (χ_m is a very small positive number).

Ferromagnetic, if $\mu_r \gg 1$ (χ_m is a large positive number).

As mentioned before, a thorough understanding of microscopic magnetic phenomena requires a knowledge of quantum mechanics. In the following we give a qualitative description of the behavior of the various types of magnetic materials based on the classical atomic model.

In a *diamagnetic* material the net magnetic moment due to the orbital and spinning motions of the electrons in any particular atom is zero in the absence of an externally applied magnetic field. As predicted by Eq. (6–4), the application of an external magnetic field to this material produces a force on the orbiting electrons, causing a perturbation in the angular velocities. As a consequence, a net magnetic moment is created. This is a process of induced magnetization. According to ***Lenz's law*** of electromagnetic induction (Section 7–2), the induced magnetic moment always *opposes* the applied field, thus reducing the magnetic flux density. The macroscopic effect of this process is equivalent to that of a negative magnetization that can be described by a negative magnetic susceptibility. This effect is usually very small, and χ_m for most known diamagnetic materials (bismuth, copper, lead, mercury, germanium, silver, gold, diamond) is of the order of -10^{-5}.

Diamagnetism arises mainly from the orbital motion of the electrons within an atom and is present in all materials. In most materials it is too weak to be of any practical importance. The diamagnetic effect is masked in paramagnetic and ferromagnetic materials. Diamagnetic materials exhibit no permanent magnetism, and the induced magnetic moment disappears when the applied field is withdrawn.

In some materials the magnetic moments due to the orbiting and spinning electrons do not cancel completely, and the atoms and molecules have a net average magnetic moment. An externally applied magnetic field, in addition to causing a very weak diamagnetic effect, tends to align the molecular magnetic moments *in the direction of* the applied field, thus increasing the magnetic flux density. The macroscopic effect is, then, equivalent to that of a positive magnetization that is described by a positive magnetic susceptibility. The alignment process is, however, impeded by the forces of random thermal vibrations. There is little coherent interaction, and the increase in magnetic flux density is quite small. Materials with this behavior are said to be *paramagnetic*. Paramagnetic materials generally have very small positive values of magnetic susceptibility, of the order of 10^{-5} for aluminum, magnesium, titanium, and tungsten.

Paramagnetism arises mainly from the magnetic dipole moments of the spinning electrons. The alignment forces, acting upon molecular dipoles by the applied field, are counteracted by the deranging effects of thermal agitation. Unlike diamagnetism, which is essentially independent of temperature, the paramagnetic effect is temperature dependent, being stronger at lower temperatures where there is less thermal collision.

The magnetization of *ferromagnetic* materials can be many orders of magnitude larger than that of paramagnetic substances. (See Appendix B–5 for typical values of relative permeability.) ***Ferromagnetism*** can be explained in terms of magnetized ***domains***. According to this model, which has been experimentally confirmed, a ferromagnetic material (such as cobalt, nickel, and iron) is composed of many small domains, their linear dimensions ranging from a few microns to about 1 mm. These domains, each containing about 10^{15} or 10^{16} atoms, are fully magnetized in the sense that they contain aligned magnetic dipoles resulting from spinning electrons even in the absence of an applied magnetic field. Quantum theory asserts that strong coupling forces exist between the magnetic dipole moments of the atoms in a domain, holding the dipole moments in parallel. Between adjacent domains there is a transition region

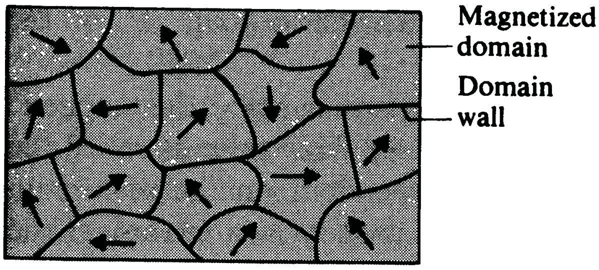


FIGURE 6–16
Domain structure of a polycrystalline ferromagnetic specimen.

about 100 atoms thick called a ***domain wall.*** In an unmagnetized state the magnetic moments of the adjacent domains in a ferromagnetic material have different directions, as exemplified in Fig. 6–16 by the polycrystalline specimen shown. Viewed as a whole, the random nature of the orientations in the various domains results in no net magnetization.

When an external magnetic field is applied to a ferromagnetic material, the walls of those domains having magnetic moments aligned with the applied field move in such a way as to make the volumes of those domains grow at the expense of other domains. As a result, magnetic flux density is increased. For weak applied fields, say up to point P_1 in Fig. 6–17, domain-wall movements are reversible. But when an applied field becomes stronger (past P_1), domain-wall movements are no longer reversible, and domain rotation toward the direction of the applied field will also occur. For example, if an applied field is reduced to zero at point P_2, the B–H relationship will not follow the solid curve P_2P_1O, but will go down from P_2 to P_2', along the lines of the broken curve in the figure. This phenomenon of magnetization lagging behind the field producing it is called ***hysteresis,*** which is derived from a Greek word meaning "to lag." As the applied field becomes even much stronger (past P_2 to P_3), domain-wall motion and domain rotation will cause essentially a total alignment of the microscopic magnetic moments with the applied field, at which point

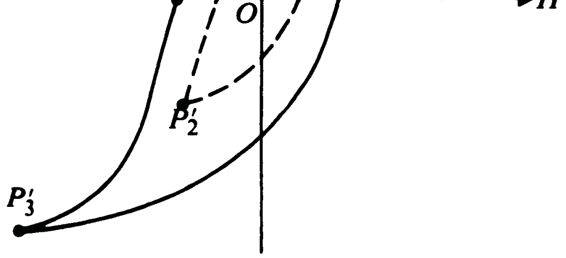


FIGURE 6–17
Hysteresis loops in the B–H plane for ferromagnetic material.

the magnetic material is said to have reached *saturation*. The curve $OP_1P_2P_3$ on the B–H plane is called the ***normal magnetization curve***.

If the applied magnetic field is reduced to zero from the value at P_3, the magnetic flux density does not go to zero but assumes the value at B_r. This value is called the ***residual*** or ***remanent flux density*** (in Wb/m^2) and is dependent on the maximum applied field intensity. The existence of a remanent flux density in a ferromagnetic material makes permanent magnets possible.

To make the magnetic flux density of a specimen zero, it is necessary to apply a magnetic field intensity H_c in the opposite direction. This required H_c is called *coercive force*, but a more appropriate name is ***coercive field intensity*** (in A/m). Like B_r, H_c also depends on the maximum value of the applied magnetic field intensity.

It is evident from Fig. 6–17 that the B–H relationship for a ferromagnetic material is nonlinear. Hence, if we write $\mathbf{B} = \mu\mathbf{H}$ as in Eq. (6–80a), the permeability μ itself is a function of the magnitude of $\mathbf{H}$. Permeability μ also depends on the history of the material's magnetization, since—even for the same $\mathbf{H}$—we must know the location of the operating point on a particular branch of a particular hysteresis loop in order to determine the value of μ exactly. In some applications a small alternating current may be superimposed on a large steady magnetizing current. The steady magnetizing field intensity locates the operating point, and the local slope of the hysteresis curve at the operating point determines the ***incremental permeability***.

Ferromagnetic materials for use in electric generators, motors, and transformers should have a large magnetization for a very small applied field; they should have tall, narrow hysteresis loops. As the applied magnetic field intensity varies periodically between $\pm H_{\max}$, the hysteresis loop is traced once per cycle. The area of the hysteresis loop corresponds to energy loss (***hysteresis loss***) per unit volume per cycle (Problem P.6–29). Hysteresis loss is the energy lost in the form of heat in overcoming the friction encountered during domain-wall motion and domain rotation. Ferromagnetic materials, which have tall, narrow hysteresis loops with small loop areas, are referred to as "soft" materials; they are usually well-annealed materials with very few dislocations and impurities so that the domain walls can move easily.

Good permanent magnets, on the other hand, should show a high resistance to demagnetization. This requires that they be made with materials that have large coercive field intensities H_c and hence fat hysteresis loops. These materials are referred to as "hard" ferromagnetic materials. The coercive field intensity of hard ferromagnetic materials (such as Alnico alloys) can be 10^5 (A/m) or more, whereas that for soft materials is usually 50 (A/m) or less.

As indicated before, ferromagnetism is the result of strong coupling effects between the magnetic dipole moments of the atoms in a domain. Figure 6–18(a) depicts the atomic spin structure of a ferromagnetic material. When the temperature of a ferromagnetic material is raised to such an extent that the thermal energy exceeds the coupling energy, the magnetized domains become disorganized. Above this critical temperature, known as the ***curie temperature***, a ferromagnetic material behaves like a paramagnetic substance. Hence, when a permanent magnet is heated above its curie temperature it loses its magnetization. The curie temperature of most ferromagnetic materials lies between a few hundred to a thousand degrees Celsius, that of iron being 770°C.

Some elements, such as chromium and manganese, which are close to ferromagnetic elements in atomic number and are neighbors of iron in the periodic table, also have strong coupling forces between the atomic magnetic dipole moments; but their coupling forces produce antiparallel alignments of electron spins, as illustrated in Fig. 6–18(b). The spins alternate in direction from atom to atom and result in no net magnetic moment. A material possessing this property is said to be ***antiferromagnetic***. Antiferromagnetism is also temperature dependent. When an antiferromagnetic material is heated above its curie temperature, the spin directions suddenly become random, and the material becomes paramagnetic.

There is another class of magnetic materials that exhibit a behavior between ferromagnetism and antiferromagnetism. Here quantum mechanical effects make the directions of the magnetic moments in the ordered spin structure alternate and the magnitudes unequal, resulting in a net nonzero magnetic moment, as depicted in Fig. 6–18(c). These materials are said to be ***ferrimagnetic***. Because of the partial cancellation, the maximum magnetic flux density attained in a ferrimagnetic substance is substantially lower than that in a ferromagnetic specimen. Typically, it is about 0.3 Wb/m^2, approximately one-tenth that for ferromagnetic substances.

Ferrites are a subgroup of ferrimagnetic material. One type of ferrites, called ***magnetic spinels***, crystallize in a complicated spinel structure and have the formula $XO \cdot Fe_2O_3$, where X denotes a divalent metallic ion such as Fe, Co, Ni, Mn, Mg, Zn, Cd, etc. These are ceramiclike compounds with very low conductivities (for instance, 10^{-4} to 1 (S/m) compared with 10^7 (S/m) for iron). Low conductivity limits eddy-current losses at high frequencies. Hence ferrites find extensive uses in such high-frequency and microwave applications as cores for FM antennas, high-frequency transformers, and phase shifters. Ferrite material also has broad applications in

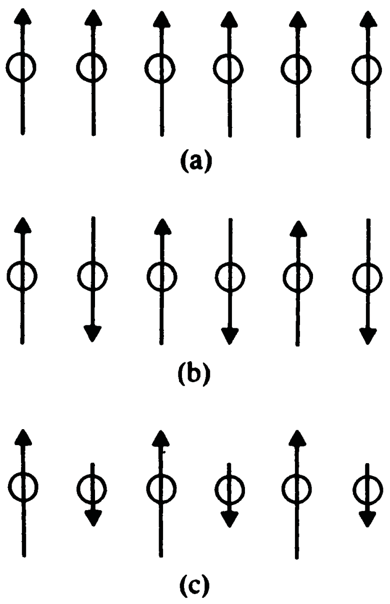


FIGURE 6–18
Schematic atomic spin structures for (a) ferromagnetic, (b) antiferromagnetic, and (c) ferrimagnetic materials.