SOLUTIONS

Module:	Telecoms Systems		
Module Code	EBU5302	Paper	A
Time allowed	2hrs	Filename	Solutions_1516-1_EBU5302_A
Rubric	ANSWER ALL FOUR QUESTIONS		
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Question 1

Let $x(t)$ be a band-limited signal to W = 2 kHz, amplitude $0 \le x(t) \le 2$ and power $P = 1$. Signal $x(t)$ is sampled at a rate 20% higher than the Nyquist rate to provide a guard band. The maximum acceptable error in the sample amplitude (the maximum quantization error) is 0.5% of the peak amplitude. The quantized samples are binary coded.

Assume "Sr" is an M=8 symbol source. Symbol A.... H represent each of the symbol amplitude values generated by the quantiser. The probability p_m of each symbol is shown in the following table:

m	A	B	C	D	E	F	G	H
P(m)	0.3	0.1	0.06	0.25	0.04	0.05	0.18	0.02

a) Using diagrams to explain why in general sampling has to meet the Nyquist sampling theorem.

[4 marks]

b) Illustrate what is the sample rate for $x(t)$.

$f_s = (1+20\%)\,2\cancel{f_m}\,W.$

[1 mark]

c) Find the minimum bandwidth of a channel required to transmit the encoded binary signal.

[6 marks]

d) If 24 such signals are time-division-multiplexed, determine the minimum transmission bandwidth required to transmit the multiplexed signal.

[1 marks]

e) What is the information content for each symbol of Sr?

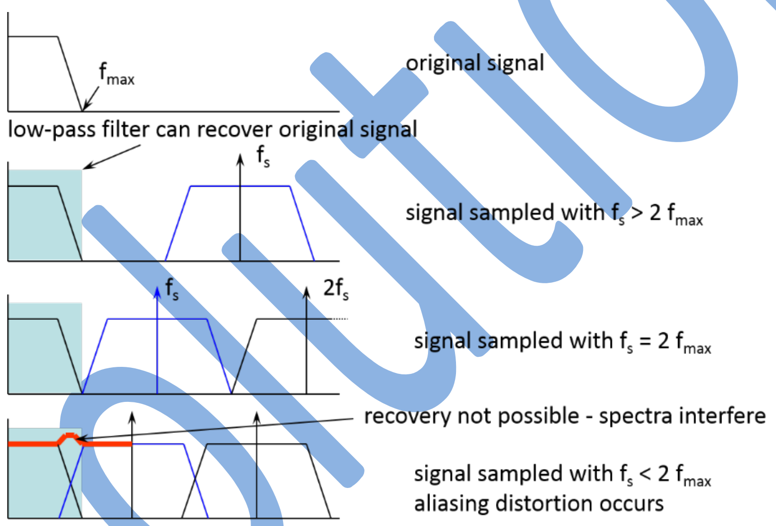
[9 marks]

f) What are the source entropy and source efficiency for Sr?

[4 marks]

Answer

a) Nyquist rate is defined as twice the signal bandwidth W for low-pass band signal. [4 marks]



b) The Nyquist sampling rate for $x(t)$ is $R_N = 2^* 2000 = 4000$ Hz (samples per second). The actual sampling rate is Rs = 4000*1.2=4800 Hz. [1 mark]

c) The quantization step is q, and the maximum quantization error is ±q/2.
Therefore

q/2 =0.5% *2, quantisation level [1 mark]

So quantization level M = 100, [1 mark] 不等同

For binary coding, L must be a power of 2. Hence, the next higher value of L that is a power of 2 is L=128. [1 mark]

So we need n=log₂ 128 = 7 bits per sample. [1 mark] x4800Hz. = — bits/s.

multilevel signal

Δ $C = 2B\log_2(M)$ $M=2$ ∴ $C=2B$

∴ $B = \frac{C}{2}$

As we required to transmit a total of C= 7*4800=33,600 bit/s. [1 mark]

Because for binary, we can transmit up to 2 bits per hertz of bandwidth, we require a minimum transmission bandwidth B_T=C/2=16.8kHz. [1 mark]

d) Multiplexed signal has a total of C_M= 24*33,600 = 0.806 Mbit/s, which requires a minimum of 0.806/2= 0.403 MHz of transmission bandwidth. [1 mark]

e) The information content I of symbol is defined as

$$I = \log_2(1/p)$$ [1 mark]

so

m	A	B	C	D	E	F	G	H
I_m	1.74	3.32	4.06	2	4.64	4.32	2.47	5.64

[8 marks]

f) The entropy is defined as

- $H = \sum_i p_i \, log_2(1/p_i)$ [1 mark]

The resulting entropy will then be H = 2.55 bits/symbol. [1 mark]

For an information source that produces 8 symbols with the same probability (uniform information source), the entropy is H_{max} = log 8 = 3. [1 mark]

So the source efficiency is 2.55/3= 85%. [1 mark]

Question 2

A digital information source produces binary sequences at a rate of ~~5 kbps~~. The probability of producing the value 0 is $p_0 = 0.2$. A Hamming code with the following parity check matrix **H** is employed to protect information against errors:

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$H = [P^t \mid I_{n-k}]$

$G = [I_k \mid P]$

The resulting binary sequences are transmitted through a wireless channel where power falloff with distance follows the formula $P_r(d) = P_t(d_0/d)^3$ for d_0=10m. Assume the channel has bandwidth B=30 kHz and AWGN with noise PSD (power spectral density) $N_0/2$, where $N_0 = 10^{-9}$ W/Hz.

a) For a transmit power of 1 W, find the capacity of this cannel for a transmit-receive distance of 100m and 1km.

[6 marks]

b) Based on the parity check matrix **H**, determine the length of the input information sequences and the length of the code words. Calculate the code rate of this Hamming code and the resulting transmission rate. **[4 marks]**

c) How can the systematic linear block code words of this Hamming code be obtained? Calculate the code words corresponding to the information sequences 0110 and 1010.

[5 marks]

d) Determine the number of errors can be detected and corrected in this Hamming code.

[5 marks]

e) Decode the following received sequence **r** = 1111010.

[5 marks]

Answer

a) The received $SNR = P_r(d)/ N_0 B$ [1 mark] and $C = B \log_2 (1+SNR)$ [1 mark]

For d_1=100m, $SNR_1 = (10/100)^3 / (10^{-9}*30*10^3)$=33= 15 dB [1 mark]

C_1 = 30000 $\log_2$ (1+33) = 152.6 kbps [1 mark]

For d_2=1km, $SNR_1 = (10/1000)^3 / (10^{-9}*30*10^3)$=0.033= -15 dB [1 mark]

C_2 = 30000 $\log_2$ (1+0.033) = 1.4 kbps [1 mark]

b) The dimensions of the parity check matrix are m x n, where n is the length of a code word, $m = n - k$, and k is the length of information sequences. [1 mark]

This is then a (7,4) Hamming code and its code rate is $R_C = 4/7$. [1 mark]

The resulting transmission rate can be obtained as R_B = 5kbps*1/R_c=5000 x 7/4= 8.75 kbps. [2 marks]

c) Based on the parity check matrix **H**, we first obtain the matrix **P**:

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \text{ [1 mark]}$$

The generator matrix **G** of a systematic linear block code will then be

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \text{ [1 mark]}$$

Code words **c** can be obtained by multiplying each 4-bit information sequence **x** by the generator matrix **G**, **c** = **xG**. [1 mark]

By using this expression, the code words corresponding to the sequences 0100 and 1000 are, respectively, 0110100 and 1010001. [2 marks]

d) Hamming codes belong to the family of linear block codes. Hence, the minimum distance can be obtained as the minimum weight (except all-zero codeword), where the weight of a code word is defined as the number of bits of value 1 in each sequence. [2 marks]

The main property of Hamming codes is that their minimum distance is always 3. [1 mark]

d_{min}>e+t+1, Since the minimum distance is 3, up to 2 errors will be detected or 1 error will be corrected. [2 marks]

e) In order to decode the received sequence, we first compute its syndrome **s** =r$\mathbf{H}^T$ =e$\mathbf{H}^T$. [1 mark]

The syndrome sequence corresponding to 1111010 is **s** = 100. [1 mark]

The error sequence corresponding to this syndrome is **e** = 0000100. [1 mark]

Hence, the transmitted code word is **c** = 1111010 + 0000100 = 1111110. [1 mark]

This code word corresponds to the information sequence **x** =1111. [1 mark]

Question 3

a) A multilevel digital communication system sends one of 16 possible levels over the channel every 0.8 ms.

- i) What is the number of bits corresponding to each level? 4
- ii) What is the baud (Symbol) rate? ~~0.8 ms~~ $1/0.8 = 1.25\,\text{k}$~~Hz~~ 1.25 kbaud.
- iii) What is the bit rate? $1.25 \times 4 =$ ~~500 kHz~~ 500 bit/s.

[6 marks]

Solutions:

(a) $L = 2^{\ell} = 16 \Rightarrow$ $\underline{\underline{\ell = 4\ \text{bits/level}}}$

(b) $D = \frac{N}{T} = \frac{1\ \text{symbol}}{0.8 \times 10^{-3}\ \text{sec}} = \underline{\underline{1,250\ \text{baud}}}$

(c) $R = \ell D = 4(1,250) = \underline{\underline{5\ \text{kbits/sec}}}$

$R = 2400$ bit/s $\quad l = 2.$

b) Multilevel data with an equivalent bit rate of 2,400 bits/s is sent over a channel using a four level line code that has a rectangular pulse shape at the output of the transmitter. The overall transmission system (i.e. the transmitter, channel and receiver) has an r=0.5 raised cosine roll-off Nyquist filter characteristic.

$D = \frac{f}{l}$ $\quad \boxed{D = \frac{2B}{1+r}}$

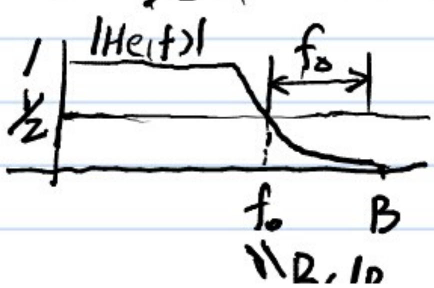
- i) Find the baud (symbol) rate of the received signal.
- ii) Find the 6-dB bandwidth for this transmission system.
- iii) Find the absolute bandwidth for the system.

[8 marks]

ANS: $L = 2^{l} = 4 \Rightarrow l = 2$ [2 marks]

i) $D = R/l = 2400/2 = 1200$ baud (

ii) $B = \frac{1}{2}(1+r)D$ where $r = f_\Delta / f_0$: [2 marks] [2 marks]



$\Rightarrow B_{6dB} = \frac{1}{2}(1+($ [2 marks]

c) The following table illustrates the operation of an FHSS system for one complete period of the PN sequence.

Time	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Input data	0	1	1	1	1	1	1	0	0	0	1	0	0	1	1	1	1	0	1	0
Frequency	f_{11}	f_2	f_{11}	f_3	f_3	f_3	f_{22}	f_{10}	f_0	f_0	f_1	f_{22}	f_9	f_1	f_{23}	f_3	f_{22}	f_{11}	f_3	f_{31}
PN Sequence	001	110	011	001	001	001	110	011	001	001	001	110	011	001	001	001	110	011	001	001

To determine:

i) What is the period of the PN sequence? $\boxed{2^4-1=15\ ?}$

ii) The system makes use of a form of FSK. What form of FSK is it? MFSK.

iii) What is the number of bits per symbol? $\boxed{L=2\ ?}$

iv) What is the number of FSK frequencies?

v) What is the length of a PN sequence per hop? 3

vi) Is this a slow or fast FH system? fast

vii) What is the total number of possible hops? $2^3=8$.

viii) Show the variation of the dehopped frequency with time.

[11 marks]

Answer:

i) Period of the PN sequence is $2^4 - 1 = 15$ [1 mark]
ii) MFSK [1 mark]
iii) $L = 2$ [1 mark]
iv) $M = 2^L = 4$ [1 mark]
v) $k = 3$ [1 mark]
vi) fast FHSS [1 mark]
vii) $2^k = 8$ [1 mark]
viii) [4 marks, each 2 for 1 mark]

Time	0	1	2	3	4	5	6	7	8	9	10	11
Input data	0	1	1	1	1	1	1	0	0	0	1	0
Frequency	f_1		f_3		f_3		f_2		f_0		f_2	

Time	12	13	14	15	16	17	18	19
Input data	0	1	1	1	1	0	1	0
Frequency	f_1		f_3		f_2		f_2	

Question 4

a) If the received signal level for a particular digital system is -151dBW and the receiver system effective noise temperature is 1500 K, what is E_b/N_0 for a link transmitting 2400bps?

$\frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{kTR}$. $S \rightarrow -151\text{dBW}$.

[2 marks]

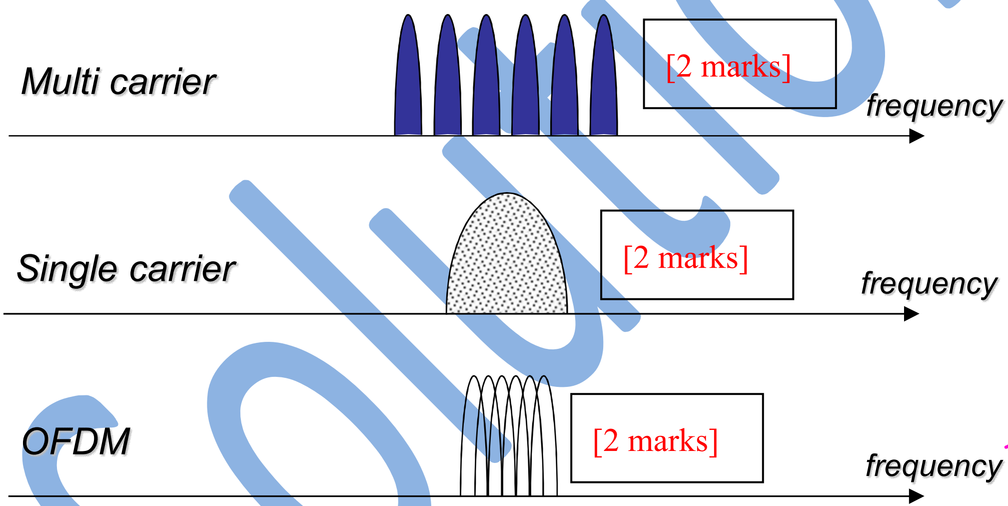
Answer:

$(E_b/N_0) = S_{dBw} - 10\log R - 10\log K - 10\log T$ **[1 mark]** $= -151$ dBW – 10 log 2400 – 10 log 1500 + 228.6 dBW = 12 dBW **[1 mark]**

b) Using diagrams and engineering terms to compare for same data rate transmission by using single carrier, multi-carrier and OFDM modulations, respectively.

[11 marks]

Solutions:



FDMA.

OFDM is multi carrier modulation [1 mark]

OFDM sub-carrier spectrum is overlapping [1 mark]

In FDMA, band-pass filter separates each transmission [1 mark]

In OFDM, each sub-carrier is separated by DFT because carriers are orthogonal [1 mark]

Each sub-carrier is modulated by PSK, QAM [1 mark]

phase shift keying.

c) Derive the power spectral density (PSD) equation for the polar NRZ signalling.

[12 marks]

[3 marks]

ANS: For polar NRZ signaling, the possible le
are +A and −A. For equally likely occurr
−A, and assumming the data are indepe
bit. $R(0) = \sum_{i=1}^{2} (a_n a_n) P_i = A^2 \frac{1}{2} + (-A)^2$

[2 marks]

[2 marks]

For $k \neq 0$,

$$R(k) = \sum_{i=1}^{4} (a_n a_{n+k}) P_i = A^2 \frac{1}{4} + (-A$$

[2 marks]

$= 0$

Thus, $R_{polar}(k) = \begin{cases} A^2 & k = 0 \\ 0 & k \neq 0 \end{cases}$

[2 marks]

[3 marks]

The general expression for the PSD of a