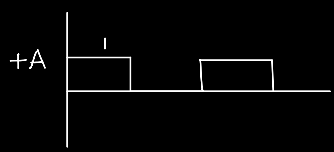
推导 PSD (五种 line-coding schemes)

$$P(f) = \frac{|F(f)|^2}{T_s}\sum_{k=-\infty}^{\infty} R(k)\, e^{\boxed{j2\pi}kfT_s}$$

$$R(k) = \sum_{i=1}^{l} (a_n a_{n+k})_i P_i$$

① Unipolar NRZ



$k=0$: $1\ 1 \rightarrow A\cdot A = A^2,\ P=\frac{1}{2}$

$0\ 0 \rightarrow 0\cdot 0 = 0,\ P=\frac{1}{2}$

$R(0) = \frac{1}{2}A^2 + 0 = \frac{1}{2}A^2$

$k\neq 0$: $1\ 1 \rightarrow A\cdot A = A^2 \quad P=\frac{1}{4}$

$0\ 1 \rightarrow 0\cdot A = 0 \quad P=\frac{1}{4}$

$1\ 0 \rightarrow A\cdot 0 = 0 \quad P=\frac{1}{4}$

$0\ 0 \rightarrow 0\cdot 0 = 0 \quad P=\frac{1}{4}$

$R(k) = \frac{1}{4}A^2$

$$R(k) = \begin{cases} \frac{1}{2}A^2, & k=0 \\ \frac{1}{4}A^2, & k\neq 0 \end{cases}$$

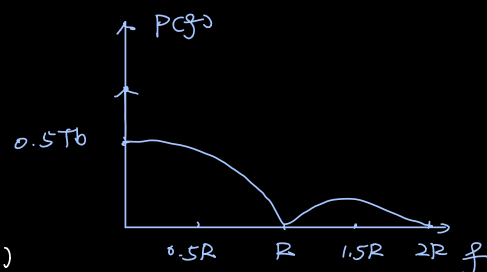
$$f(t) = \begin{cases} 1, & |t| < \frac{T_b}{2} \\ 0, & |t| > \frac{T_b}{2} \end{cases} \xleftrightarrow{\mathcal{F}(t)} \frac{2}{\omega}\sin\left(\omega\frac{T_b}{2}\right) = \frac{T_b}{\pi f T_b}\cdot \sin(\pi f T_b)$$

$$F(f) = T_b \cdot \frac{\sin(\pi f T_b)}{\pi f T_b}$$

$$P(f) = T_b \cdot \left|\frac{\sin(\pi f T_b)}{\pi f T_b}\right|^2 \sum_{k=-\infty}^{+\infty} R(k)\cdot e^{-j2\pi k f T_s}$$

$$\sum_{k=-\infty}^{+\infty} R(k)\cdot e^{-j\pi f k T_s}$$

$$= \frac{1}{4}A^2 \Big(1 + \underbrace{\sum_{k=-\infty}^{+\infty} e^{-j\pi f k T_b}}_{\displaystyle \| \atop \displaystyle \frac{1}{T_b}\cdot\sum_{k=-\infty}^{+\infty}\delta\left(f - \frac{k}{T_b}\right)}\Big)$$



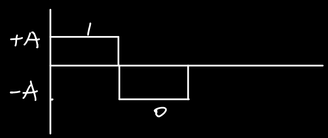
$k\neq 0,\ F(f) = 0 \Rightarrow P(f) = 0$

$$= \frac{1}{4}A^2\left(1 + \frac{1}{T_b}\cdot\delta(f)\right)$$

$$P(f) = T_b\cdot\left|\frac{\sin \pi f T_b}{\pi f T_b}\right|^2 \cdot \frac{1}{4}A^2 \left[1 + \frac{1}{T_b}\delta(f)\right]$$

$$= \frac{A^2 T_b}{4}\cdot\left|\frac{\sin(\pi f T_b)}{\pi f T_b}\right|^2\cdot\left[1 + \frac{1}{T_b}\cdot\delta(f)\right]$$

② Polar NRZ



$$P(f) = \frac{|F(f)|^2}{T_s} \sum_K R(K)\, e^{-j2\pi K f T_s}$$

$$R(K) = \sum_i (a_n a_{n+K})_i P_i$$

$R(K)$: $K=0$

1	1	$A \times A$		$\frac{1}{2}$
0	0	$-A \times -A$		$\frac{1}{2}$

$$R(0) = \frac{1}{2}A^2 + \frac{1}{2}A^2 = A^2$$

$K \neq 0$

1	1	A	A	$\frac{1}{4}$
1	0	A	$-A$	$\frac{1}{4}$
0	1	$-A$	A	$\frac{1}{4}$
0	0	$-A$	$-A$	$\frac{1}{4}$

$$R(K) = \frac{1}{4}A^2 - \frac{1}{4}A^2 - \frac{1}{4}A^2 + \frac{1}{4}A^2 = 0$$

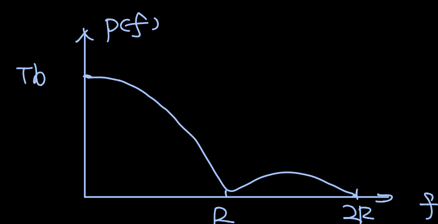
$$R(K) = 0$$

$$f(t) = \begin{cases} 1 & |t| < \frac{T_b}{2} \\ 0 & |t| > \frac{T_b}{2} \end{cases} \xleftrightarrow{\mathcal{F}} F(f) = T_b \cdot \frac{\sin(\pi f T_b)}{\pi f T_b}$$

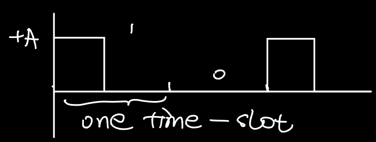
$$P(f) = T_b \cdot \left(\frac{\sin(\pi f T_b)}{\pi f T_b}\right)^2 \cdot \sum_K R(K) \cdot e^{-j2\pi K f T_b}$$

$$\sum_K R(K) \cdot e^{-j2\pi K f T_b} = A^2$$

$$P(f) = A^2 T_b \cdot \left(\frac{\sin(\pi f T_b)}{\pi f T_b}\right)^2$$



③ Unipolar RZ



$$P(f) = \frac{|F(f)|^2}{T_s} \cdot \sum_{k=-\infty}^{+\infty} R(k) \cdot e^{-j2\pi k f T_s}$$

$$R(k) = \sum_i (a_n a_{n+k})_i \, P_i$$

$R(k)$ $k=0$

1	1	$\rightarrow A \times A$	$\frac{1}{2}$
0	0	$\rightarrow 0 \times 0$	$\frac{1}{2}$

$R(0) = \frac{1}{2}A^2$

$k \neq 0$

1	1	$\rightarrow A \times A$	$\frac{1}{4}$
1	0	$\rightarrow A \times 0$	$\frac{1}{4}$
0	1	$\rightarrow 0 \times A$	$\frac{1}{4}$
0	0	$\rightarrow 0 \times 0$	$\frac{1}{4}$

$R(k) = \frac{1}{4}A^2$

$$R(k) = \begin{cases} \frac{1}{4}A^2 & k \neq 0 \\ \frac{1}{2}A^2 & k = 0 \end{cases}$$

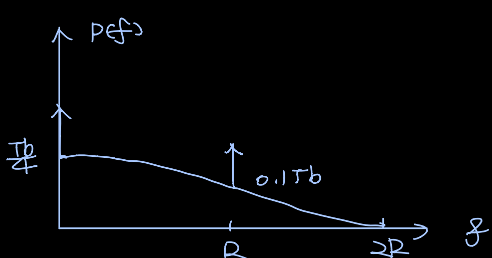
$$\sum_{k=-\infty}^{+\infty} R(k)\, e^{-j2\pi k f T_b} = \frac{1}{4}A^2 \Big(1 + \underbrace{\sum_{k=-\infty}^{+\infty} e^{-j2\pi k f T_b}}_{= \frac{1}{T_b}\sum_{k=-\infty}^{+\infty} \delta(f - \frac{k}{T_b})}\Big)$$

$$f(t) = \begin{cases} 1 & |t| < \frac{T_b}{4} \\ 0 & |t| > \frac{T_b}{4} \end{cases}$$

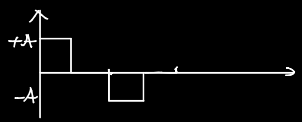
$$F(f) = \frac{2}{\omega}\sin\left(\omega \cdot \frac{T_b}{4}\right) = \frac{T_b}{\pi f T_b} = \sin\left(\pi f \frac{T_b}{2}\right) = \frac{T_b}{2} \cdot \frac{\sin(\pi f \frac{T_b}{2})}{(\pi f \frac{T_b}{2})}$$

$$P(f) = \frac{T_b}{4} \cdot \left(\frac{\sin(\frac{\pi f T_b}{2})}{\frac{\pi f T_b}{2}}\right)^2 \cdot \frac{1}{4}A^2 \left(1 + \frac{1}{T_b} \cdot \sum_{k=-\infty}^{+\infty} \delta(f - \frac{k}{T_b})\right)]$$

$$P(f) = \frac{A^2 T_b}{16} \left(\frac{\sin(\frac{\pi f T_b}{2})}{\frac{\pi f T_b}{2}}\right)^2 \cdot \left(1 + \frac{1}{T_b} \sum_{k=-\infty}^{+\infty} \delta(f - \frac{k}{T_b})\right)$$



④ Bipolar RZ



$$P(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{+\infty} R(k) \cdot e^{-j2\pi k f T_s}$$

$$R(k) = \sum_i (a_n a_{n+k})_i \, P_i$$

R(k) : $k = 0$

1	1	$A \times A$	$\frac{1}{4}$
		$(-A) \times (-A)$	$\frac{1}{4}$
0	0	0×0	0

$$R(0) = \frac{1}{4}A^2 + \frac{1}{4}A^2 = \frac{1}{2}A^2$$

$|k| = 1$

1	1	$A \times (-A)$	$\frac{1}{4}$
		$(-A) \times A$	$\frac{1}{4}$
1	0	$\longrightarrow$	0
0	1	$\longrightarrow$	0
0	0	$\longrightarrow$	0

$$R(k) = -\frac{1}{4}A^2 + (-\frac{1}{4}A^2) = -\frac{1}{2}A^2$$

k = others

1	1	$A \times A$	$\frac{1}{8}$
		$A \times (-A)$	$\frac{1}{8}$
		$(-A) \times A$	$\frac{1}{8}$
		$(-A) \times (-A)$	$\frac{1}{8}$

$$\left.\begin{matrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{matrix}\right\} \; 0$$

$$R(k) = \frac{1}{8}A^2 - \frac{1}{8}A^2 - \frac{1}{8}A^2 + \frac{1}{8}A^2 = 0$$

$$R(k) = \begin{cases} \frac{1}{2}A^2 & k=0 \\ -\frac{1}{4}A^2 & |k| = 1 \\ 0 & \text{others} \end{cases}$$

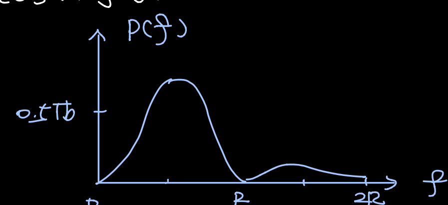
$$f(t) = \begin{cases} 1 & |t| < \frac{T_b}{4} \\ 0 & |t| > \frac{T_b}{4} \end{cases} \longleftrightarrow F(f) = \frac{T_b}{2} \cdot \left(\frac{\sin \frac{\pi f T_b}{2}}{\frac{\pi f T_b}{2}} \right)$$

$$P(f) = \frac{T_b}{4} \left(\frac{\sin \frac{\pi f T_b}{2}}{\frac{\pi f T_b}{2}} \right)^2 \underbrace{\sum_{k=-\infty}^{+\infty} R(k) \cdot e^{-j2\pi k f T_s}}_{=}$$

$$\frac{1}{2}A^2 - \frac{1}{4}A^2 (e^{-j2\pi f T_b} + e^{j2\pi f T_b})$$

$$= \frac{1}{2}A^2 (1 - \cos(2\pi f T_b))$$

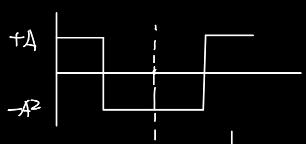
$$P(f) = \frac{A^2 T_b}{8} \left(\frac{\sin \frac{\pi f T_b}{2}}{\frac{\pi f T_b}{2}} \right)^2 (\sin^2(\pi f T_b))$$

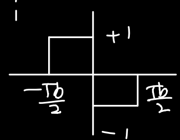


⑤ Manchester NRZ

$$P(f) = \frac{|F(f)|^2}{T_b} \cdot \sum_{k=-\infty}^{+\infty} R(k) \cdot e^{-j2\pi k f T_b}$$

$$R(k) = \sum_{\varepsilon} (a_n a_{n+k})_\varepsilon P_\varepsilon$$



$f(t) =$ 

$$F(f) = \int_{-\infty}^{+\infty} f(t) \cdot e^{-j2\pi f t}\, dt$$

$$= \int_{-\frac{T_b}{2}}^{0} e^{-j2\pi f t}\, dt + \int_{0}^{\frac{T_b}{2}} -e^{-j2\pi f t}\, dt$$

$$= -\frac{1}{j2\pi f}\left(1 - e^{-j2\pi f \cdot (-\frac{T_b}{2})}\right) + \frac{1}{j2\pi f}\left(e^{-j2\pi f (-\frac{T_b}{2})} - 1\right)$$

$$= -\frac{1}{j2\pi f}\left(2 - \underbrace{(e^{j\pi f T_b} + e^{j\pi f T_b})}_{2\cos(\pi f T_b)}\right)$$

$$= \frac{-1}{j\pi f}\left(\underbrace{1 - \cos(\pi f T_b)}_{2\sin^2\frac{\pi f T_b}{2}}\right)$$

$$= \frac{j}{\pi f} \cdot 2\sin^2\frac{\pi f T_b}{2}$$

$$= 2j \cdot \frac{T_b}{2} \cdot \frac{\sin^2\frac{\pi f T_b}{2}}{\pi f \frac{T_b}{2}}$$

$$= jT_b \cdot \frac{\sin\frac{\pi f T_b}{2}}{\pi f \frac{T_b}{2}} \cdot \sin\frac{\pi f T_b}{2}$$

$$\frac{|F(f)|^2}{T_b} = T_b \cdot \left(\frac{\sin\frac{\pi f T_b}{2}}{\frac{\pi f T_b}{2}}\right)^2 \cdot \sin^2\left(\frac{\pi f T_b}{2}\right)$$

$R(k)$: $k=0$

1	1	$A \times A = A^2$	$\frac{1}{2}$
0	0	$(-A) \times (-A) = A^2$	$\frac{1}{2}$

$$R(0) = \frac{1}{2}A^2 + \frac{1}{2}A^2 = A^2$$

$k \neq 0$

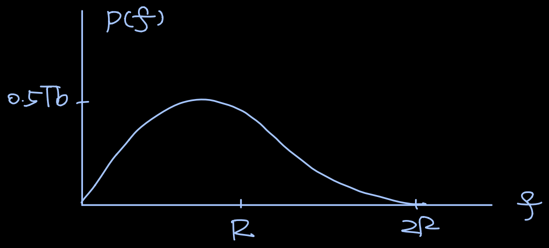
1	1	A^2	$\frac{1}{4}$
0	1	$-A^2$	$\frac{1}{4}$
1	0	$-A^2$	$\frac{1}{4}$
0	0	A^2	$\frac{1}{4}$

$$R(k) = \begin{cases} A^2, & k=0 \\ 0, & k \neq 0 \end{cases}$$

$$R(k) = \frac{1}{4}A^2 - \frac{1}{4}A^2 - \frac{1}{4}A^2 + \frac{1}{4}A^2 = 0$$

$$P(f) = T_b \cdot \left(\frac{\sin \frac{\pi f T_b}{2}}{\frac{\pi f T_b}{2}} \right)^2 \cdot \sin^2\left(\frac{\pi f T_b}{2}\right) \cdot \frac{\sum_K R(K) \cdot e^{-j2\pi K f T_b}}{A^2}$$

$$P(f) = A^2 T_b \left(\frac{\sin \frac{\pi f T_b}{2}}{\frac{\pi f T_b}{2}} \right)^2 \cdot \sin^2\left(\frac{\pi f T_b}{2} \right)$$



Power spectra for multilevel polar NRZ signal.

$L = 8$ - level , $l = 3$ - bit

$$P(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{+\infty} R(k) \cdot e^{-j2\pi k f T_s} \qquad (T_s = 3T_b \text{ in this case})$$

DAC code table

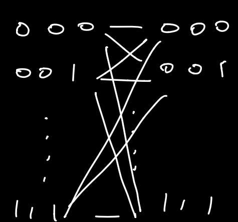
000	+7	100	-1
001	+5	101	-3
010	+3	110	-5
011	+1	111	-7

$R(k)$: $k=0$

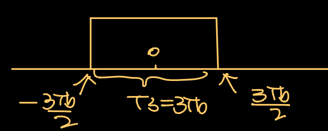
000	000	$+7 \times +7$	$\frac{1}{8}$
001	001	$+5 \times +5$	$\frac{1}{8}$
⋮			⋮
111	111	$-7 \times (-7)$	$\frac{1}{8}$

$$R(k) = \frac{1}{8}(7^2+5^2+3^2+1^2+1^2+3^2+5^2+7^2) = 21$$

$k \neq 0$



$R(k) = 0$



$$f(t) = \begin{cases} 1 & |t| < \frac{3T_b}{2} \\ 0 & \text{others} \end{cases}$$

$$R(k) = \begin{cases} 21 & k=0 \\ 0 & k \neq 0 \end{cases}$$

$$F(f) = \frac{2}{w}\sin\left(w \cdot \frac{3T_b}{2}\right) = \frac{1}{\pi f} \cdot \sin(3\pi f T_b) = 3T_b \cdot \frac{\sin(3\pi f T_b)}{3\pi f T_b}$$

$$P(f) = \frac{9T_b^2}{3T_b} \cdot \left(\frac{\sin 3\pi f T_b}{3\pi f T_b}\right)^2 \cdot \underbrace{\sum_{k=-\infty}^{+\infty} R(k) \cdot e^{-j2\pi k f T_s}}_{= 21}$$

$$P(f) = 63T_b \cdot \left(\frac{\sin 3\pi f T_b}{3\pi f T_b}\right)^2$$

Bandwidth : $\frac{R_b}{3}$.
(first null)