EBU6018 Advanced Transform Methods

Class Test 22nd October 2020

45 Minutes, Closed Book, Calculators can be used. Write your answer under the question.

Student Name (pinyin):

Student Signature (Chinese):

BUPT Number: **QMUL Number:**

Question 1

Does the following set of three vectors form an orthonormal basis for R^3? Show all your working.

$$v_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \quad v_3 = \begin{bmatrix} \frac{-1}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix}$$

Answer:

$$< v_1, v_1 > = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ 0 \end{bmatrix} = 1 \qquad < v_2, v_2 > = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} = 1 \qquad < v_3, v_3 > = \begin{bmatrix} \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \frac{-1}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix} = 1$$

$$< v_1, v_2 > = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} = 0 \qquad < v_1, v_3 > = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} \frac{-1}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix} = 0 \qquad < v_2, v_3 > = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \frac{-1}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix} = 0$$

So the vectors form an orthonormal basis for R^3

[Total 7 marks: 1 for each dot product and 1 for conclusion]

Question 2

Estimate the approximate number of computations required to perform an 8-point DFT.

An FFT is a fast algorithm for implementing a DFT.

Estimate the approximate number of computations that are required to perform the FFT of an 8-point sequence.

One FFT structure is radix-2 decimation-in-time. In the context of this structure, explain what is meant by bit reversal.

Illustrate this FFT structure using the following 8-point sequence:

S[n] = [-3, 6, -5, 7, 9, 4, 1, 2]

Answer:

For N-point DFT, number of computations is O($N^2) = 64$. [1 mark]

For N-point FFT, number of computations is O(N$log_2 N) = 8x3 = 24$ [1 mark]

Bit reversal means that the order of the sequence elements in the output is the order of the input sequence elements with the bits of the element position reversed.
[1 mark]

Input sequence: [-3, 6, -5, 7, 9, 4, 1, 2]

Re-order even and odd positions: [-3, -5, 9, 1][6, 7, 4, 2]

Again: [-3, 9][-5, 1][6, 4][7, 2]

Again for FFT: [-3][9][-5][1][6][4][7][2] [3 marks: 1 for each row]

Example of bit reversal: take the element in input position 001 [6]

take the element in output position 100 [6]

[1 mark for any correct example]

[Total 7 marks]

Question 3

The 1D Type II DCT is defined as:

$$DCT[k] = c(k)\sum_{n=0}^{N-1} s[n]\cos\frac{\pi(2n+1)k}{2N} \qquad c(k)=\begin{cases}\sqrt{1/N} & k=0\\ \sqrt{2/N} & k\neq 0\end{cases}$$

Calculate the coefficients of a 4x4 DCT transform matrix and hence evaluate the DCT of the 4-point input sequence:

s[n] = [6, -2, 3, 5]

Answer:

1-Dimensional DCT

Example. N = 4 (input is a 4-point sequence)

For each value of k = 0….N-1, insert n = 0….N-1:

$$\psi_0 = (1,1,1,1)/2$$

$$\psi_1 = \sqrt{1/2}(\cos(\pi/8),\cos(3\pi/8),\cos(5\pi/8),\cos(7\pi/8))$$

$$\psi_2 = \sqrt{1/2}(\cos(\pi/4),\cos(3\pi/4),\cos(5\pi/4),\cos(7\pi/4))$$

$$\psi_3 = \sqrt{1/2}(\cos(3\pi/8),\cos(9\pi/8),\cos(15\pi/8),\cos(5\pi/8))$$

$$DCT[0] = \frac{1}{\sqrt{N}}\sum_{n=0}^{3} s[n] \qquad DCT[1] = \sqrt{\frac{2}{N}}\sum_{n=0}^{3} s[n]\cos\frac{\pi(2n+1)}{8}$$

$$DCT[2] = \sqrt{\frac{2}{N}}\sum_{n=0}^{3} s[n]\cos\frac{\pi(2n+1)}{4} \qquad DCT[3] = \sqrt{\frac{2}{N}}\sum_{n=0}^{3} s[n]\cos\frac{\pi(2n+1)3}{8}$$

EBU6018 9

4x4 DCT Basis Matrix

$$\Psi=\begin{bmatrix}0.50 & 0.50 & 0.50 & 0.50\\ 0.65 & 0.27 & -0.27 & -0.65\\ 0.50 & -0.50 & -0.50 & 0.50\\ 0.27 & -0.65 & 0.65 & -0.27\end{bmatrix}$$

EBU6018 12

$$\text{DCT} = \begin{bmatrix}0.50 & 0.50 & 0.50 & 0.50\\ 0.65 & 0.27 & -0.27 & -0.65\\ 0.50 & -0.50 & -0.50 & 0.50\\ 0.27 & -0.65 & 0.65 & -0.27\end{bmatrix}\begin{bmatrix}6\\ -2\\ 3\\ 5\end{bmatrix} = \begin{bmatrix}6\\ -0.7\\ 5\\ 3.52\end{bmatrix}$$

[1 mark for each row of cosine form of basis functions,

1 mark for each row of numerical form of basis function,

2 marks for correct answer.   Total 10 marks]