

# Histograms

Histogram: maps the **quantization levels** into the frequency of each quantization level in the image

Histogram → pixel density distribution

Normalized histogram → probabilities of intensity levels

Histogram applications:

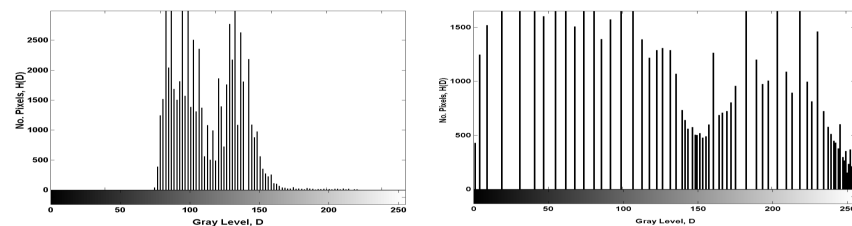
- Over and under exposure
- Brightness
- Contrast
- Dynamic range

Contrast: difference between average grey level of an object and surroundings

Brightness range: brightness span of the grey scale of an image

Low contrast → concentrated near a narrow range

Higher contrast & brightness range → change the intensity value distribution to cover a wide range

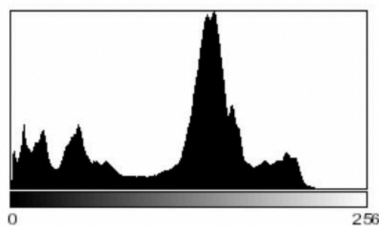


Exposure

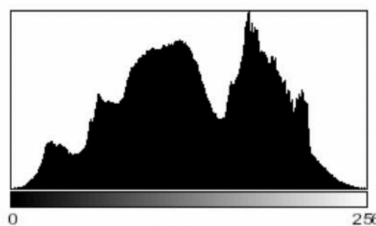
Underexposed: intensity values bunched up at dark areas

Properly exposed: spread out

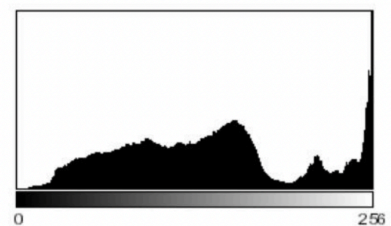
Overexposed: intensity values spread



Underexposed



Properly  
Exposed



Overexposed

Histogram 性质

- **many-one-mapping** (non-invertible)
- Invariant to geometric image (certain): rotation, ...

## Thresholding

For an object on a contrasting background: bimodal (two peaks)

Area of the segmented object: area A of objects

$$A = \sum_{D=0}^T H(D)$$

## Histogram equalization (intensity的转换, pixel value)

To map grey levels of an image into a target image with an even distribution of grey levels  
Transformation of probability density

### Continuous histogram equalization

#### Continuous histogram equalization

- Suppose we have a transformation  $g = T(f)$
- Let  $p(f)$  and  $p(g)$  be the densities (histograms) of  $f$  and  $g$
- If  $T(f)$  is a monotonic function, then  $p(f)df = p(g)dg$
- Set  $p(g) = 1$  to make the output distribution uniform
- Then  $p(f) = dg/df$  from before. We want to find  $g = T(f)$
- Integrate both sides:  $g = \int_0^f p(x)dx = P(f)$
- Where  $P(f)$  is the cumulative distribution function of  $f$
- If  $p(f)$  is high, then  $P(f)$  is steep so  $dg$  is wide and  $p(g)$  is low
- If  $p(f)$  is low, then  $P(f)$  is shallow so  $dg$  is narrow and  $p(g)$  is high
- The histogram is evened-out, so that  $P(g)$  is linear

### Discrete histogram equalization

The discrete version of the transformation function (in continuous case) is:

$$\begin{aligned} s_k &= T(r_k) \\ &= (L-1) \sum_{j=0}^k p_r(r_j) \end{aligned}$$

where,  $r_k = 0, 1, \dots, L-1$

$p_r$  — the normalized histogram

### Color image histogram

- Intensity histogram: convert color to grey scale image and display histogram of the grey image
- Individual color histogram

## Practice

a) This question is about **image histograms**.

[11 marks]

i) Why do we use histograms in image processing? Explain in your own words.

Also, how do the image intensity values appear in histograms for low contrast images? Sketch a histogram for a low-contrast image.

(6 marks)

ii) Consider the greyscale chess-board image in **Figure 1** (made with black and white squares).



Figure 1

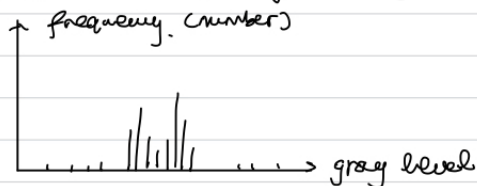
Draw a labelled sketch of the image histogram, where the intensity value 0 represents black and the intensity value 255 represents white.

Now, consider rotating the chess-board image (**Figure 1**) by 90 degrees. What effect, if any, would this have on the histogram? Explain your answer.

(5 marks)

(1) The histogram encapsulates many information such as contrast, brightness, whether the image is over/under exposure and so on which is significantly useful for image processing task such as image segmentation/equalization. The number of the intensity values only concentrate in a narrow range of intensity values.

Low-contrast image histogram:



The histogram will remain the same, since the rotation does not change the pixel value of the pixel, which will not change the gray level distribution.

a) This question is about image histograms.

[15 marks]

i) Why do we use histograms in image processing? Explain in your own words.

Also, how do the image intensity values appear in histograms for good contrast images? Sketch a histogram for a good-contrast image.

(6 marks)

ii) Figure 1 shows a histogram of an image that can be used to reveal the image contrast. What does this histogram reveal about the image?

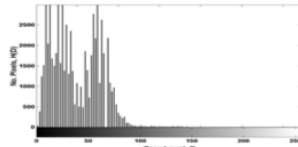


Figure 1

(2 marks)

iii) Use the information from the image file shown in Figure 2 to compute the frequency of each pixel and draw the corresponding histogram.

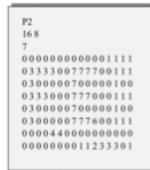


Figure 2

(5 marks)

iv) How do you represent a colour image histogram using the intensity histogram?

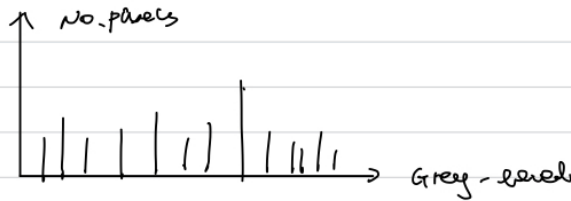
(2 marks)

(2) This is a dark image with low contrast. It is also under-exposure. Since the values are concentrated narrowly in the dark level (regions).

(3) 0 : 81      4 : 2  
1 : 18      5 : 0  
2 : 1      6 : 1  
3 : 13      7 : 12



(1) ... Good contrast image : the intensity value distributed in a wide range,



(4) First convert the color image to the gray scale image, then create the histogram of the gray scale image