

## Interest points

### Properties of good features

- Local: robust to occlusion and clutter
- Accurate: precise localization
- Covariant
- Robust: noise, blur, compression
- Efficient: close to real-time performance

### Use interest point to build panorama

1. Detect **corners** in both images
2. Find corresponding corner pairs by comparing the corner descriptors
3. Use these pairs to align images

## Moravec operator

Given a grayscale image  $I(x,y)$

- Centre a 3x3 pixel array on each pixel in turn
- Define another array one pixel up to the right of the first array
- Calculate the **absolute differences** between the values of the corresponding pairs of pixels in the two arrays
- **Sum all these absolute differences values (9 values)**
- Repeat for arrays moved one pixel relative to the initial array up, down, right, left, diagonally up left, diagonally down left and diagonally down right.
- Take the **minimum** of these 8 values and this is the new value for the pixel in the Moravec image  $M(i,j)$
- Threshold the new image  $M(i,j)$  to isolate corners in the image

### Moravec operator features:


1. Maximal in pixels with **high contrast**
2. **Simple** implementation
3. **Diagonal edges** will falsely be detected as corners
4. Computationally **efficient** —> works well for real-time applications

### Issue:

1. Use a discrete **rectangular window**
2. Use a simple **min** function

### Example:

[Week8/9-Quiz] Given an image below, apply the Moravec operator to the centre and type the value of the Moravec image

  $\begin{bmatrix} 10 & 10 & 20 \\ 15 & 10 & 20 \\ 20 & 10 & 20 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 5 & 5 \end{bmatrix} - 0 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 5 & 5 \end{bmatrix} \Rightarrow 20$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 5 & 5 \end{bmatrix} - 0 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 5 & 5 \end{bmatrix} \Rightarrow 10$

☐ 10

☐ 15

☒ 1

☐ 20

☐ 20

☐ 10

☐ 20


$\min \begin{bmatrix} 10 & 10 & 20 \\ 15 & 10 & 20 \\ 20 & 10 & 20 \end{bmatrix} = 10$

If threshold  $\geq 5$ , then it is a corner ( $10 > 5$ )

## Harris corner detector (good for rotation)

Basic idea, should easily recognize the point by looking through a small window, **shifting** a window **in any direction** should give a **large change in intensity**

Change of intensity for the shift  $[u, v]$ :

$$E(u, v) = \sum_{x, y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$


Window function: 1 in window, 0 outside window (rectangular)

$$E(u, v) \cong [u, v] M \begin{bmatrix} u \\ v \end{bmatrix} \quad M = \sum_{x, y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Measure of corner response:

$$R = \det M - k (\text{trace } M)^2$$

$$\det M = \lambda_1 \lambda_2$$

$$\text{trace } M = \lambda_1 + \lambda_2$$

( $k$  – empirical constant,  $k = 0.04-0.06$ )

Classification:

- $R > 0 \rightarrow$  corner ( $\lambda_1, \lambda_2$  are large)
- $R < 0 \rightarrow$  edge ( $\lambda_1 \gg \lambda_2$  or  $\lambda_1 \ll \lambda_2$ )
- $|R|$  is small  $\rightarrow$  flat region ( $\lambda_1, \lambda_2$  are small)

Procedure:

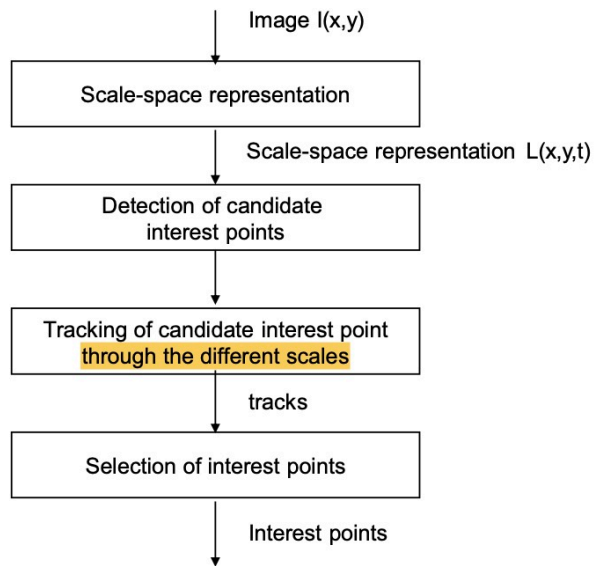
1. Calculate  $I_x, I_y, I_x I_y \rightarrow M$
2. Calculate  $R$  according to  $\lambda_1 \lambda_2 - k (\lambda_1 + \lambda_2)^2$
3. Corner point  $\rightarrow$  large positive  $R$

(Note: 注意  $I_x I_y$  相乘是指对应元素相乘,  $I_x I_x, I_y I_y$  同理)

Evaluation of interest points

- Repeatability: **stability** of interest points when the view point **changes** or **geometric transformations**
- Accuracy: high **accuracy in localization** (the position of the interest point), compare the estimated interest point and real interest point

Improvements on Moravec —> **scale-space approach**



**Criteria: life-span of a track**

Repeatability evaluation: (# of interest points repeated) / (total number of interest points)  
Rotate the image, interest points should always be identified —> not influenced by geometric change

Accuracy: rotate the objects, measure the position. Calculate the error between positions  
use RMSE

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{n=1}^N [(x_{n0} - x_n)^2 + (y_{n0} - y_n)^2]}$$

## Practice

b) This question is about **Interest Points**.

[7 marks]

- i) Give the Harris matrix  $M$  and the corner response measure  $R$  in mathematical terms for the Harris detector with image  $f(x, y)$ .

(4 marks)

- ii) Fill the following table in the three cases of  $R$  for the classification of image points.

(3 marks)

corner response measure $R$	classification of image points
$ R  < 1$	
$R \leq -1$	
$R \geq 1$	

(1)

$$M = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$R = \det(M) - k \cdot (\text{trace}(M))^2 \quad k \text{ usually ranges in } (0.04, 0.06)$$

$$= \lambda_1 \lambda_2 - k \cdot (\lambda_1 + \lambda_2)^2$$

(2)

Flat region
Edge
Corner

b) This question is about **Interest Points**.

[7 marks]

- i) By using Harris detector with  $3 \times 3$  window of equal weighting, the empirical constant  $k = 0.05$ , and differentiation kernel below ( $d/dx$  and  $d/dy$ ), 1) find the Harris matrix, and 2) the corner response for the centre of the following image  $I$ , and 3) determine whether the point is flat, edge, or corner:

(7 marks)

$$d/dx = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \quad d/dy = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 \\ 1 & 1 & 2 & 7 & 7 \\ 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 \end{bmatrix}$$

① Find the Harris matrix  $M$

$$M = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

First calculate  $I_x$ ,  $I_y$

$$I_x: \text{ apply } \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 \\ 1 & 1 & 2 & 7 & 7 \\ 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 \end{bmatrix} \Rightarrow I_x = \begin{bmatrix} -5 & -4 & 1 \\ 5 & 4 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

calculate this part

$$6 \times 0 + 6 \times (-1) + 1 \times 1 = -5$$

$$6 \times 0 + 6 \times (-1) + 2 \times 1 = -4$$

$$I_y: \text{ (same calculation)} \Rightarrow I_y = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$I_x^2 = (-5)^2 + (-4)^2 + 1^2 + 5^2 + 4^2 + (-1)^2 = (25 + 16 + 1) \times 2 = 84$$

↑  
各元素平方再相加

$$I_y^2 = 1^2 + 5^2 = 26$$

$$I_x I_y = 5 \times 1 + 4 \times 5 = 25$$

$$M = \begin{bmatrix} 84 & 25 \\ 25 & 26 \end{bmatrix}$$

$$\textcircled{2} R = \lambda_1 \lambda_2 - k \cdot (\lambda_1 + \lambda_2)^2$$

$$\lambda_1 \lambda_2 = 84 \times 26 - 25 \times 25 = 1559$$

$$\lambda_1 + \lambda_2 = 84 + 26 = 110$$

$$R = 1559 - 0.05 \cdot 110^2 = 954$$

③  $R$  is relatively big and  $R > 0 \Rightarrow$  this is a corner

$$\begin{cases} \lambda_1 \lambda_2 = |M| \Rightarrow \text{行列式} \\ \lambda_1 + \lambda_2 = \text{特征值之和} \end{cases}$$