Morphology

Morphology:
- Remove imperfections introduced during segmentation

Foreground (FG), Background (BG)
Support of Image: the set of foreground pixel locations
The complement of support —> background

SE (Structuring Element) — a small image used as a moving window

Reflected Structuring Elements

$$\tilde{s}(x, y) = s(-x, -y)$$

is the reflected structuring element.

$\tilde{s}$ is s rotated by 180º around its origin.

Fitting, Hitting

Fundamental Operations
- Erosion
- Dilation

Erosion

$$f \ominus s$$

$$g(x, y) = \begin{cases} 1 \text{ if } s \text{ fits } f \\ 0 \text{ otherwise} \end{cases}$$

Erosion 作用:
1. **Split** joined objects (separate touching objects)
2. Strip away **extrusions**
3. **Remove small spurious bright spots (salt noise)**

Dilation

$$f \oplus s$$

$$g(x, y) = \begin{cases} 1 \text{ if } s \text{ hits } f \\ 0 \text{ otherwise} \end{cases}$$

Dilation作用:
1. **Repair** breaks
2. Repair **intrusions**
3. **Remove small spurious holes (pepper noise)**

Erosion from Dilation / Dilation from Erosion

$$f \oplus s = f^c \ominus \tilde{s}$$

Compound Operations
- Opening
- Closing

Opening

$$f \circ s = (f \ominus s) \oplus s$$

Closing

$$f \bullet s = (f \oplus s) \ominus s$$

Morphological Algorithms
- Boundary extraction
- Region filling

Boundary Extraction (2)

$$\beta(A) = A - (A \ominus B)$$

$$(A \oplus B) - A$$

Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1,2,3.....$$

X0: simple starting point <u>inside the boundary</u>
B: SE
A: the original boundary
Stop criteria: until $X_k = X_{k-1}$

Binary reconstruction: used after opening to grow back pieces of the original image that are connected to the opening (not important)

Practice

c) This question is about **Image Morphology**.

[6 marks]

i) By using erosion and dilation operators, give the mathematical representation of morphological opening and morphological closing for the structuring element *S* and image *M*.

(3 marks)

ii) By using erosion and dilation operators, give the mathematical representation of morphological inside and outside boundary extraction for the structuring element *S* and image *M*.

(3 marks)

(1) Declaration for signatures:

$\circ$: opening.

$\bullet$: closing

$\oplus$: Dilation $g(x,y) = \begin{cases} 1, & \text{if } S \text{ hits } M \\ 0, & \text{otherwise} \end{cases}$

$\ominus$: Erosion $g(x,y) = \begin{cases} 1, & \text{if } S \text{ fits } M \\ 0, & \text{otherwise} \end{cases}$

Morphological opening: $M \circ S = (M \ominus S) \oplus S$

Morphological closing: $M \bullet S = (M \oplus S) \ominus S$

(2) $\beta(M)$ represent the boundary for M:

① using erosion $\ominus$:

$\beta(M) = M - (M \ominus S)$

② using dilation $\oplus$:

$\beta(M) = (M \oplus S) - M$

c) This question is about **Image Morphology**.

[8 marks]

i) Give the mathematical representation of morphological erosion and morphological dilation for the structuring element S and image M.

(2 marks)

ii) Show the relationship between erosion and dilation.

(2 marks)

iii) By using the following structuring element S, apply erosion and dilation operators to the image M and give the results.

$$S = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(4 marks)

只需验证有1的地方与M有无“1”

(1) morphological erosion: $M \ominus S = \begin{cases} 1 & \text{if } S \text{ fits } M \\ 0 & \text{otherwise} \end{cases}$

morphological dilation: $M \oplus S = \begin{cases} 1 & \text{if } S \text{ hits } M \\ 0 & \text{otherwise} \end{cases}$

(2) The relationship between erosion and Dilation

$\tilde{S}$ means the reflected of the structure element S (rotation by 180°)

$M \oplus S = M^c \ominus \tilde{S}$ ⟺

Dilation with the reflected SE of the complement is the complement of erosion.

Erosion with the reflected SE of the complement is the complement of dilation.

(3) ① Errosion:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

② Dilation:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$