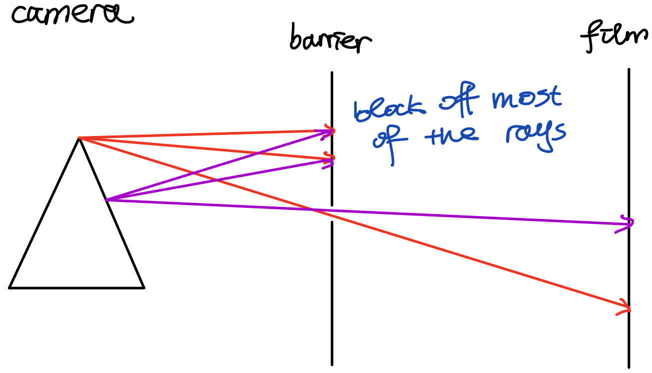
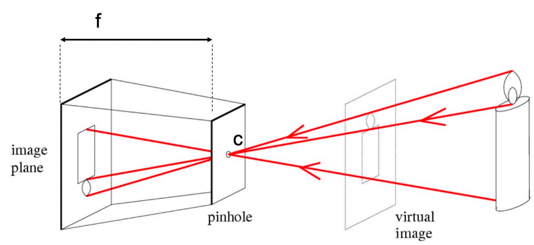
Pinhole camera



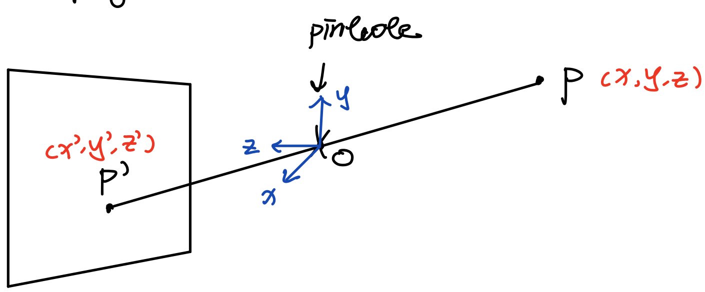
Capture pencil of rays

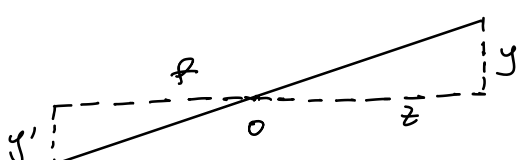
- all rays through a single point



f = focal length
c = center of the camera

Pinhole projection model.





$f : y' = z : y$

$y' = f \cdot \frac{y}{z}$

$x' = f \cdot \frac{x}{z}$

Relationship :

$(x, y, z) \rightarrow (f\frac{x}{z}, \ f\frac{y}{z})$

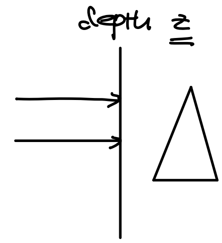
focal length is large ⇒ point has large value

Dimensionality reduction (3D → 2D image)

- preserved : straight lines , incidence
- not preserved : angles , lengths

Fronto – parallel planes

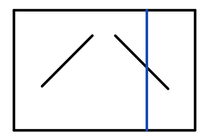
- All points are at fixed depth z
- scaled $\frac{f}{z}$. angles & ratios of length / region preserved

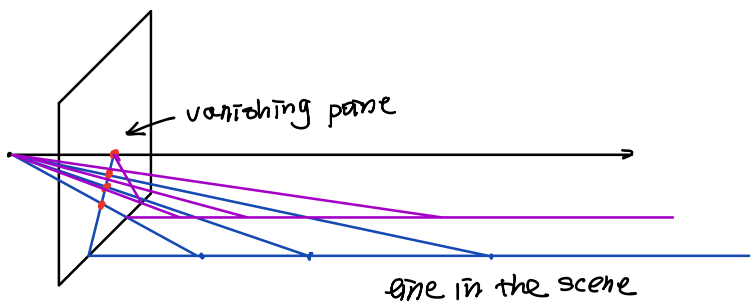


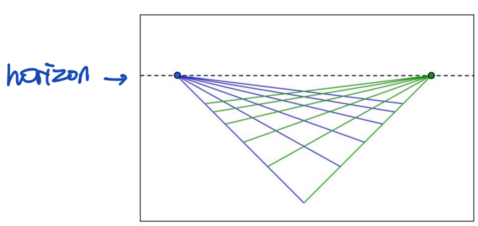
Vanishing points

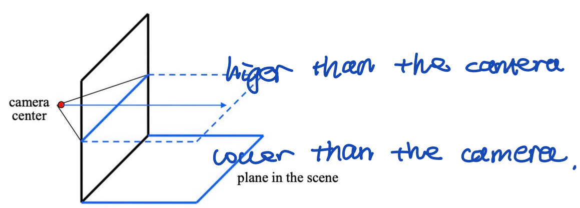
All parallel lines converge to a vanishing point

Exception : directions parallel to the image plane

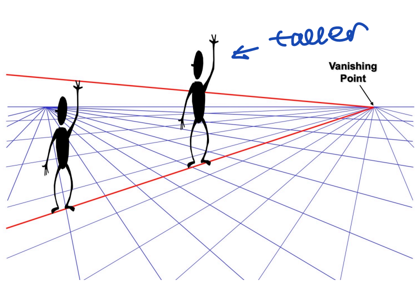








Comparing heights / Measure heights



Homogeneous coordination

- Nonlinearity : $(x, y, z) \longrightarrow \left(f\frac{x}{z},\ f\frac{y}{z}\right)$

division z is non-linear

$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ (image) $\qquad (x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ (scene)

$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow \left(\frac{x}{w}, \frac{y}{w}\right) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow \left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w}\right)$

- invariant to scaling

$$k\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \end{bmatrix}$$

Perspective Projection Matrix.

Projection: a matrix multiplication with homogeneous coordinates

$$\left(\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \Rightarrow \left(f\frac{x}{z},\ f\frac{y}{z}\right)$$

$\Downarrow$

projection matrix

- Orthographic projection (parallel projection)

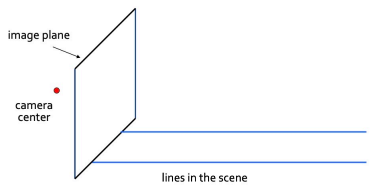
$$\left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Quiz-01) 3D-to-2D

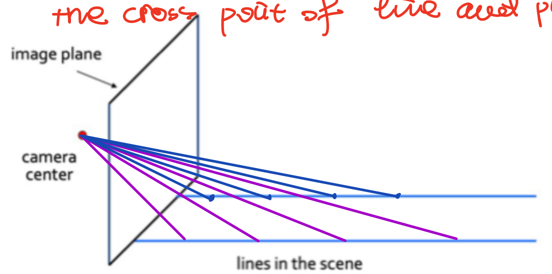
- When the 3D world dimension reduces to 2D, what properties are preserved?
  - Straight lines
  - Incidence
  - Angles
  - Lengths

Quiz-02) Constructing the vanishing point of a line

- In the figure below, there are two blue parallel lines, perpendicular to the image plane. Draw how the lines look like in the image plan and discuss the result

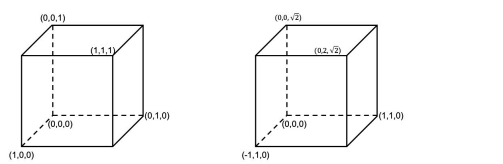


Q : How to determine which point is the cross point of line and plane?

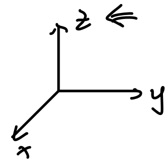


Quiz-03) Orthographic projection

- Perform Orthograpic projection to each 3D cube in 3D below and show the result in 2D



$z \Leftarrow$ from z look to the image



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow (0,0)$$

$(1,1,1) \Rightarrow (1,1)$

$(0,0,0) \Rightarrow (0,0)$

$(1,0,0) \Rightarrow (1,0)$

$(0,1,0) \Rightarrow (0,1)$

$(0,0,\sqrt{2}) \Rightarrow (0,0)$

$(0,2,\sqrt{2}) \Rightarrow (0,2)$

$(-1,1,0) \Rightarrow (-1,1)$

$(1,1,0) \Rightarrow (1,1)$

Quiz-04) Projection matrix (2D-to-2D) ✓

- Given two point sets:
  - $\boldsymbol{x} = \{x_1, \dots, x_4\} = \{(u_1, v_1), \dots, (u_4, v_4)\} = \{(0,260), (640,260), (0,400), (640,400)\}$
  - $\boldsymbol{x}' = \{x'_1, \dots, x'_4\} = \{(u'_1, v'_1), \dots, (u'_4, v'_4)\} = \{(0,0), (400,0), (0,640), (400,640)\}$

Find the perspective projection matrix $\boldsymbol{P}$ such that $\boldsymbol{x}' = \boldsymbol{P}\boldsymbol{x}$

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{cases} x_1 = \frac{5}{8} \\ x_2 = 0 \\ x_3 = 0 \end{cases} \qquad \begin{cases} x_4 = 0 \\ x_5 = 4 \\ x_6 = -960 \end{cases}$$

$0 \cdot x_1 + 260 x_2 + x_3 = 0$

$640 x_1 + 260 x_2 + x_3 = 400$

$640 x_1 = 400$

$x_1 = \frac{40}{64} = \frac{5}{8}$

$$P = \begin{bmatrix} \frac{5}{8} & 0 & 0 \\ 0 & 4 & -960 \\ 0 & 0 & 1 \end{bmatrix}$$

$0 \cdot x_1 + 400 x_2 + x_3 = 0 \Rightarrow x_2 = 0$

$640 x_1 + 400 x_2 + x_3 = 400 \Rightarrow x_3 = 400 - 640 x_1 = 0$

$260 x_5 + x_6 = 0$

$640 x_4 + 260 x_5 + x_6 = 0$

$x_4 = 0$

$400 x_5 + x_6 = 640 \Rightarrow x_5 = \frac{640}{160} = 4 \qquad x_6 = -960$

$640 x_4 + 400 x_5 + x_6 = 640$

$$
\begin{bmatrix} C_{00} & C_{01} & C_{02} \\ C_{10} & C_{11} & C_{12} \\ C_{20} & C_{21} & C_{22} \end{bmatrix}
\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}
=
\begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}
$$

4 1

$$C_{00}u + C_{01}v + C_{02} = u'$$

$$C_{10}u + C_{11}v + C_{12} = v' \quad \Rightarrow \quad \frac{C_{00}u + C_{01}v + C_{02}}{C_{20}u + C_{21}v + C_{22}} = u'$$

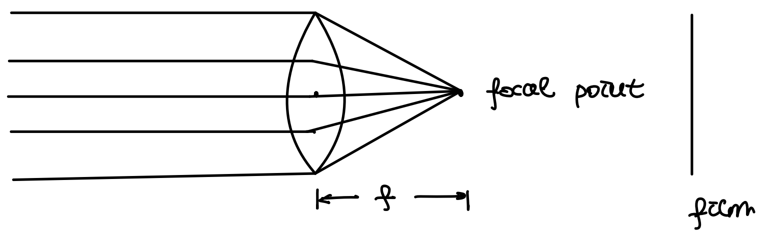
$$C_{20}u + C_{21}v + C_{22} = 1 \qquad\qquad \frac{C_{10}u + C_{11}v + C_{12}}{C_{20}u + C_{21}v + C_{22}} = v'$$

Shrinking the aperture

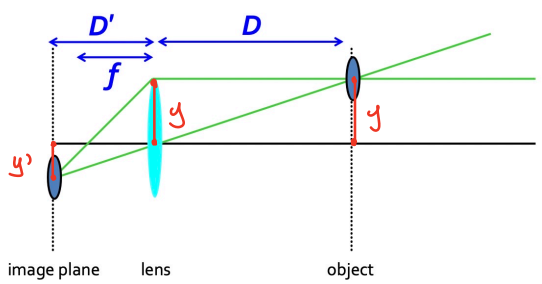
- aperture smaller ⇒ clearer image
- aperture too small : { less lights go through / diffraction effects

Thin lens model :

- rays passing through the center are not deviated
  (pinhole model still holds)
- all rays // optical axis pass through focal point
- all // rays converge to points on the focal plane



Thin lens formula

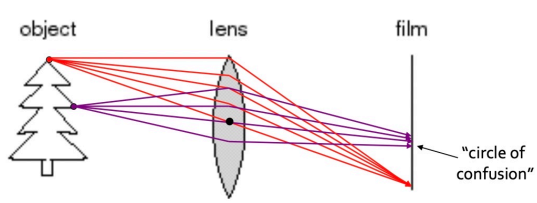


$$\begin{cases} \dfrac{y'}{y} = \dfrac{D'}{D} \\[2ex] \dfrac{y'}{y} = \dfrac{D'-f}{f} \end{cases}$$

$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

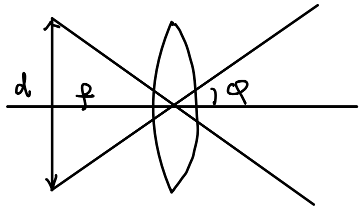
Depth of field (DoF)

- a specific distance at which objects are "in focus"



{ large aperture = small DoF / small aperture = large DoF

Field of View (FOV)



$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$

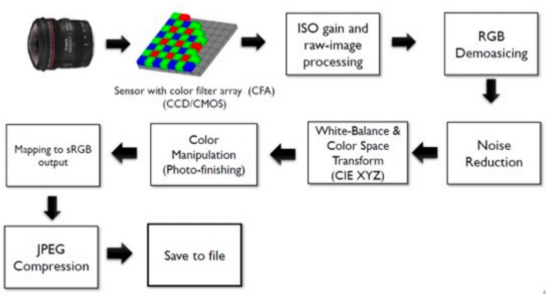
The dolly zoom

- adjusting the focal length where the camera moves away from the subject

Lens flaws

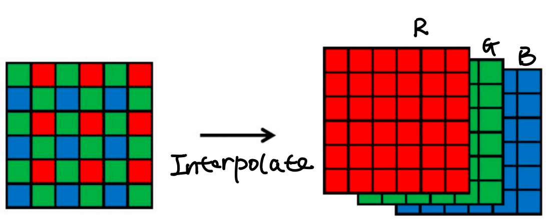
- spherical aberration
- chromatic aberration

Modern photography pipeline



Demosaicing : estimation of missing components from neighboring values

- produce full RGB image from mosaiced output

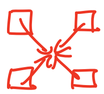
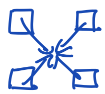


Bilinear neighboring :

① green channel

$$\begin{array}{ccc} & G_2 & \\ & \downarrow & \\ G_1 \rightarrow & G? & \leftarrow G_3 \\ & \uparrow & \\ & G_4 & \end{array} \qquad G? = \frac{G_1 + G_2 + G_3 + G_4}{4}$$

② Red / Blue channel

Digital Camera artifacts

- noise
- in-camera processing
- compression
- blooming
- color artifacts

How to remove ?

filtering

A Bayer filter mosaic is a color filter array (CFA) for arranging RGB color filters on a square grid of photosensors. Figure 1 shows a raw image obtained by the Bayer pattern CFA where each pixel includes the R or G or B value. Perform image demosaicing to the image region in orange box using bilinear interpolation and provide each pixel's RGB value, (R, G, B) of the 3×3 image.

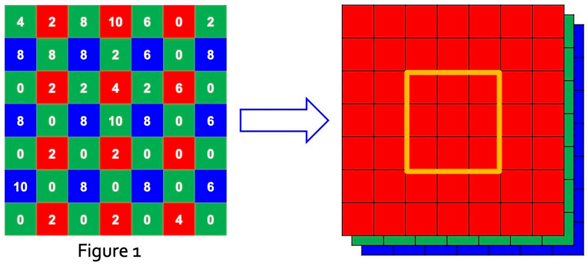


Figure 1

2 10 0
4.5 5
2 3.25 4 4.5 6
2.5 2.875 3
2 2.125 2 1.75 0
2 2
2 2 4

$$\Rightarrow \begin{bmatrix} 3.25 & 4 & 4.5 \\ 2.5 & 2.875 & 3 \\ 2.125 & 2 & 1.75 \end{bmatrix}$$

8 8 6 8
8 7.785 7.5 7.125 7
8 8 7.875 8 6
8.5 8.125 8 7.75 7
10 8 8 6

$$\Rightarrow \begin{matrix} 7.785 & 7.5 & 7.125 \\ 8 & 7.875 & 8 \\ 8.125 & 8 & 7.75 \end{matrix}$$

2
2 4 2
0 3 10 3 0
0 2.5 0
0

$$\Rightarrow \begin{matrix} 2 & 4 & 2 \\ 3 & 10 & 3 \\ 0 & 2.5 & 0 \end{matrix}$$