3D Graphics Programming Tools

Projection and Rasterisation (Revision 2)

(Go to **www.menti.com** and use the code **2516 6979** to ask me questions)

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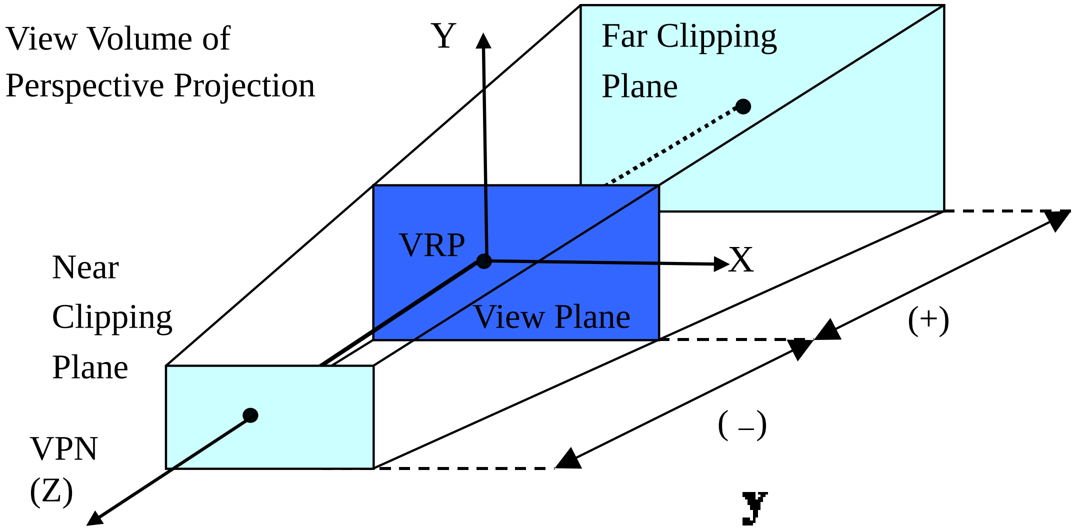
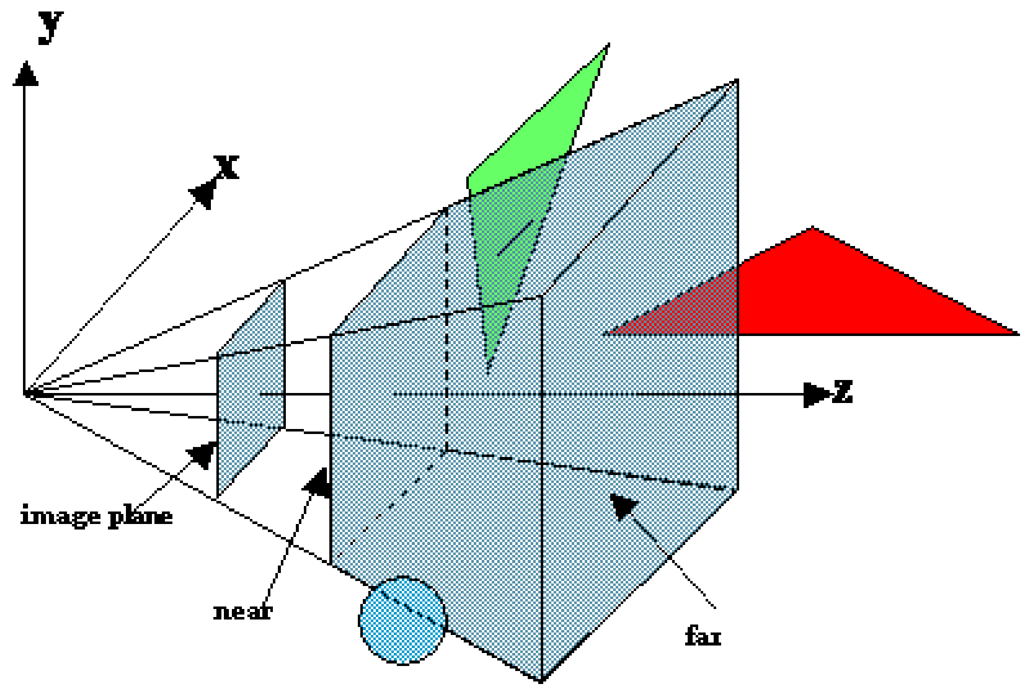
The rendering pipeline: 3D

Object geometry
↓
Modelling Transforms
↓
Lighting Calculations
↓
Viewing Transform
↓
Clipping
↓
Projection Transform
↓
Rasterisation

Clipping: remove geometry that is out of view

View Volume and 3D Clipping





The rendering pipeline: 3D

Object geometry

↓

Modelling Transforms

↓

Lighting Calculations

↓

Viewing Transform

↓

Clipping

↓

Projection Transform

↓

Rasterisation

Projection:

View reference coordinates (VRC) → screen coordinates

(3D coordinates → 2D coordinates)

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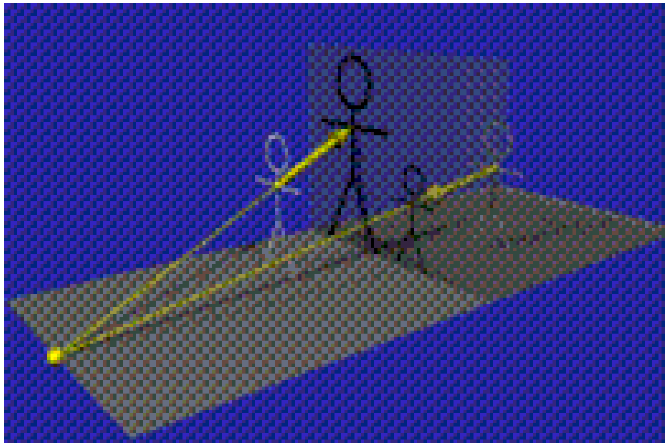
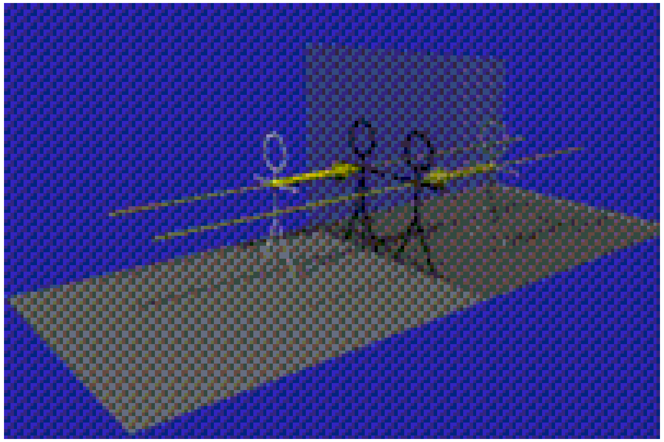
Projection Transformations

- Perspective camera
  (Perspective projection)



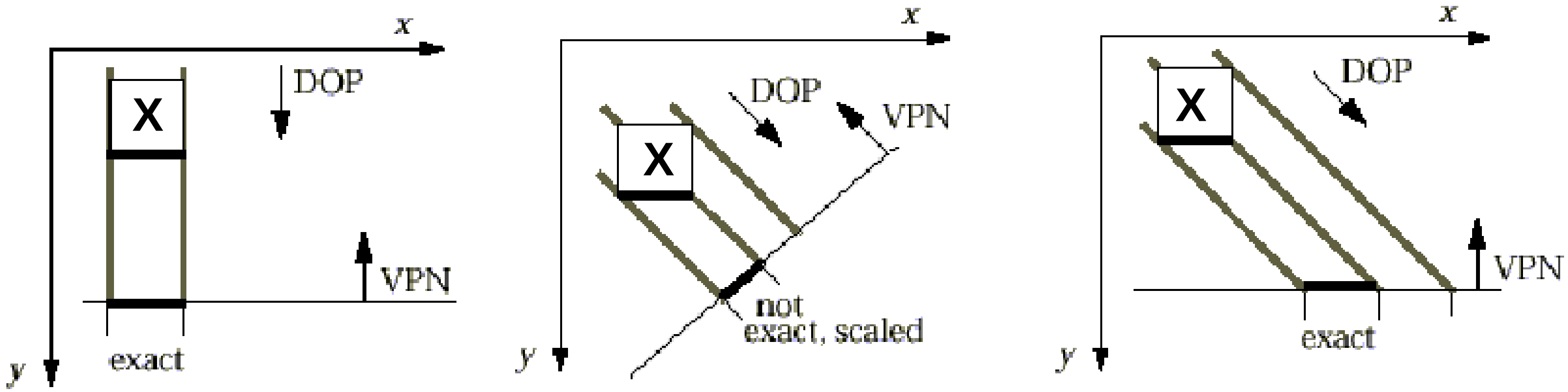
- Orthographic camera
  (Parallel projection)



Parallel Projections

Generally, we wish that the object face of interest lies in principal plane, i.e., parallel to *xy*, *yz*, or *zx* planes.

DOP = Direction of Projection, VPN = View Plane Normal



Multiview Orthographic
– VPN is parallel to a principal coordinate axis
– DOP is parallel to VPN
– shows single face, exact measurements

Axonometric
– VPN is NOT parallel to a principal coordinate axis
– DOP is parallel to VPN
– adjacent faces, none exact, uniformly foreshortened (as a function of angle between

Oblique
– VPN is parallel to a principal coordinate axis
– DOP is NOT parallel to VPN
– adjacent faces, one exact, others uniformly foreshortened

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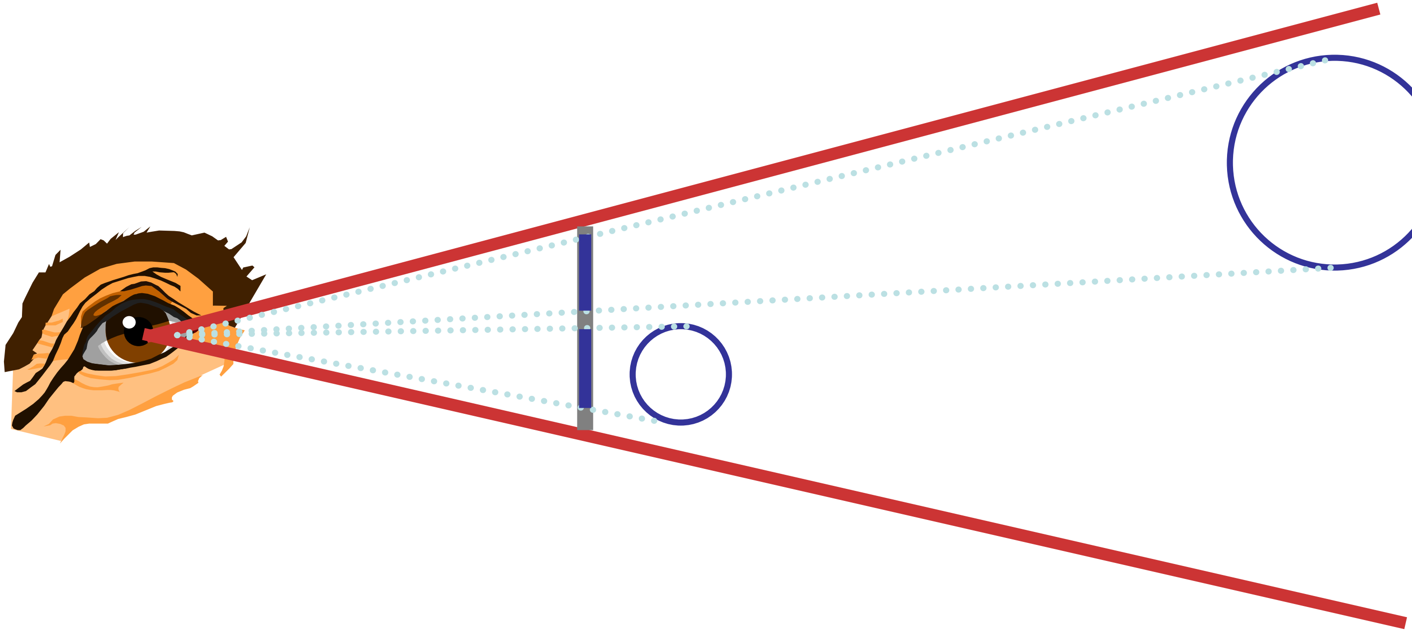
Projection Transformation

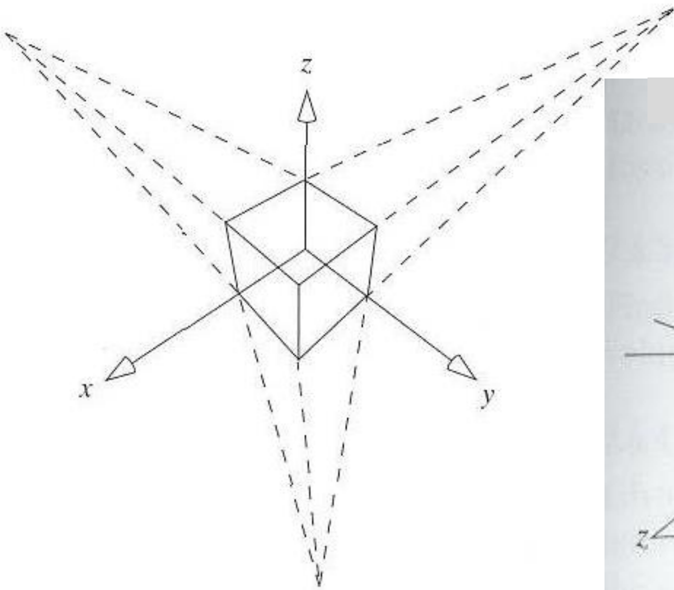
- Projection transform
  - Apply perspective foreshortening
    - Distant = small: the pinhole camera model
  - View reference coordinates (VRC, 3D) → screen coordinates (2D)



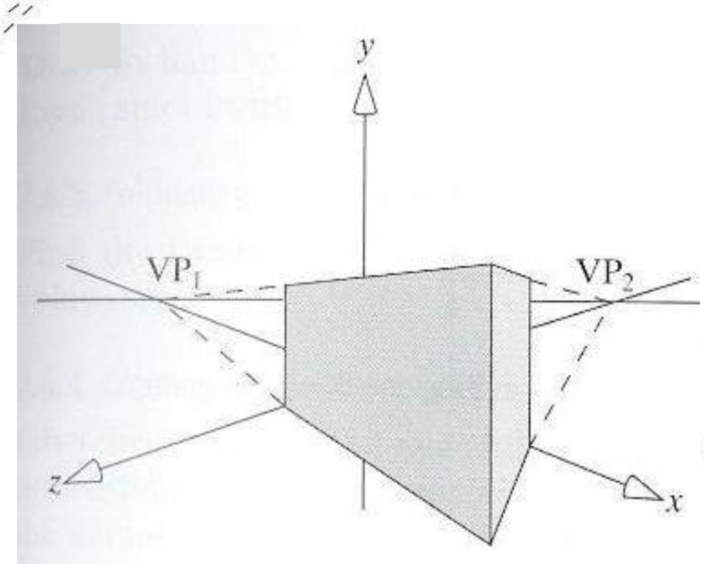
Perspective projection

How many axis vanishing points?

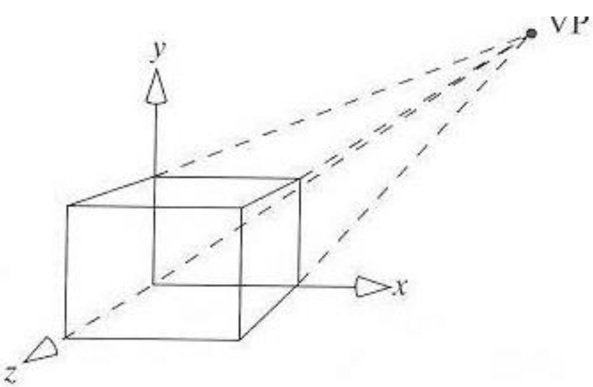
- ***Axis vanishing point*: vanishing point of lines parallel to one of three principle axes. At most 3: x-axis vanishing point, y-axis vanishing point, and z-axis vanishing point.**
- **The number of axis vanishing points can be used to categorize perspective projections.**



3-point perspective



2-point perspective



1-point perspective

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Parallel / Perspective Projection Matrix

- Parallel: Centre of Projection (COP) is taken as at infinity

$$P_{parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Perspective: Centre of Projection (COP) gives the distance to the view plane, *d*.

origin = COP   origin in view plane

$$P_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \text{ or } P'_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}$$

Parallel projection from perspective projection

- Centre of Projection (COP) is taken as at infinity.
- Then we can find the parallel projection matrix with the perspective projection matrix if the origin in the view plane :

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \quad \text{or} \quad P' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}$$

$$d \to \infty$$

$$P_{parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Constructing viewing transformation

1. **Translation of the coordinate system to the origin in homogeneous matrix form is:**

$$T = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. **Using the unit vectors of the coordinate axes, the resulting rotation matrix is:**

$$R = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. **Combination for the single transformation matrix (parallel):**

$$M = R \cdot T = \begin{bmatrix} u_x & u_y & u_z & -(u_x \cdot VRP_x + u_y \cdot VRP_y + u_z \cdot VRP_z) \\ v_x & v_y & v_z & -(v_x \cdot VRP_x + v_y \cdot VRP_y + v_z \cdot VRP_z) \\ n_x & n_y & n_z & -(n_x \cdot VRP_x + n_y \cdot VRP_y + n_z \cdot VRP_z) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transform world coordinate into VRC

4. Parallel or Perspective projection:

$$P_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \text{ or } P'_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix} \text{ or } P''_{parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Combination for the single transformation matrix :

$$M = P \cdot R \cdot T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix} \cdot \begin{bmatrix} u_x & u_y & u_z & -(u_x \cdot VRP_x + u_y \cdot VRP_y + u_z \cdot VRP_z) \\ v_x & v_y & v_z & -(v_x \cdot VRP_x + v_y \cdot VRP_y + v_z \cdot VRP_z) \\ n_x & n_y & n_z & -(n_x \cdot VRP_x + n_y \cdot VRP_y + n_z \cdot VRP_z) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

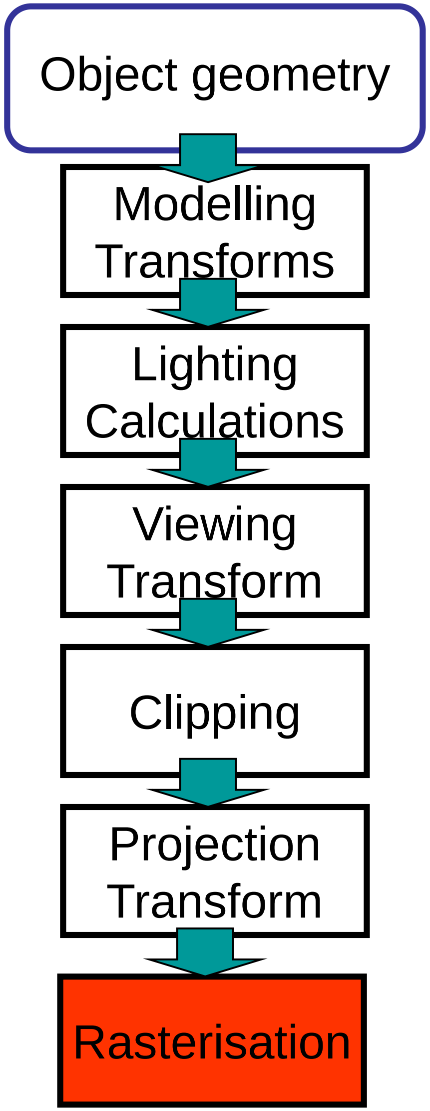
$$= \begin{bmatrix} u_x & u_y & u_z & -(u_x \cdot VRP_x + u_y \cdot VRP_y + u_z \cdot VRP_z) \\ v_x & v_y & v_z & -(v_x \cdot VRP_x + v_y \cdot VRP_y + v_z \cdot VRP_z) \\ n_x & n_y & n_z & -(n_x \cdot VRP_x + n_y \cdot VRP_y + n_z \cdot VRP_z) \\ \frac{n_x}{d} & \frac{n_y}{d} & \frac{n_z}{d} & -\frac{n_x \cdot VRP_x + n_y \cdot VRP_y + n_z \cdot VRP_z}{d} \end{bmatrix}$$

(perspective projection with origin=COP, at d distance from the view plane)

$or\ M' = P' \cdot R \cdot T$ (perspective projection with origin=VRP, in the view plane)

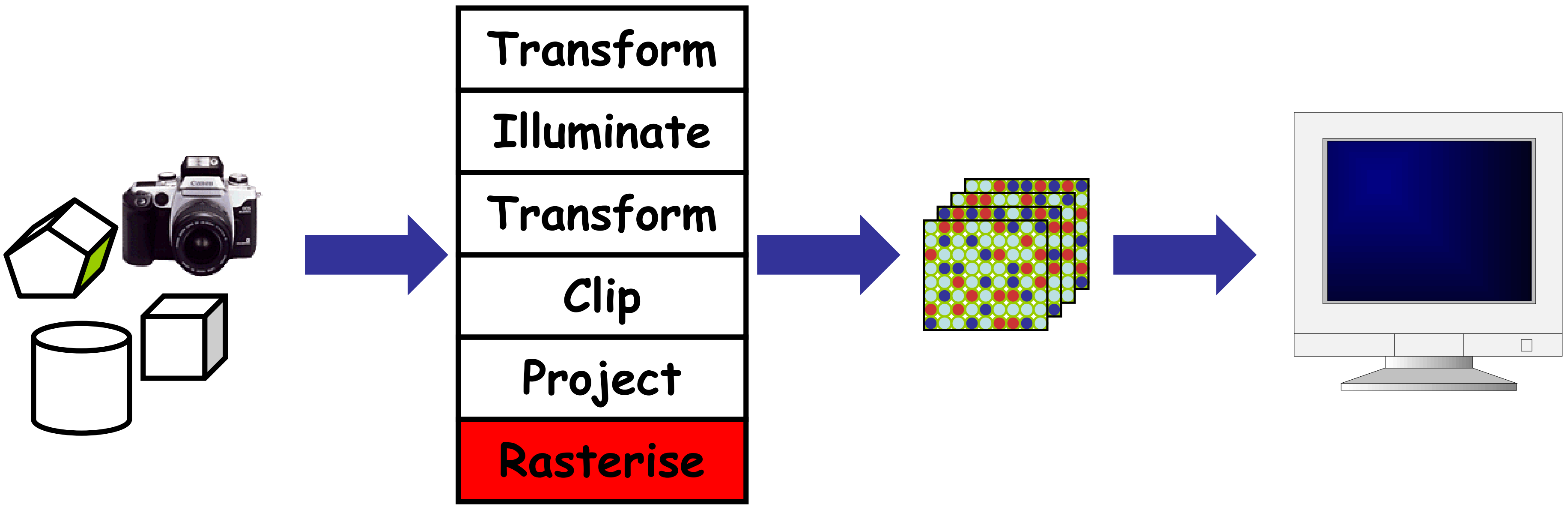
$or\ M'' = P'' \cdot R \cdot T$ (parallel projection)

The rendering pipeline: 3D



Rasterisation: 2D coordinates => pixels

Rendering 3D scenes



model & camera parameters

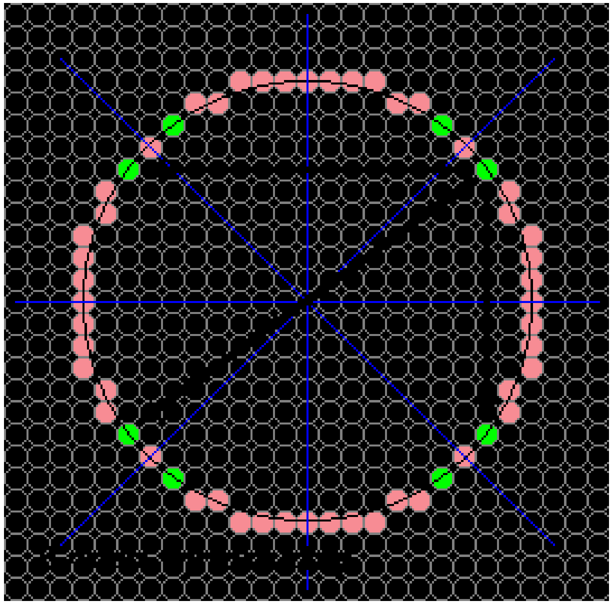
rendering pipeline

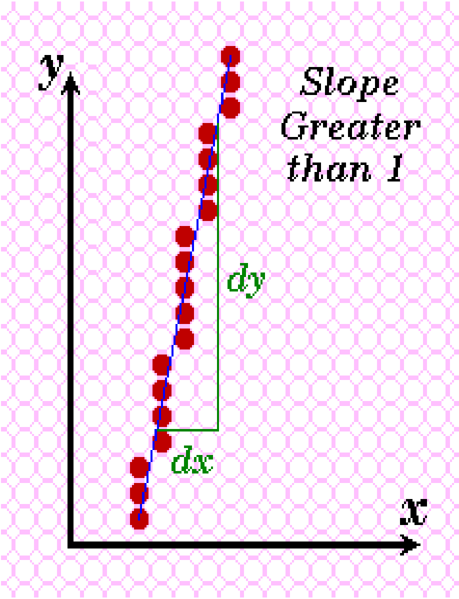
framebuffer

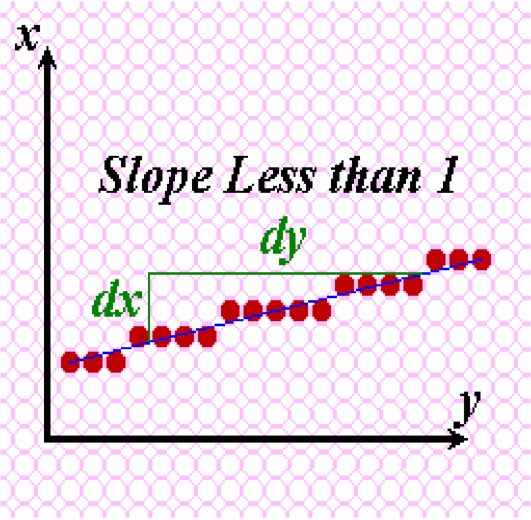
display

Rasterise

- Convert screen coordinates to pixel colors
- Fast implementation:
  - –Use integer calculations
  - –Avoid divides and multiplies
  - –Use incremental computations
  - –Use spatial coherence

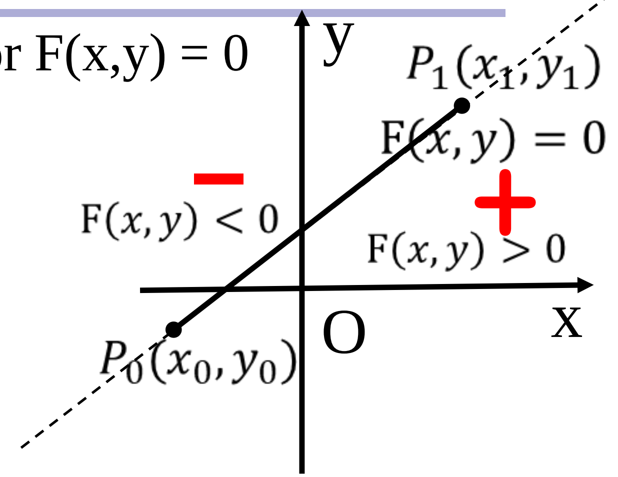






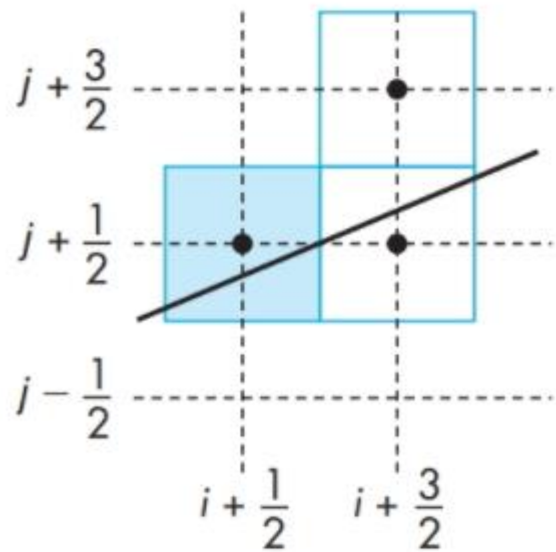
Line Generation with Midpoints

Line equation: (x1 – x0)(y – y0) = (y1 – y0)(x – x0) or F(x,y) = 0
F(x,y) = (y1 – y0)(x – x0) – (x1 – x0)(y – y0)
= (y1–y0)x–(x1–x0)y – (y1–y0)x0+(x1–x0)y0
F(x+1, y)   – F(x,y) = (y1–y0)
F(x+1, y+1)   – F(x,y) = (y1–y0) – (x1–x0)
F(x+1, y+1/2) – F(x,y) = (y1–y0) – (x1–x0)/2



If point P(x,y) drawn, the next point is either P(x+1,y) or P(x+1,y+1)
To decide which point, use the relative position of the midpoint M = (x+1, y+1/2) with respect to the line, which half-plane it is, positive or negative.

We use 2F(x,y) = 0, dx=x1–x0, dy=y1–y0, then we have:
d(x0, y0) = 2dy – dx
d(x+1, y+1) = 2F(x+1, y+1) – 2F(x,y) = 2dy – 2dx
d(x+1, y) = 2F(x+1, y) – 2F(x,y) = 2dy
as the updating for every move.

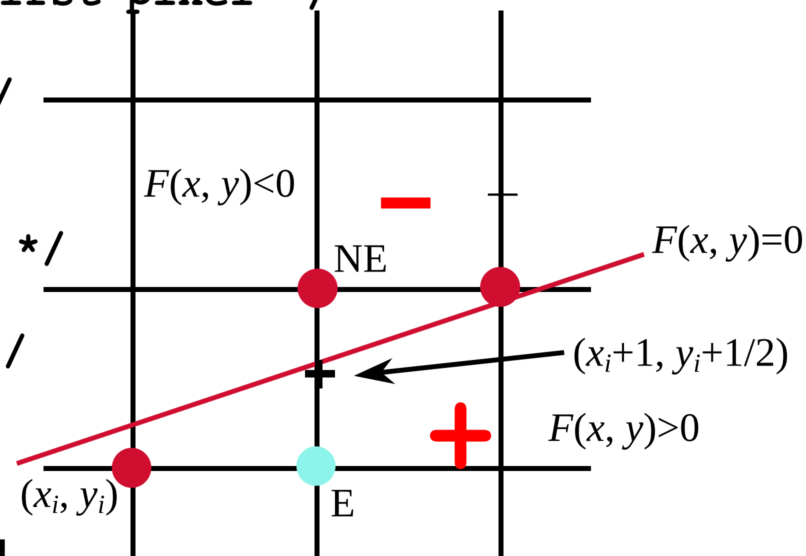


Bresenham's Midpoint Line Algorithm

Bresenham's algorithm:

```
void MidpointLine(int x0, int y0, int x1, int y1)
{  int dx,dy,incrE,incrNE,d,x,y;
   dx=x1-x0;  dy=y1-y0;
   d=2*dy-dx;          /* initial value of d */
   incrE=2*dy;         /* increment for move to E */
   incrNE=2*dy-2*dx;   /* increment for move to NE */
   x=x0;  y=y0;
   DrawPixel(x,y)         /* draw the first pixel */
   while (x<x1){
        if (d<=0){         /* choose E */
              d+=incrE;
              x++;        /* move E */
        } else {           /* choose NE */
              d+=incrNE;
              x++;  y++; /* move NE */
        }
        SetPixel (x,y);
   }
}
```

Cost: 1 integer add per pixel



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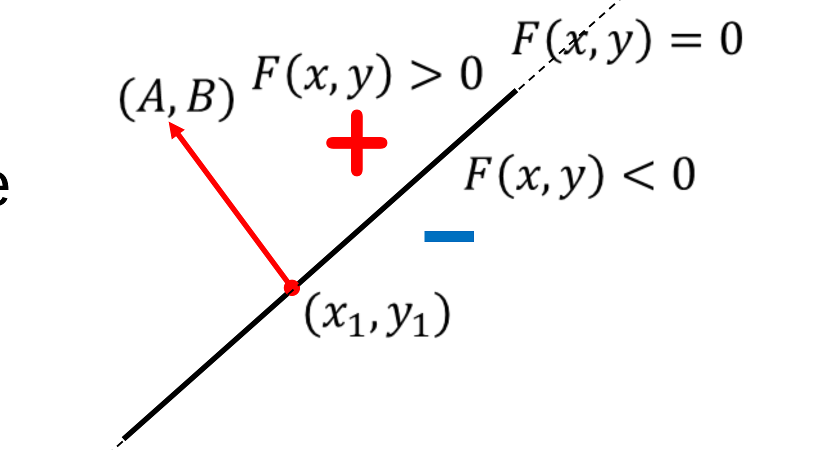
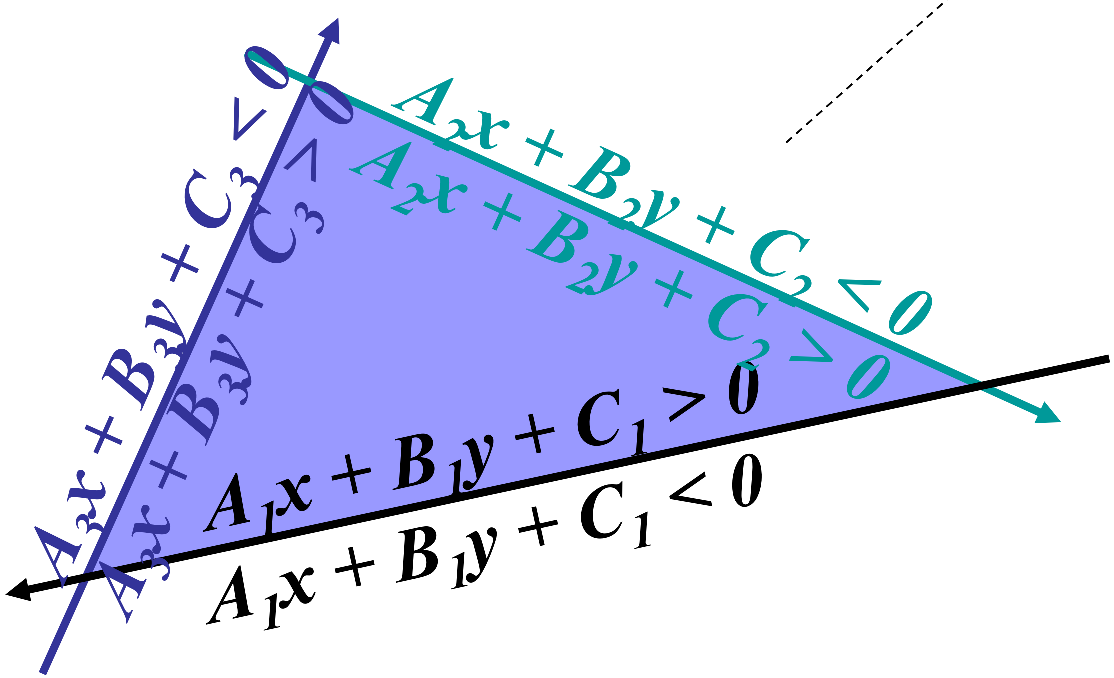
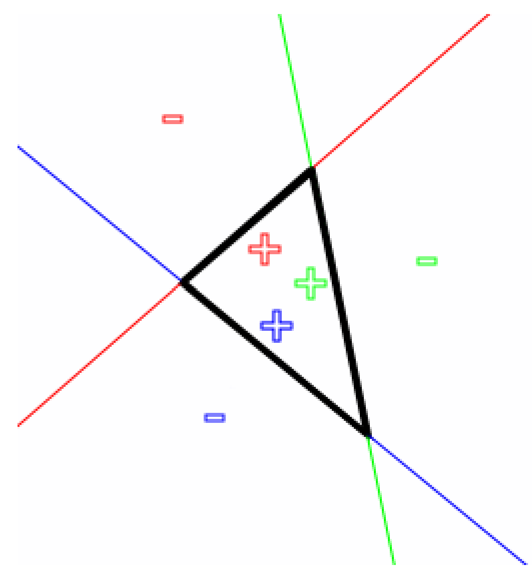
Polygon Filling with edge equations

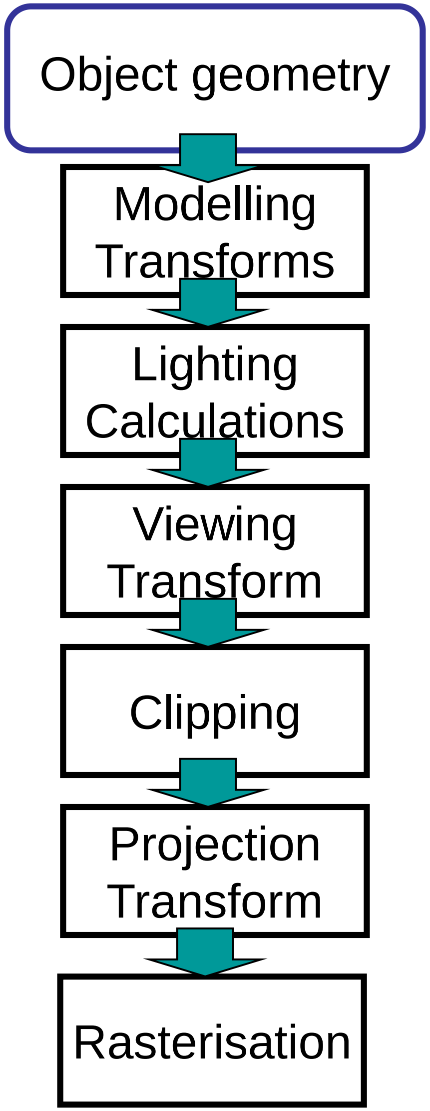
- For an edge of 2 vertices, take the third vertex as in the positive half-space, a triangle can be defined as the intersection of three positive half-spaces







The rendering pipeline



- Object/Camera models
- Modelling transforms
- Lighting
- Viewing transforms
- Clipping
- Projection transforms
- Rasterise

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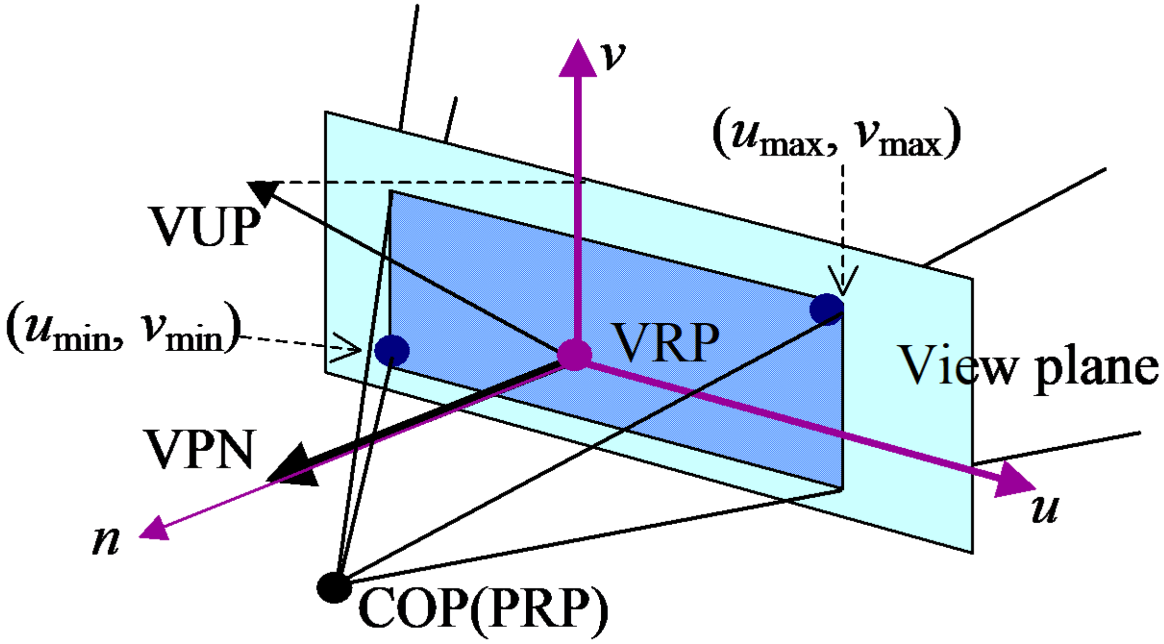
Find viewing transformation

- Translation to align the origin with VRP
- Find the axes of the view reference coordinate system: n, u, v
- Rotation to align the axes with (u, v, n)
- Composition of the 3D transformations: R.T



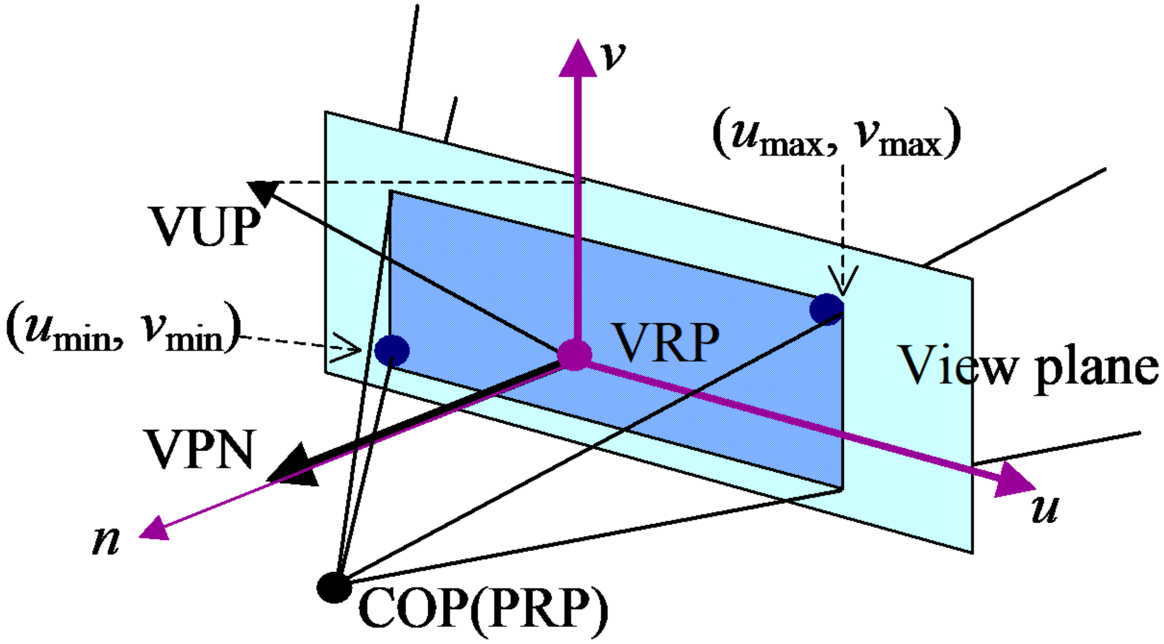
Find viewing transformation

- **Translation to align the origin with VRP:**

If the view reference point (VRP) is given in the world coordinate system, and is taken as the origin in the view reference coordinate (VRC) system

We find the translation matrix first:

$$T\left(-VRP_x, -VRP_y, -VRP_z\right) = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



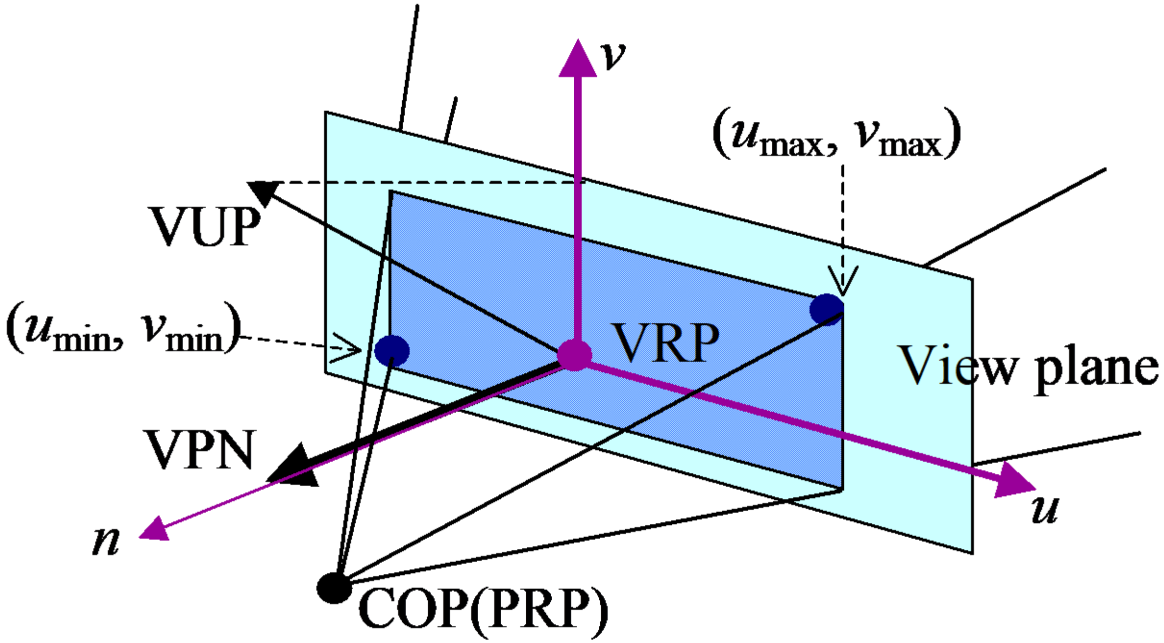
Find viewing transformation

- **Translation to align the origin with VRP:**

$$T(-VRP_x, -VRP_y, -VRP_z) = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Find the axes of the view reference coordinate system with view plane normal (VPN) and view-up vector (VUP):**

$$n = \frac{VPN}{|VPN|}, u = \frac{VUP \times VPN}{|VUP \times VPN|}, v = n \times u$$



Find viewing transformation

- **Translation to align the origin with VRP:**

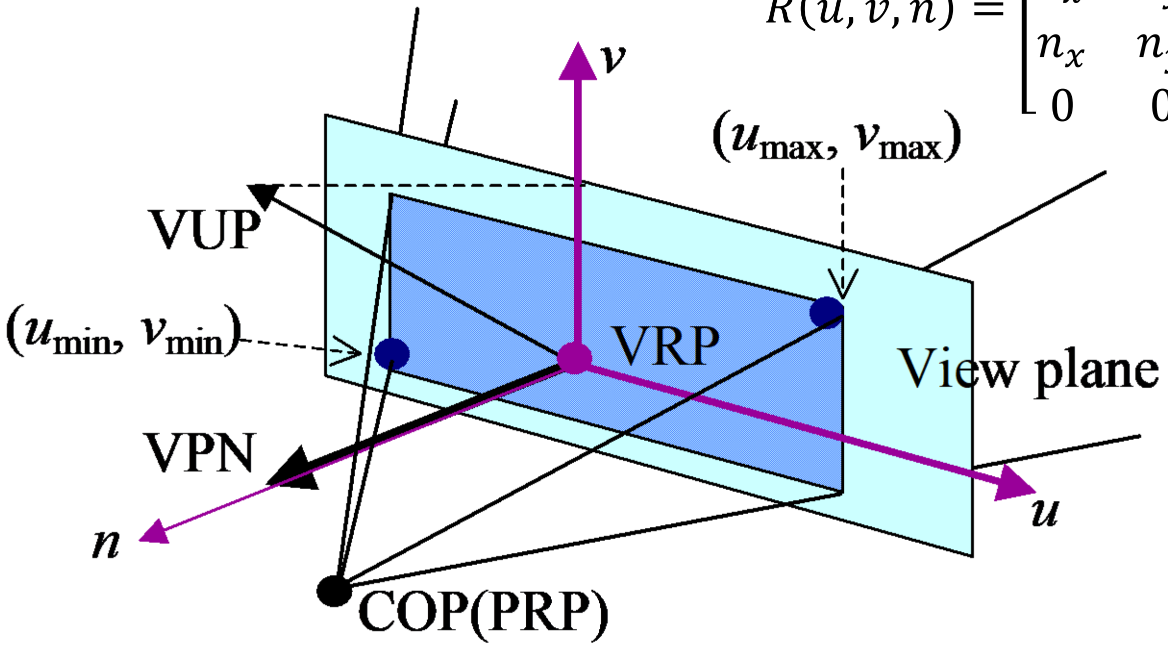
$$T(-VRP_x, -VRP_y, -VRP_z) = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Find the axes of the view reference coordinate system:**

$$n = \frac{VPN}{|VPN|}, u = \frac{VUP \times VPN}{|VUP \times VPN|}, v = n \times u$$

- **Rotation to align the axes with (u, v, n):**

$$R(u, v, n) = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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Find viewing transformation with projection

- **Translation to align the origin with VRP:**

$$T(-VRP_x, -VRP_y, -VRP_z) = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Find the axes of the view reference coordinate system:**

$$n = \frac{VPN}{|VPN|}, u = \frac{VUP \times VPN}{|VUP \times VPN|}, v = n \times u$$

- **Rotation to align the axes with (u, v, n):**

$$R(u, v, n) = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Parallel or Perspective projection :**

$$P_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \text{ or } P'_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix} \text{ or } P''_{parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Composition of the 3D transformations:**

$M = P \cdot R \cdot T$ (perspective projection with origin at COP, d from the view plane)

$or\ M' = P' \cdot R \cdot T$ (perspective projection with origin at VRP, in the view plane)

$or\ M'' = P'' \cdot R \cdot T$ (parallel projection)

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-VRP_x,-VRP_y,-VRP_z)=\begin{bmatrix}1&0&0&-VRP_x\\0&1&0&-VRP_y\\0&0&1&-VRP_z\\0&0&0&1\end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n=\frac{VPN}{|VPN|},u=\frac{VUP\times VPN}{|VUP\times VPN|},v=n\times u$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n)=\begin{bmatrix}u_x&u_y&u_z&0\\v_x&v_y&v_z&0\\n_x&n_y&n_z&0\\0&0&0&1\end{bmatrix}$$

- **Parallel or Perspective projection :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/d&0\end{bmatrix}\text{ or }P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/d&1\end{bmatrix}\text{ or }P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

- **Composition of the 3D transformations:**

$M=P\cdot R\cdot T$ (perspective projection with origin at COP, d from the view plane)
$or\ M'=P'\cdot R\cdot T$ (perspective projection with origin at VRP, in the view plane)
$or\ M''=P''\cdot R\cdot T$ (parallel projection)

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3)=\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n=\frac{VPN}{|VPN|},u=\frac{VUP\times VPN}{|VUP\times VPN|},v=n\times u$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n)=\begin{bmatrix}u_x&u_y&u_z&0\\v_x&v_y&v_z&0\\n_x&n_y&n_z&0\\0&0&0&1\end{bmatrix}$$

- **Parallel or Perspective projection :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/d&0\end{bmatrix} \text{ or } P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/d&1\end{bmatrix} \text{ or } P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

- **Composition of the 3D transformations:**

$M=P\cdot R\cdot T$ (perspective projection with origin at COP, d from the view plane)
$or\ M'=P'\cdot R\cdot T$ (perspective projection with origin at VRP, in the view plane)
$or\ M''=P''\cdot R\cdot T$ (parallel projection)

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3)=\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n=\frac{(2,-1,2)}{|(2,-1,2)|},u=\frac{VUP\times VPN}{|VUP\times VPN|},v=n\times u$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n)=\begin{bmatrix}u_x&u_y&u_z&0\\v_x&v_y&v_z&0\\n_x&n_y&n_z&0\\0&0&0&1\end{bmatrix}$$

- **Parallel or Perspective projection :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/d&0\end{bmatrix} \text{ or } P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/d&1\end{bmatrix} \text{ or } P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

- **Composition of the 3D transformations:**

$M=P\cdot R\cdot T$ (perspective projection with origin at COP, d from the view plane)
$or\ M'=P'\cdot R\cdot T$ (perspective projection with origin at VRP, in the view plane)
$or\ M''=P''\cdot R\cdot T$ (parallel projection)

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3)=\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n=\frac{(2,-1,2)}{3},u=\frac{VUP\times VPN}{|VUP\times VPN|},v=n\times u$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n)=\begin{bmatrix}u_x&u_y&u_z&0\\v_x&v_y&v_z&0\\n_x&n_y&n_z&0\\0&0&0&1\end{bmatrix}$$

- **Parallel or Perspective projection :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/d&0\end{bmatrix}\text{ or }P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/d&1\end{bmatrix}\text{ or }P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

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Find viewing transformation – example

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$$T(-1,-2,-3)=\begin{bmatrix}1 & 0 & 0 & -1\\0 & 1 & 0 & -2\\0 & 0 & 1 & -3\\0 & 0 & 0 & 1\end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n=\frac{(2,-1,2)}{3},u=\frac{(0,1,0)\times(2,-1,2)}{|VUP\times VPN|},v=n\times u$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n)=\begin{bmatrix}u_x & u_y & u_z & 0\\v_x & v_y & v_z & 0\\n_x & n_y & n_z & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

- **Parallel or Perspective projection :**

$$P_{perspective}=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 1/d & 0\end{bmatrix}\text{ or }P'_{perspective}=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 1/d & 1\end{bmatrix}\text{ or }P''_{parallel}=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

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Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3)=\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n=\frac{(2,-1,2)}{3},u=\frac{(2,0,-2)}{2\sqrt{2}},v=n\times u$$

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$$R(u,v,n)=\begin{bmatrix}u_x&u_y&u_z&0\\v_x&v_y&v_z&0\\n_x&n_y&n_z&0\\0&0&0&1\end{bmatrix}$$

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Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n = \frac{(2,-1,2)}{3}, u = \frac{(1,0,-1)}{\sqrt{2}}, v = n \times u$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n) = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Parallel or Perspective projection :**

$$P_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \text{ or } P'_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix} \text{ or } P''_{parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n)=\begin{bmatrix}u_x&u_y&u_z&0\\v_x&v_y&v_z&0\\n_x&n_y&n_z&0\\0&0&0&1\end{bmatrix}$$

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Find viewing transformation – example

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$$T(-1,-2,-3) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n = \frac{(2,-1,2)}{3}, u = \frac{(1,0,-1)}{\sqrt{2}}, v = \frac{(1,4,1)}{3\sqrt{2}}$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n) = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Parallel or Perspective projection :**

$$P_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \text{ or } P'_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix} \text{ or } P''_{parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$or\ M'' = P'' \cdot R \cdot T$ (parallel projection)

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3)=\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

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$$R(u,v,n)=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}$$

- **Parallel or Perspective projection :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/d&0\end{bmatrix}\text{ or }P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/d&1\end{bmatrix}\text{ or }P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

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$$R(u,v,n)=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}$$

- **Projection, d=5 for distance from COP to view plane is 5 :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/5&0\end{bmatrix}\text{ or }P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/5&1\end{bmatrix}\text{ or }P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

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- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n=\frac{(2,-1,2)}{3},u=\frac{(1,0,-1)}{\sqrt{2}},v=\frac{(1,4,1)}{3\sqrt{2}}$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n)=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}$$

- **Projection, d=5 for distance from COP to view plane is 5 :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/5&0\end{bmatrix}\text{ or }P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/5&1\end{bmatrix}\text{ or }P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

- **Composition of 3D transformations** (perspective projection, COP=origin) **:**

$$M=P\cdot R\cdot T=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/5&0\end{bmatrix}\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3)=\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n=\frac{(2,-1,2)}{3},u=\frac{(1,0,-1)}{\sqrt{2}},v=\frac{(1,4,1)}{3\sqrt{2}}$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n)=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}$$

- **Projection, d=5 for distance from COP to view plane is 5 :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/5&0\end{bmatrix}\text{ or }P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/5&1\end{bmatrix}\text{ or }P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

- **Composition of 3D transformations** (perspective projection, COP=origin) **:**

$$M=P\cdot R\cdot T=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&\sqrt{2}\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&-2\sqrt{2}\\2/3&-1/3&2/3&-2\\2/15&-1/15&2/15&-2/5\end{bmatrix}$$

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3)=\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

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$$n=\frac{(2,-1,2)}{3},u=\frac{(1,0,-1)}{\sqrt{2}},v=\frac{(1,4,1)}{3\sqrt{2}}$$

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$$R(u,v,n)=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}$$

- **Projection, d=5 for distance from COP to view plane is 5 :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/5&0\end{bmatrix} \text{ or } P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/5&1\end{bmatrix} \text{ or } P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

- **3D transformation and perspective projection for given point (3,1,5) :**

$$M\cdot\begin{bmatrix}3\\1\\5\\1\end{bmatrix}=P\cdot R\cdot T\cdot\begin{bmatrix}3\\1\\5\\1\end{bmatrix}=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&\sqrt{2}\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&-2\sqrt{2}\\2/3&-1/3&2/3&-2\\2/15&-1/15&2/15&-2/5\end{bmatrix}\begin{bmatrix}3\\1\\5\\1\end{bmatrix}=\begin{bmatrix}0\\0\\3\\3/5\end{bmatrix}=\begin{bmatrix}0\\0\\5\\1\end{bmatrix}$$ (origin=COP)

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3)=\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n=\frac{(2,-1,2)}{3},u=\frac{(1,0,-1)}{\sqrt{2}},v=\frac{(1,4,1)}{3\sqrt{2}}$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n)=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}$$

- **Projection, d=5 for distance from COP to view plane is 5 :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/5&0\end{bmatrix} \text{ or } P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/5&1\end{bmatrix} \text{ or } P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

- **Composition of 3D transformations** (perspective, origin=VRP) **:**

$$M'=P'\cdot R\cdot T=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/5&1\end{bmatrix}\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3)=\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n=\frac{(2,-1,2)}{3},u=\frac{(1,0,-1)}{\sqrt{2}},v=\frac{(1,4,1)}{3\sqrt{2}}$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n)=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}$$

- **Projection, d=5 for distance from COP to view plane is 5 :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/5&0\end{bmatrix}\text{ or }P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/5&1\end{bmatrix}\text{ or }P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

- **Composition of 3D transformations** (perspective, origin=VRP) **:**

$$M'=P'\cdot R\cdot T=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&\sqrt{2}\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&-2\sqrt{2}\\0&0&0&0\\2/15&-1/15&2/15&3/5\end{bmatrix}$$

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3)=\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

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$$R(u,v,n)=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}$$

- **Projection, d=5 for distance from COP to view plane is 5 :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/5&0\end{bmatrix}\text{ or }P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/5&1\end{bmatrix}\text{ or }P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

- **3D transformation and perspective projection for given point (3,1,5) :**

$$M'\cdot\begin{bmatrix}3\\1\\5\\1\end{bmatrix}=P'\cdot R\cdot T\cdot\begin{bmatrix}3\\1\\5\\1\end{bmatrix}=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&\sqrt{2}\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&-2\sqrt{2}\\0&0&0&0\\2/15&-1/15&2/15&3/5\end{bmatrix}\begin{bmatrix}3\\1\\5\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\\8/5\end{bmatrix}=\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\text{(origin=VRP)}$$

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3)=\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n=\frac{(2,-1,2)}{3},u=\frac{(1,0,-1)}{\sqrt{2}},v=\frac{(1,4,1)}{3\sqrt{2}}$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n)=\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}$$

- **Projection, d=5 for distance from COP to view plane is 5 :**

$$P_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/5&0\end{bmatrix}\text{ or }P'_{perspective}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&1/5&1\end{bmatrix}\text{ or }P''_{parallel}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$$

- **Composition of the 3D transformations** (parallel projection) **:**

$$M''=P''\cdot R\cdot T=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\sqrt{2}/2&0&-\sqrt{2}/2&0\\\sqrt{2}/6&2\sqrt{2}/3&\sqrt{2}/6&0\\2/3&-1/3&2/3&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&-1\\0&1&0&-2\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

$$n = \frac{(2,-1,2)}{3}, u = \frac{(1,0,-1)}{\sqrt{2}}, v = \frac{(1,4,1)}{3\sqrt{2}}$$

- **Rotation to align the axes with (u, v, n):**

$$R(u,v,n) = \begin{bmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/6 & 2\sqrt{2}/3 & \sqrt{2}/6 & 0 \\ 2/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Projection, d=5 for distance from COP to view plane is 5 :**

$$P_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/5 & 0 \end{bmatrix} \text{ or } P'_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/5 & 1 \end{bmatrix} \text{ or } P''_{parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Composition of the 3D transformations** (parallel projection) **:**

$$M'' = P'' \cdot R \cdot T = \begin{bmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 & \sqrt{2} \\ \sqrt{2}/6 & 2\sqrt{2}/3 & \sqrt{2}/6 & -2\sqrt{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find viewing transformation – example

- **Translation to align the origin with VRP=(1,2,3):**

$$T(-1,-2,-3) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Find the axes of VRC system, VPN=(2,−1,2), VUP=(0,1,0):**

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- **Projection, d=5 for distance from COP to view plane is 5 :**

$$P_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/5 & 0 \end{bmatrix} \text{ or } P'_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/5 & 1 \end{bmatrix} \text{ or } P''_{parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **3D transformation and projection for a given point (3,1,5) :**

$$M'' \cdot \begin{bmatrix} 3 \\ 1 \\ 5 \\ 1 \end{bmatrix} = P'' \cdot R \cdot T \cdot \begin{bmatrix} 3 \\ 1 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 & \sqrt{2} \\ \sqrt{2}/6 & 2\sqrt{2}/3 & \sqrt{2}/6 & -2\sqrt{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$ (parallel)

ry

3D Graphics Programming Tools

Projection and Rasterisation (Revision 2)

(Go to **www.menti.com** and use the code **2516 6979** to ask me questions)

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