EBU7240
Computer Vision
- Camera: Perspective projection -

Semester 1, 2021

Changjae Oh

Announcement of
Pope Benedict XVI (2005),
Pope Francis (2013)

Content

- **The pinhole projection model**
  - Geometric properties
  - Perspective projection matrix
- **Cameras with lenses**
  - Depth of focus
  - Field of view
  - Lens aberrations
- **Digital sensors**

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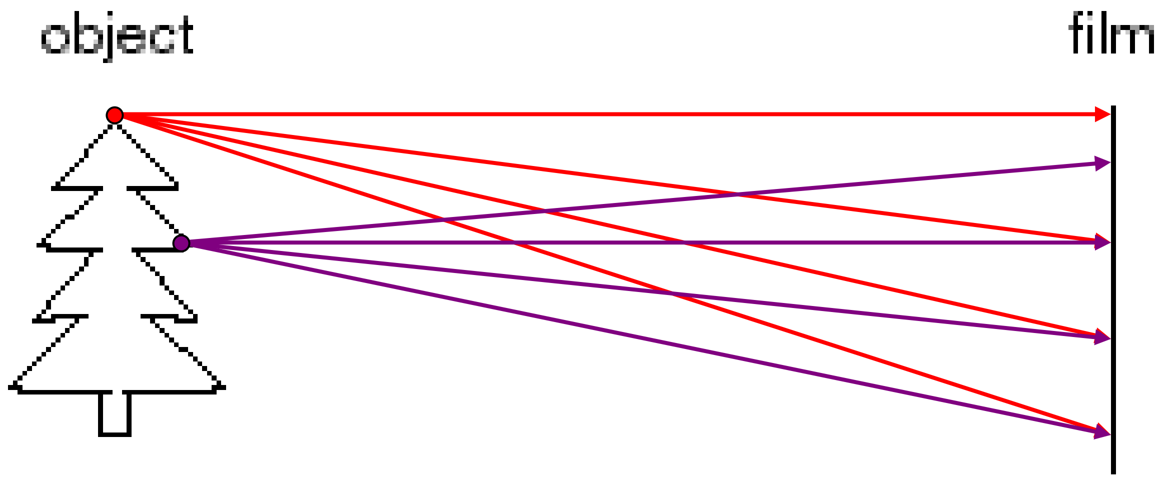
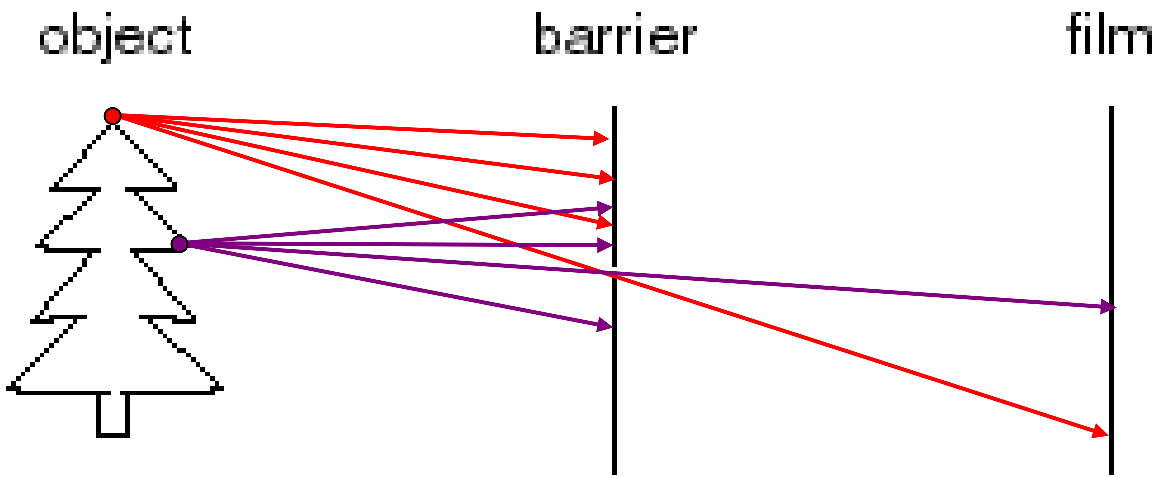
Let's design a camera



- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

Slide by Steve Seitz

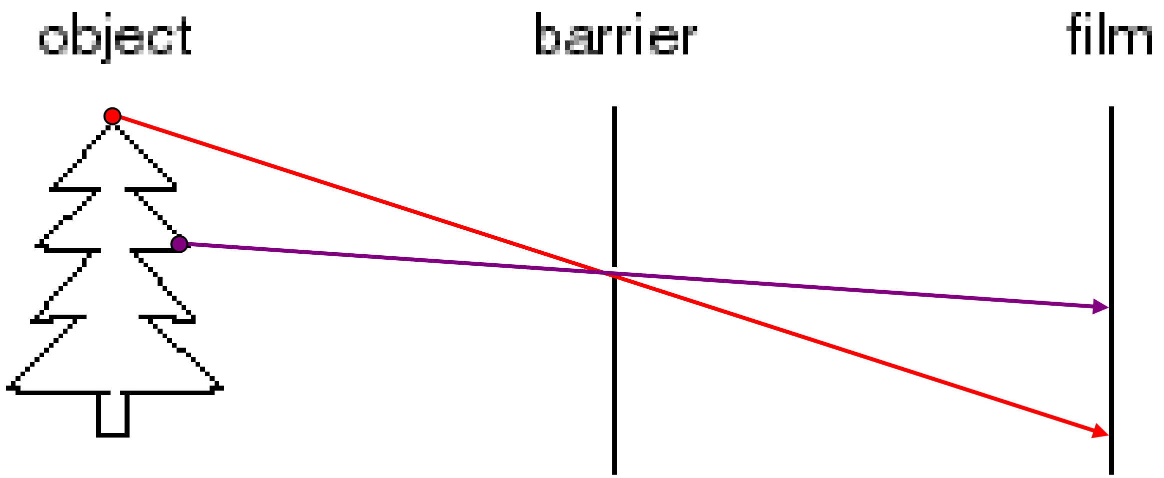
Pinhole camera



- **Add a barrier to block off most of the rays**
  - Tiny aperture without lens
  - Light from a scene passes through the aperture and projects an inv erted image on the film

Slide by Steve Seitz

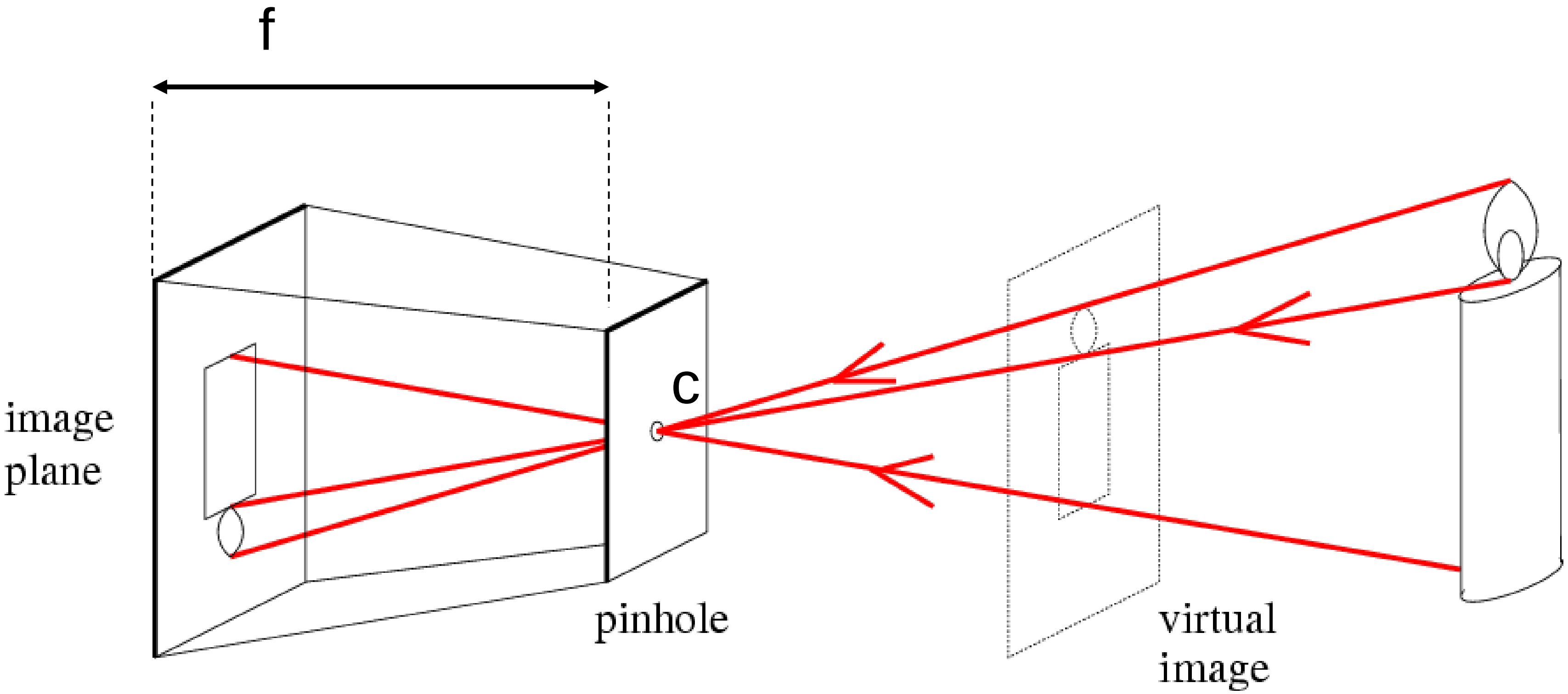
Pinhole camera



- **Captures pencil of rays**
  - All rays through a single point: aperture, center of projection, optical center, focal point, camera center
- **The image is formed on the image plane (film)**

Slide by Steve Seitz

Pinhole camera



f = focal length
c = center of the camera

Slide from Forsyth

Pinhole cameras are everywhere

Tree shadow during a solar eclipse

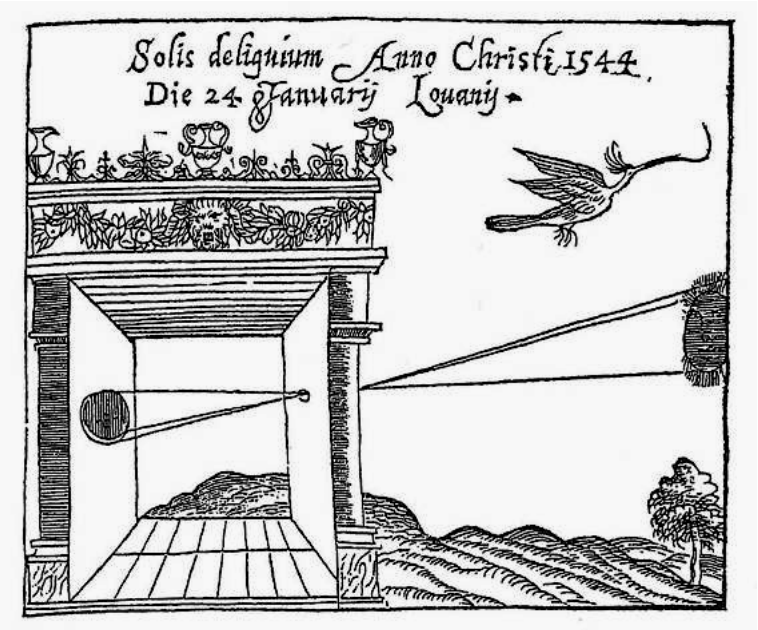
photo credit: Nils van der Burg

http://www.physicstogo.org/index.cfm

Credit: Steve Seitz

Camera obscura

- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)



Gemma Frisius, 1558

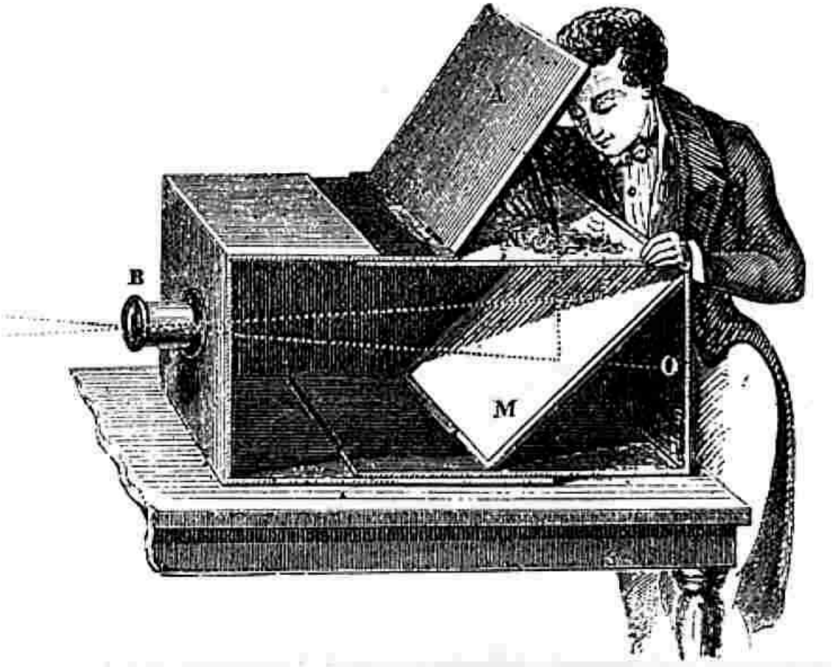
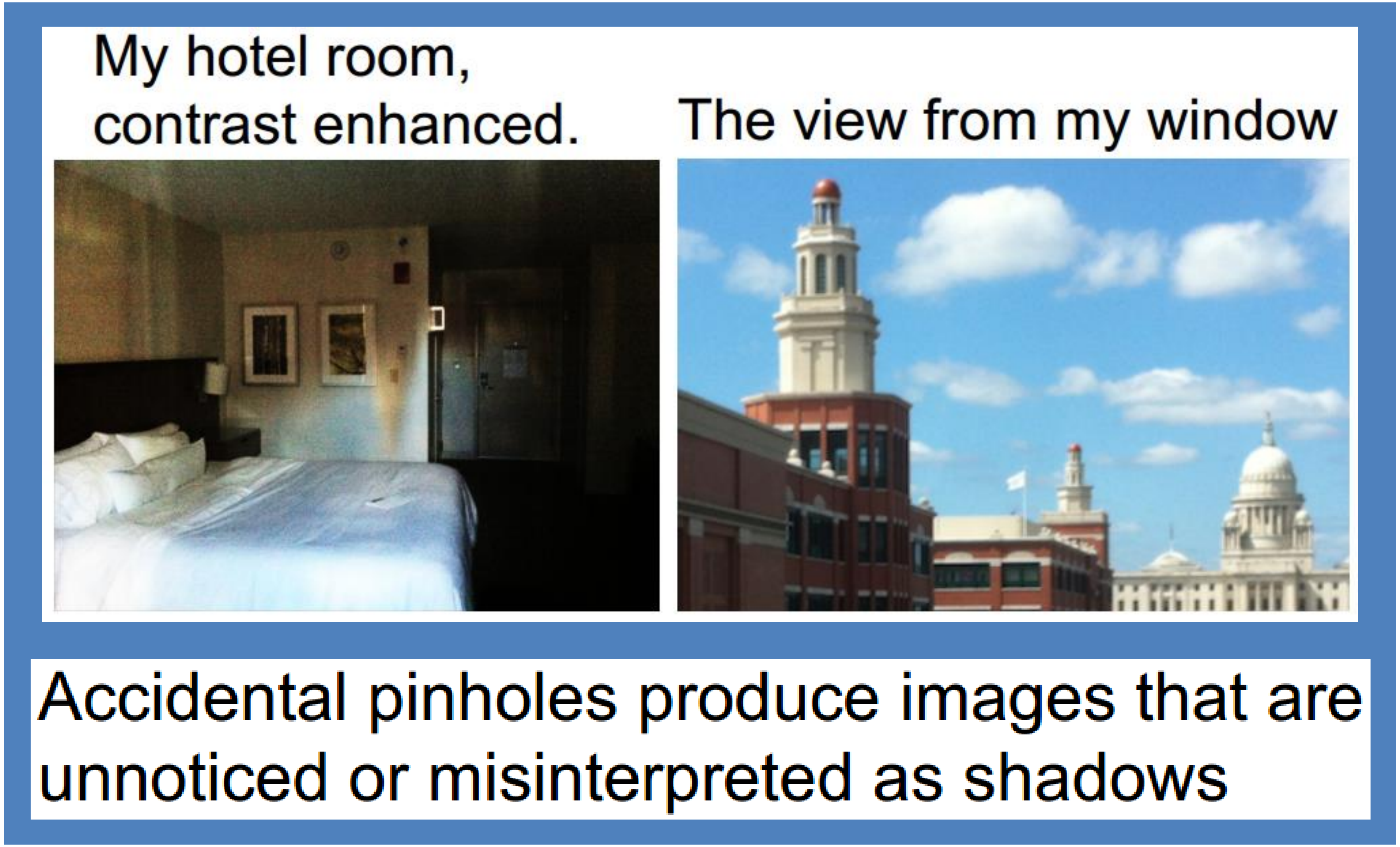


Image source

Slide from Lazebnik

Turning a room into a camera obscura

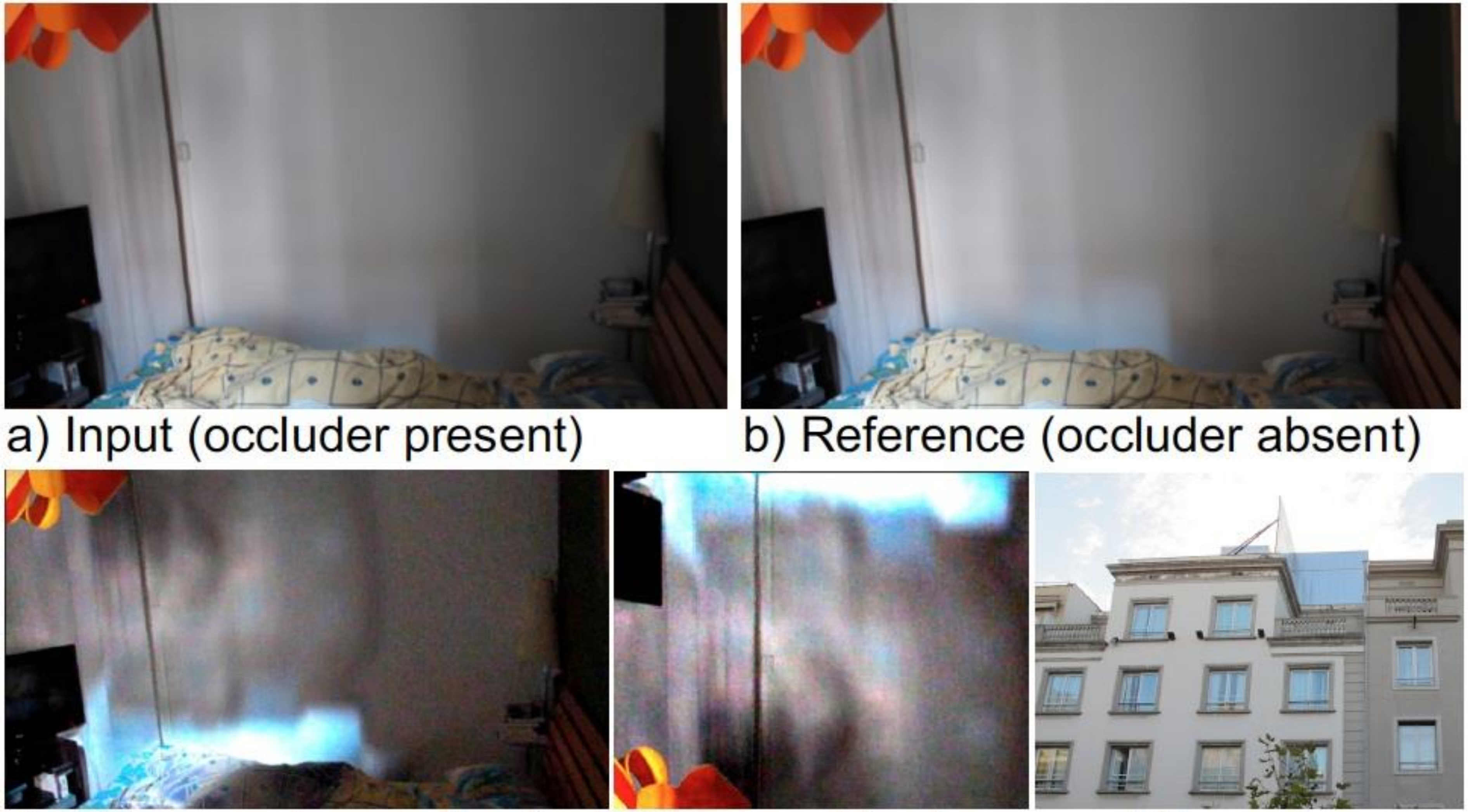


- A. Torralba and W. Freeman, Accidental Pinhole and Pinspeck Cameras, CVPR 2012

Turning a room into a camera obscura

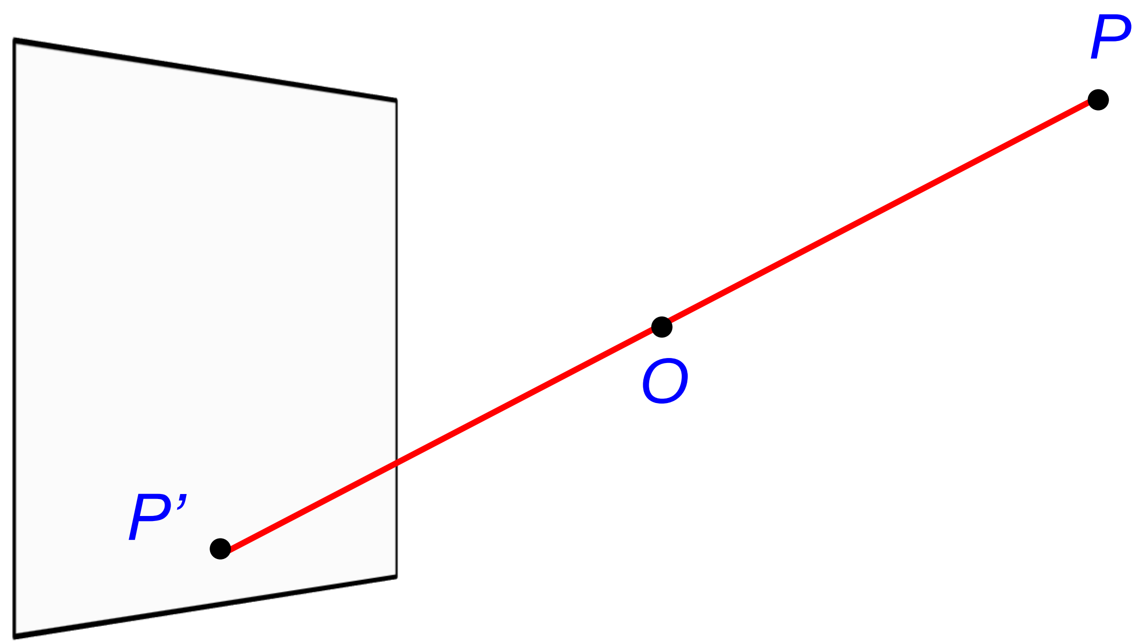
Accidental Pinhole and Pinspeck Cameras Revealing the scene outside the picture. Antonio Torralba, William T. Freeman

Turning a room into a camera obscura



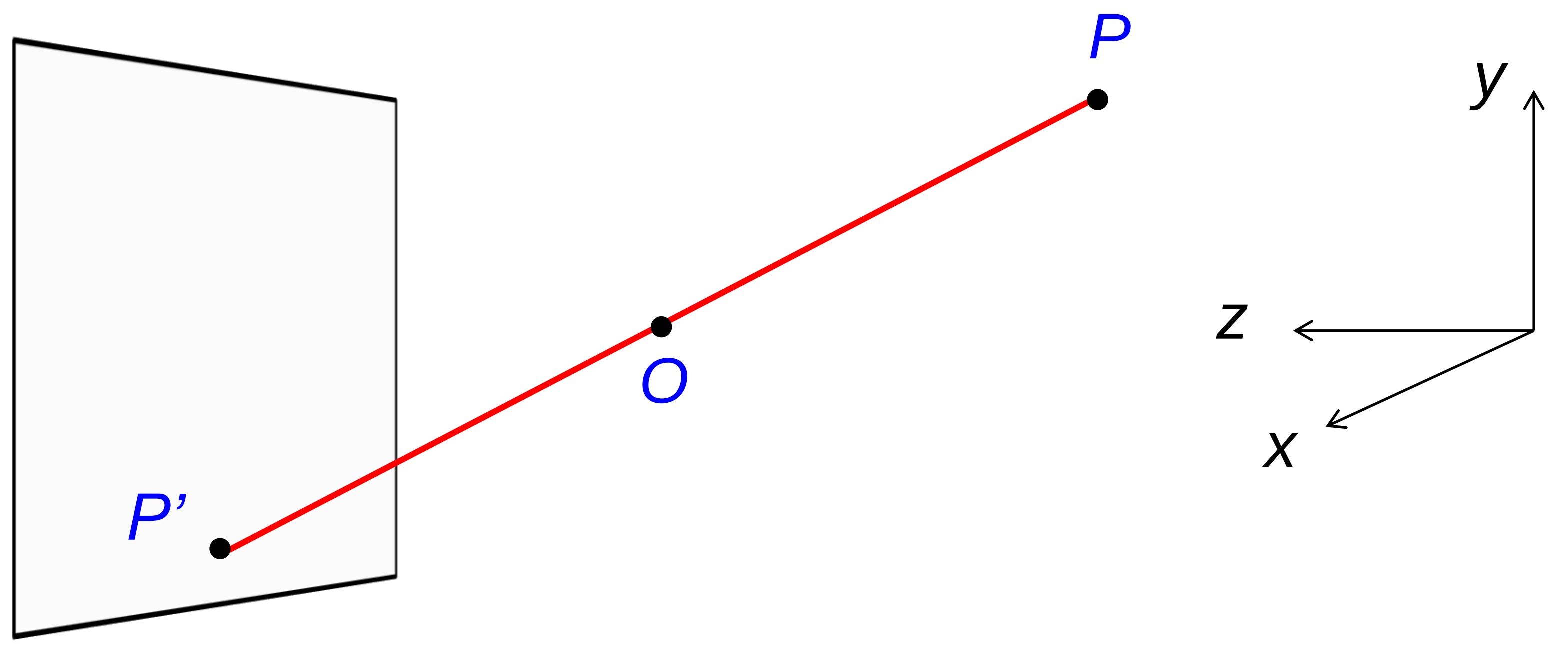
a) Input (occluder present)
b) Reference (occluder absent)
c) Difference image (b-a)
d) Crop upside down
e) True view

Pinhole projection model



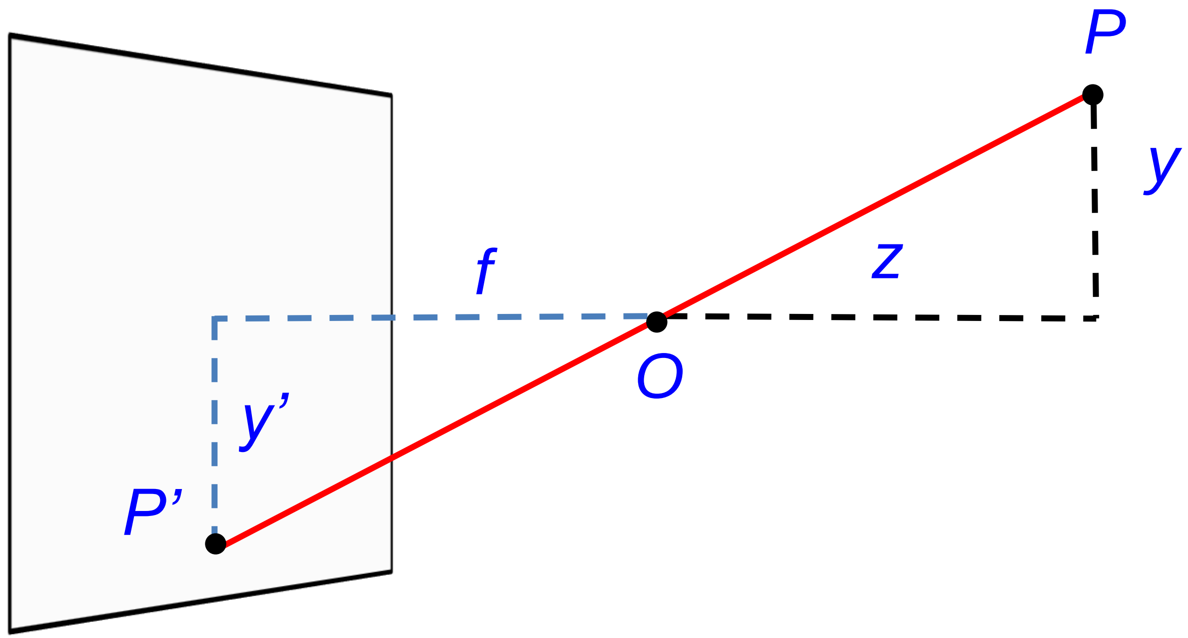
- To compute the projection *P'* of a scene point *P*,
  - Form the visual ray connecting *P* to the camera center *O* and find where it intersects the image plane

Pinhole projection model



- The coordinate system
  - The optical center (***O***) is at the origin
  - The image plane is parallel to xy-plane or perpendicular to the z-axis, which is the *optical axis*

Pinhole projection model

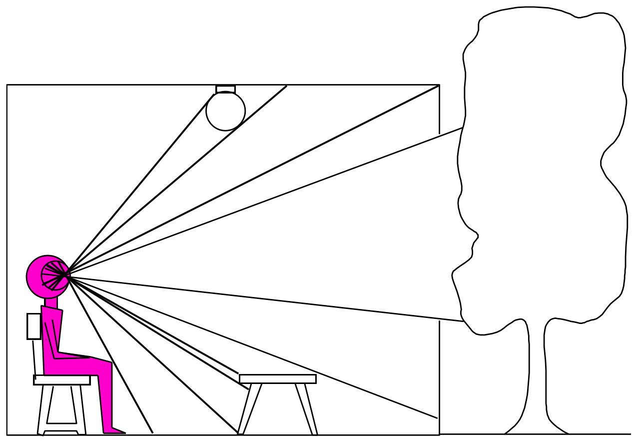


- Projection equations
  - Derived using similar triangles $(x, y, z) \rightarrow (f\,\frac{x}{z}, f\,\frac{y}{z})$

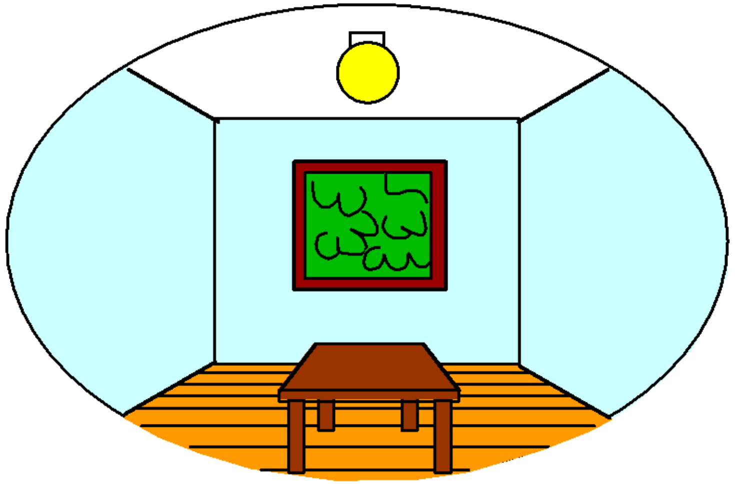
Dimensionality reduction: from 3D to 2D

3D world

2D image



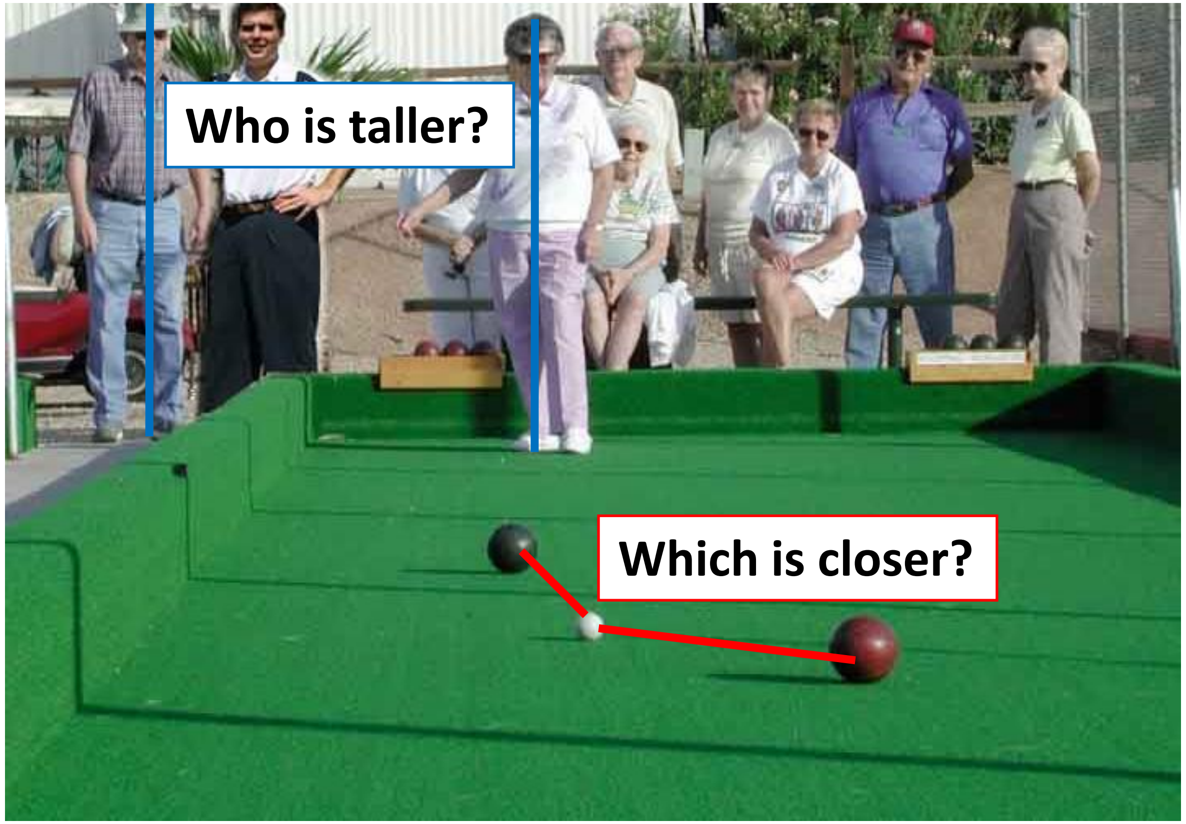
Point of observation



- What properties of the world are preserved?
  - Straight lines, incidence
- What properties are not preserved?
  - Angles, lengths

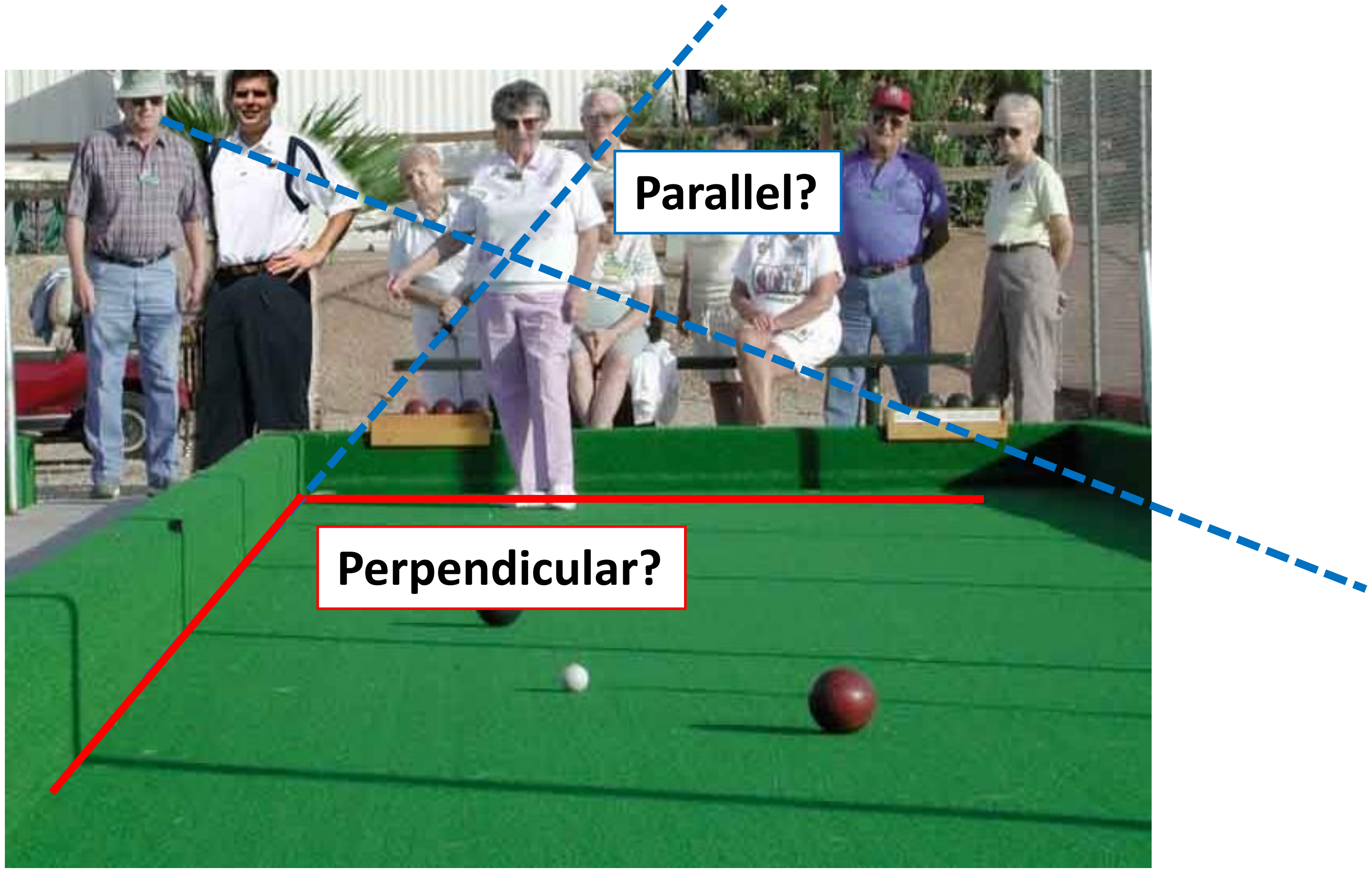
Slide by A. Efros

Properties of projection



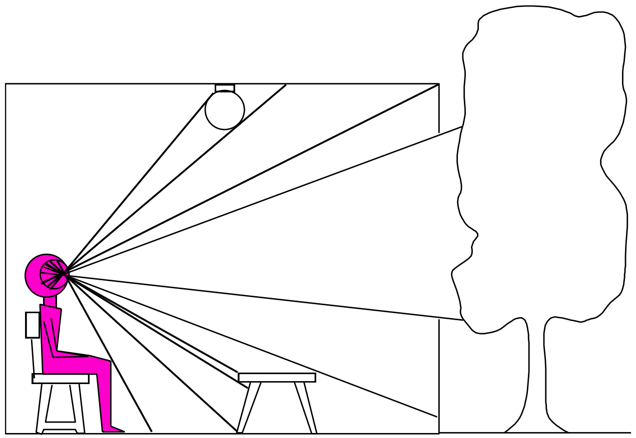
Slide by Derek Hoiem

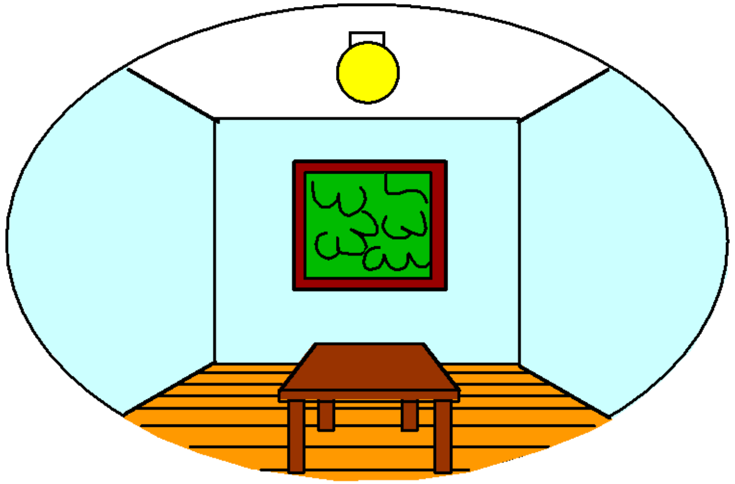
Properties of projection



Fronto-parallel planes

- **What happens to the projection of a pattern on a plane parallel to the image plane?**
  - All points on that plane are at a fixed *depth z*
  - The pattern gets scaled by a factor of *f* / *z*, but angles and ratios of lengths/areas are preserved





$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Fronto-parallel planes

- **What happens to the projection of a pattern on a plane parallel to the image plane?**
  - All points on that plane are at a fixed *depth* z
  - The pattern gets scaled by a factor of f / z, but angles and ratios of lengths/areas are preserved

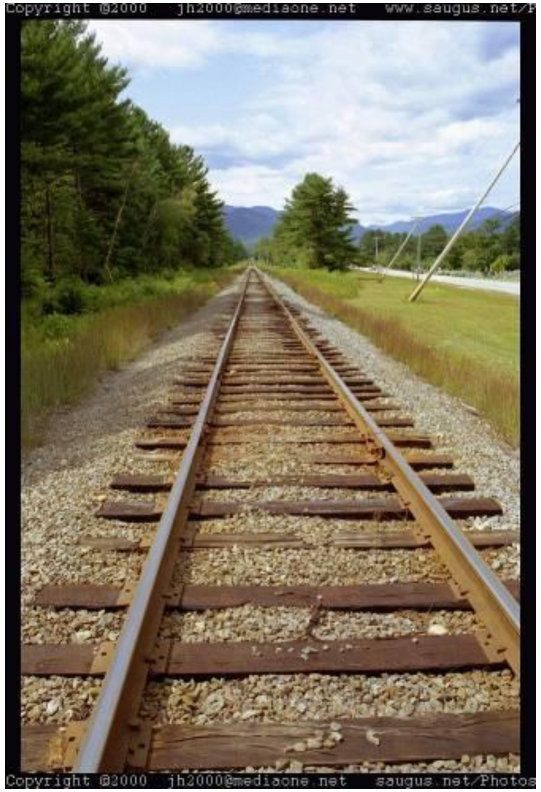
Piero della Francesca, *Flagellation of Christ*, 1455-1460

Jan Vermeer, *The Music Lesson*, 1662-1665

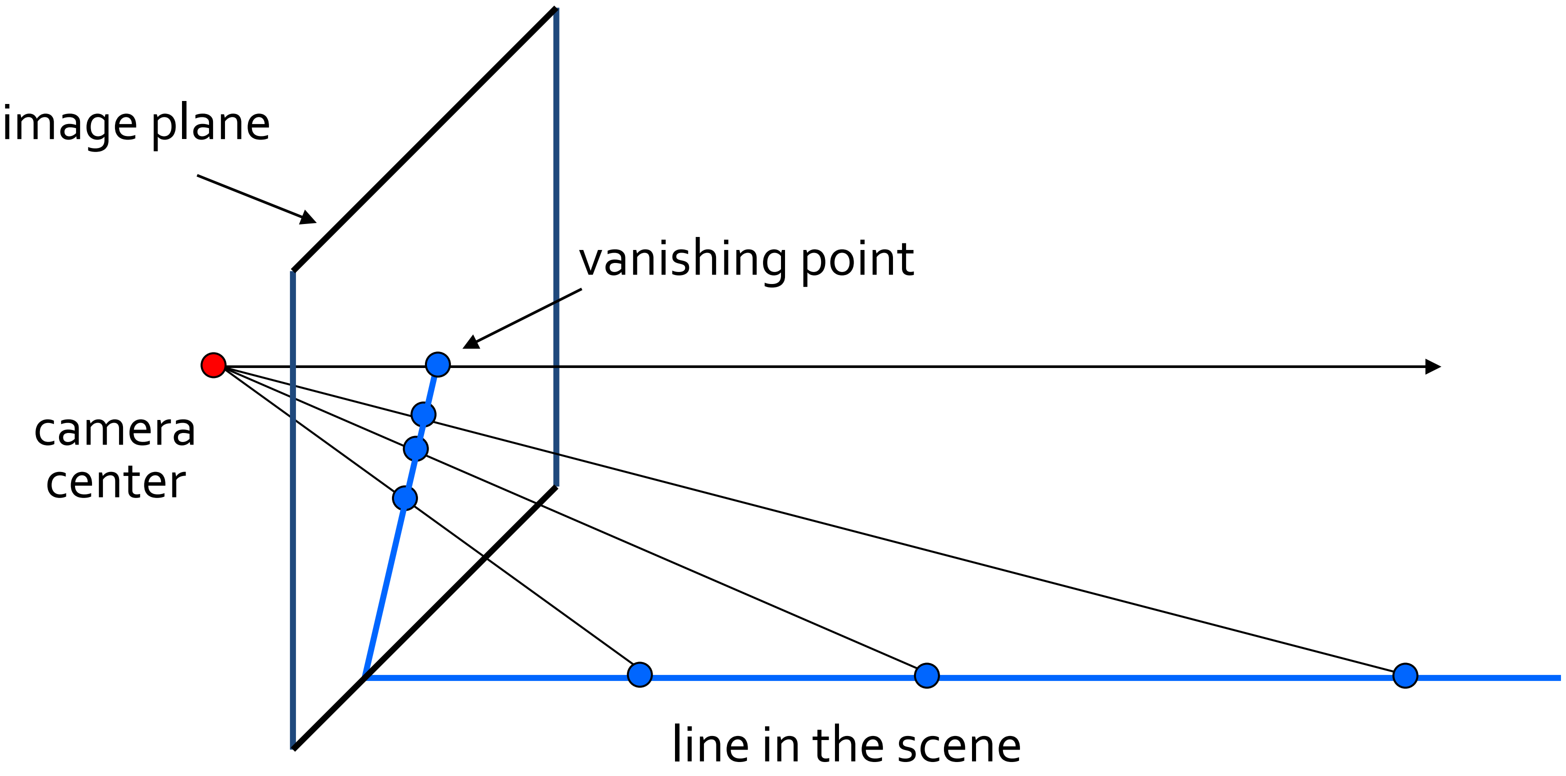
Slide from S. Lazebnik

Vanishing points

- **All parallel lines converge to a vanishing point**
  - Each direction in space is associated with its own vanishing point
  - Exception: directions parallel to the image plane



Constructing the vanishing point of a line



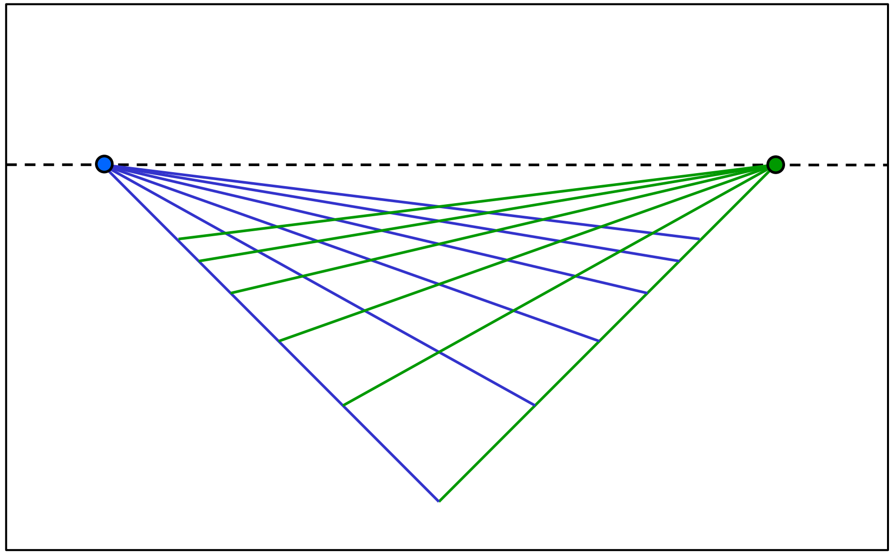
Slide from S. Lazebnik

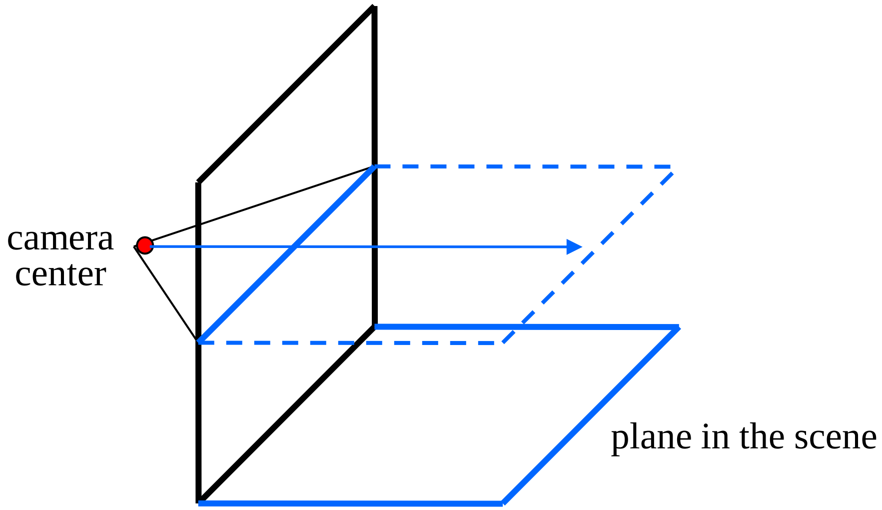
Vanishing lines of planes

How do we construct the vanishing line of a plane?

Image source

Vanishing lines of planes





- *Horizon*: vanishing line of the ground plane
  - All points at the same height as the camera project to the horizon
  - Points higher (resp. lower) than the camera project above (resp. b elow) the horizon
  - Provides way of comparing height of objects

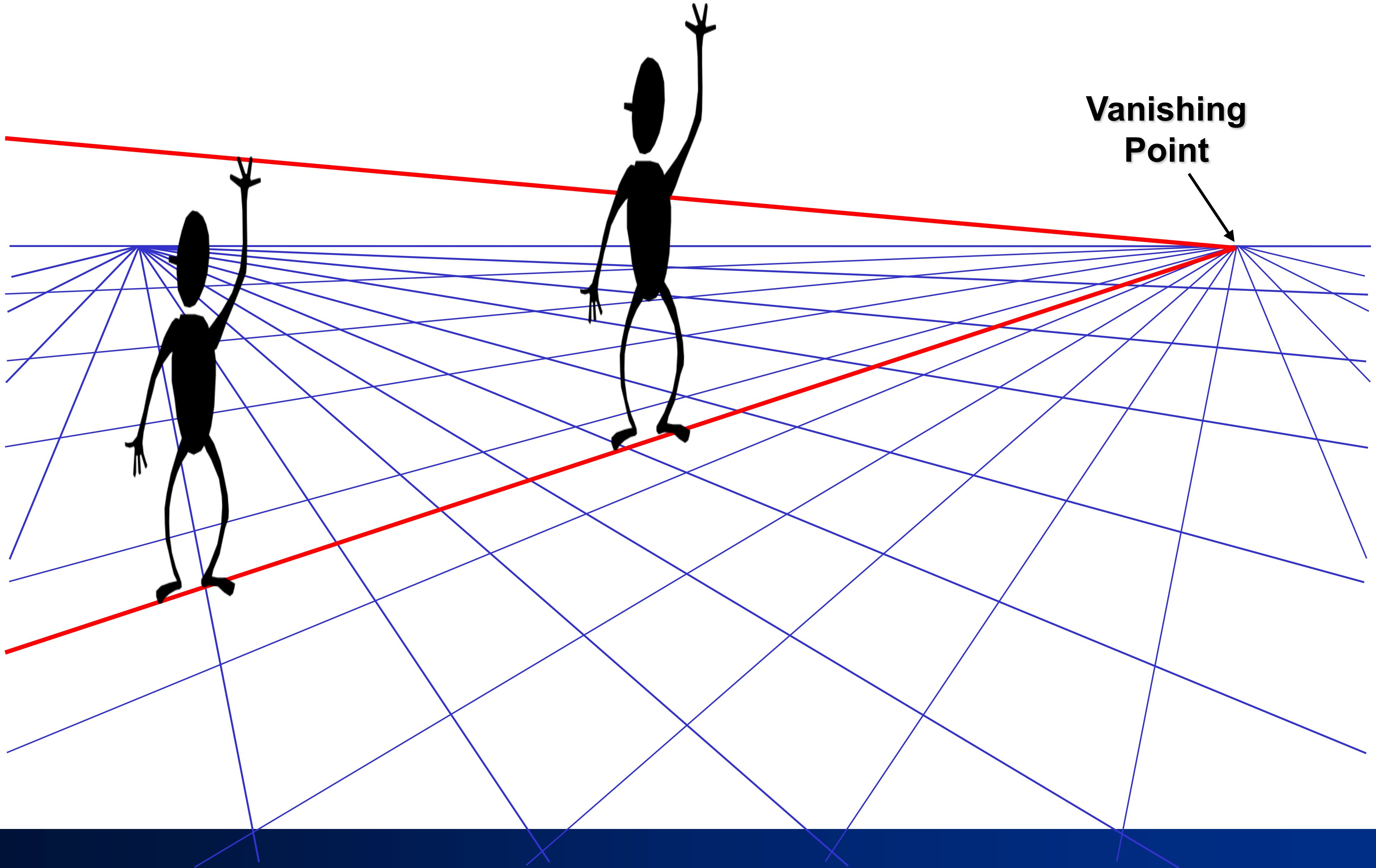
Slide by S. Seitz

Vanishing lines of planes

Is the parachutist above or below the camera?

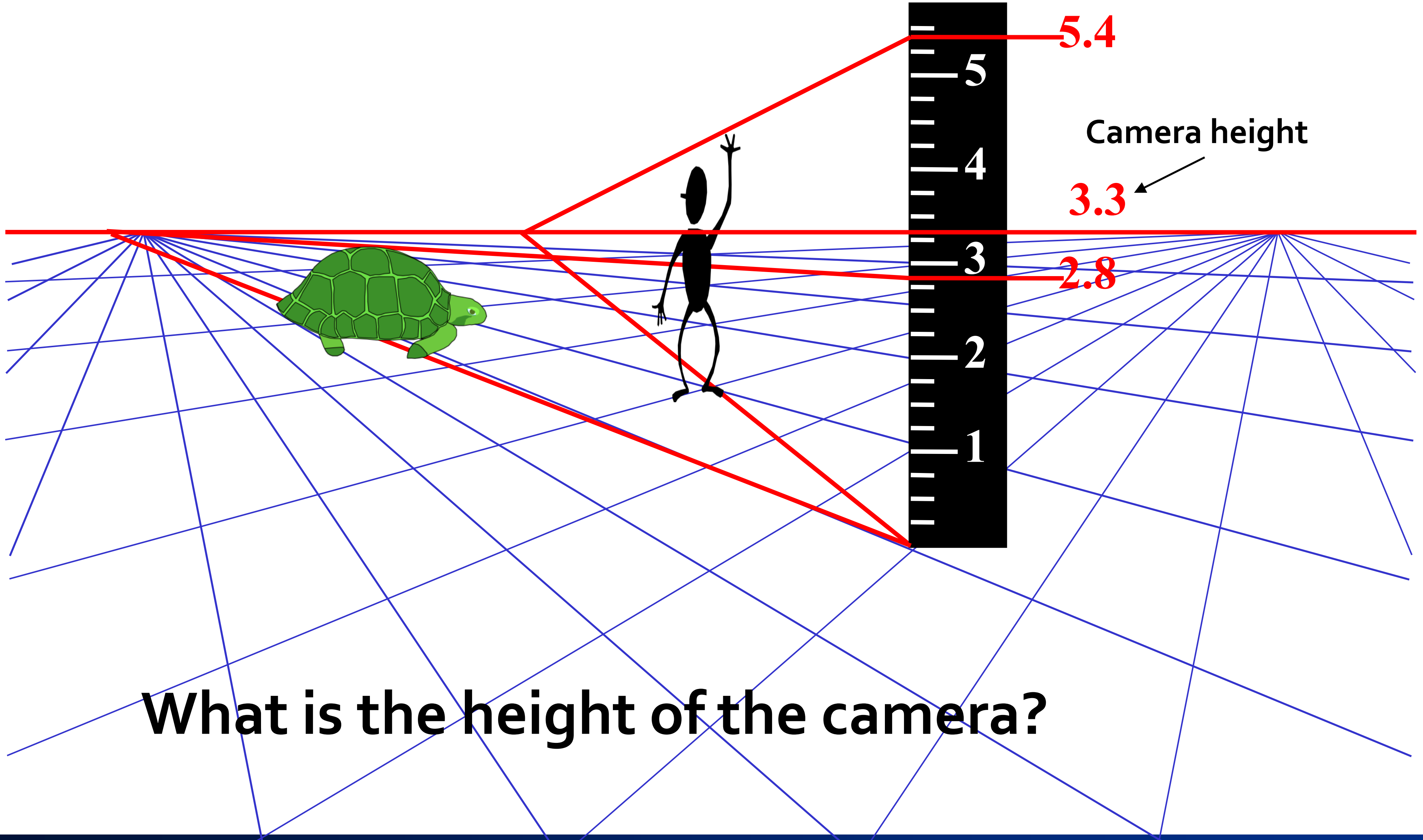
Slide by S. Seitz

Comparing heights



Slide by S. Seitz

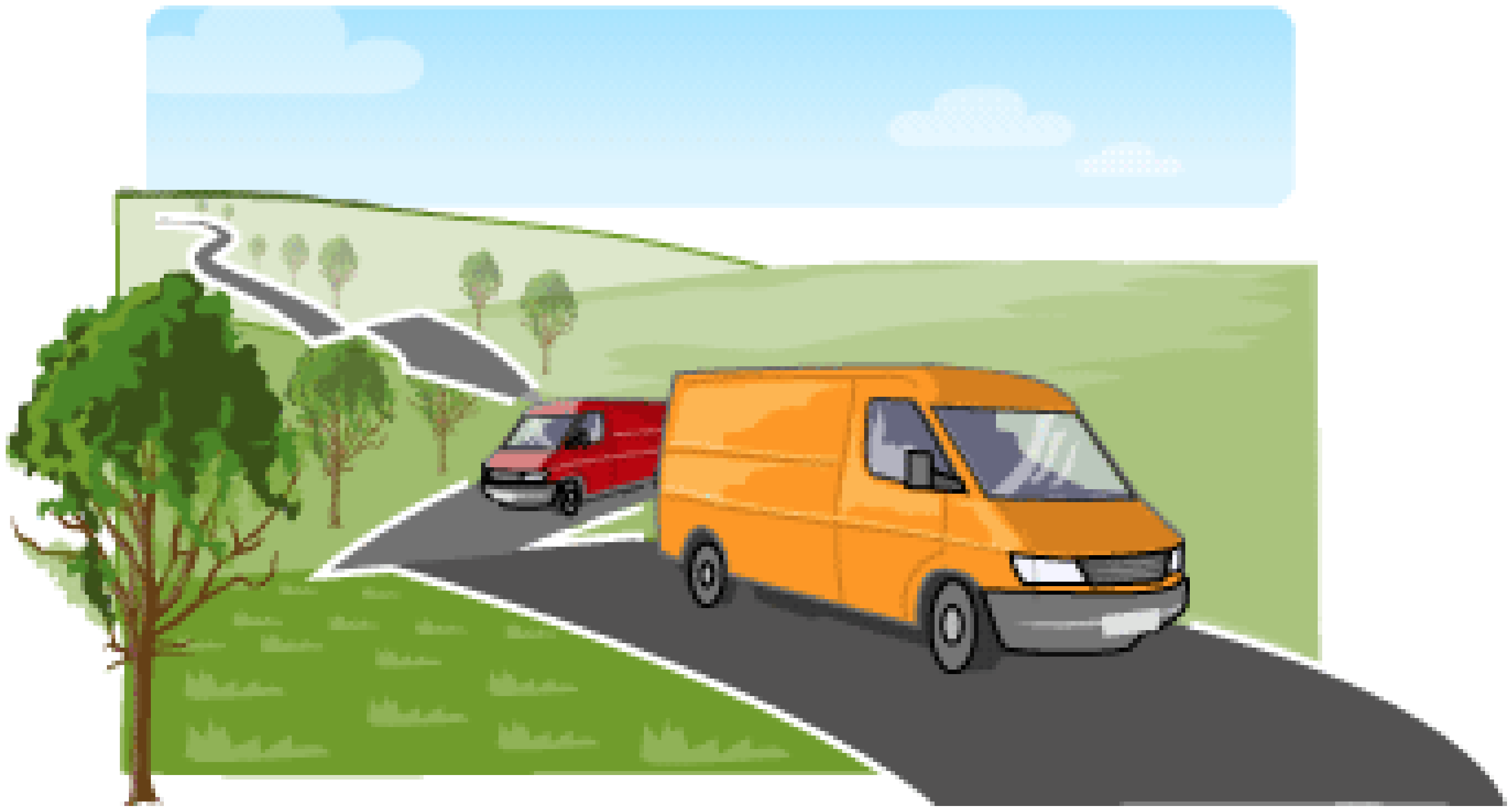
Measuring height

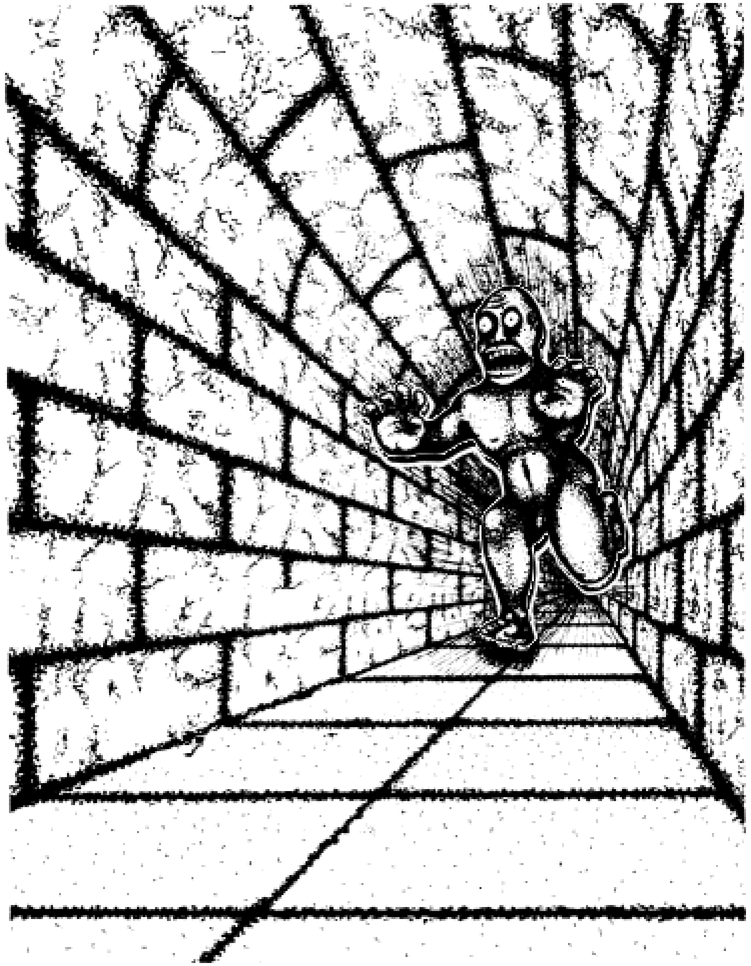


What is the height of the camera?

Slide by S. Seitz

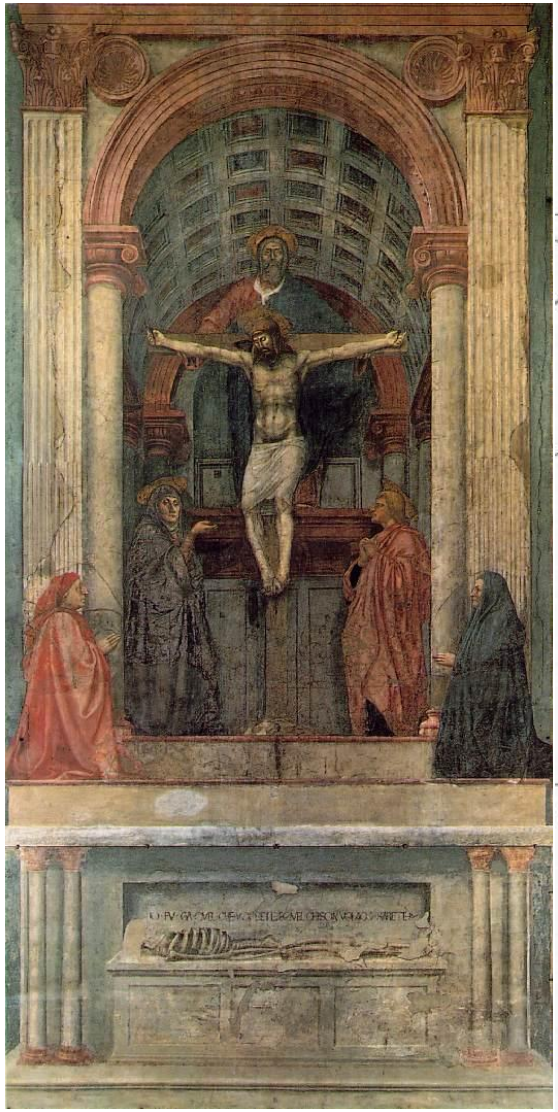
Perspective geometry

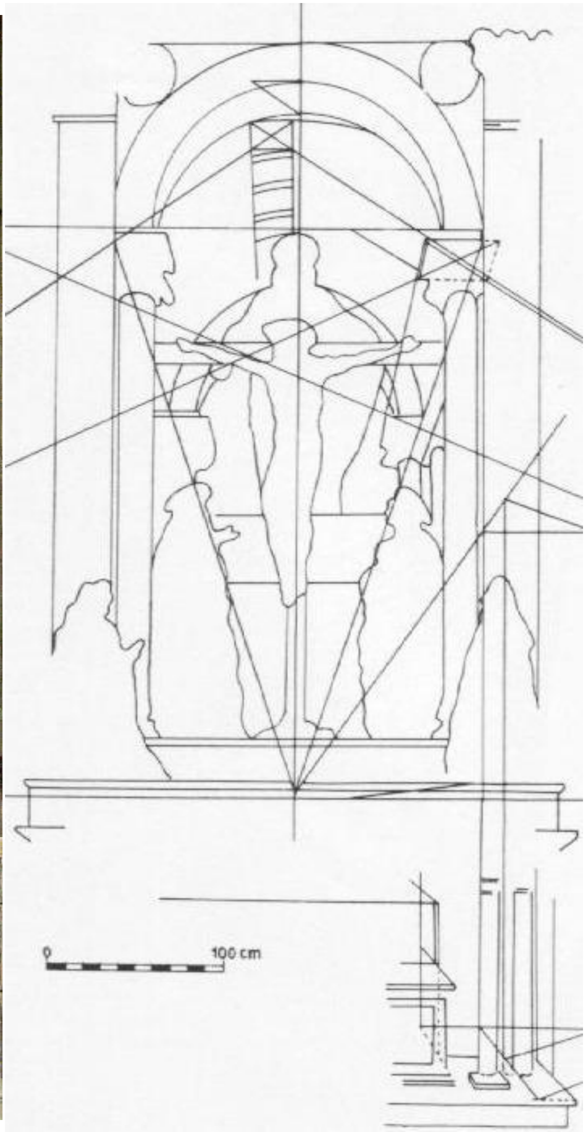




Illusion Credit: RN Shepard, Mind Sights: Original Visual Illusions, Ambiguities, and other Anomalies

Perspective cues in art





- Masaccio, *Trinity*, Santa Maria N ovella, Florence, 1425-28

- One of the first consistent uses o f perspective in Western art

Perspective distortion

- **What is the shape of the projection of a sphere?**

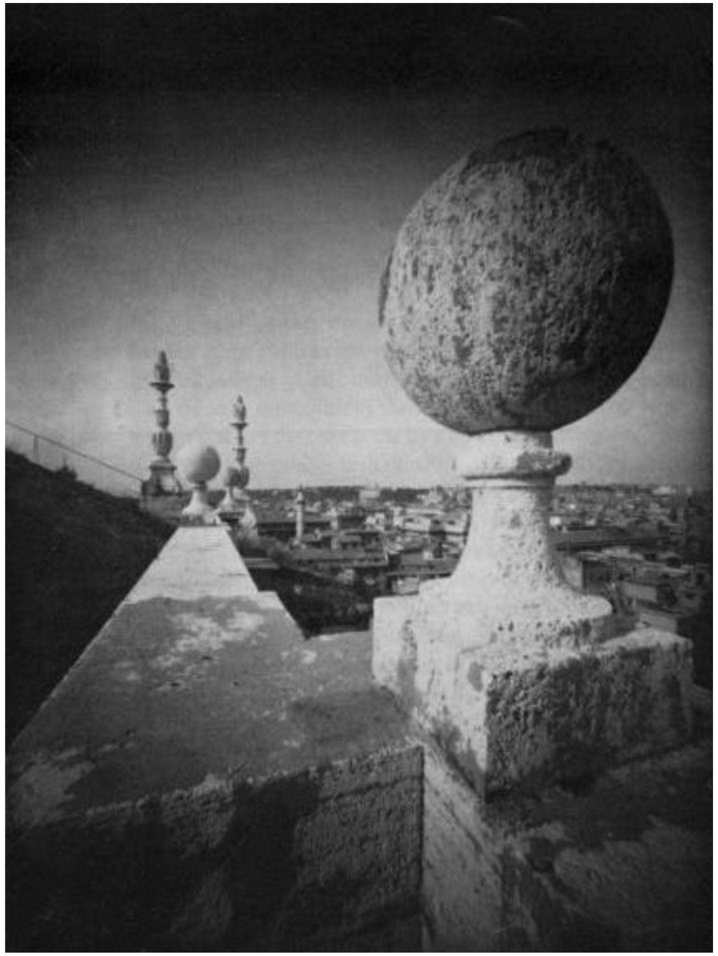
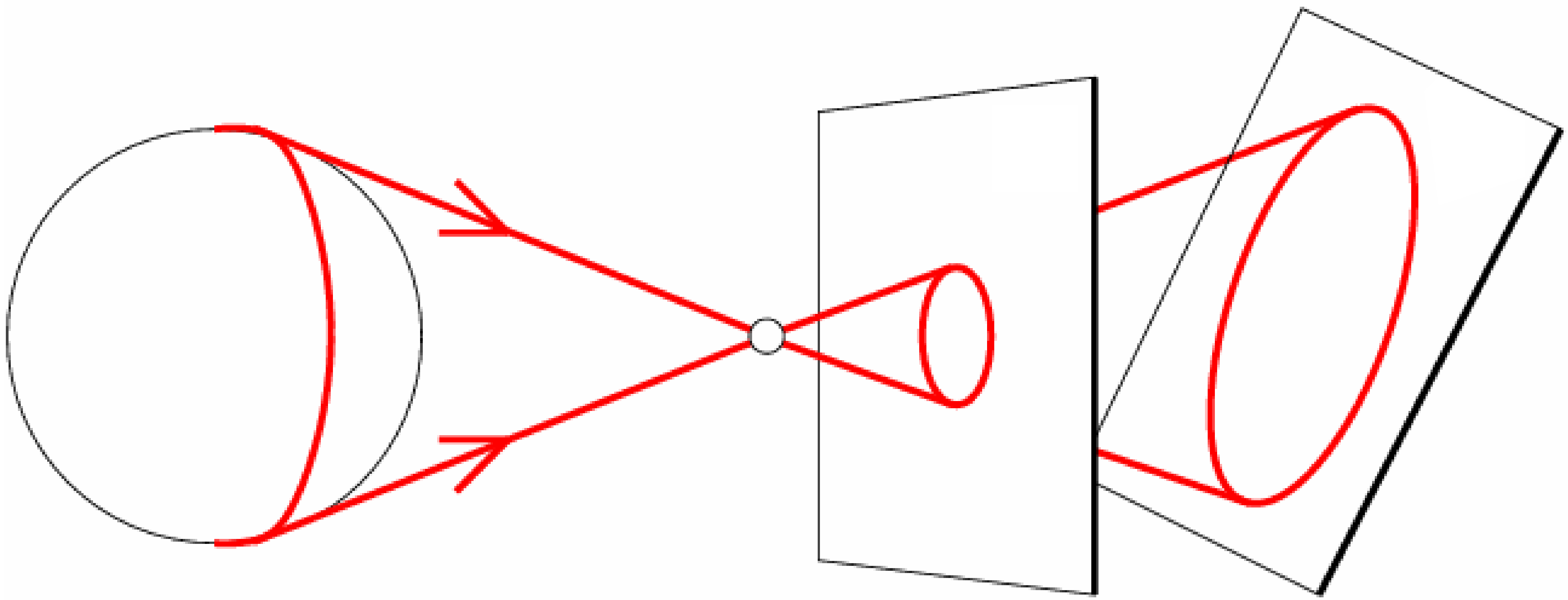


Image source: F. Durand

Perspective distortion

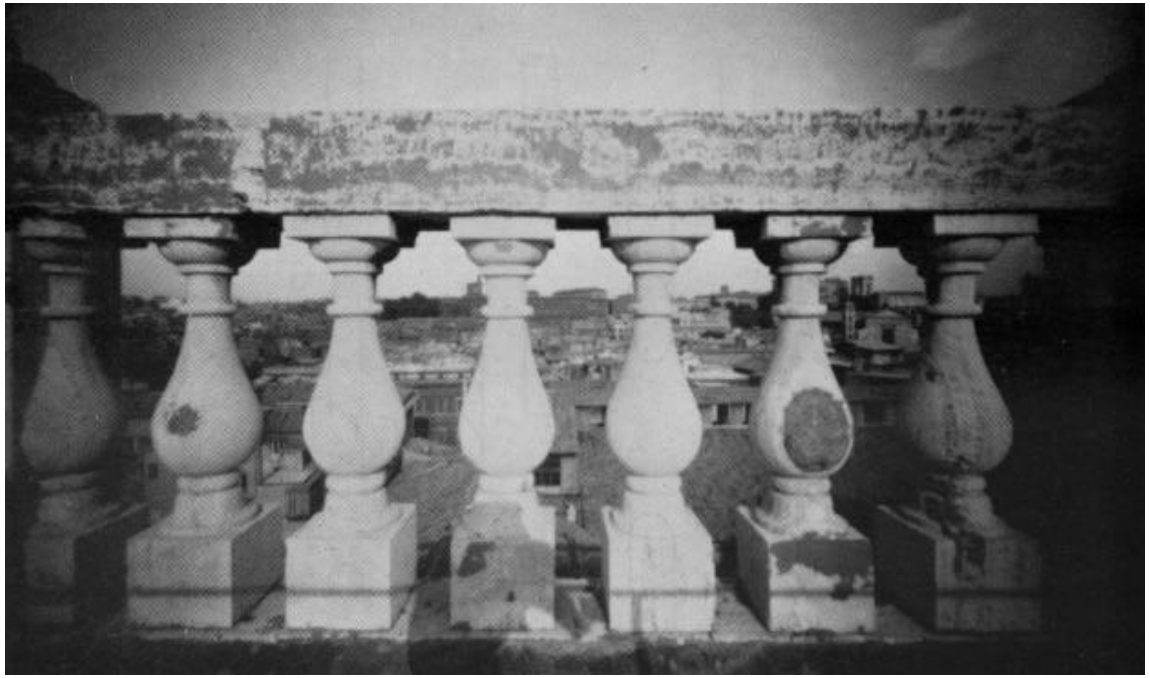
- What is the shape of the projection of a sphere?

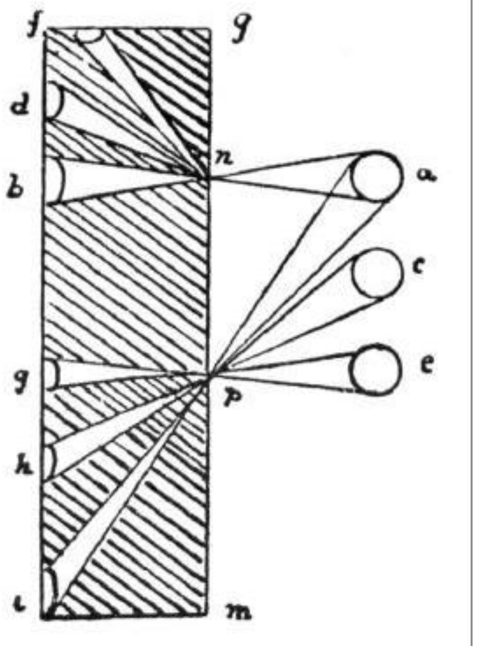


Slide from S. Lazebnik

Perspective distortion

- **Are the widths of the projected columns equal?**
  - The exterior columns are wider
  - This is not an optical illusion, and is not due to lens flaws
  - Phenomenon pointed out by Da Vinci



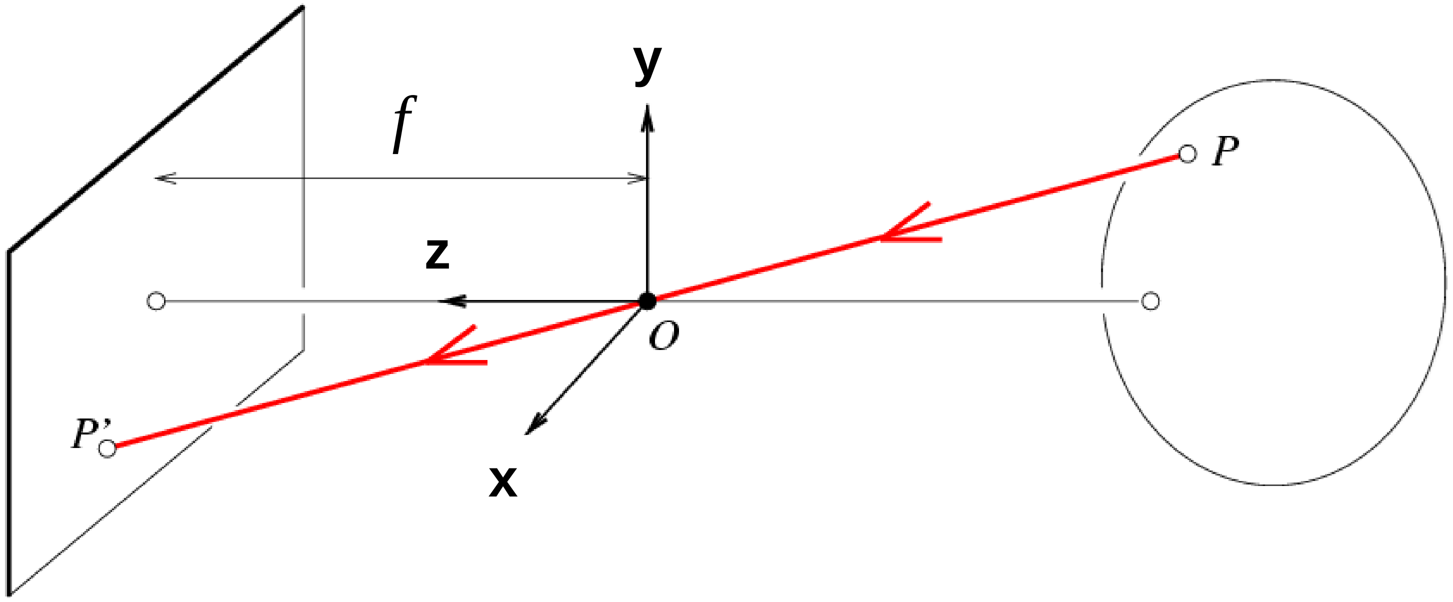


Source: F. Durand

Perspective distortion: People

Slide from S. Lazebnik

Modelling projection: world to image



- **Projection equation:** $(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$

Note: instead of dealing with an image that is upside down, most of the time we will pretend **that the image plane is in front of the camera center.**

Source: J. Ponce, S. Seitz

Homogeneous coordinate

- Nonlinearity in the projection equation

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$: division by z is nonlinear

- Add one more coordinate for linear transformation

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous coordinate

- **Why does this matter?**
  - Homogeneous coordinates can handle general cases
  - Invariant to scaling
  - Point in Cartesian is ray in Homogeneous

$$k\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \end{bmatrix}$$

Homogeneous Coordinates

Cartesian Coordinates

Credit: J. Hays

Photo Tourism

Exploring photo collections in 3D

Noah Snavely Steven M. Seitz Richard Szeliski

University of Washington *Microsoft Research*

SIGGRAPH 2006

Homogeneous coordinate

- **EBU6230 Image and Video Processing**
  - Week 2 day 1: image_transformations
- **https://pages.mtu.edu/~shene/COURSES/cs3621/NOTES/geometry/homo-coor.html**

Perspective projection matrix

- **Projection:**
  - a matrix multiplication with homogeneous coordinates

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} \quad \Rightarrow (f\,\frac{x}{z}, f\,\frac{y}{z})$$

divide by the third co ordinate

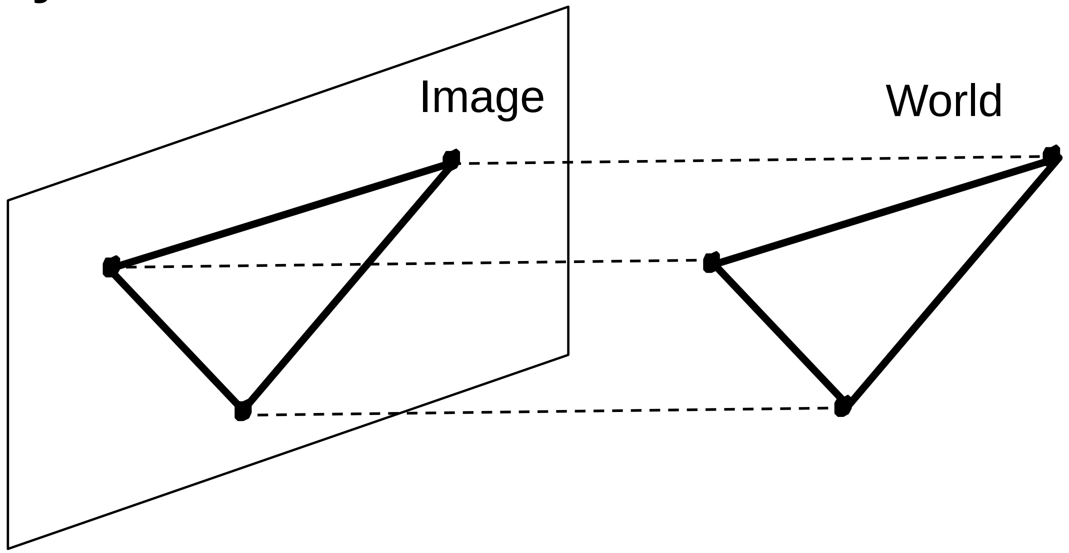
In practice: lots of coordinate transformations…

$$\begin{pmatrix} \text{2D} \\ \text{point} \\ (3\text{x}1) \end{pmatrix} = \begin{pmatrix} \text{Camera to} \\ \text{pixel coord.} \\ \text{trans. matrix} \\ (3\text{x}3) \end{pmatrix} \begin{pmatrix} \text{Perspective} \\ \text{projection matrix} \\ (3\text{x}4) \end{pmatrix} \begin{pmatrix} \text{World to} \\ \text{camera coord.} \\ \text{trans. matrix} \\ (4\text{x}4) \end{pmatrix} \begin{pmatrix} \text{3D} \\ \text{point} \\ (4\text{x}1) \end{pmatrix}$$

Slide from S. Lazebnik

Orthographic Projection

- **Special case of perspective projection**
  - Distance from center of projection to image plane is infinite
  - Also called "parallel projection"



Slide by Steve Seitz

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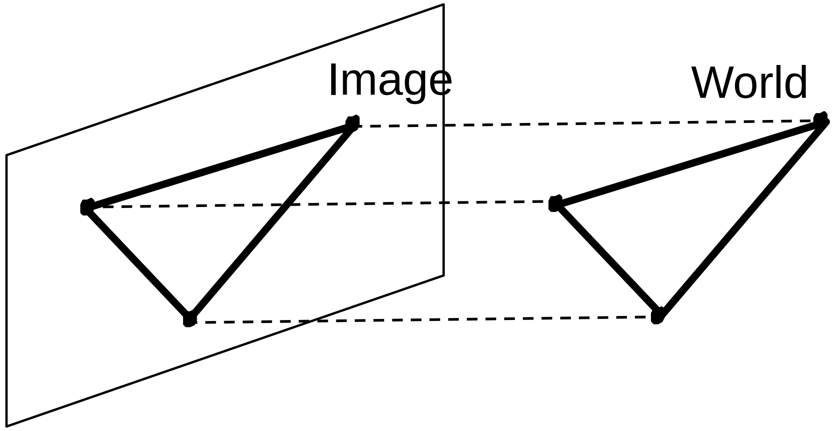


  - What’s the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

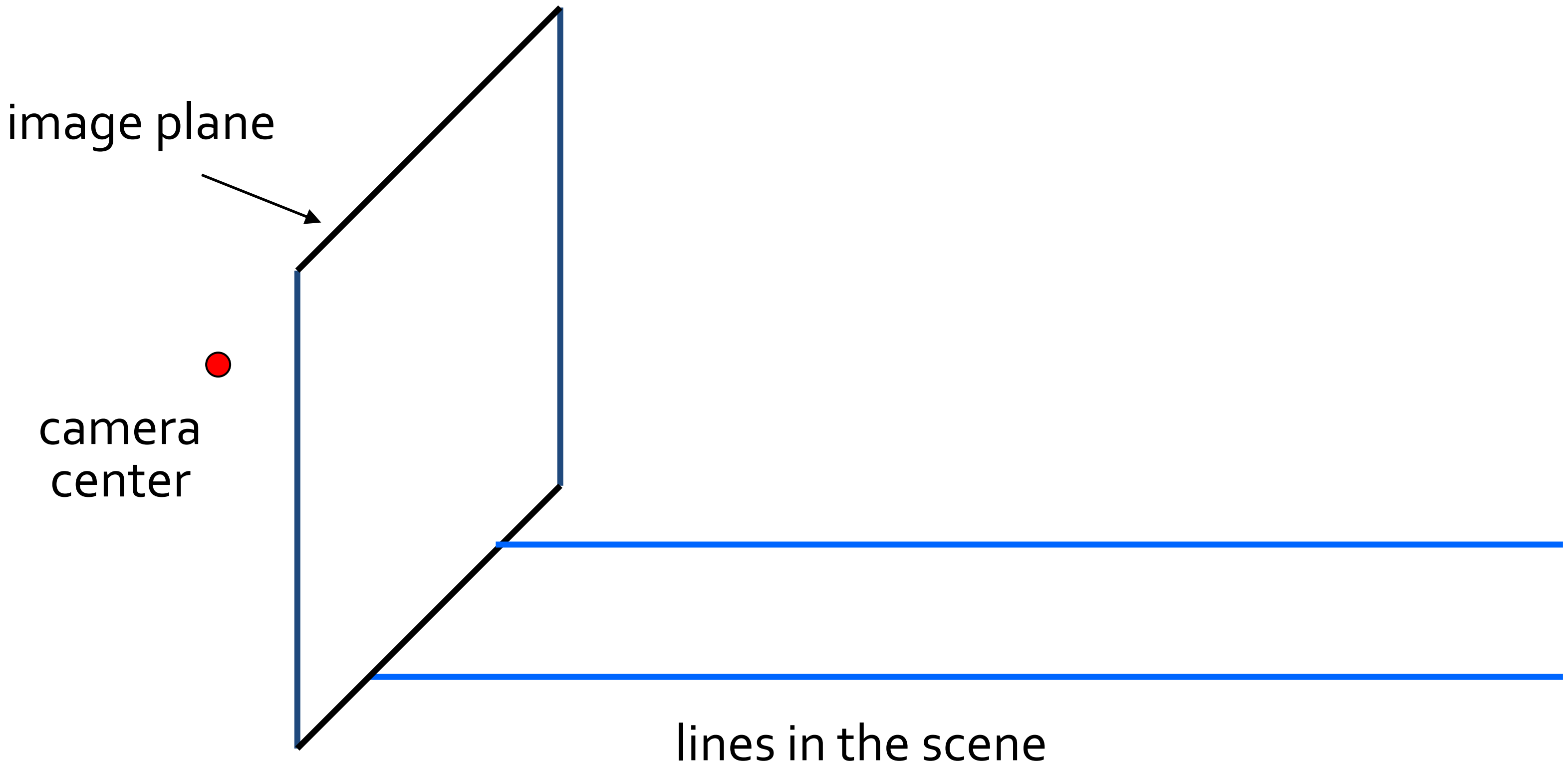
Slide by Steve Seitz

Quiz-01) 3D-to-2D

- When the 3D world dimension reduces to 2D, what properties are preserved?
  – Straight lines
  – Incidence
  – Angles
  – Lengths

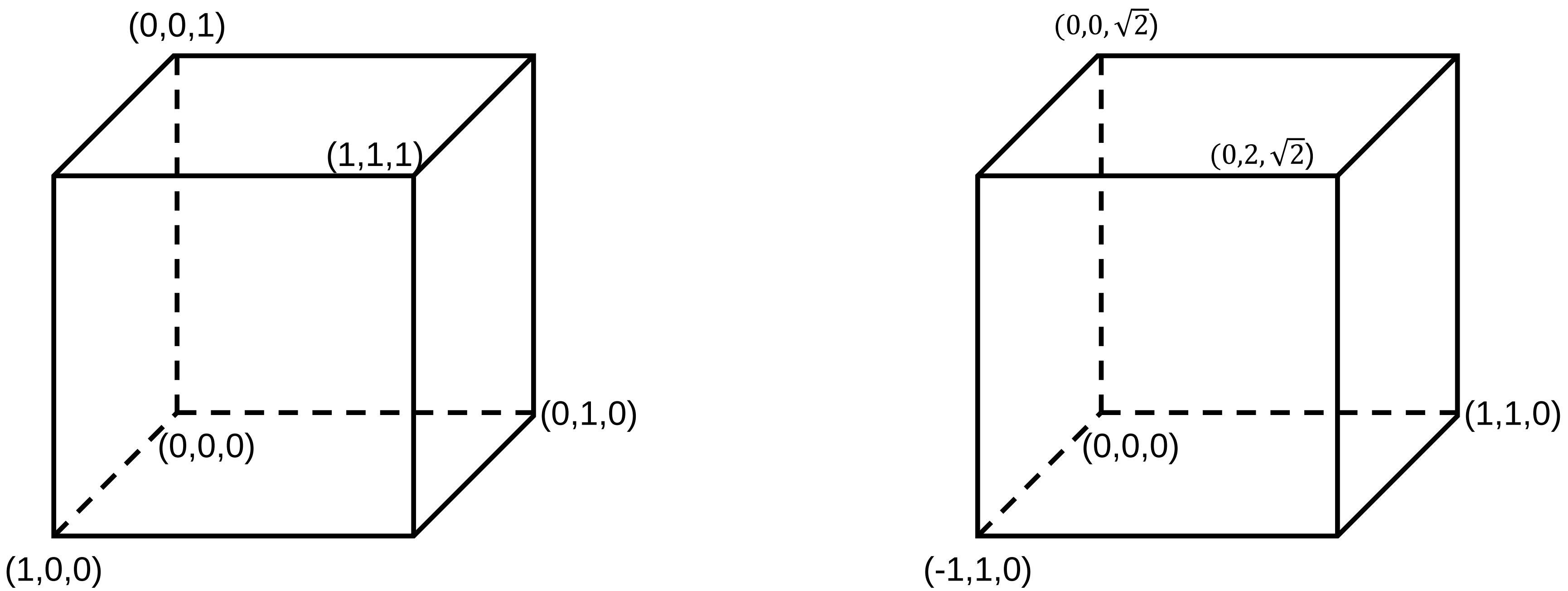
Quiz-02) Constructing the vanishing point of a line

- In the figure below, there are two blue parallel lines, perpendicular to the image plane. Draw how the lines look like in the image plan and discuss the result



Quiz-03) Orthographic projection

- Perform Orthograpic projection to each 3D cube in 3D below and show the result in 2D



Quiz-04) Projection matrix (2D-to-2D)

- Given two point sets:
  - $\boldsymbol{x} = \{\boldsymbol{x}_1, \dots, \boldsymbol{x}_4\} = \{(u_1, v_1), \dots, (u_4, v_4)\} = \{(0,260), (640,260), (0,400), (640,400)\}$
  - $\boldsymbol{x}' = \{\boldsymbol{x}'_1, \dots, \boldsymbol{x}'_4\} = \{(u'_1, v'_1), \dots, (u'_4, v'_4)\} = \{(0, 0), (400, 0), (0,640), (400,640)\}$

  Find the perspective projection matrix $\boldsymbol{P}$ such that $\boldsymbol{x}' = \boldsymbol{P}\boldsymbol{x}$

EBU7240
Computer Vision
- Camera: lenses & digital sensors -

Semester 1, 2021

Changjae Oh

Content

- **The pinhole projection model**
  - Geometric properties
  - Perspective projection matrix
- **Cameras with lenses**
  - Depth of focus
  - Field of view
  - Lens aberrations
- **Digital sensors**

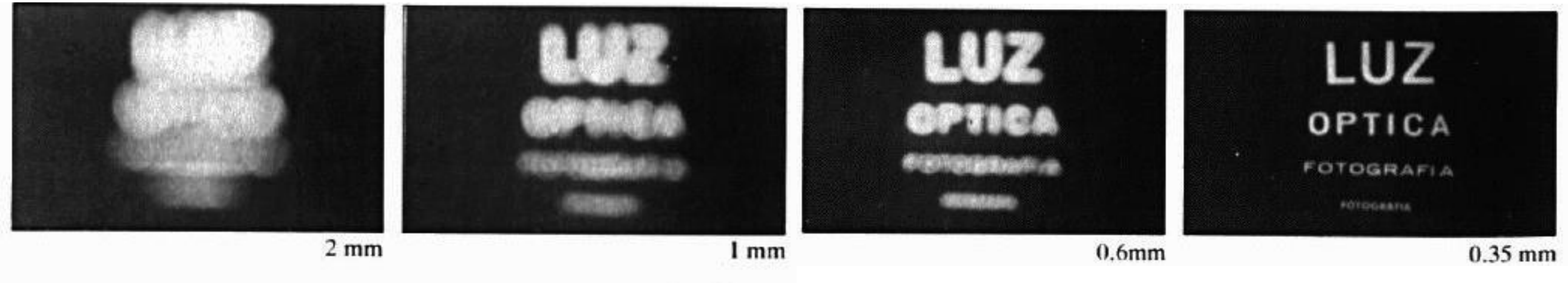
Building a Real Camera

Home-made pinhole camera

http://www.debevec.org/Pinhole/

Shrinking the aperture

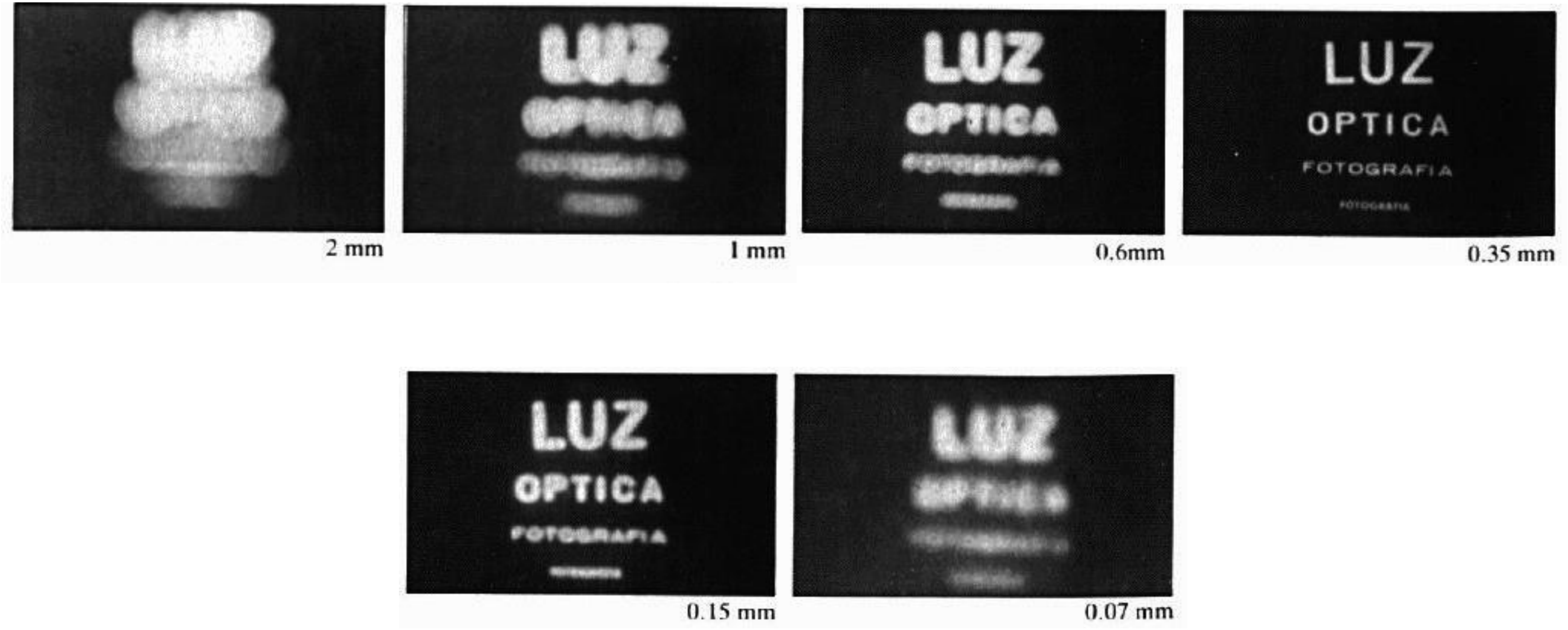
- **Images with varying the aperture size**



- **Why not make the aperture as small as possible?**
  - Less light gets through
  - Diffraction effects...

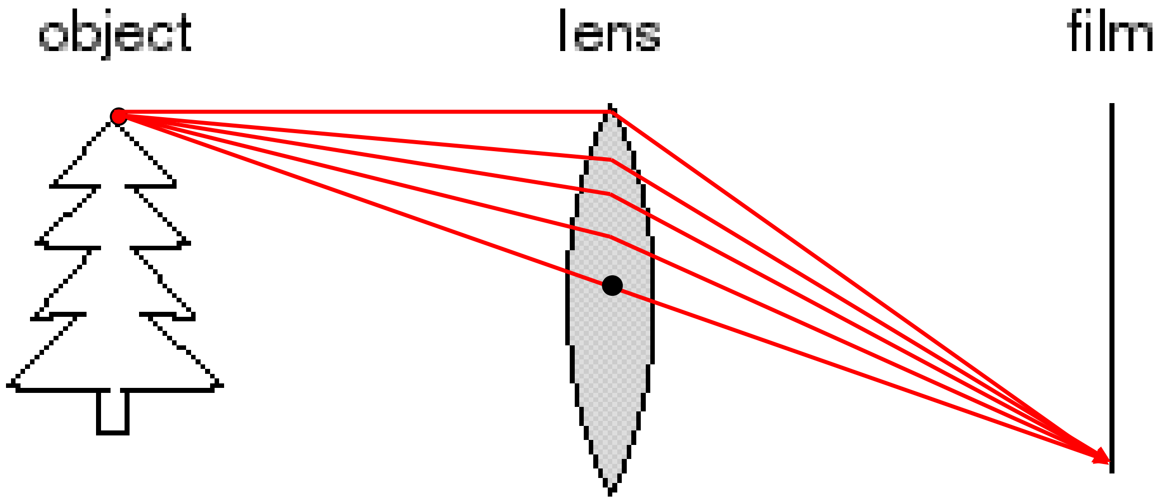
Shrinking the aperture

- Images with varying the aperture size



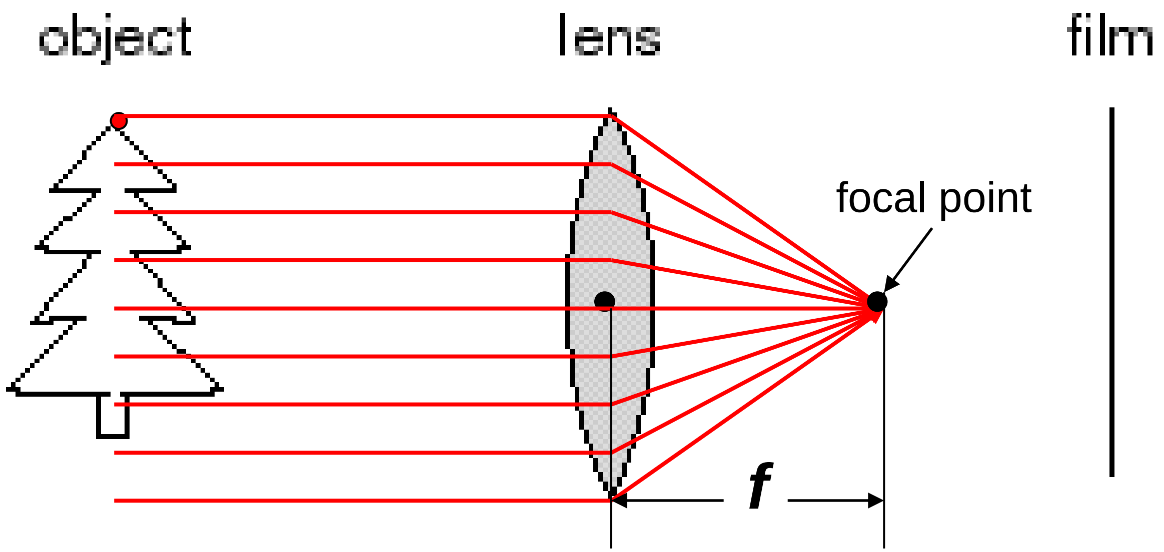
Adding a lens

- **A lens focuses light onto the film**
  - Thin lens model:
    - Rays passing through the center are not deviated (pinhole projection model still holds)



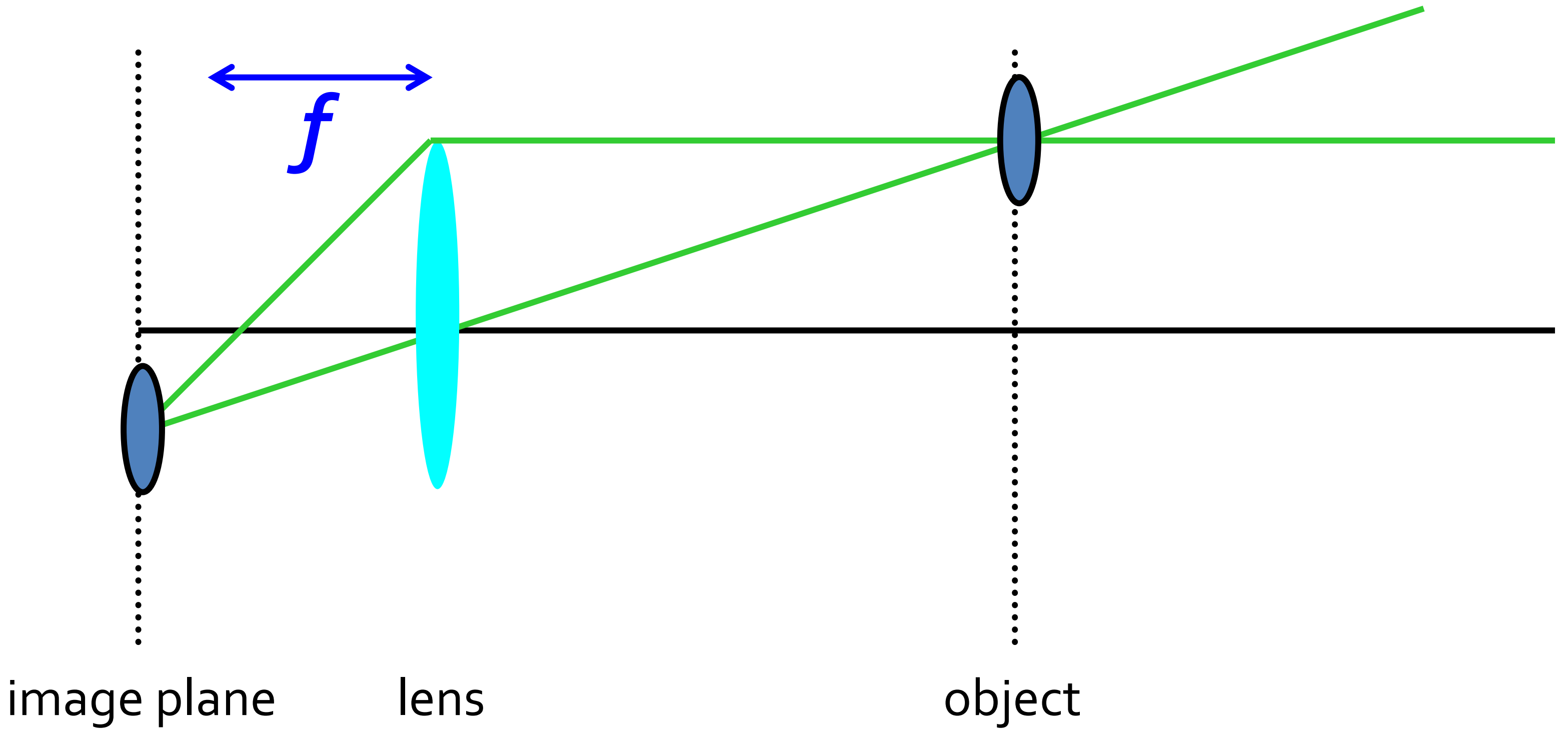
Adding a lens

- **A lens focuses light onto the film**
  - Thin lens model:
    - Rays passing through the center are not deviated (pinhole projection model still holds)
    - All rays parallel to the optical axis pass through the *focal point*
    - All parallel rays converge to points on the *focal plane*



Thin lens formula

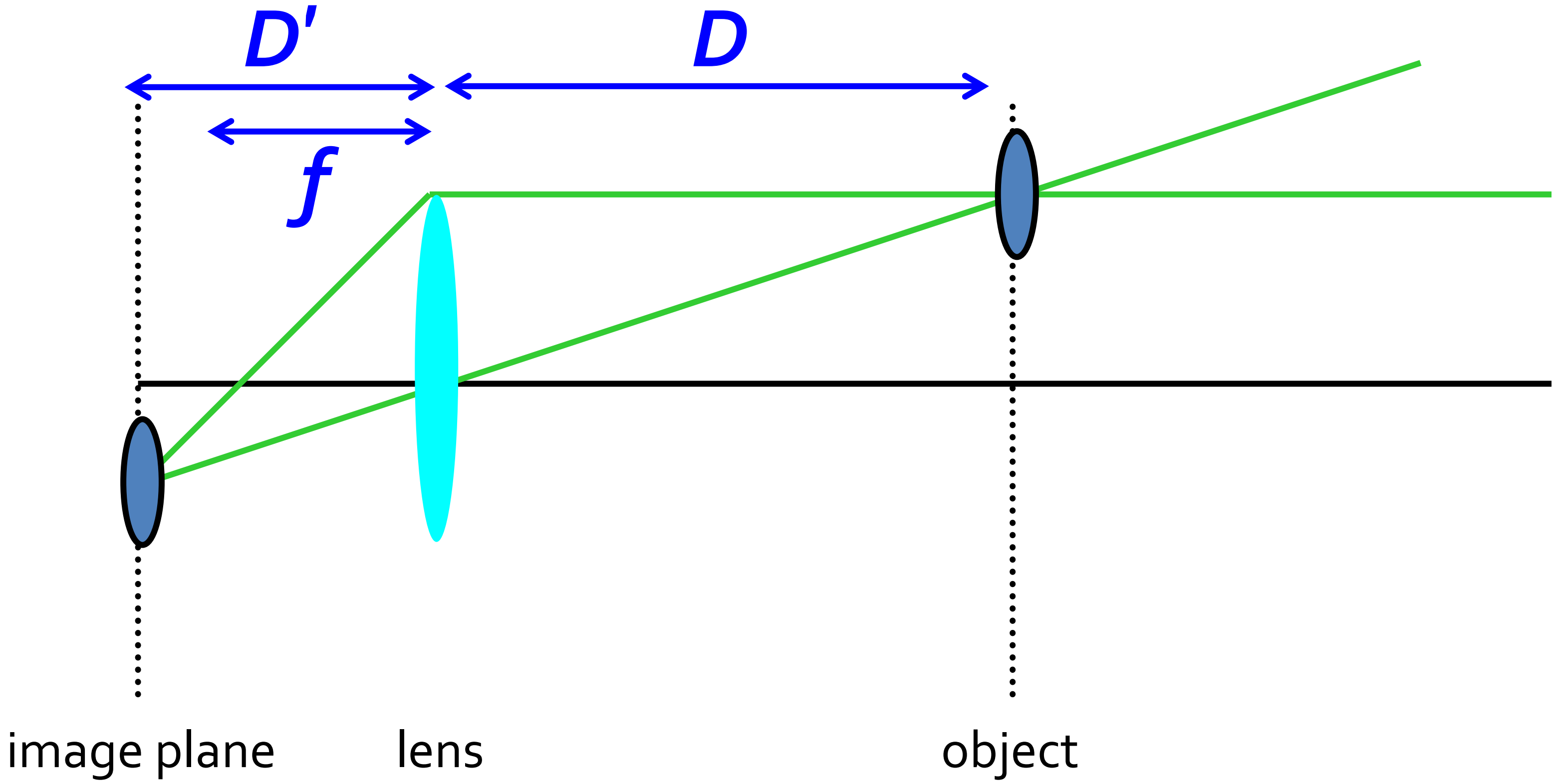
- Where does the lens focus the rays coming from a given point in the scene?



Slide by F. Durand

Thin lens formula

- What is the relation between the focal length (f), the distance of the object from the optical center (D), and the distance at which the object will be in focus (D')?

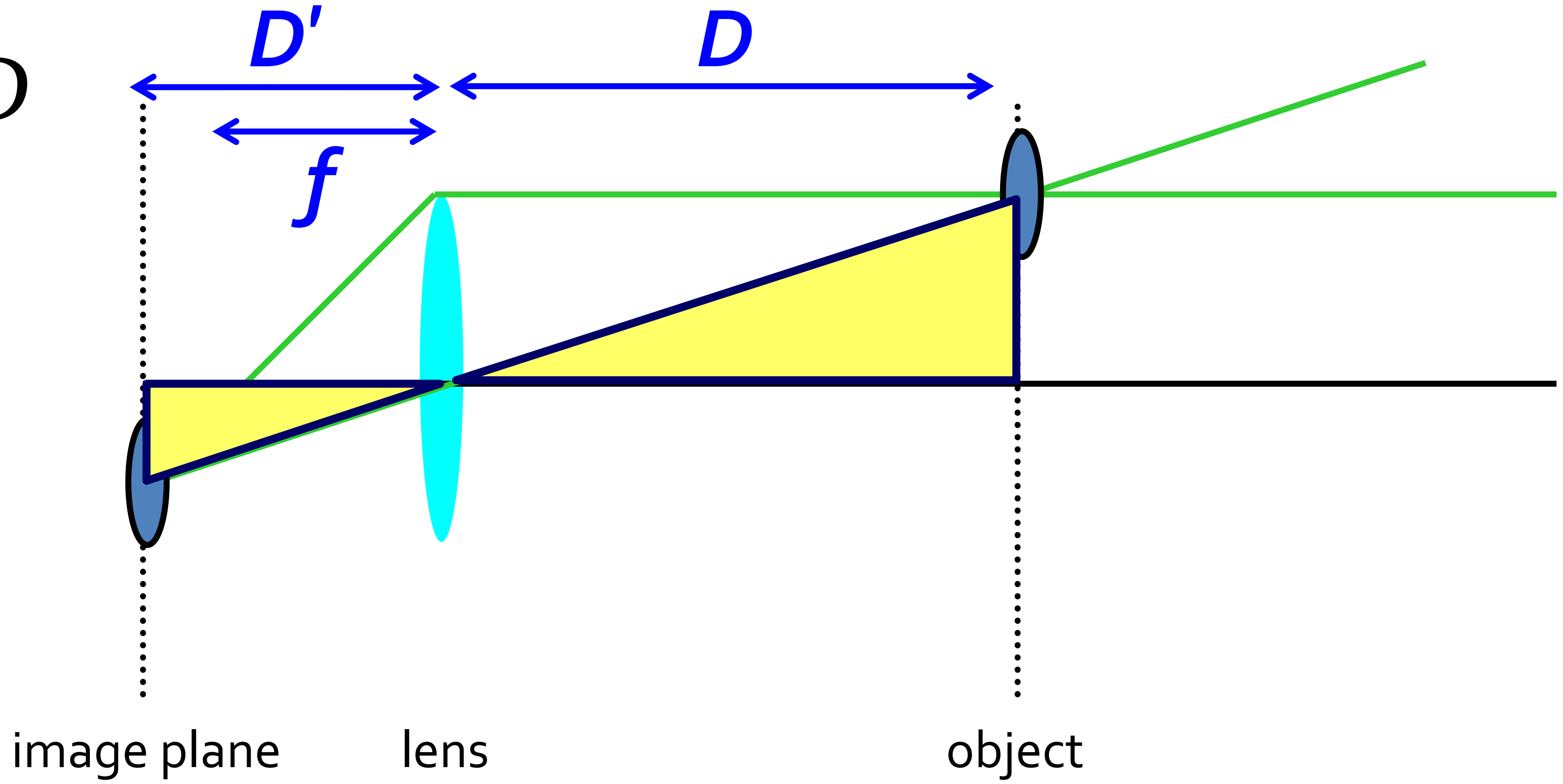


Slide by F. Durand

Thin lens formula

- **Similar triangles everywhere!**

$$y'/y = D'/D$$



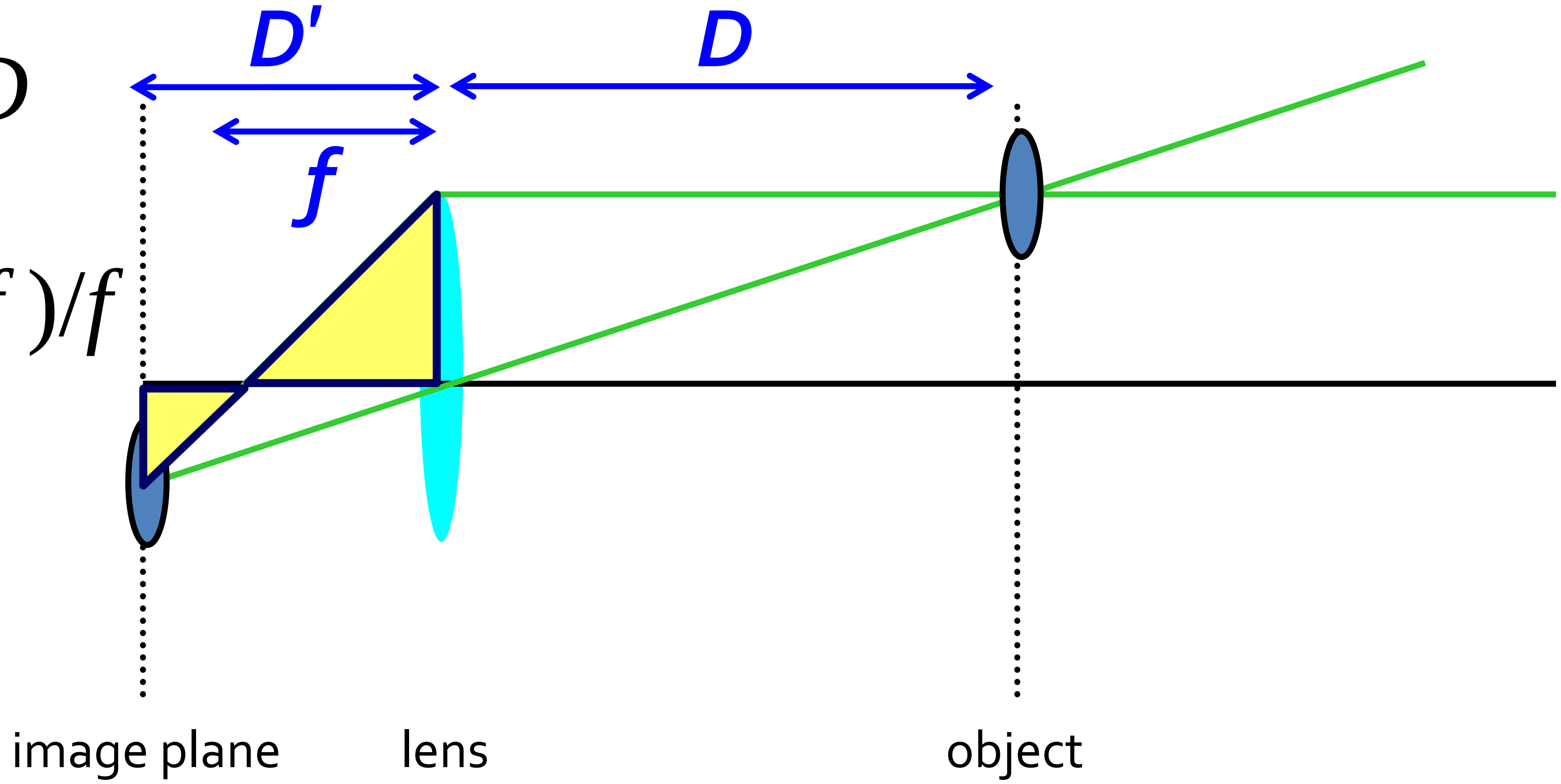
Slide by F. Durand

Thin lens formula

- **Similar triangles everywhere!**

$$y'/y = D'/D$$

$$y'/y = (D' - f\,)/f$$



Slide by F. Durand

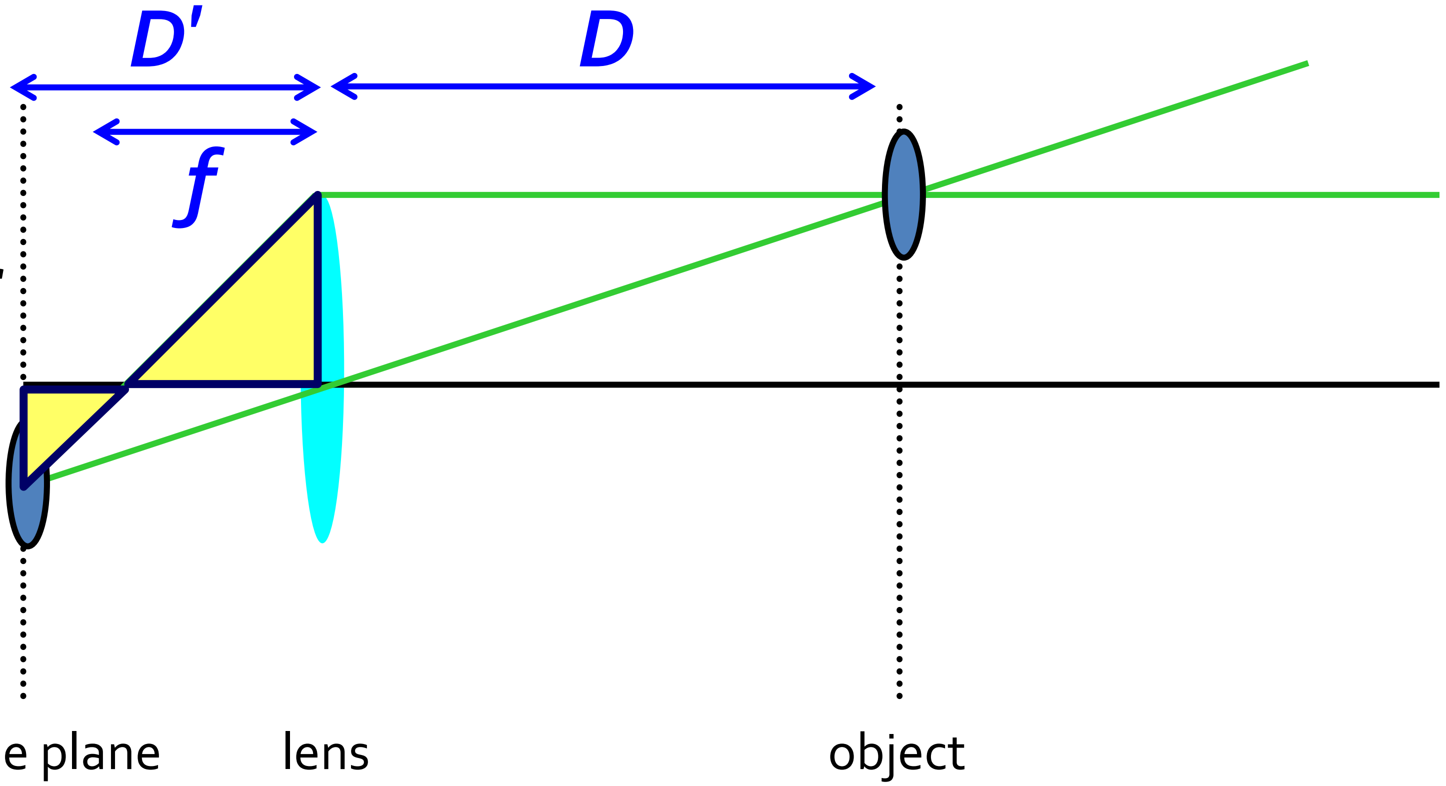
Thin lens formula

- Any point satisfying the thin lens equation is in focus.
- What happens when *D* is very large?

$$y'/y = D'/D$$

$$y'/y = (D'-f)/f$$

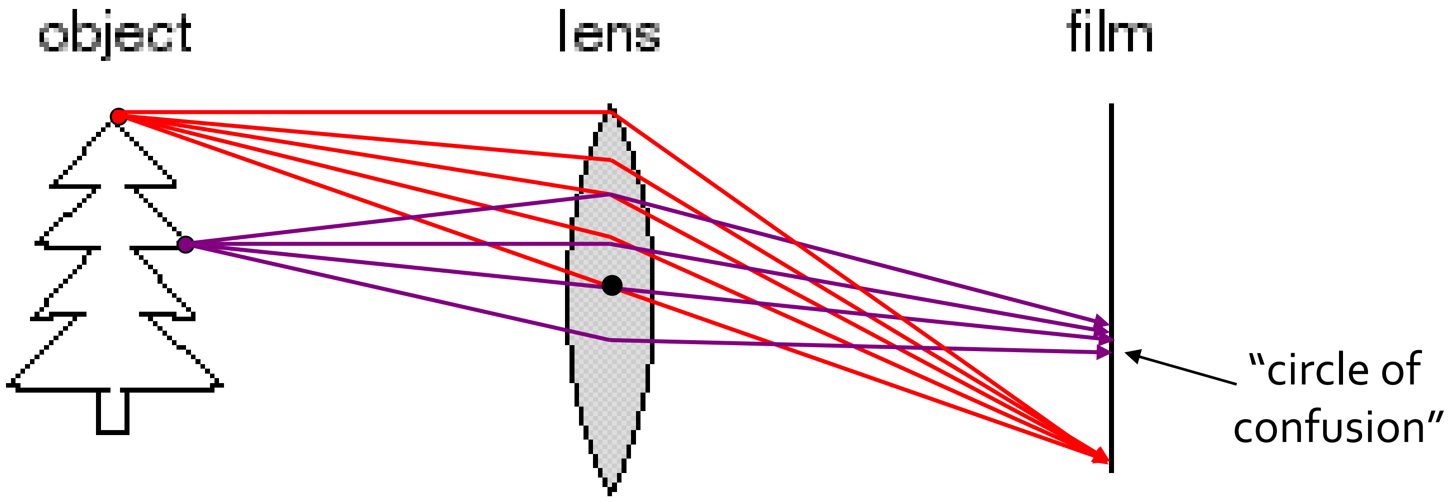
$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$



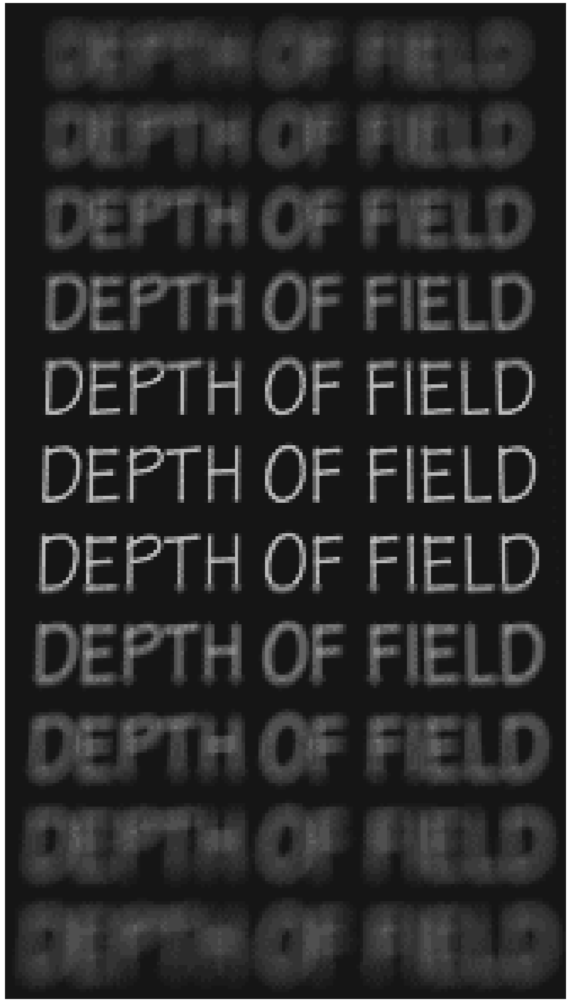
Slide by F. Durand

Depth of Field

- **For a fixed focal length, there is a specific distance at which objects are "in focus"**
  - Other points project to a "circle of confusion" in the image



Depth of Field

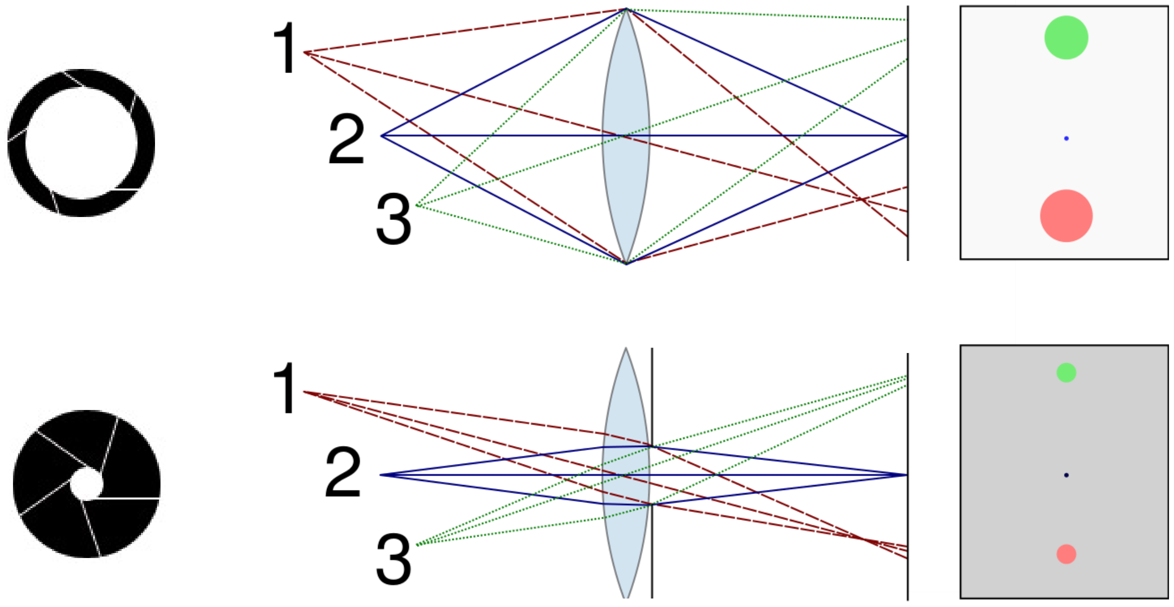


http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Slide by A. Efros

Controlling depth of field

- **Changing the aperture size affects depth of field**
  - A smaller *aperture* increases the range in which the object is approximately in focus
  - But small aperture reduces amount of light – need to increase *exposure*



http://en.wikipedia.org/wiki/File:Depth_of_field_illustration.svg

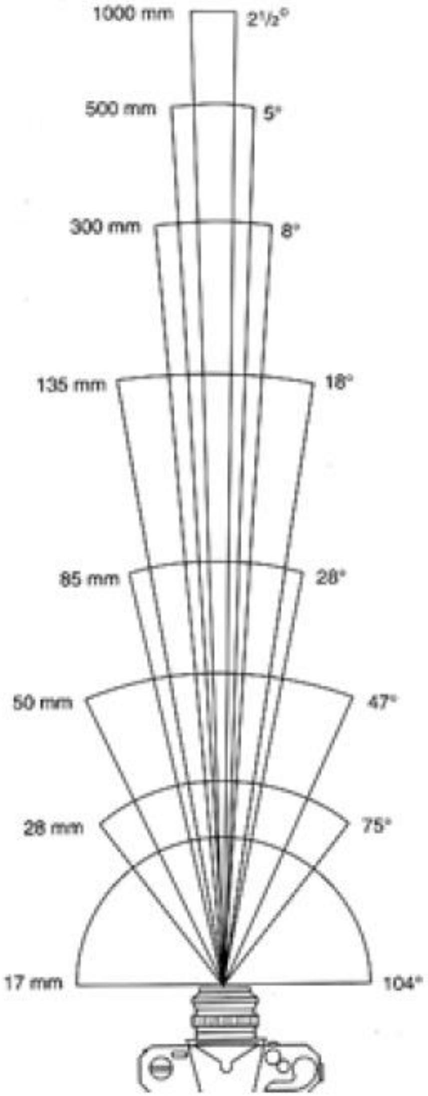
Varying the aperture

Large aperture = small DOF

Small aperture = large DOF

Slide by A. Efros

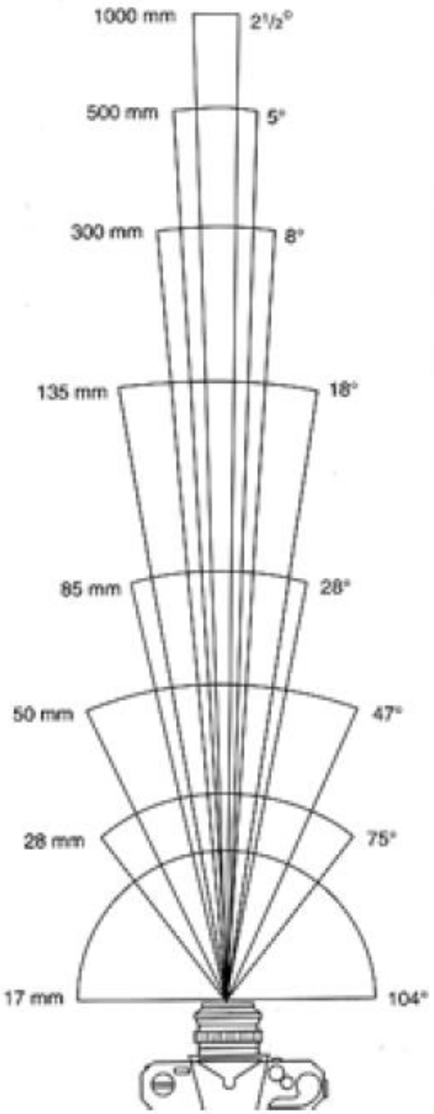
Field of View

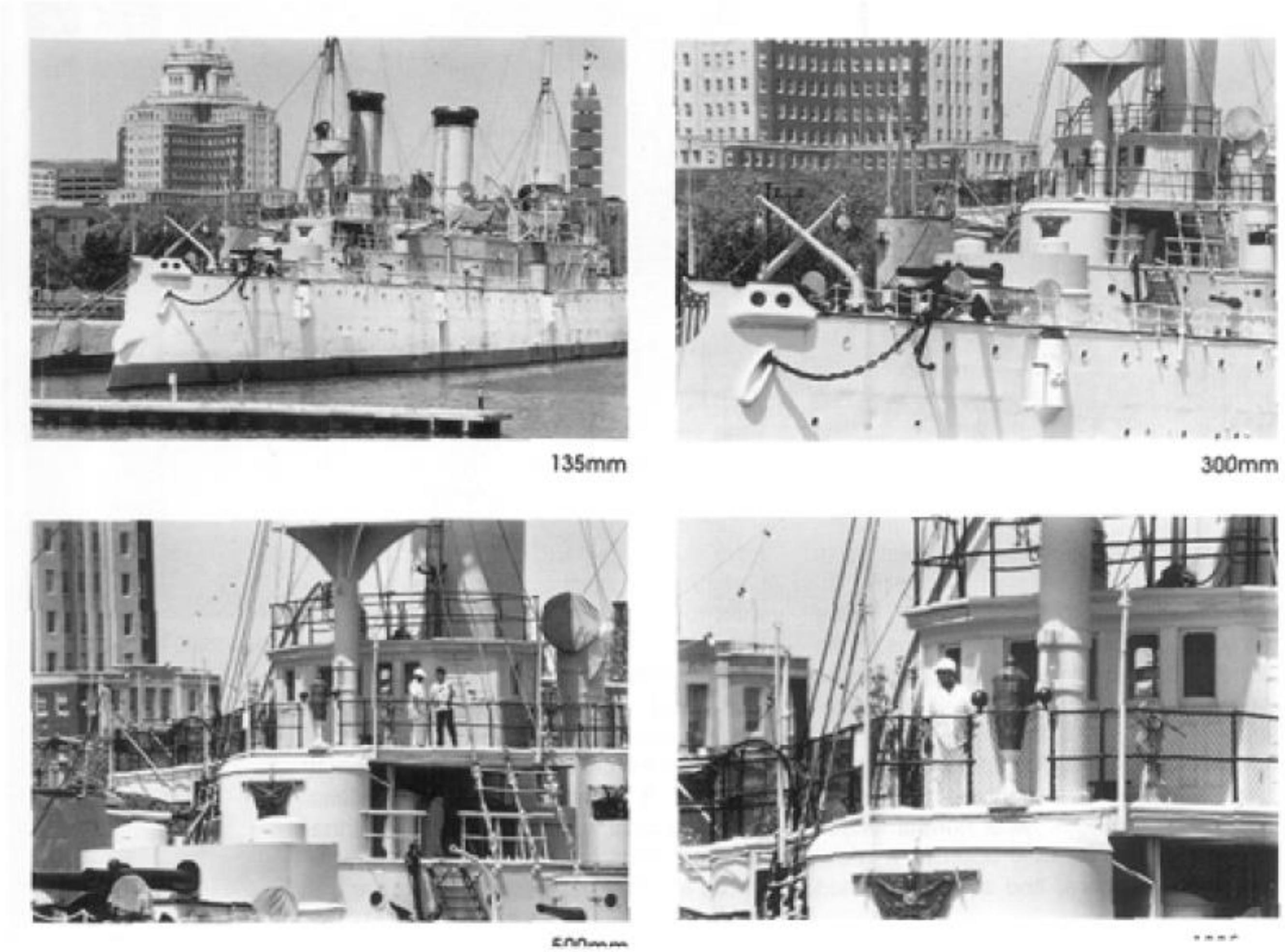


From London and Upton

Slide by A. Efros

Field of View

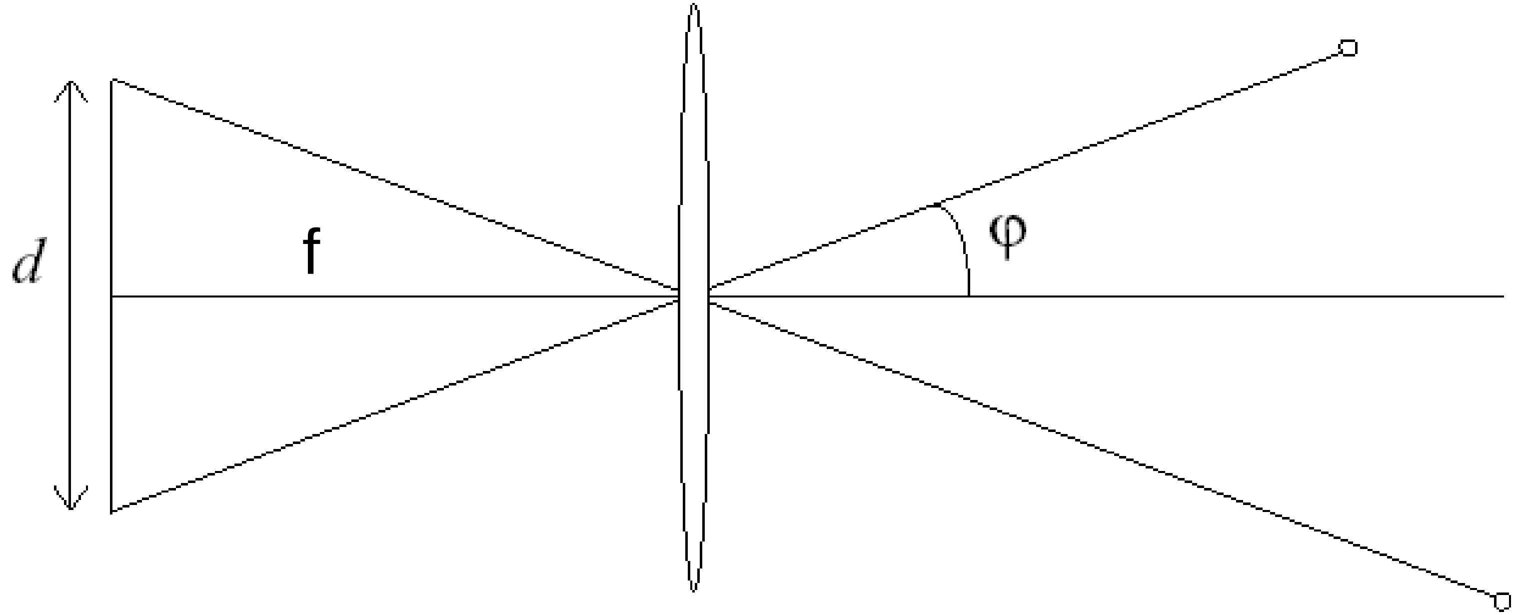




From London and Upton

Slide by A. Efros

Field of View



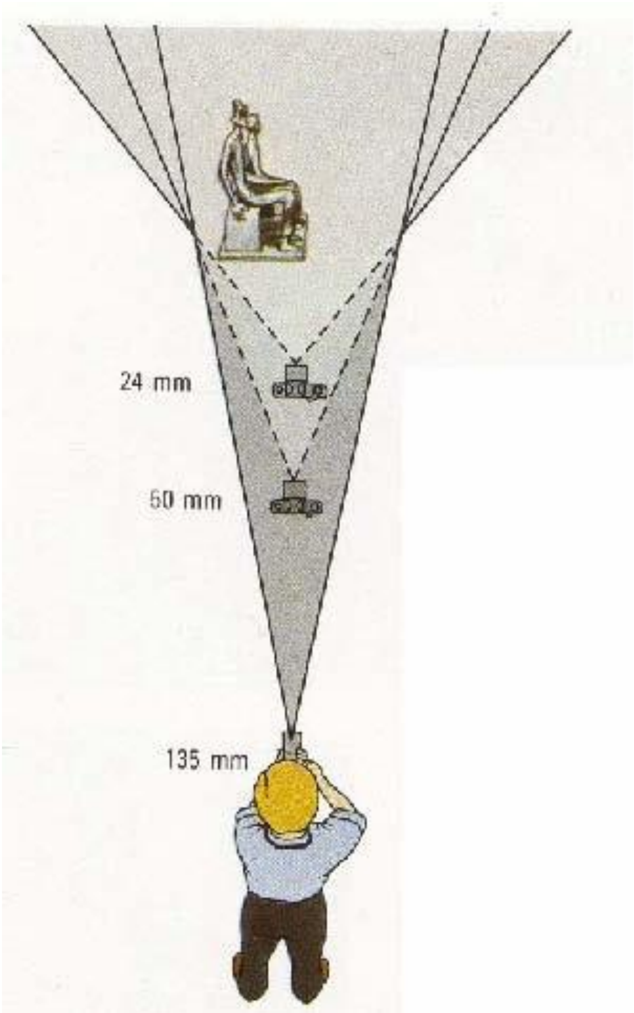
FOV depends on focal length and size of the camera retina

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Larger focal length = smaller FOV

Slide by A. Efros

Field of View / Focal Length



Large FOV, small *f*
Camera close to car

Small FOV, large *f*
Camera far from the car

Sources: A. Efros, F. Durand

Same effect for faces

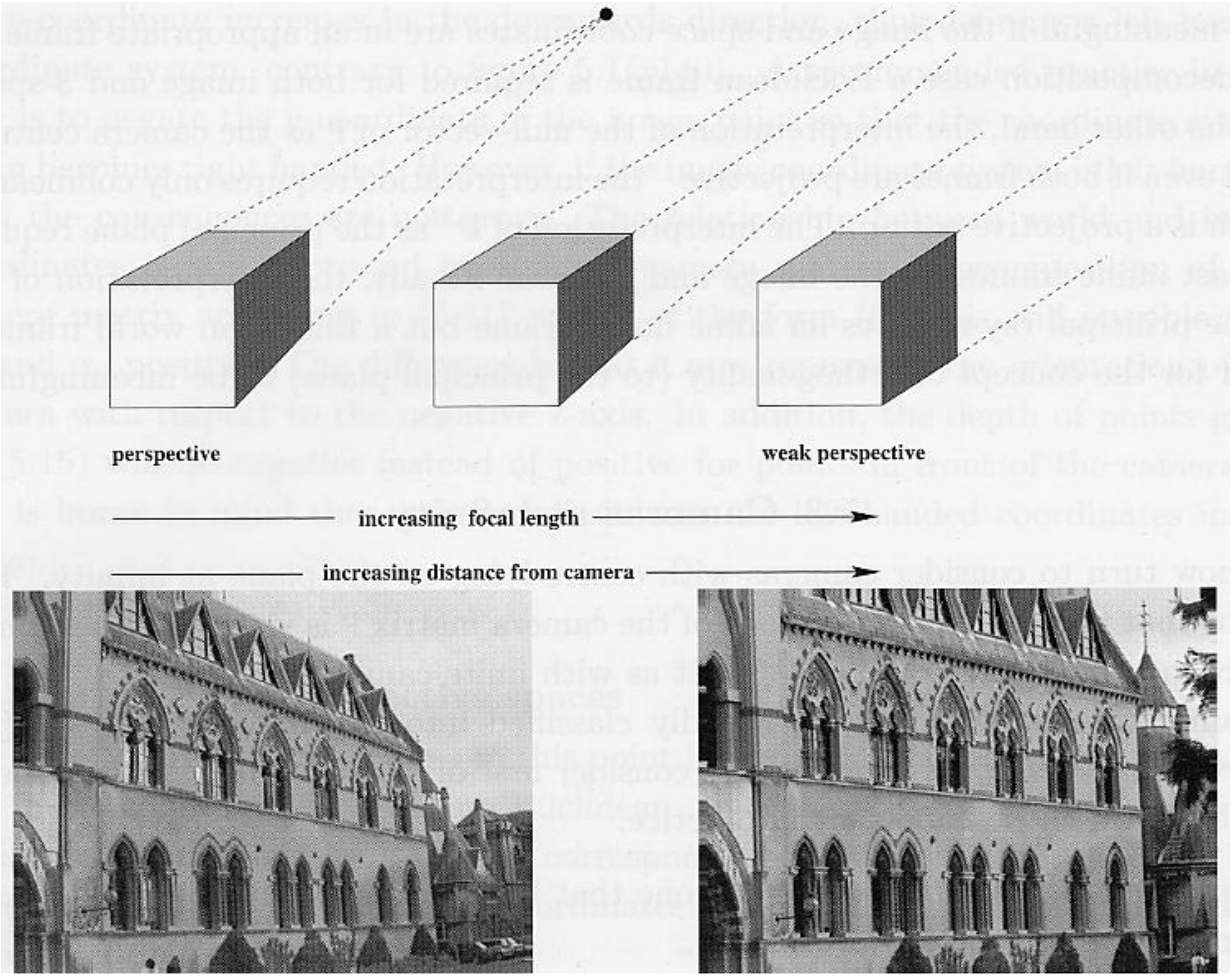
wide-angle

standard

telephoto

Source: F. Durand

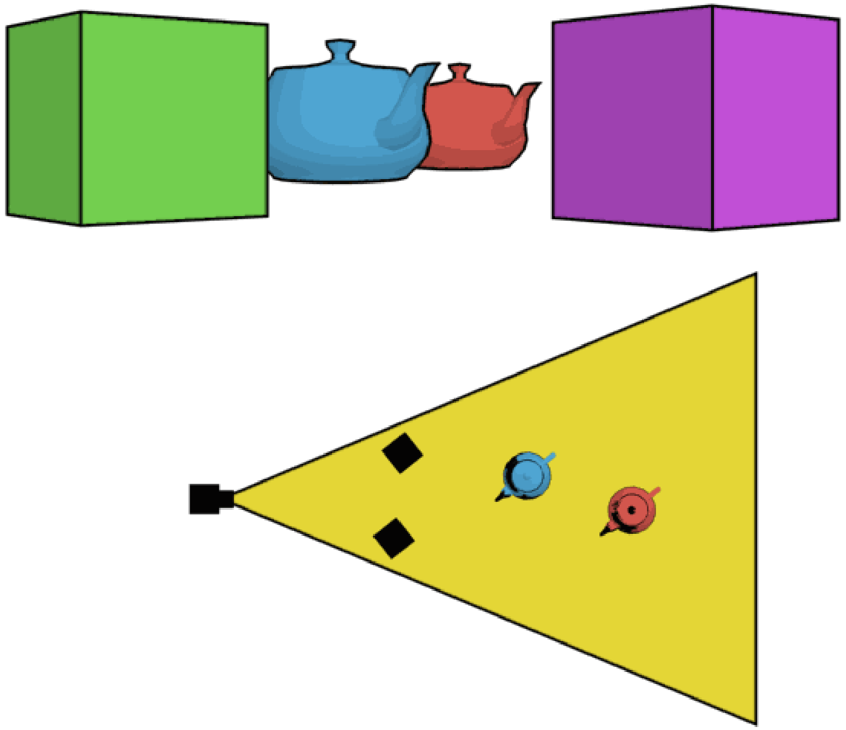
Approximating an orthographic camera



Source: Hartley & Zisserman

The dolly zoom

- Continuously adjusting the focal length while the camera moves away fr om (or towards) the subject



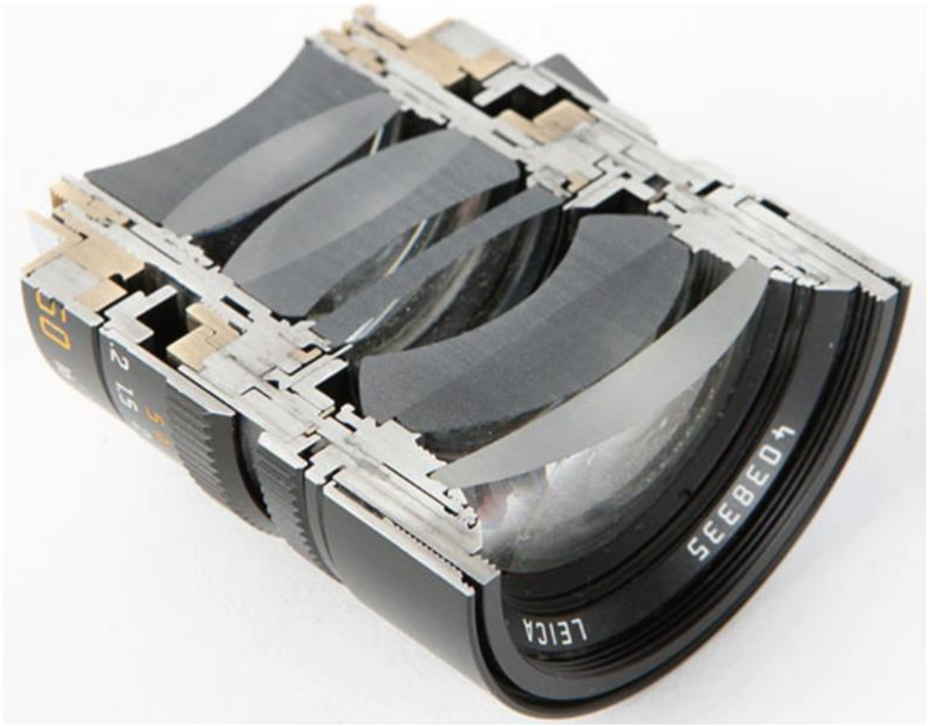
http://en.wikipedia.org/wiki/Dolly_zoom

The dolly zoom

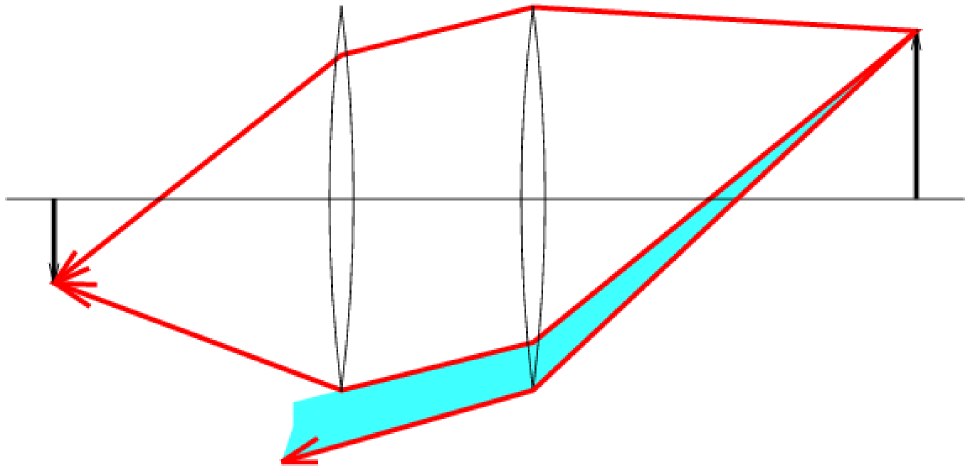
- **Continuously adjusting the focal length while the camera moves away from (or towards) the subject**

[Example of dolly zoom from *Goodfellas*] (YouTube)

Real lenses

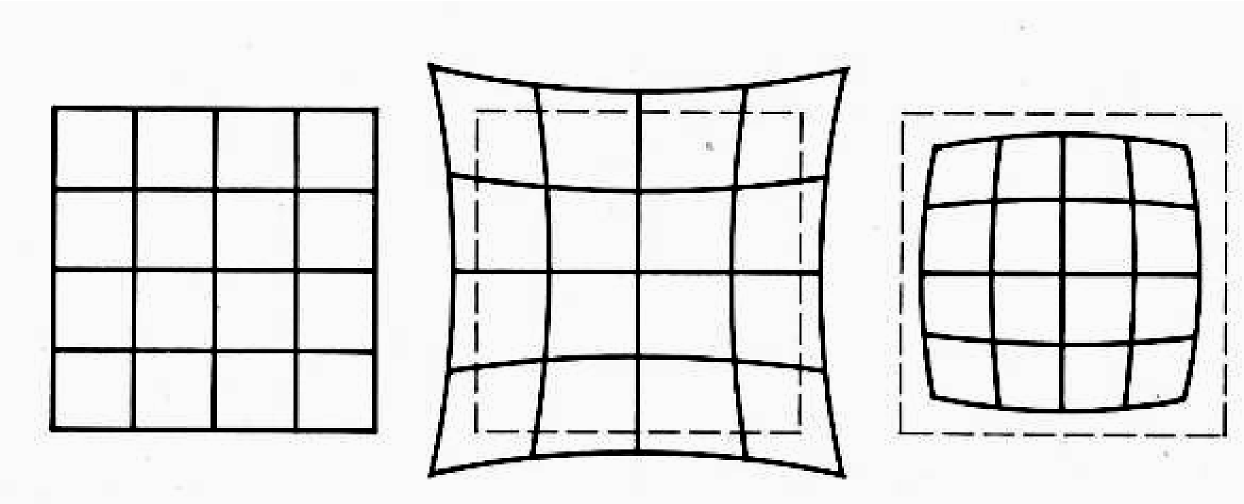


Lens flaws: Vignetting



Radial Distortion

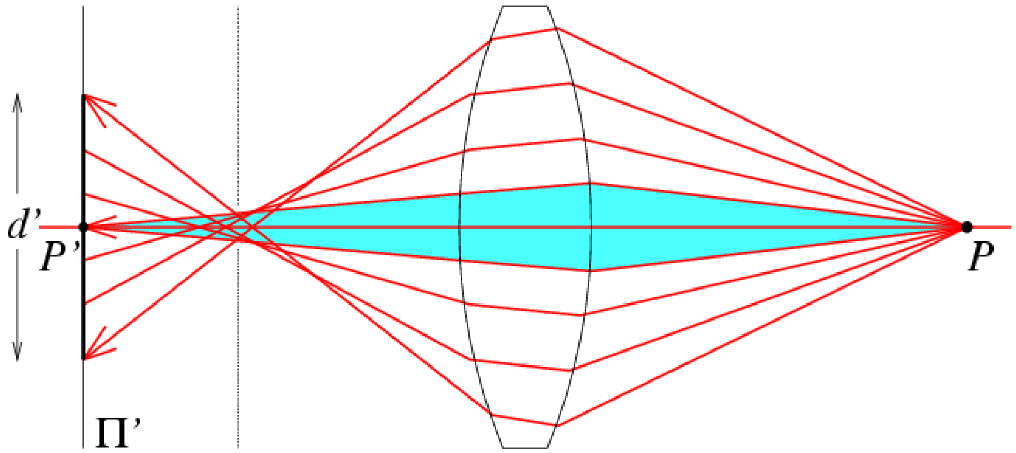
- **Caused by imperfect lenses**
- **Deviations are most noticeable near the edge of the lens**



No distortion Pin cushion Barrel

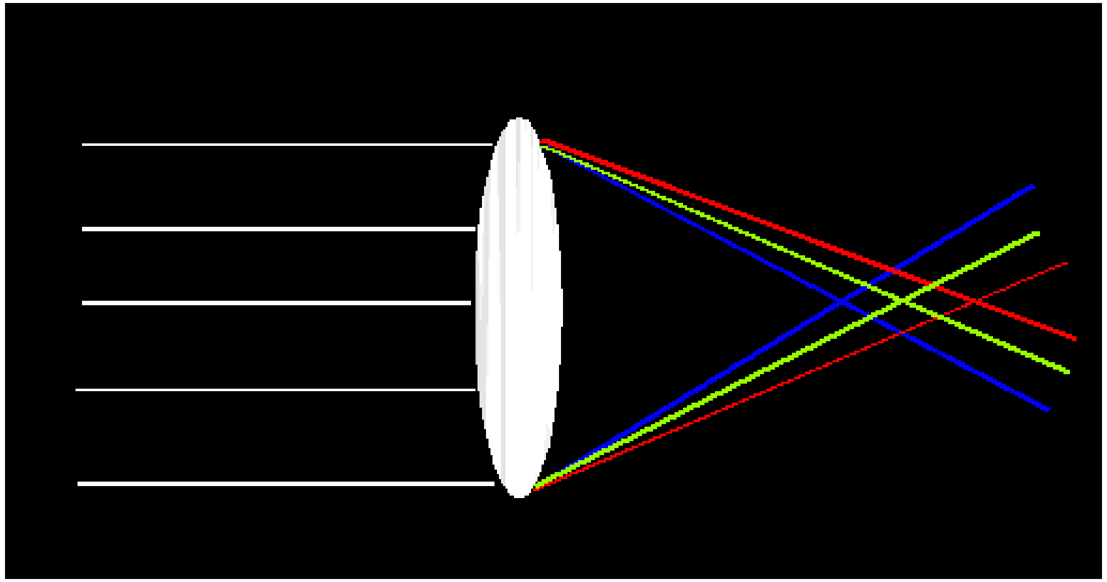
Lens flaws: Spherical aberration

- Spherical lenses don't focus light perfectly
- Rays farther from the optical axis focus closer

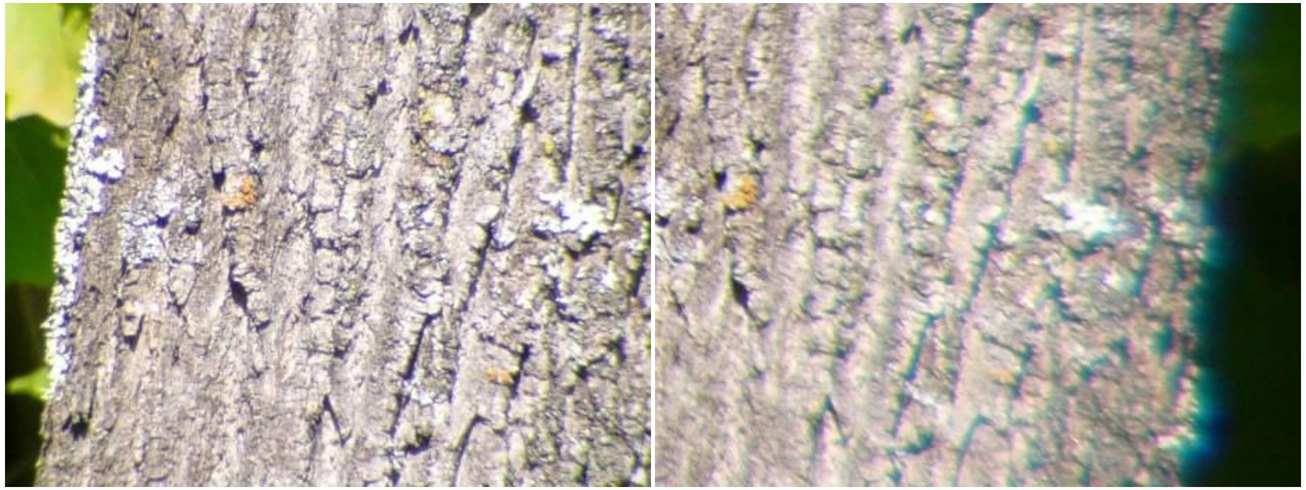


Lens flaws: Chromatic Aberration

- Lens has different refractive indices for different wavelengths: causes co lor fringing



Near Lens Center | Near Lens Outer Edge



Content

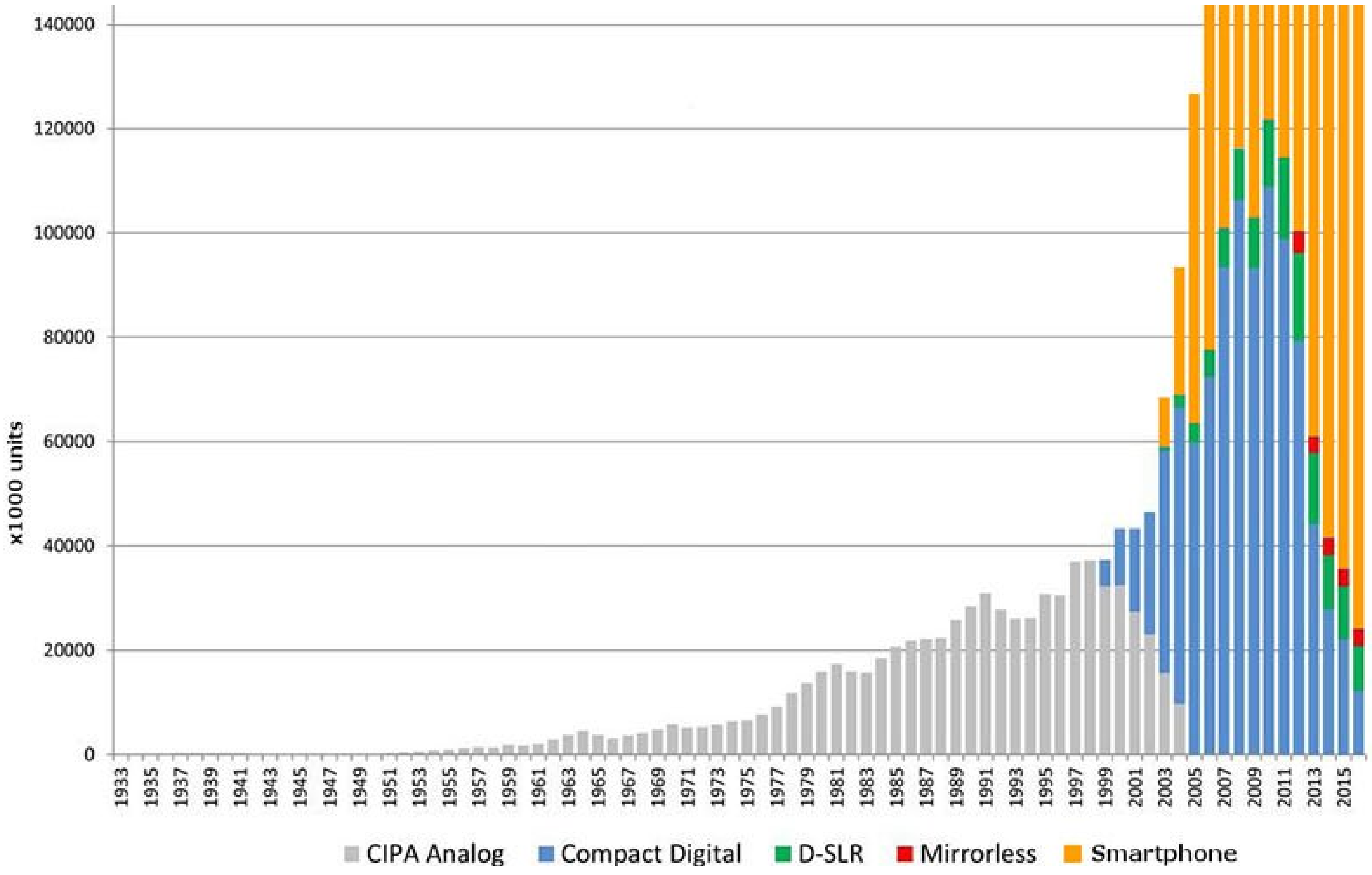
- **The pinhole projection model**
  - Geometric properties
  - Perspective projection matrix
- **Cameras with lenses**
  - Depth of focus
  - Field of view
  - Lens aberrations
- **Digital sensors**

First digitally scanned photograph

- 1957, 176x176 pixels

http://listverse.com/history/top-10-incredible-early-firsts-in-photography/

Camera sales over time



[Source]

Camera sales over time

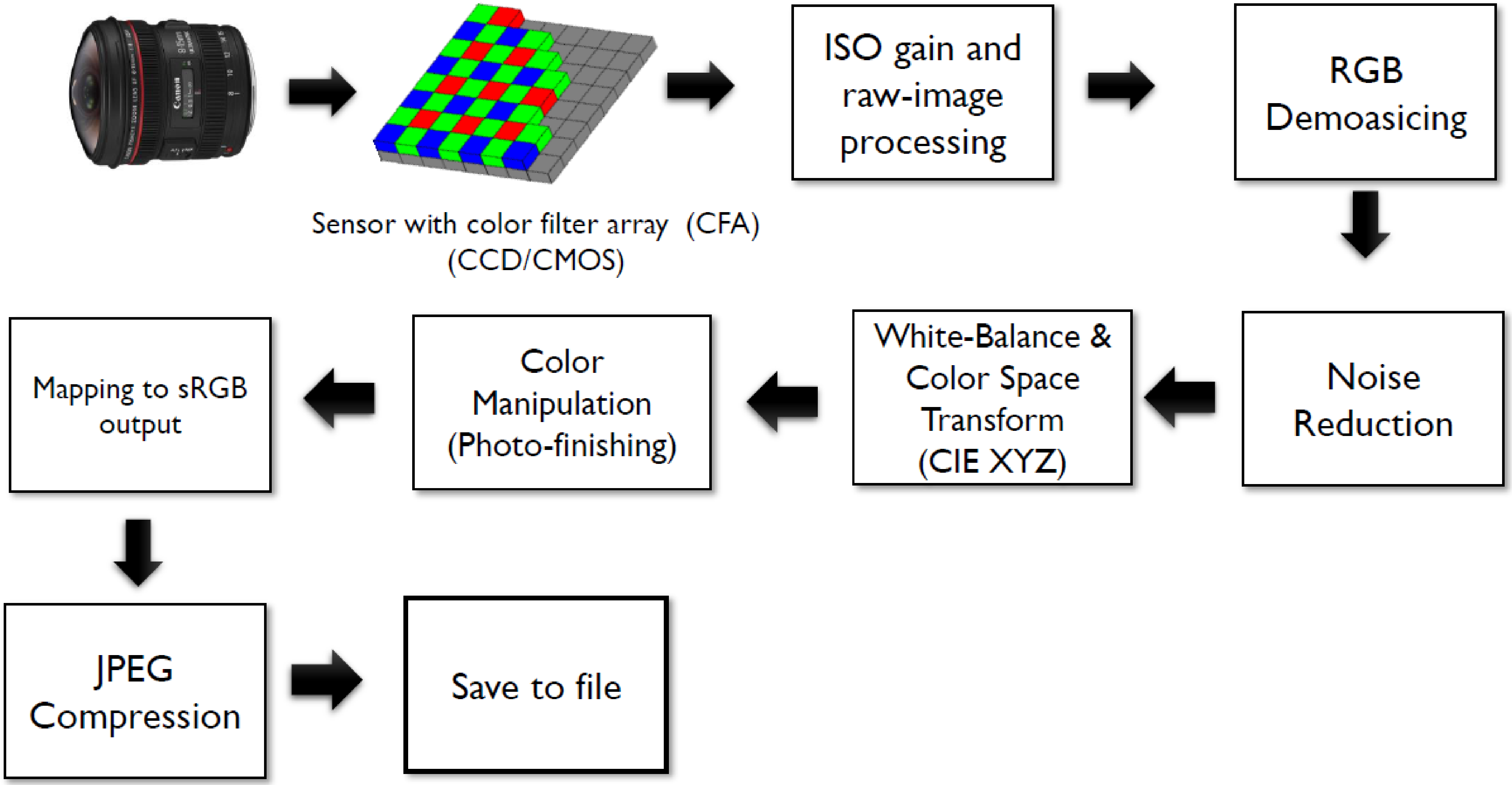
- **The full chart…**

[Source]

Digital cameras

- Digital cameras are not designed to be light measuring devices
- They are designed to produce visually pleasing photographs
- There is a great deal of processing (photo finishing) applied in the camera h ardware

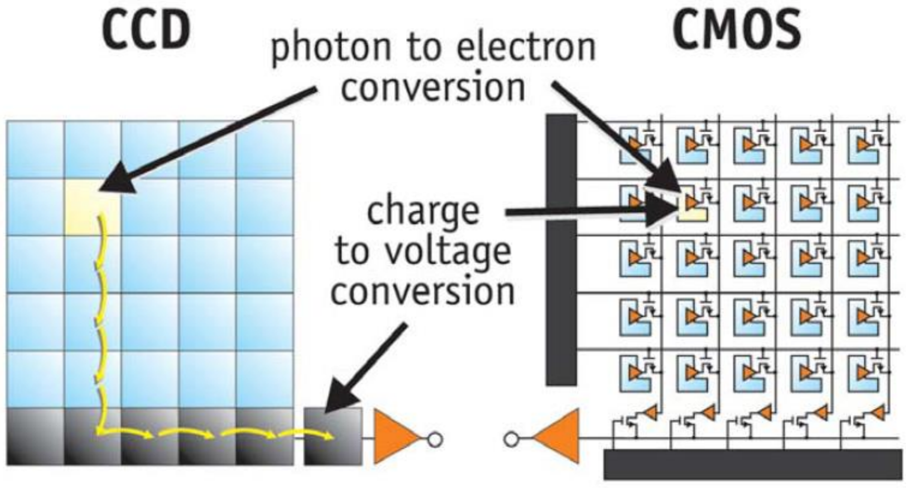
Modern photography pipeline



Source: M. Brown

Digital camera sensors

- **Each cell in a sensor array is a light-sensitive diode that converts photons to electrons**
  - Dominant in the past: Charge Coupled Device (CCD)
  - Dominant now: Complementary Metal Oxide Semiconductor (CMOS)



CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.

http://electronics360.globalspec.com/article/9464/ccd-vs-cmos-the-shift-in-image-sensor-technology

What does a raw image look like?

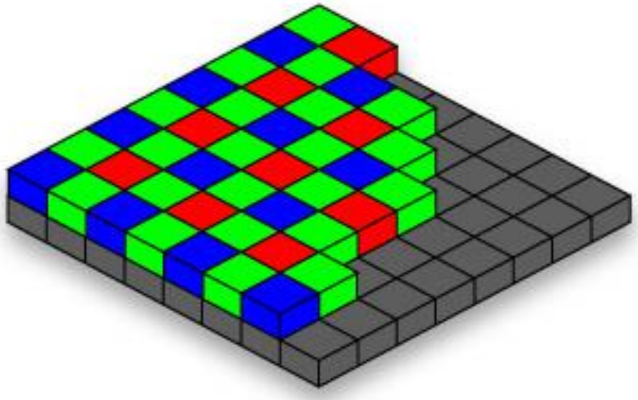
lots of noise

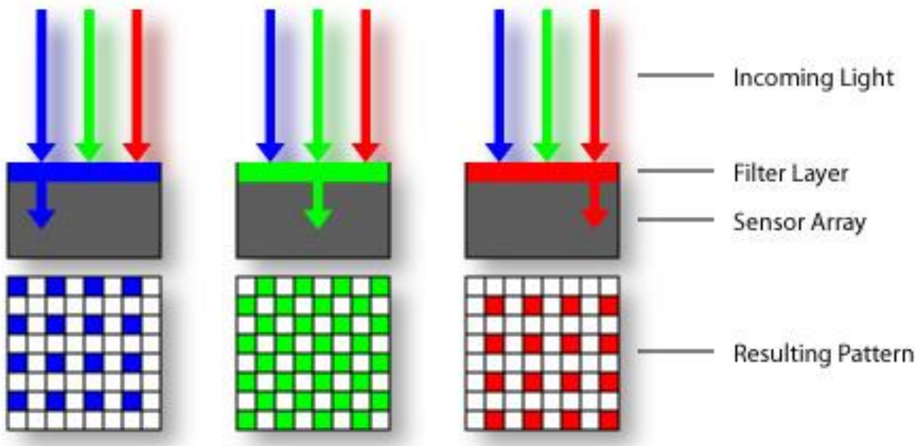
mosaicking artifacts

- Kind of disappointing.
- We call this the *RAW* image.

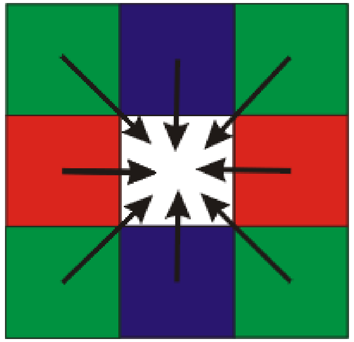
Color filter arrays

Bayer grid

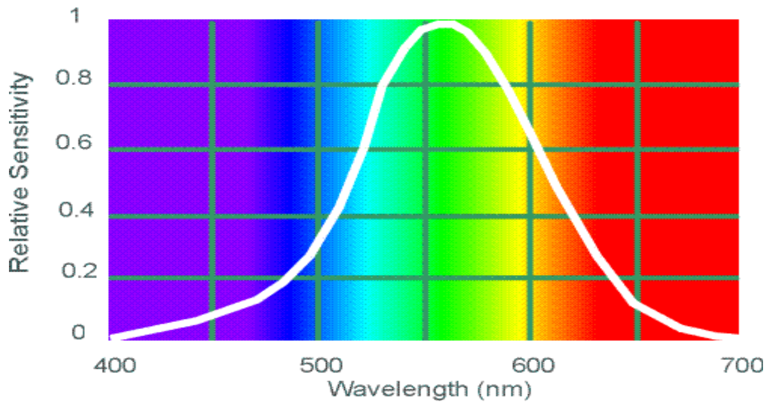




Demosaicing:
Estimation of missing components from nei ghboring values



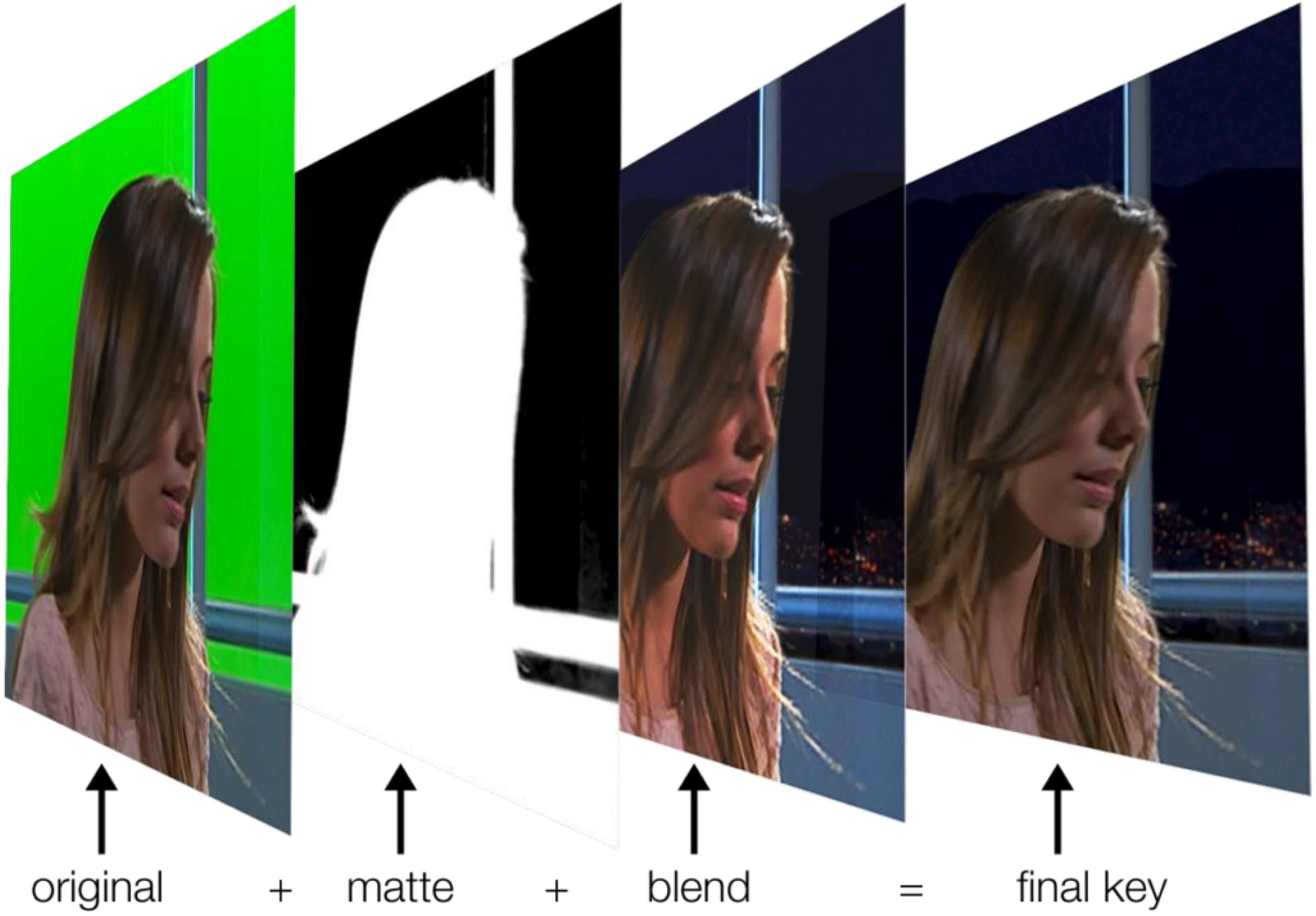
Why more green?



Human Luminance Sensitivity Function

Source: Steve Seitz

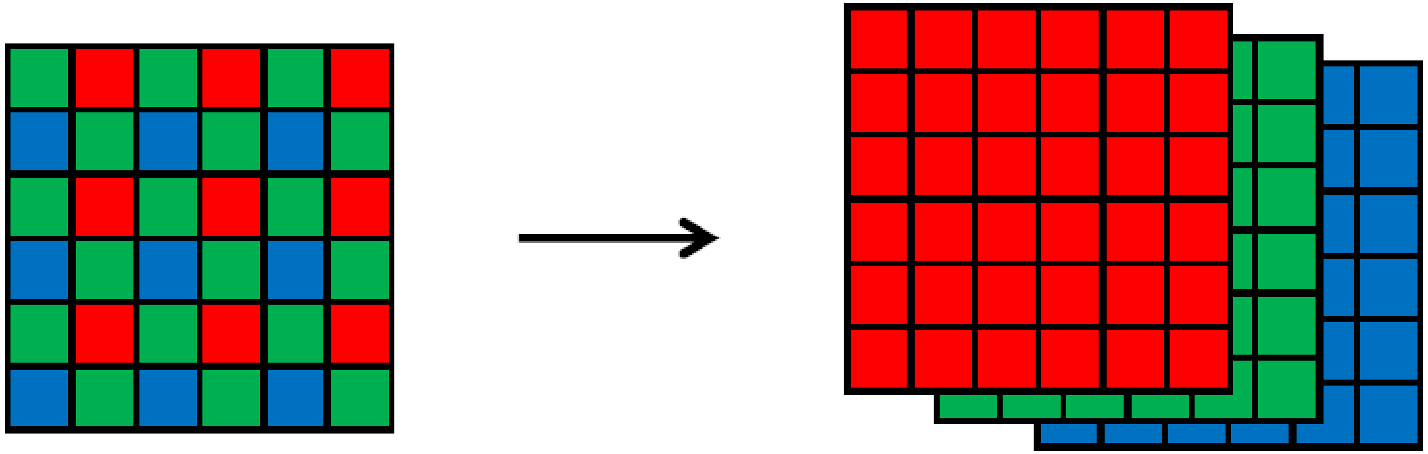
Virtual Background



https://infocusfilmschool.com/filming-green-screen-guide/

Demosaicing

- **Producing full RGB image from mosaiced sensor output**

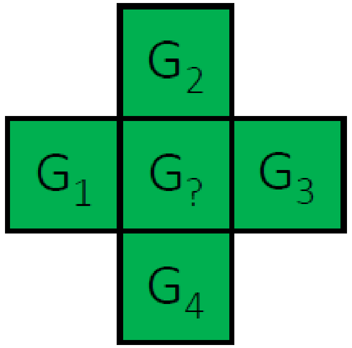


  – Interpolate from neighbors:
    - Bilinear interpolation (needs 4 neighbors).
    - Bicubic interpolation (needs more neighbors, may overblur).
    - Edge-aware interpolation.
    - Large area of research.

Slide credit: I. Gkioulekas

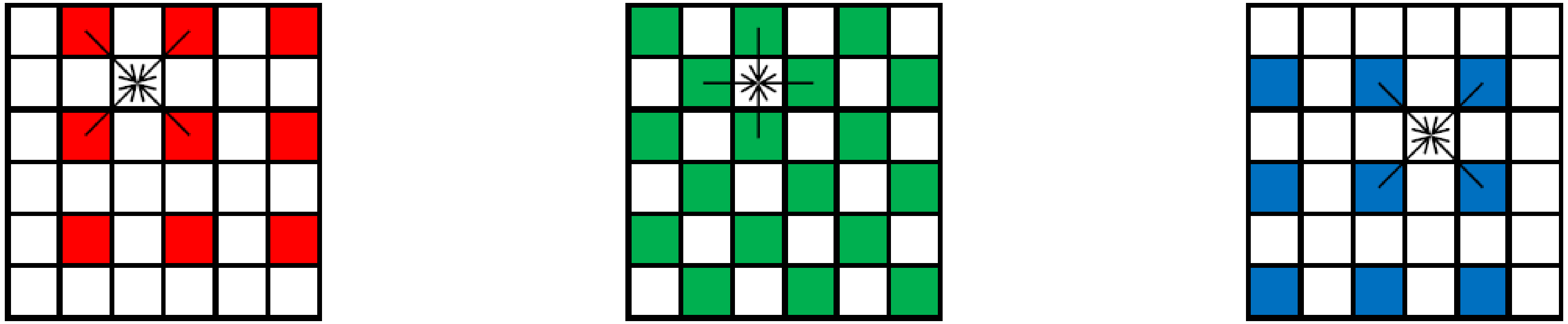
Demosaicing

- **Bilinear interpolation: Simply average your 4 neighbors.**



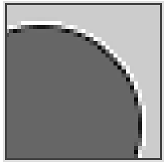
$$G_? = \frac{G_1 + G_2 + G_3 + G_4}{4}$$

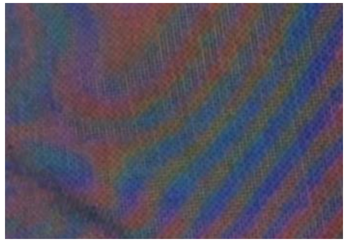
- **Neighborhood changes for different channels:**



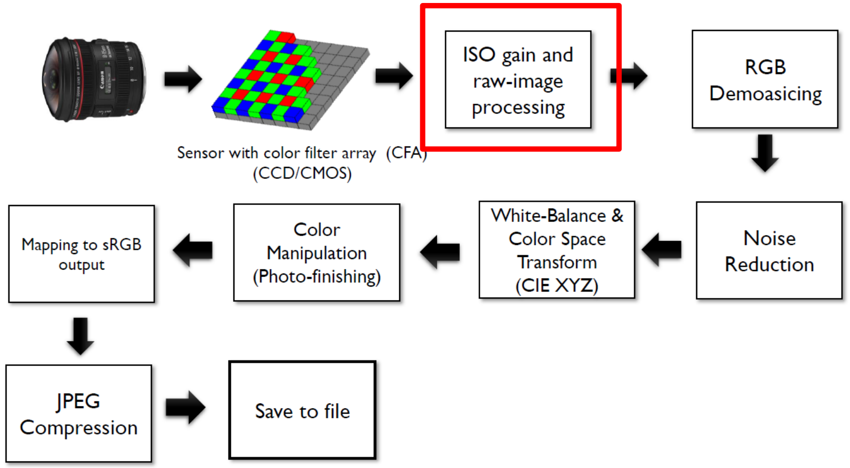
Digital camera artifacts

- **Noise**
  - low light is where you most notice noise
  - light sensitivity (ISO) / noise tradeoff
  - stuck pixels
- **In-camera processing**
  - oversharpening can produce halos
- **Compression**
  - JPEG artifacts, blocking
- **Blooming**
  - CCD charge overflowing into neighboring pixels
- **Color artifacts**
  - Color moire
  - Purple fringing from microlenses

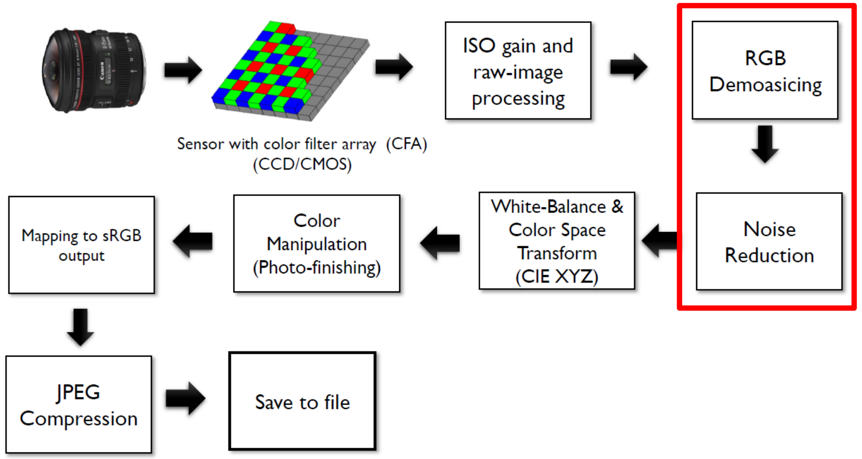




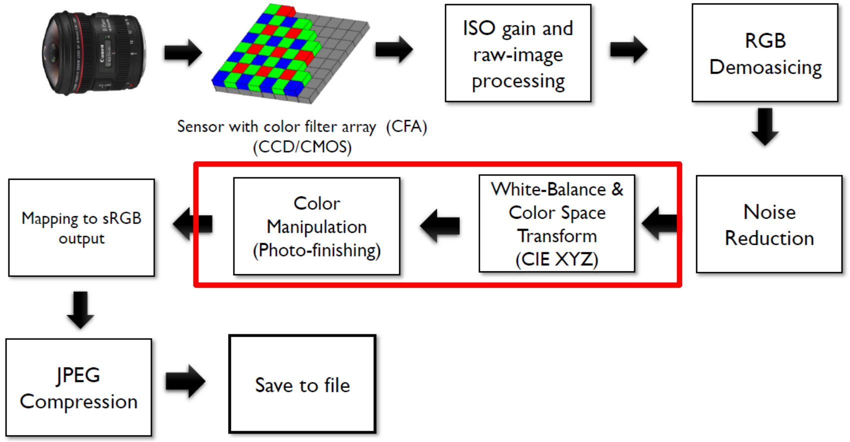
Walking through the pipeline



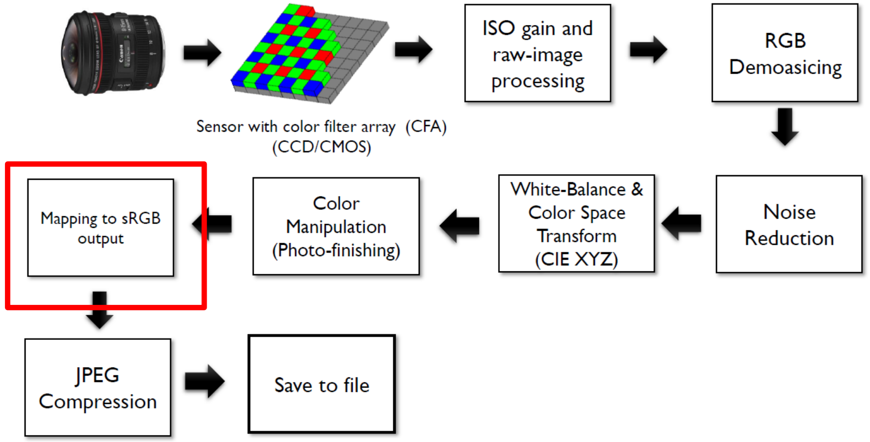
Walking through the pipeline



Walking through the pipeline



Walking through the pipeline

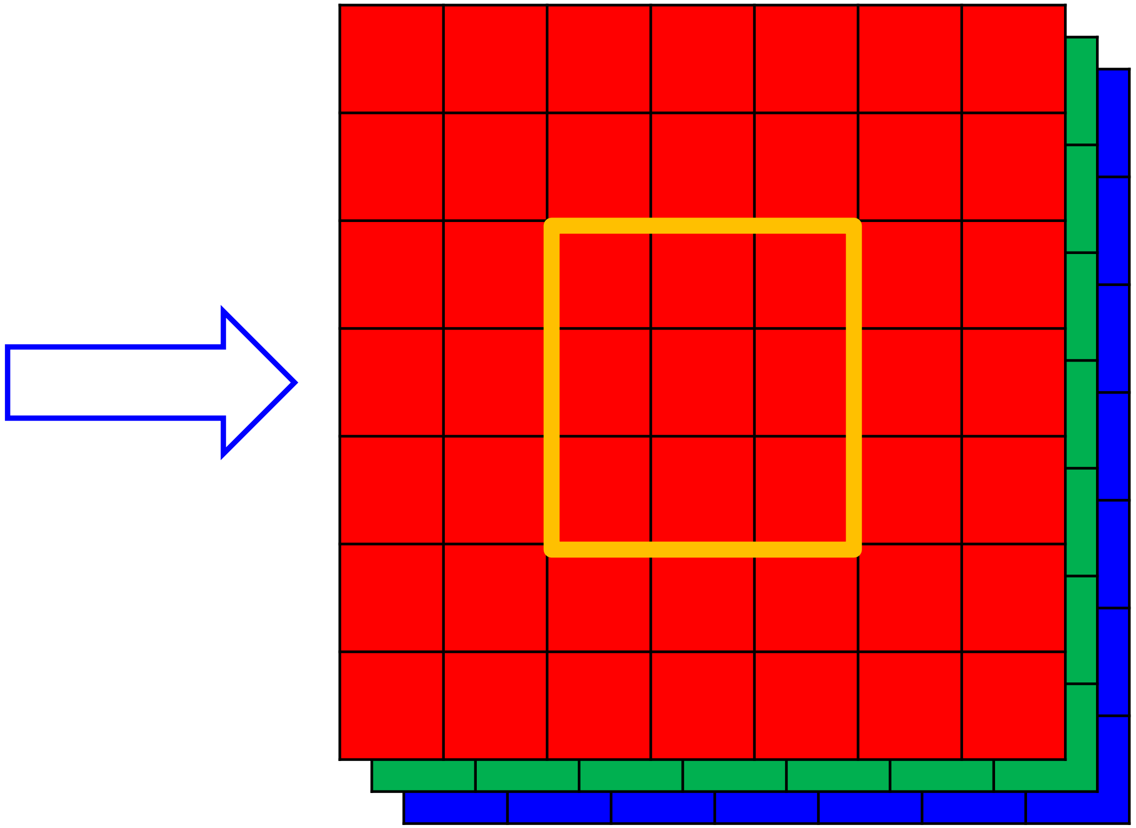


Quiz

- A Bayer filter mosaic is a color filter array (CFA) for arranging RGB color filters on a square grid of photose nsors. Figure 1 shows a raw image obtained by the Bayer pattern CFA where each pixel includes the R or G or B value. Perform image demosaicing to the image region in orange box using bilinear interpolation and provide each pixel's RGB value, (R, G, B) of the 3 × 3 image.

4	2	8	10	6	0	2
8	8	8	2	6	0	8
0	2	2	4	2	6	0
8	0	8	10	8	0	6
0	2	0	2	0	0	0
10	0	8	0	8	0	6
0	2	0	2	0	4	0

Figure 1



Next topic

- **How can we remove artifacts in images?**
  - Prerequisite
    - Review EBU6230 Image/Video Processing - Week2: Image Filtering