3D Graphics Programming Tools

Projection

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The 3D rendering pipeline

3D geometric primitives

↓

Modelling Transformation	Transform individual object coordinates into 3D world coordinate system
Lighting	Illuminate according to lighting and reflectance
Viewing Transformation	Transform 3D scene into 3D camera coordinate system
Projection Transformation	Transform 3D coordinates into 2D screen coordinate system
Clipping	Clip primitives outside camera’s view
Scan Conversion	Draw pixels (includes texturing, hidden surface, ...)

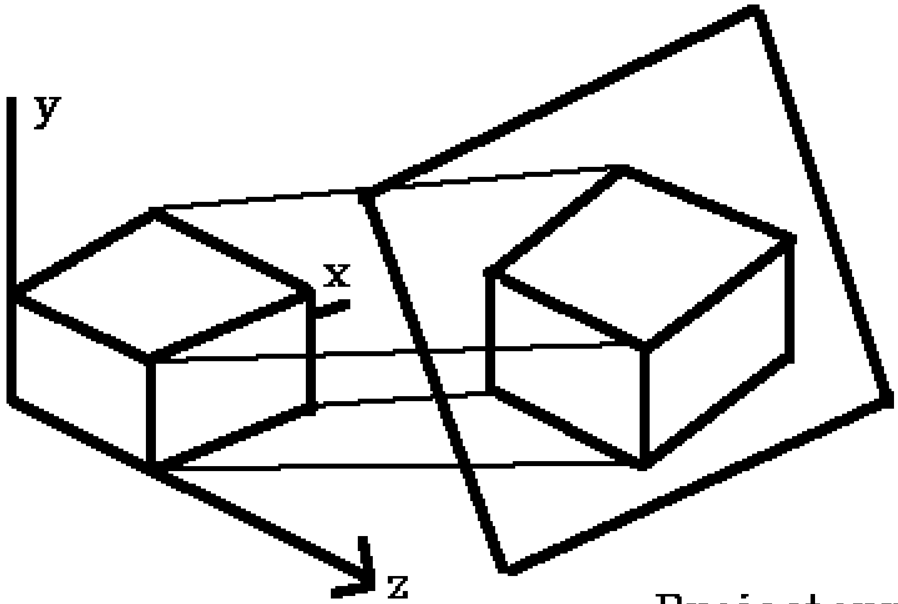
↓

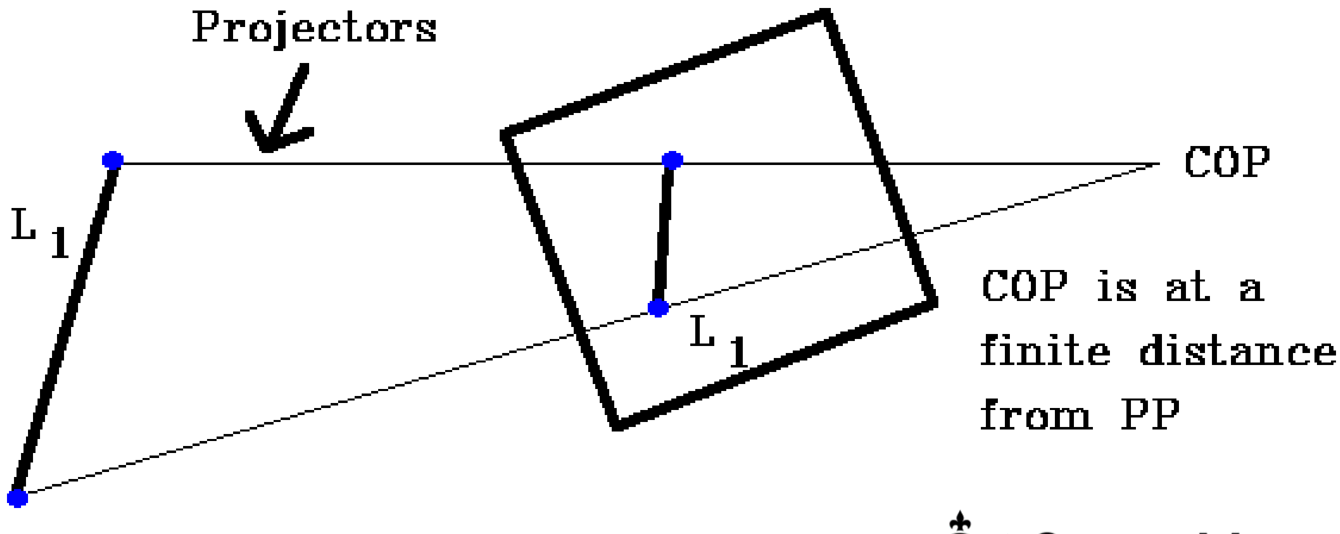
Image

Today’s agenda

- Taxonomy of projections
- Parallel projection
- Perspective projection

Planar Geometric Projection





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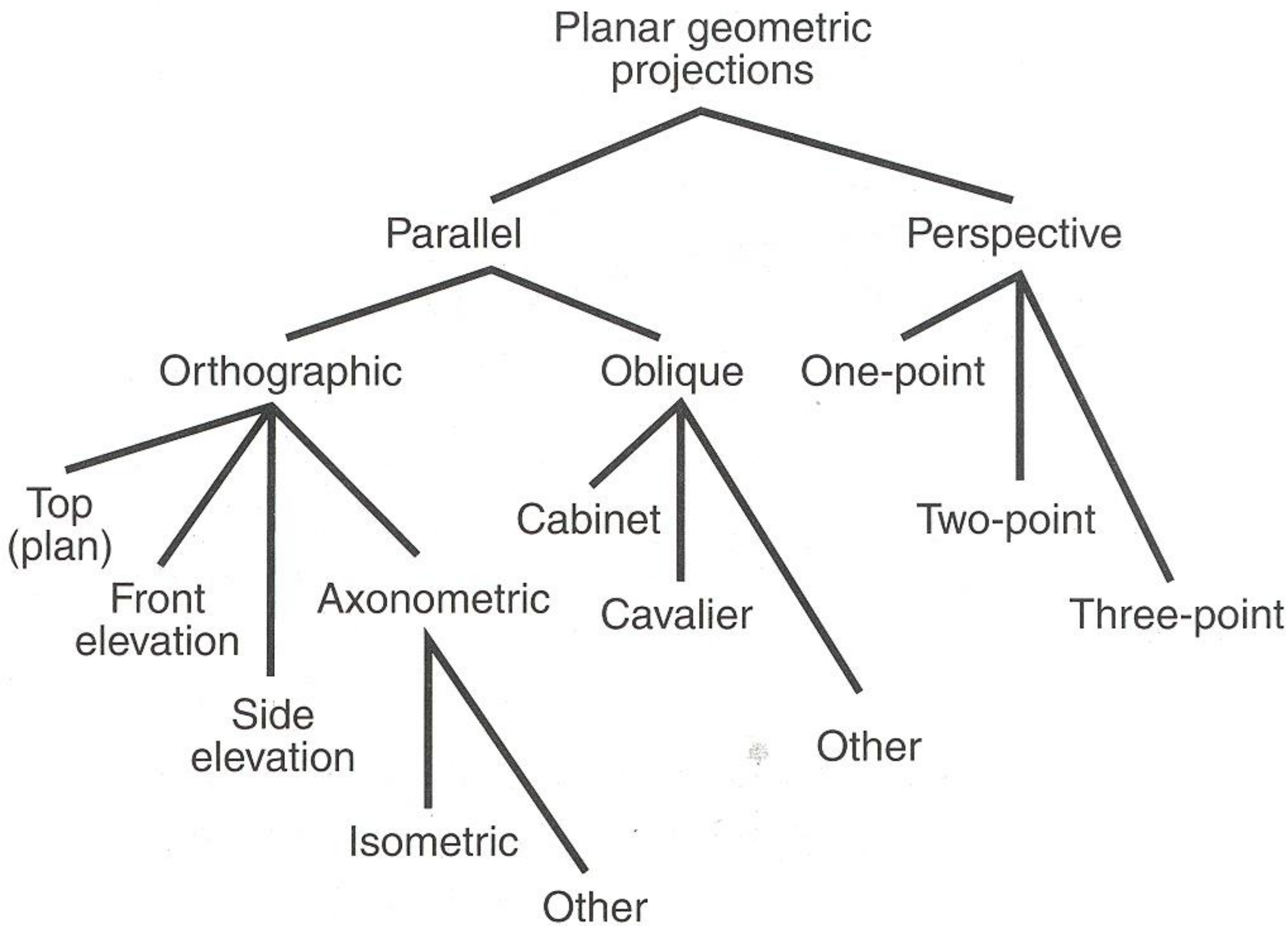
Taxonomy of projections



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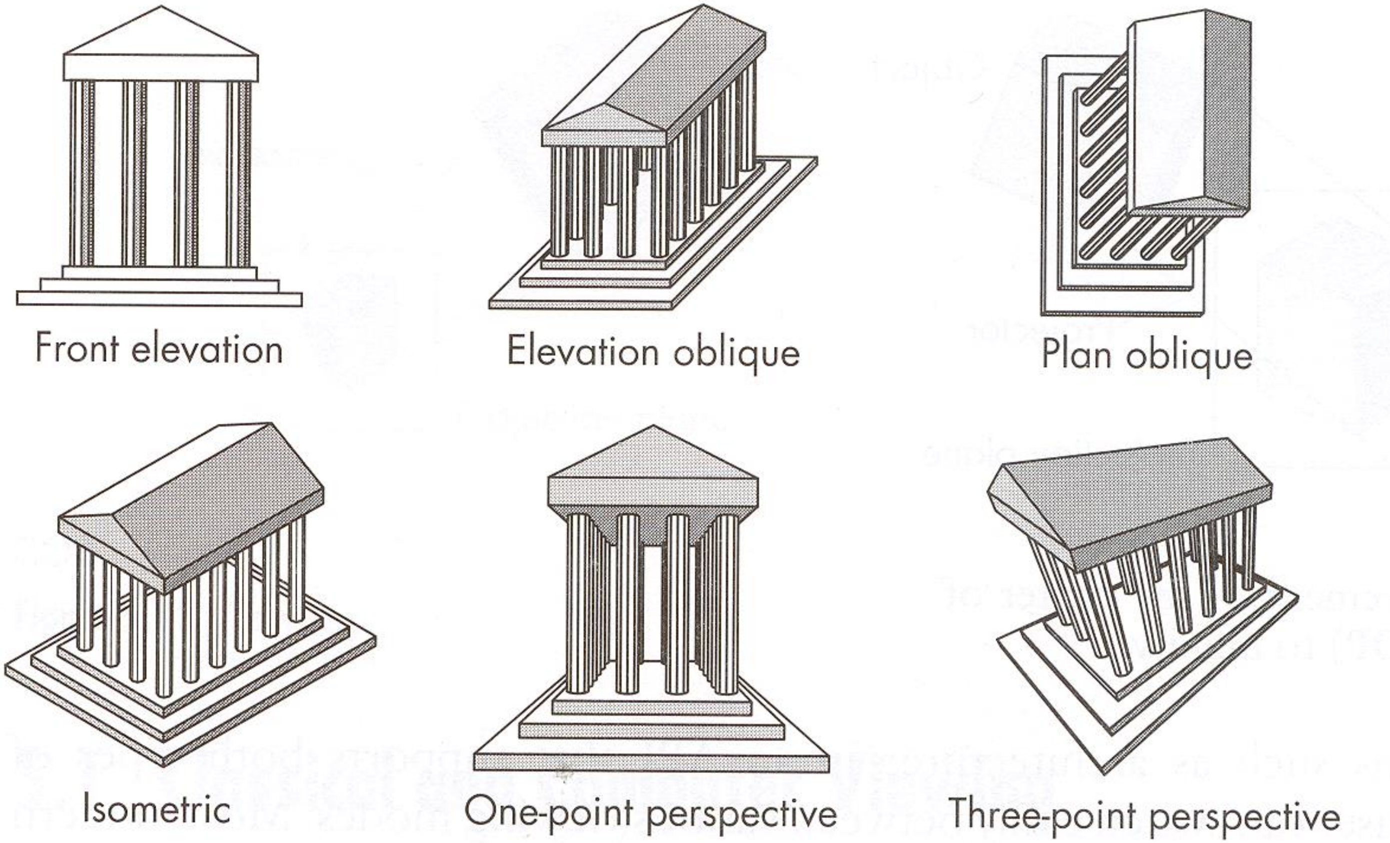
Classical projections



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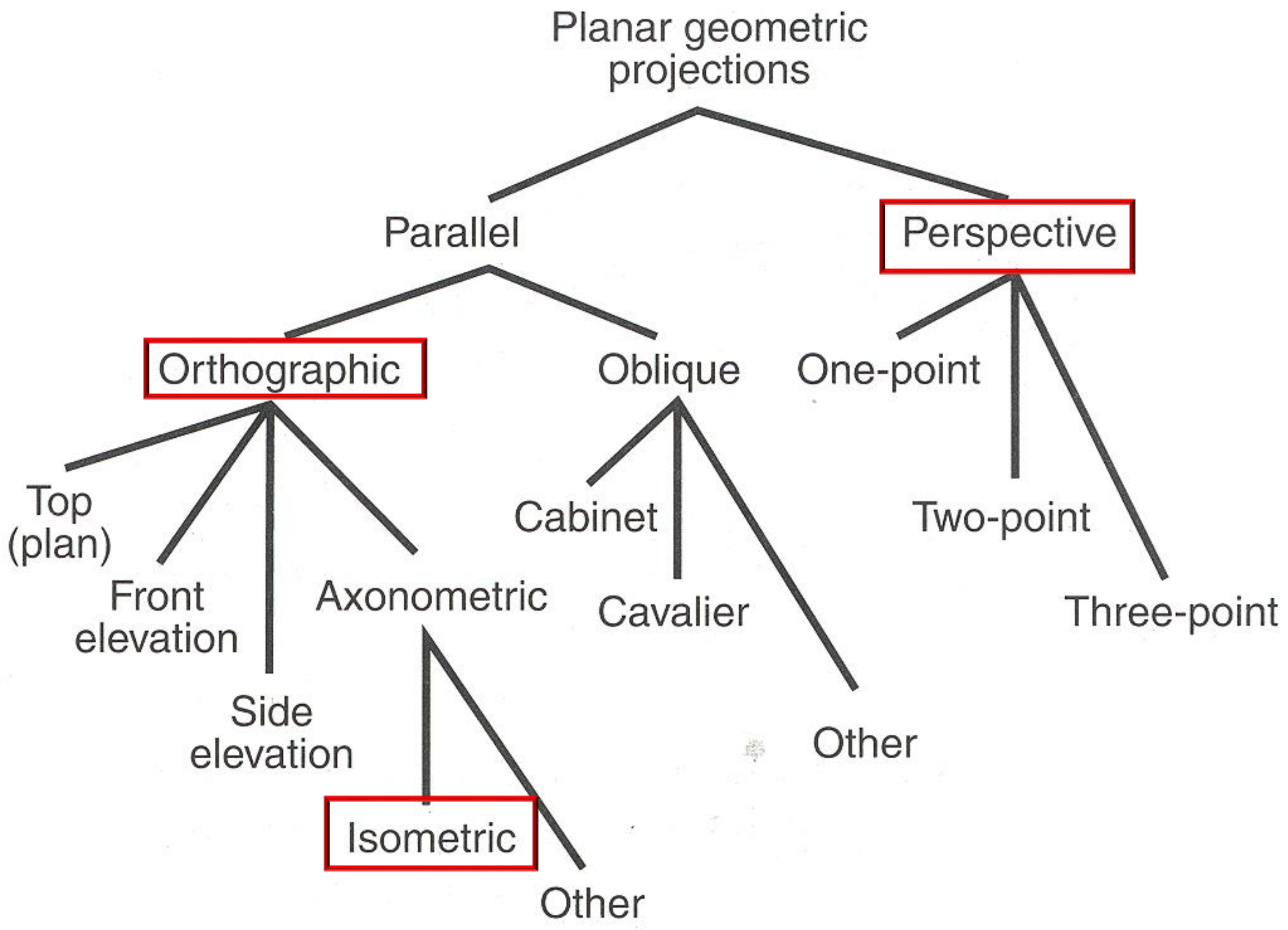
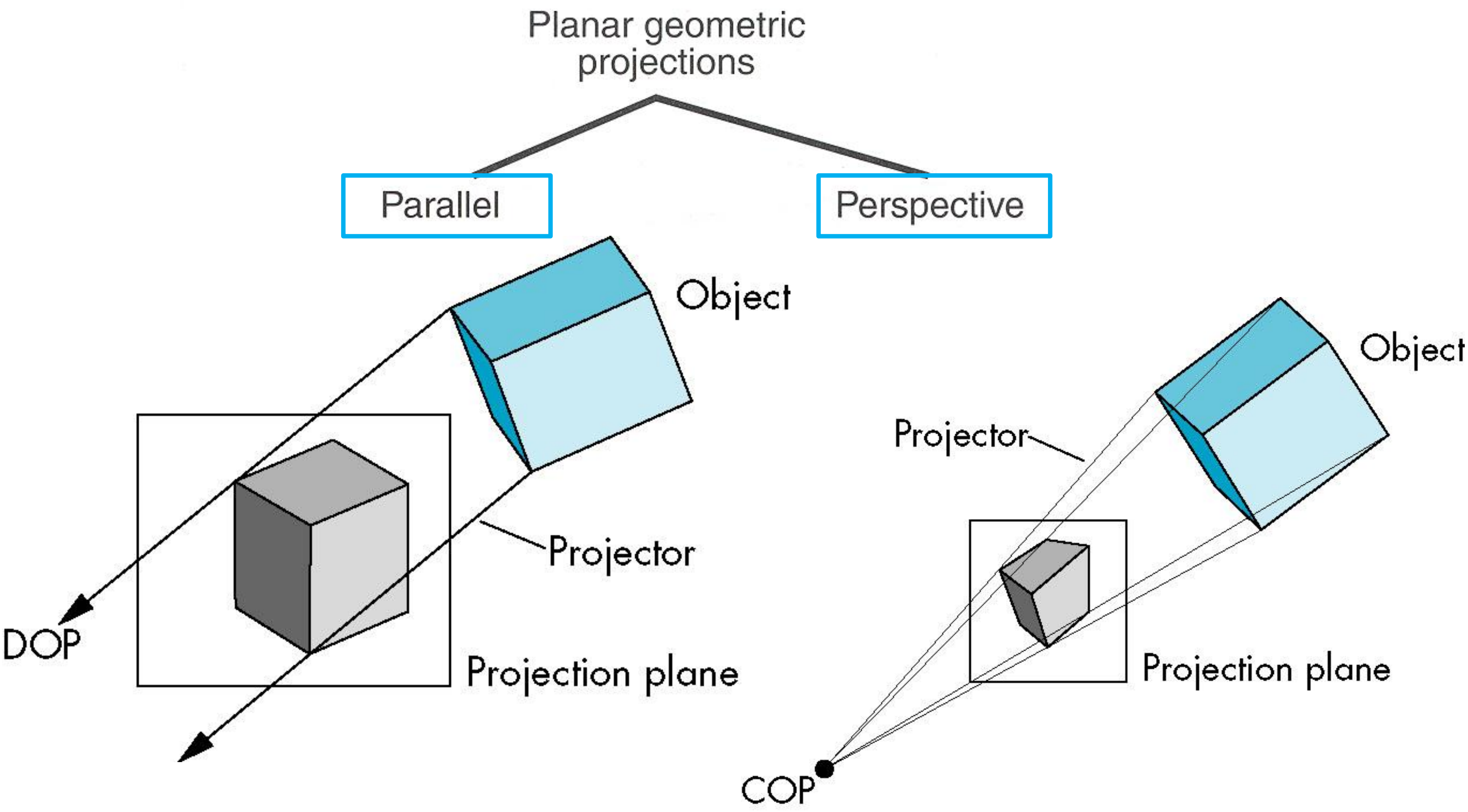
Taxonomy of projections



Planar geometric projections



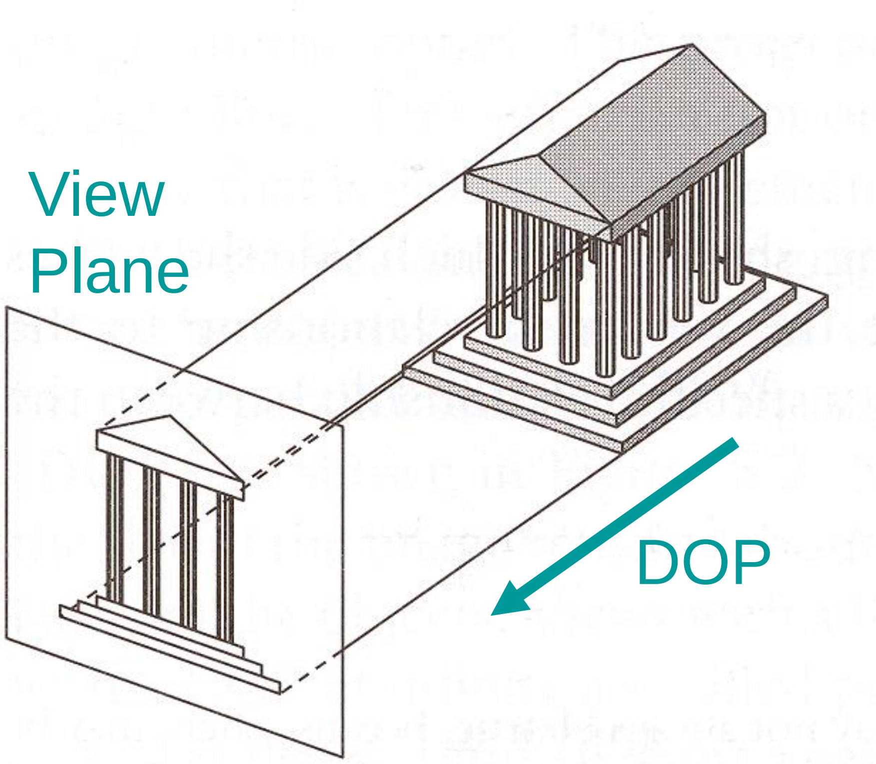
Today's agenda

- Taxonomy of projections
- Parallel projection
- Perspective projection

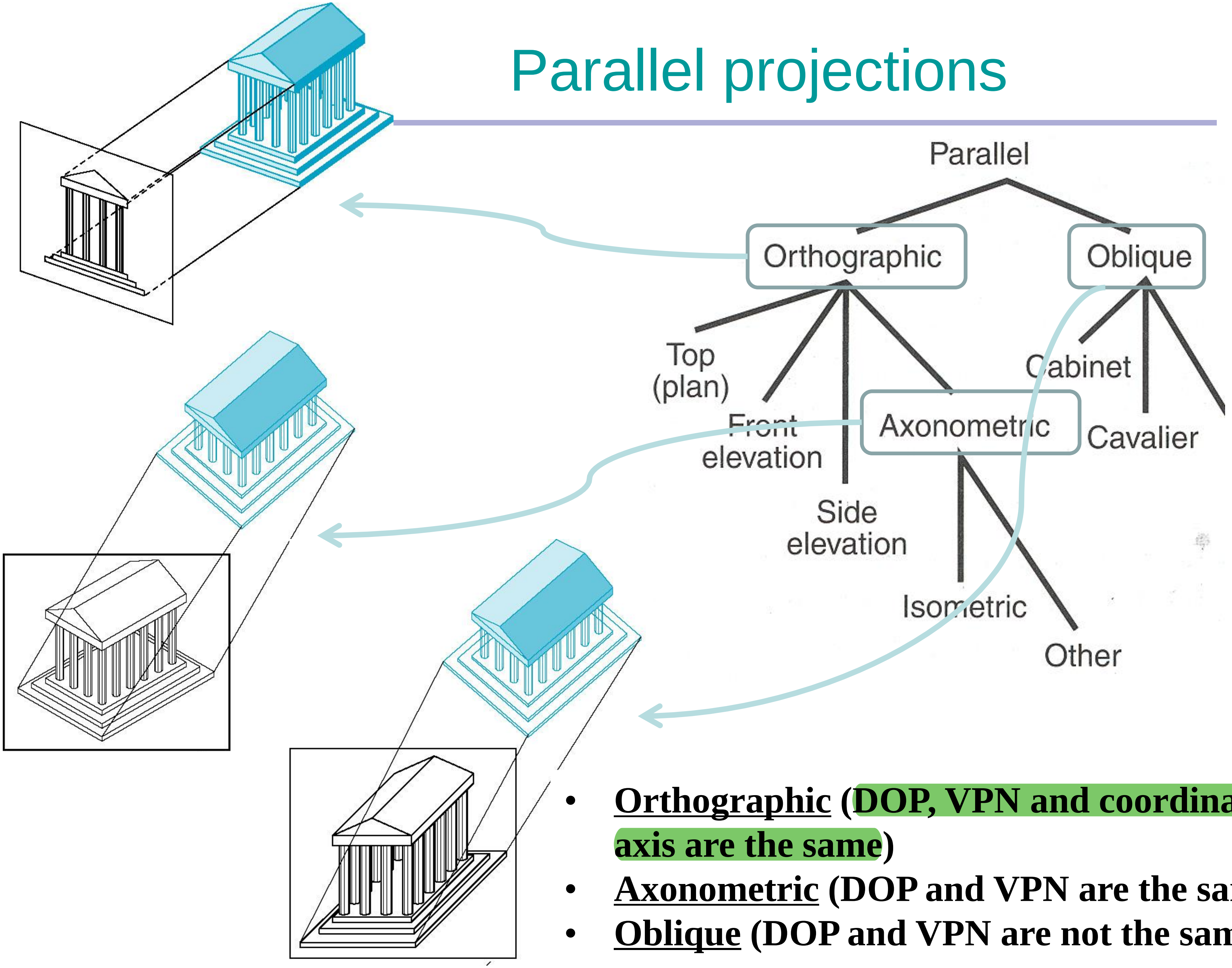
Parallel projection

Center of projection is at infinity

- Direction of projection (DOP) is the same for all points



Parallel projections



- **<u>Orthographic</u> (DOP, VPN and coordinate axis are the same)**
- **<u>Axonometric</u> (DOP and VPN are the same)**
- **<u>Oblique</u> (DOP and VPN are not the same).**

Orthographic projections

DOP//VPN//Axis are parallel (the same)

DOP is perpendicular to the view plane

Used for:

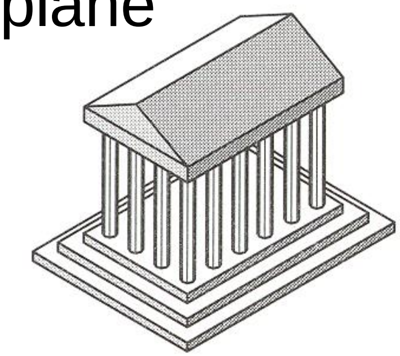
- **engineering drawings**
- **working architectural drawings**

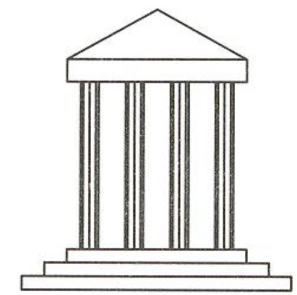
Pros:

- **accurate measurement possible**
- **all views are at same scale**

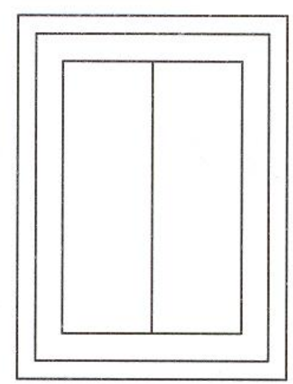
Cons:

- **does not appear natural (i.e. they lack perspective foreshortening)**
- **does not provide “realistic” view or sense of 3D form, usually needs multiple views to get a 3D feeling for object**
- **hard to deduce 3D nature**

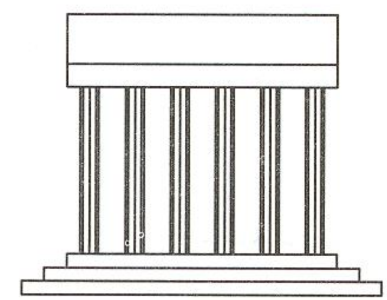




Front

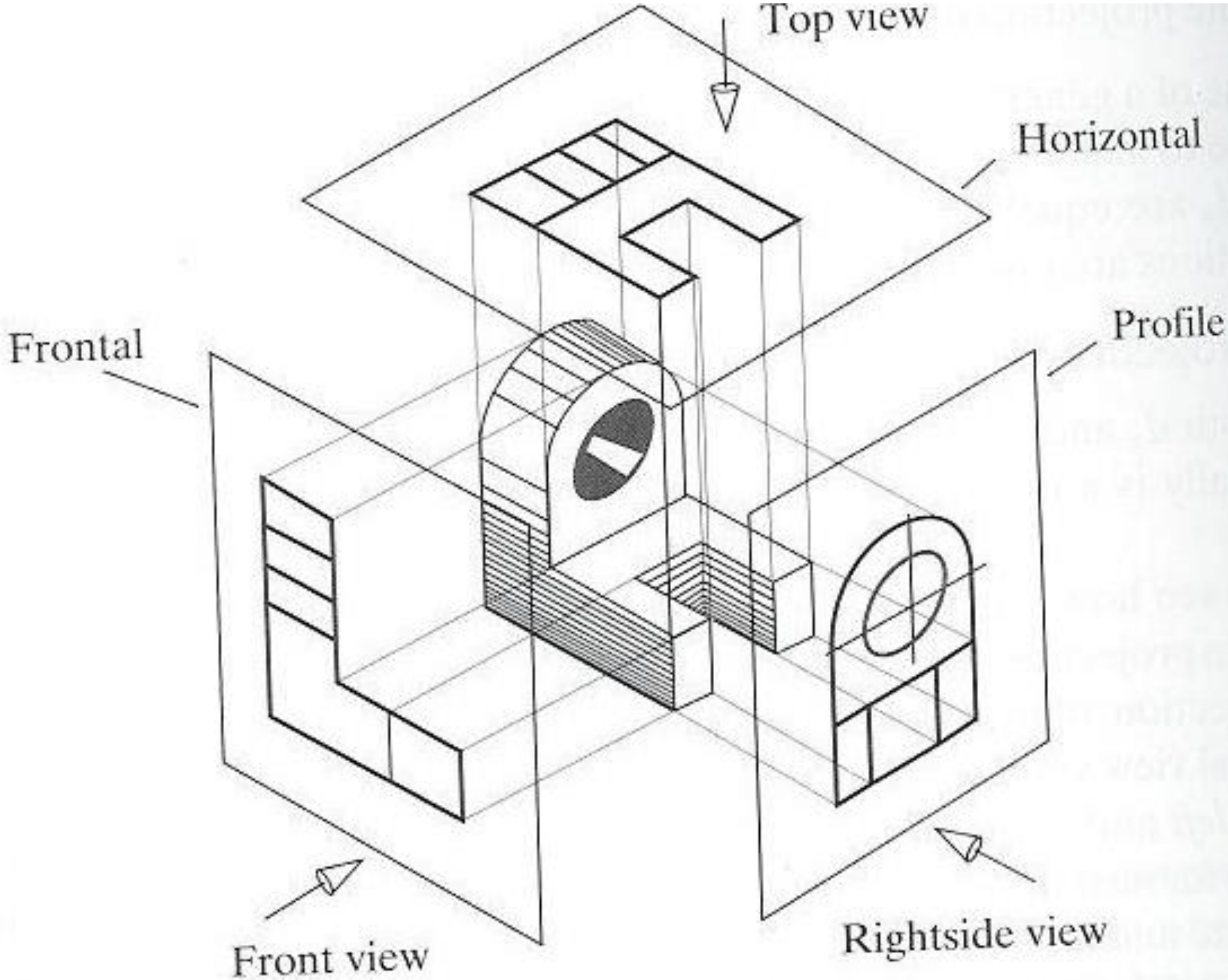


Top



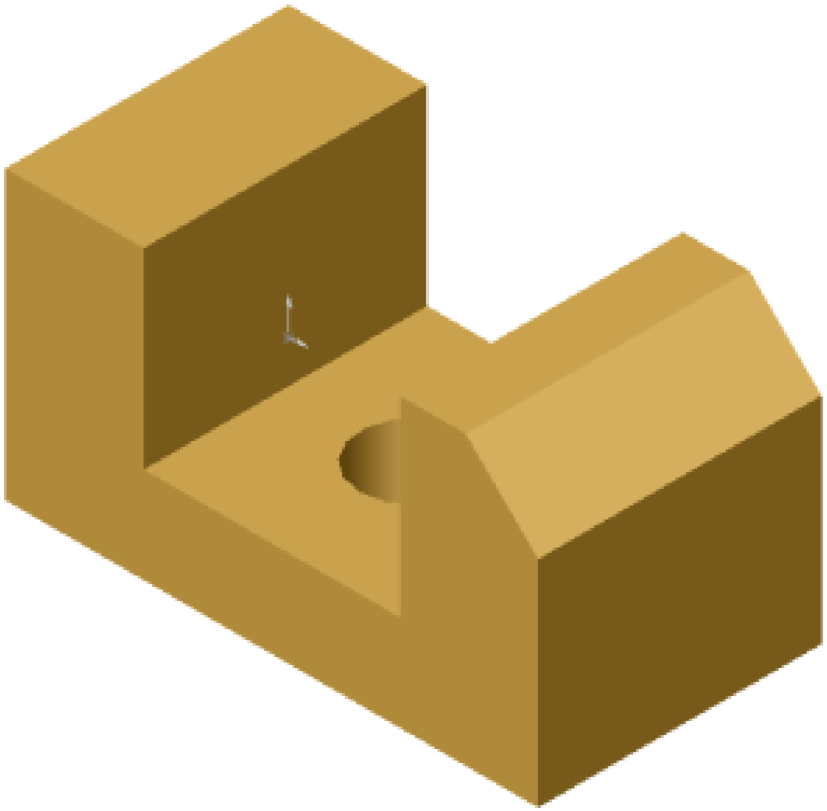
Side

Orthographic projection

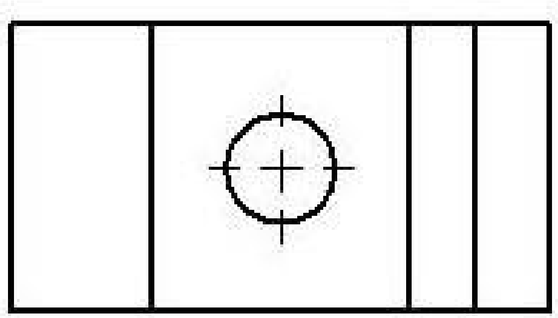


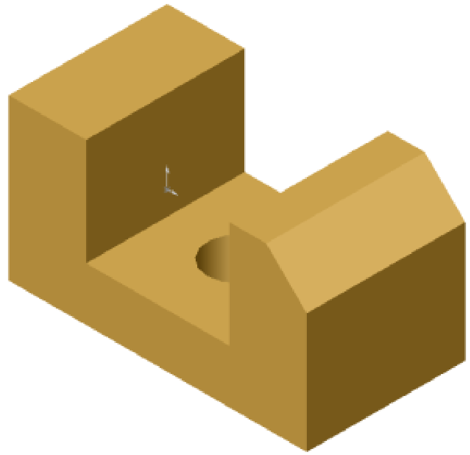
Exercise

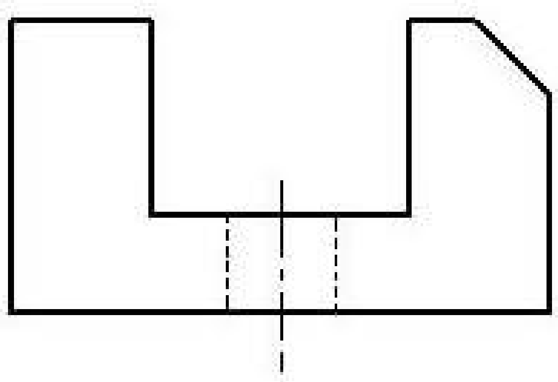
Draw the top, front and right side views

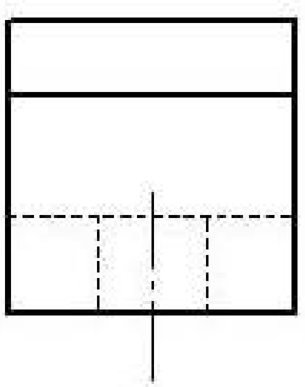


Exercise

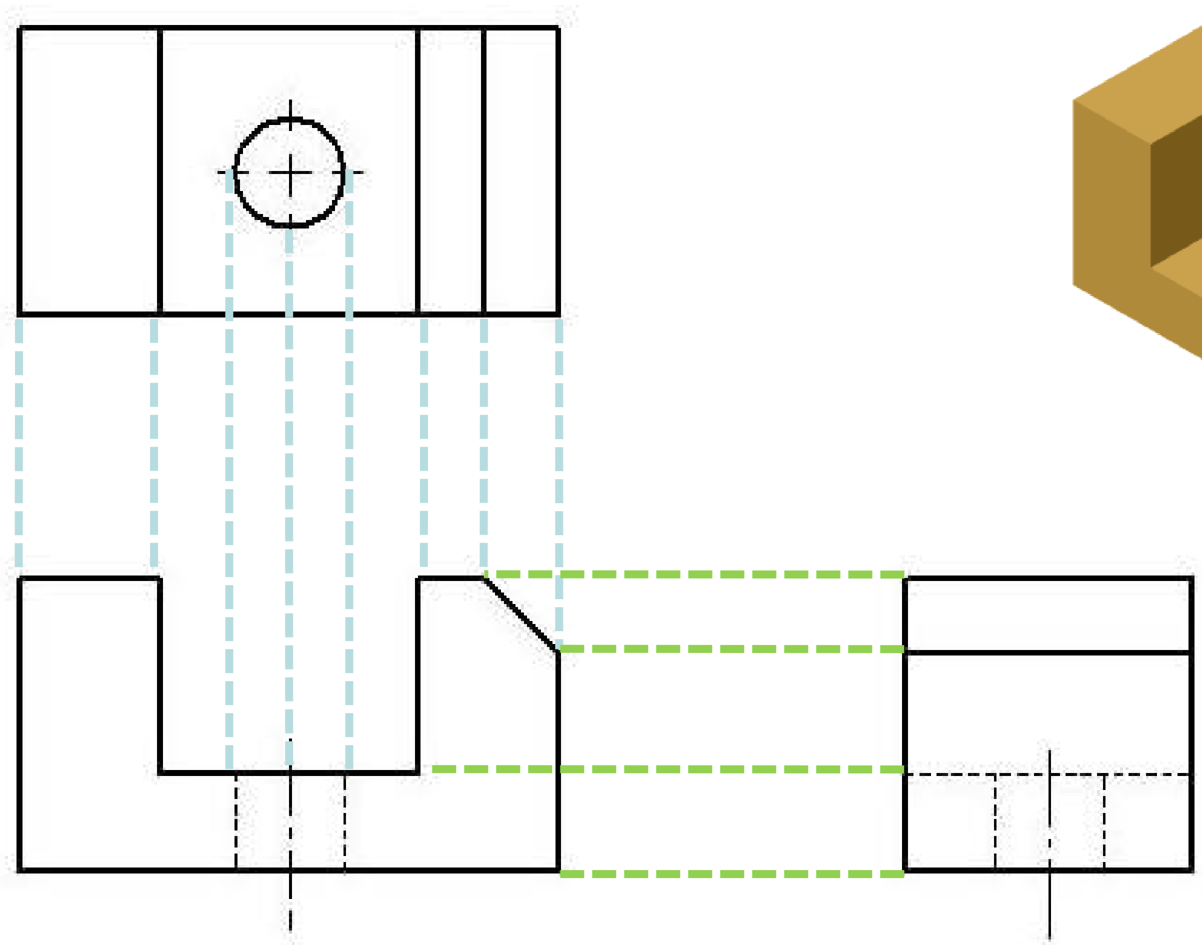


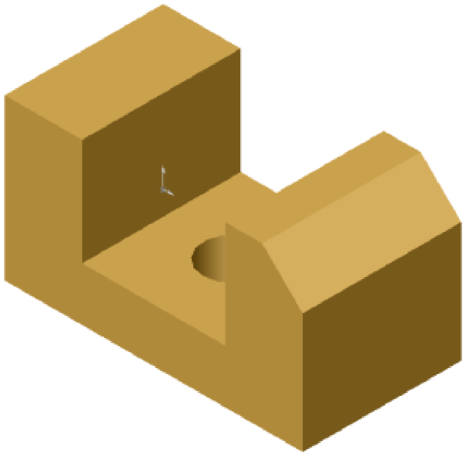






Exercise





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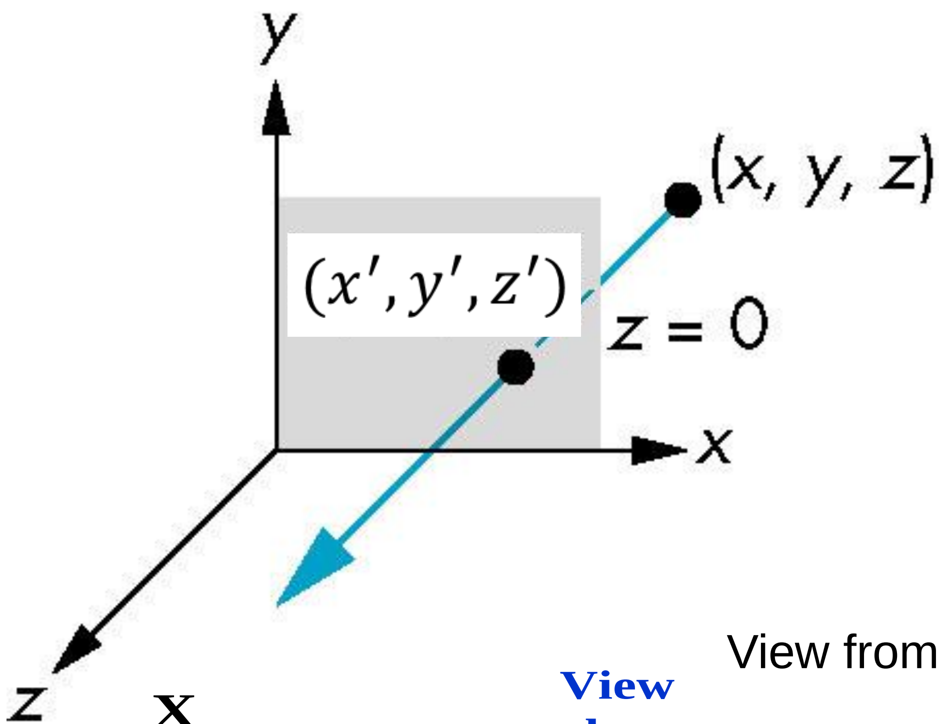
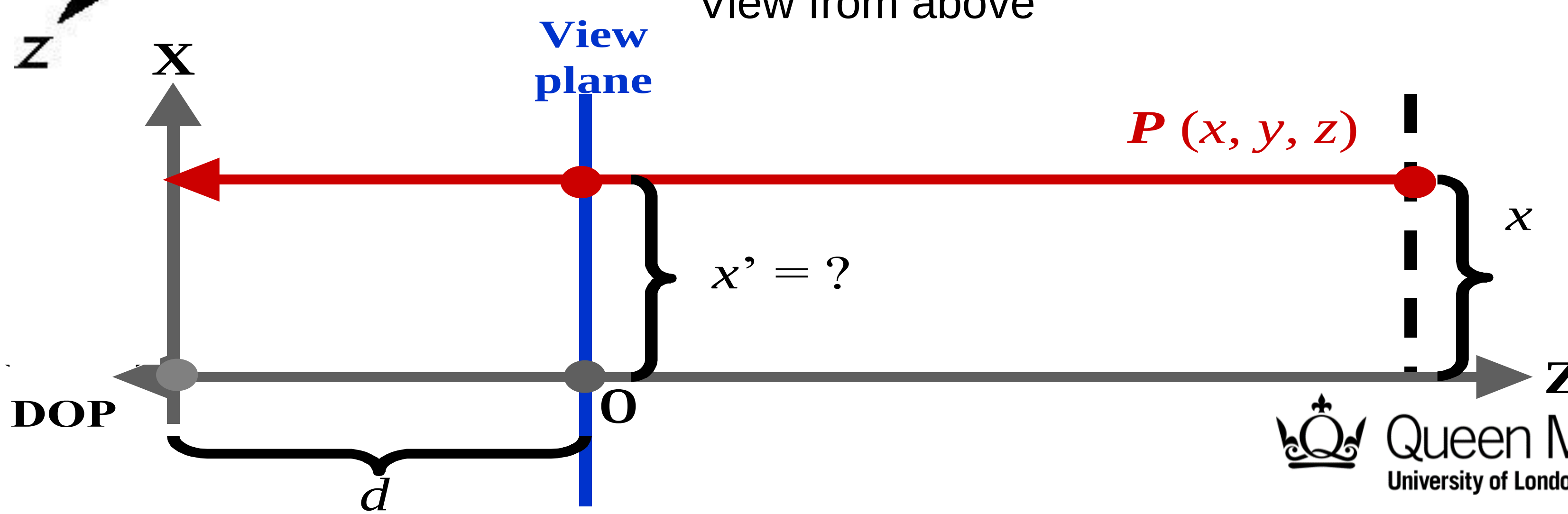
Orthographic projection



$$\begin{cases} x' = x \\ y' = y \\ z' = 0 \end{cases}$$

View from above



Orthographic projection

- Simple orthographic transformation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Notice that the parallel lines of the tiled floor remain parallel after orthographic projection.

Orthographic projection

- Simple orthographic transformation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Notice that the parallel lines of the tiled floor remain parallel after orthographic projection.

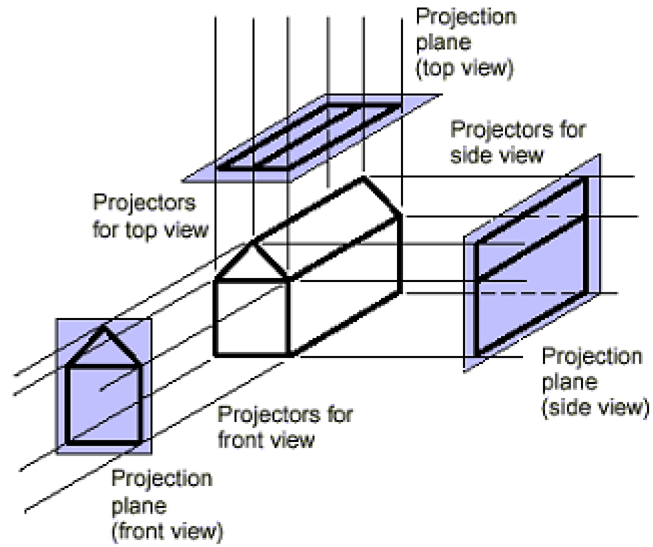
Multiview Orthographic

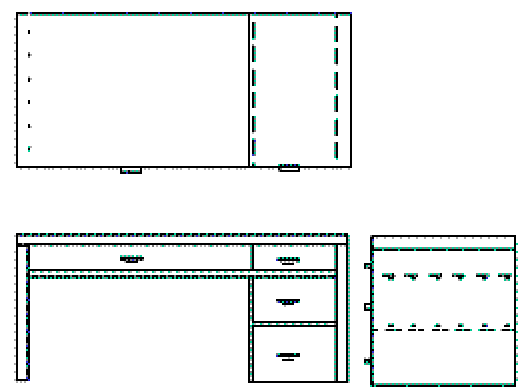
Transform matrices

$$\text{Front-view: } M_{front} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Top-view: } M_{top} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Side-view: } M_{side} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$





multiview orthographic

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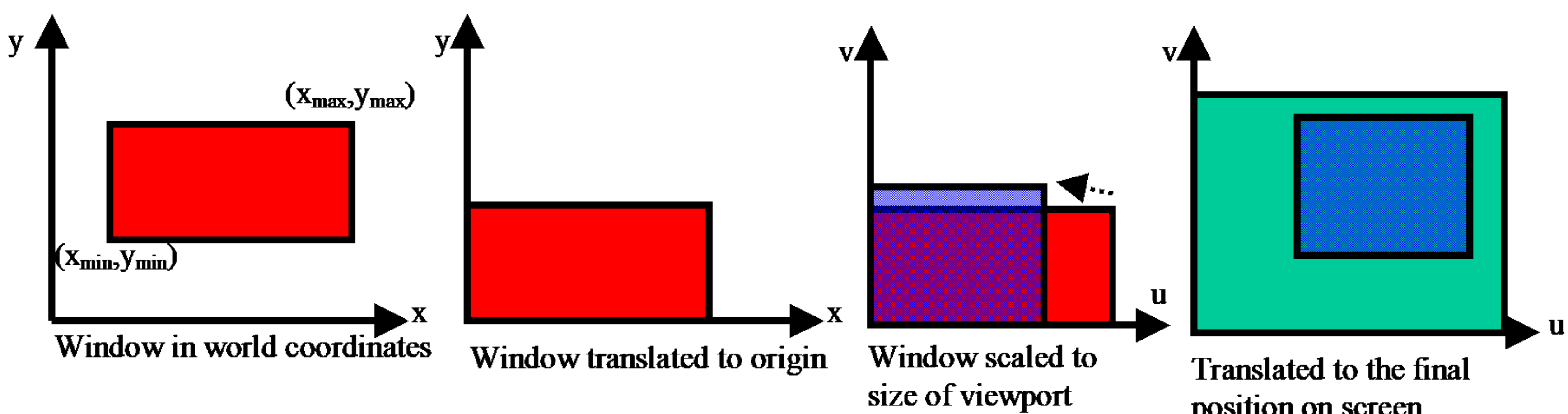
View volume window to viewport transformation

- **Window : Rectangular region in world coordinate system**
- **Viewport : Rectangular region in screen coordinates on the computer screen**
- **Window to Viewport Transformation:**
  1. **Translate the window to the origin of word coordinates.**
  2. **Scale the size of the window to be equal to the size of the viewport.**
  3. **Translate it to the final position of screen coordinates.**

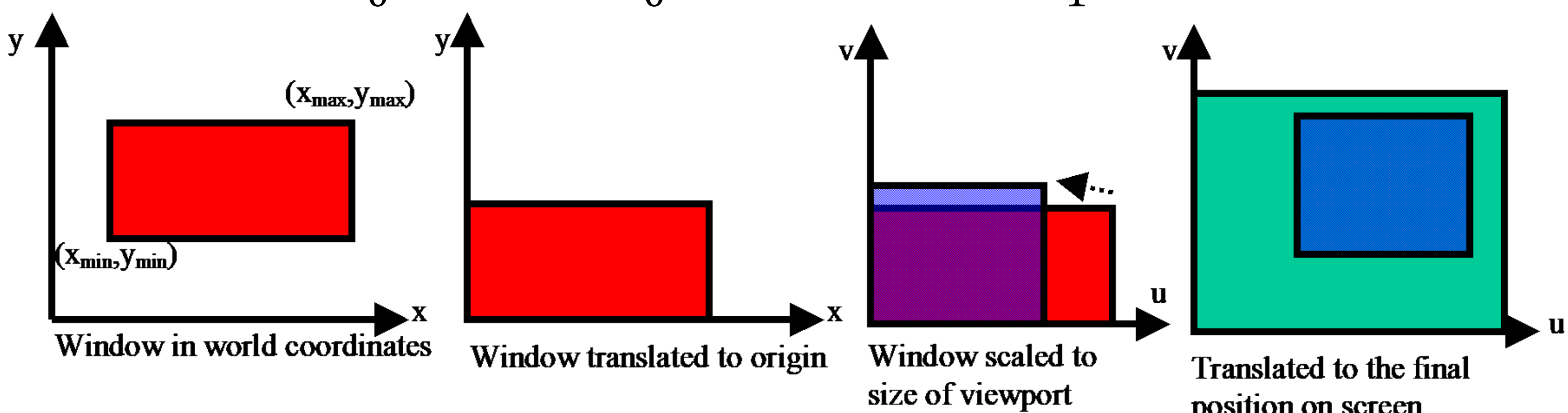


View volume window to viewport transformation

$$M_{W2V} = T(u_{min}, v_{min}) \cdot S\left(\frac{u_{max}-u_{min}}{x_{max}-x_{min}}, \frac{v_{max}-v_{min}}{y_{max}-y_{min}}\right) \cdot T(-x_{min}, -y_{min})$$

$$= \begin{bmatrix} 1 & 0 & u_{min} \\ 0 & 1 & v_{min} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{u_{max}-u_{min}}{x_{max}-x_{min}} & 0 & 0 \\ 0 & \frac{v_{max}-v_{min}}{y_{max}-y_{min}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_{min} \\ 0 & 1 & -y_{min} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{u_{max}-u_{min}}{x_{max}-x_{min}} & 0 & -x_{min} \cdot \frac{u_{max}-u_{min}}{x_{max}-x_{min}} + u_{min} \\ 0 & \frac{v_{max}-v_{min}}{y_{max}-y_{min}} & -y_{min} \cdot \frac{v_{max}-v_{min}}{y_{max}-y_{min}} + v_{min} \\ 0 & 0 & 1 \end{bmatrix}$$



With $(x_{min}, y_{min}) = (-10, -20)$, $(x_{max}, y_{max}) = (110, 220)$, $(u_{min}, v_{min}) = (30, 40)$, $(u_{max}, v_{max}) = (80, 140)$, we have

$$M_{W2V} = T(u_{min}, v_{min}) \cdot S\left(\frac{u_{max} - u_{min}}{x_{max} - x_{min}}, \frac{v_{max} - v_{min}}{y_{max} - y_{min}}\right) \cdot T(-x_{min}, -y_{min})$$

$$= \begin{bmatrix} 1 & 0 & u_{min} \\ 0 & 1 & v_{min} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{u_{max} - u_{min}}{x_{max} - x_{min}} & 0 & 0 \\ 0 & \frac{v_{max} - v_{min}}{y_{max} - y_{min}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_{min} \\ 0 & 1 & -y_{min} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{u_{max} - u_{min}}{x_{max} - x_{min}} & 0 & -x_{min} \cdot \frac{u_{max} - u_{min}}{x_{max} - x_{min}} + u_{min} \\ 0 & \frac{v_{max} - v_{min}}{y_{max} - y_{min}} & -y_{min} \cdot \frac{v_{max} - v_{min}}{y_{max} - y_{min}} + v_{min} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{80 - 30}{110 - (-10)} & 0 & -(-10) \cdot \frac{80 - 30}{110 - (-10)} + 30 \\ 0 & \frac{140 - 40}{220 - (-20)} & -(-20) \cdot \frac{140 - 40}{220 - (-20)} + 40 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{12} & 0 & 10 \cdot \frac{5}{12} + 30 \\ 0 & \frac{5}{12} & 20 \cdot \frac{5}{12} + 40 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{12} & 0 & \frac{205}{6} \\ 0 & \frac{5}{12} & \frac{145}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

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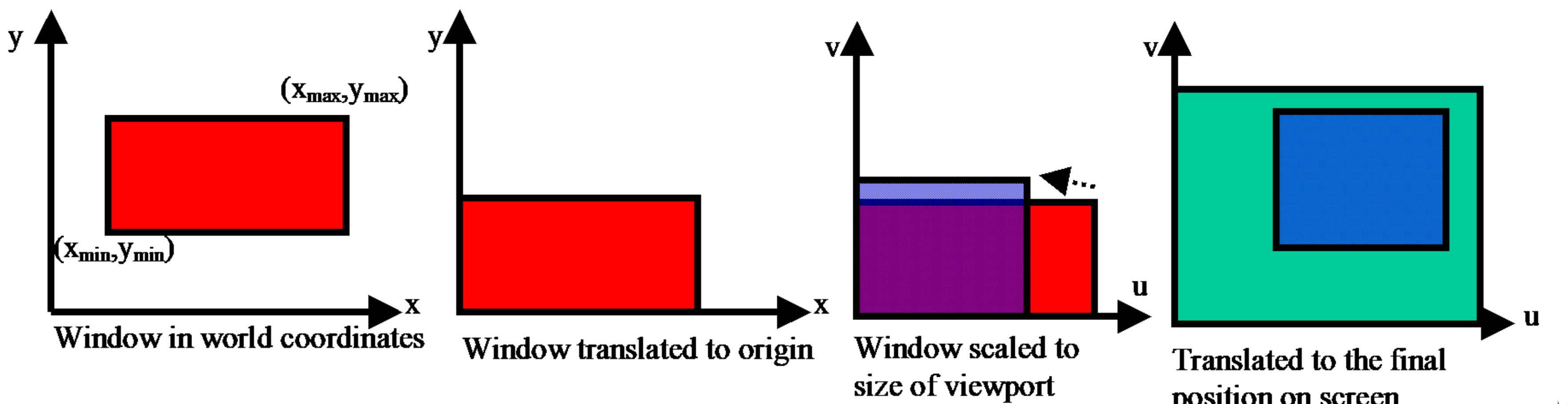
View volume window to viewport transformation

$$P = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \boldsymbol{M}_{W2V} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \dfrac{u_{\max} - u_{\min}}{x_{\max} - x_{\min}} & 0 & -x_{\min} \cdot \dfrac{u_{\max} - u_{\min}}{x_{\max} - x_{\min}} + u_{\min} \\ 0 & \dfrac{v_{\max} - v_{\min}}{y_{\max} - y_{\min}} & -y_{\min} \cdot \dfrac{v_{\max} - v_{\min}}{y_{\max} - y_{\min}} + v_{\min} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} u_{\min} + \dfrac{u_{\max} - u_{\min}}{x_{\max} - x_{\min}} \cdot (x - x_{\min}) & v_{\min} + \dfrac{v_{\max} - v_{\min}}{y_{\max} - y_{\min}} \cdot (y - y_{\min}) & 1 \end{bmatrix}^T$$



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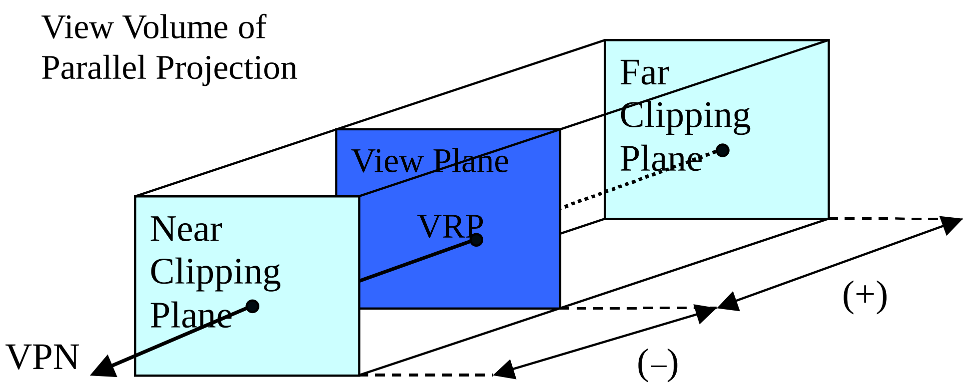
Screen space transformation

The transformation can also be done in 3D normalised viewing space :



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{width}{right-left} & 0 & 0 & \frac{-left \times width}{right-left} \\ 0 & \frac{height}{bottom-top} & 0 & \frac{-top \times height}{bottom-top} \\ 0 & 0 & \frac{z_{max}}{far-near} & \frac{-near \times z_{max}}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- This matrix scales and translates to accomplish the transition in units
  - Left, right, top, bottom refer to the viewing frustum (view volume) in modelling coordinates
  - width and height are in pixel units (viewport)

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Isometric Projection

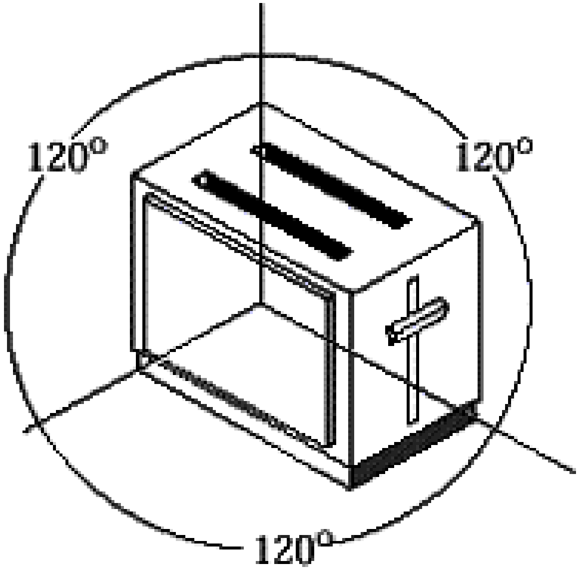
DOP//VPN are parallel (the same), VPN= (1, 1, 1)

$$M = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Used for:
- catalogue illustrations
- patent office records
- furniture design
- structural design

Pros:
- don't need multiple views
- illustrates 3D nature of object
- measurements can be made to scale along principal axes

Cons:
- lack of foreshortening creates distorted appearance
- more useful for rectangular than curved shapes



Oblique Projections

DOP<>VPN, DOP: $\boldsymbol{D} = \begin{bmatrix} D_x & D_y & D_z \end{bmatrix}^T$

$$M = \begin{bmatrix} 1 & 0 & -\frac{D_x}{D_z} & 0 \\ 0 & 1 & -\frac{D_y}{D_z} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Used for skyscrapers.

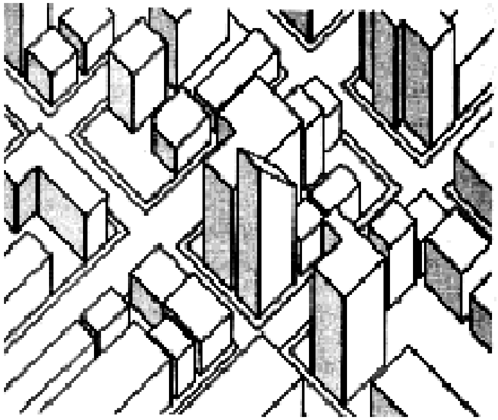
Pros:

- **can present the exact shape of one face of an object (can take accurate measurements)**
- **makes comparison of sizes easier, no perspective foreshortening**
- **displays some of object's 3D appearance**

Cons:

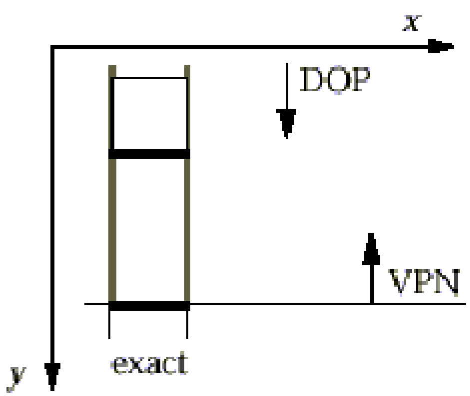
- **objects can look distorted if careful choice not made about position of projection plane (e.g., circles become ellipses)**
- **lack of foreshortening (not realistic looking)**

Plan oblique projection of a city

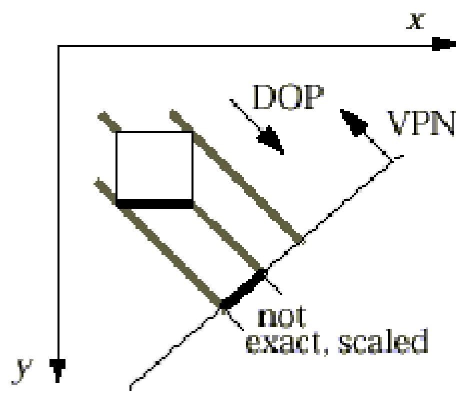


Summary of Parallel Projections

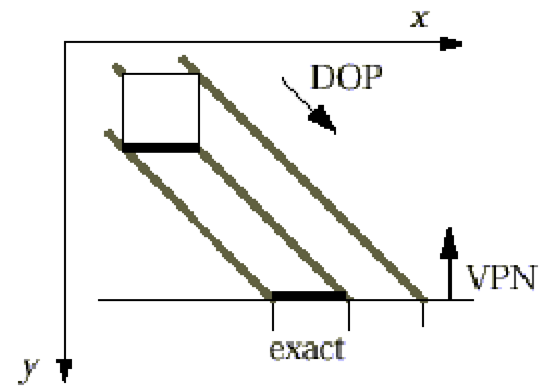
Assume object face of interest lies in principal plane, i.e., parallel to *xy*, *yz*, or *zx* planes.
DOP = Direction of Projection, VPN = View Plane Normal



Multiview Orthographic
– VPN is parallel to a principal coordinate axis
– DOP is parallel to VPN
– shows single face, exact measurements



Axonometric
– VPN is NOT parallel to a principal coordinate axis
– DOP is parallel to VPN
– adjacent faces, none exact, uniformly foreshortened (as a function of angle between



Oblique
– VPN is parallel to a principal coordinate axis
– DOP is NOT parallel to VPN
– adjacent faces, one exact, others uniformly foreshortened

Today's agenda

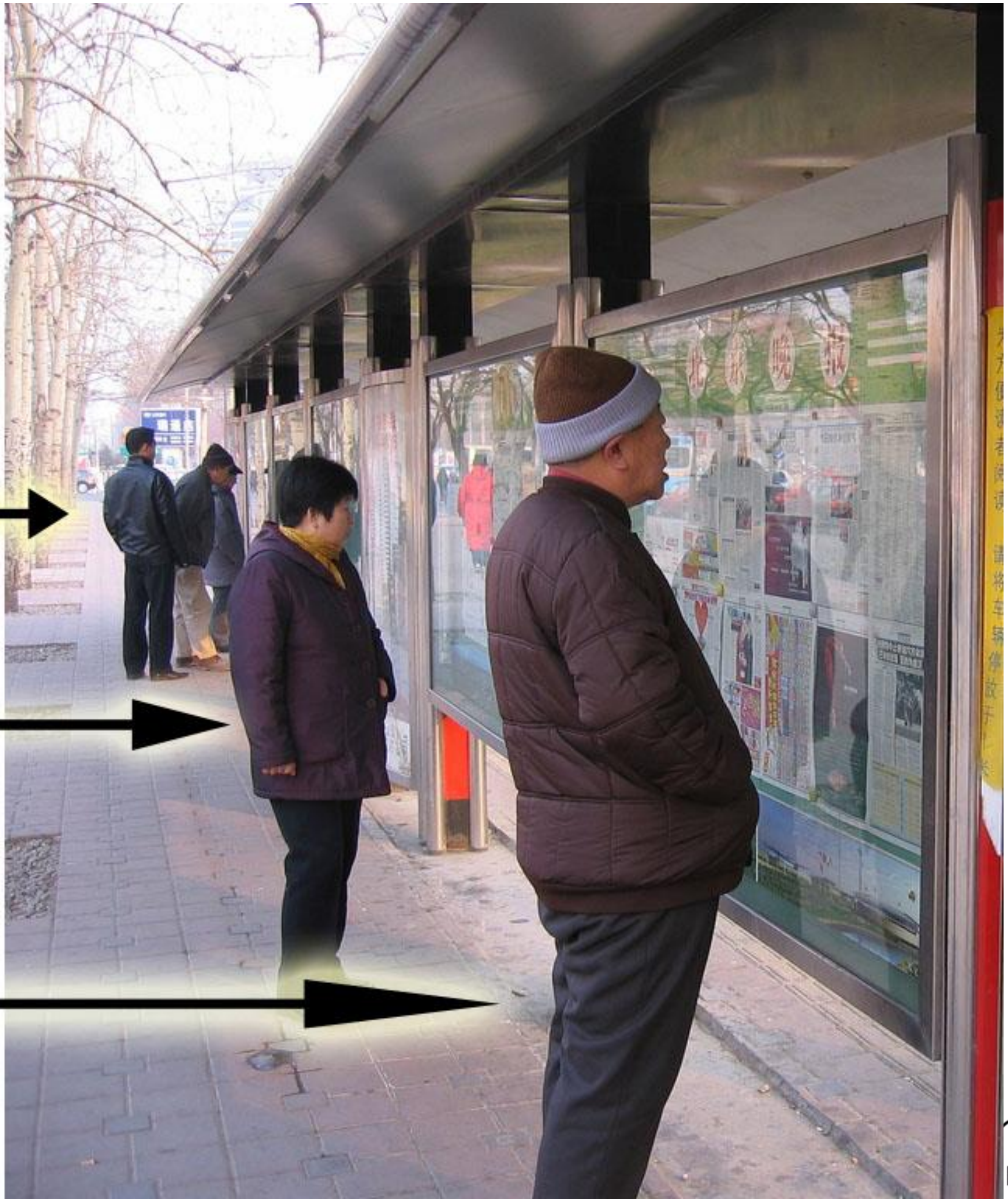
- Taxonomy of projections
- Parallel projection
- Perspective projection

Perspective

166 pixels tall

370 pixels tall

600 pixels tall



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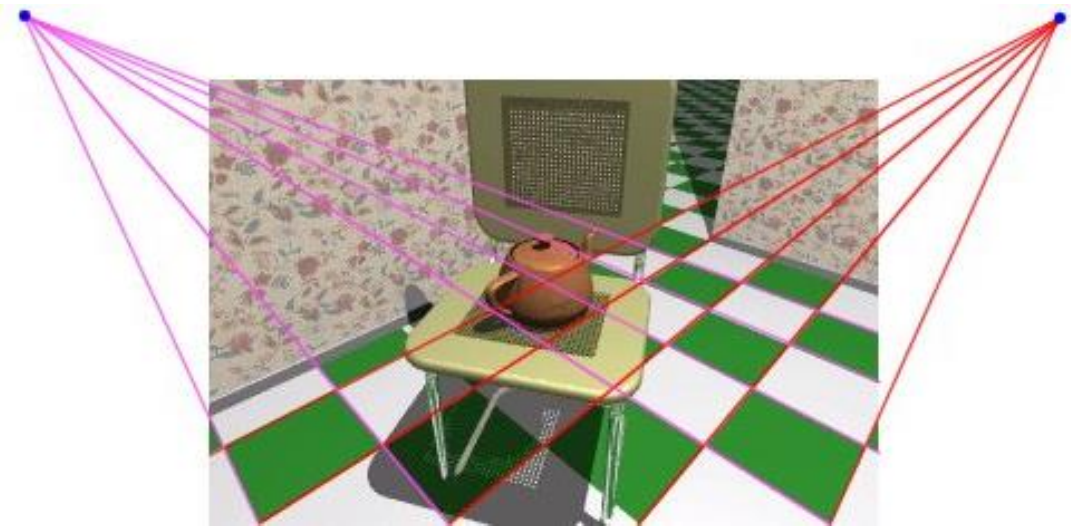
Perspective projection

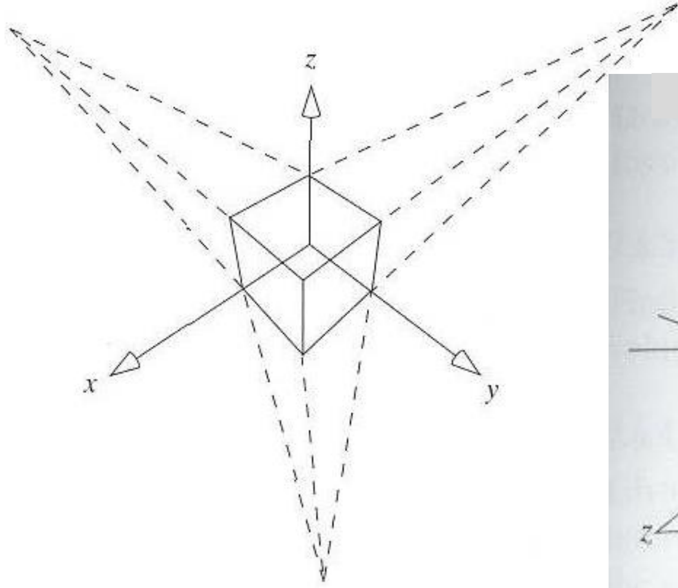
- In the real world, objects exhibit perspective foreshortening
  - distant objects appear smaller
  - objects closer to viewer look larger
- Parallel lines appear to converge to single point (vanishing point)
- First discovered by Donatello, Brunelleschi, and Da Vinci during Renaissance



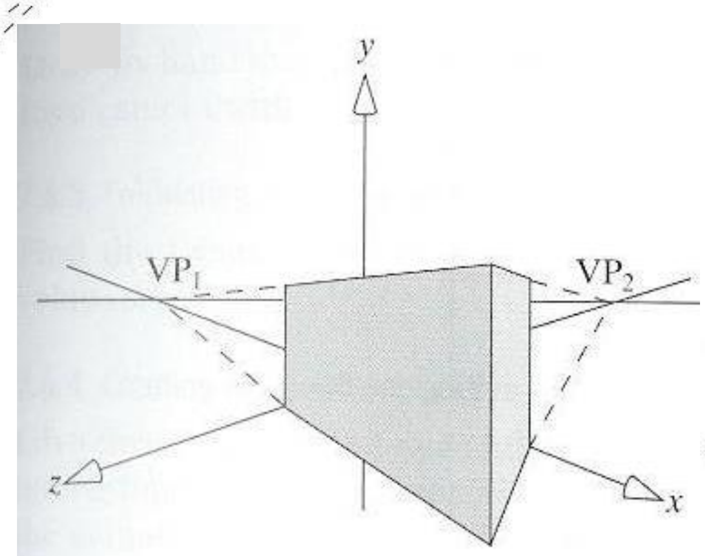
Perspective projection

How many axis vanishing points?

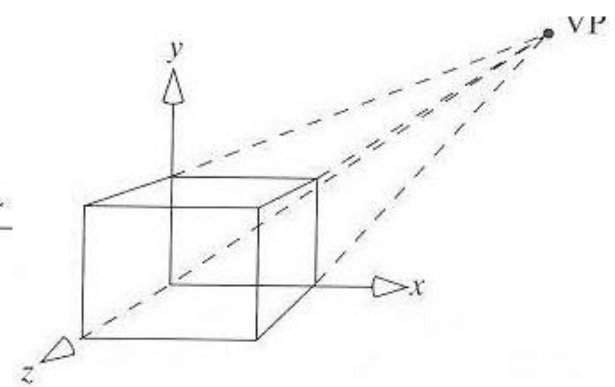
- ***Axis vanishing point*****: vanishing point of lines parallel to one of three principle axes. At most 3: x-axis vanishing point, y-axis vanishing point, and z-axis vanishing point.**
- **The number of axis vanishing points can be used to categorize perspective projections.**



3-point perspective



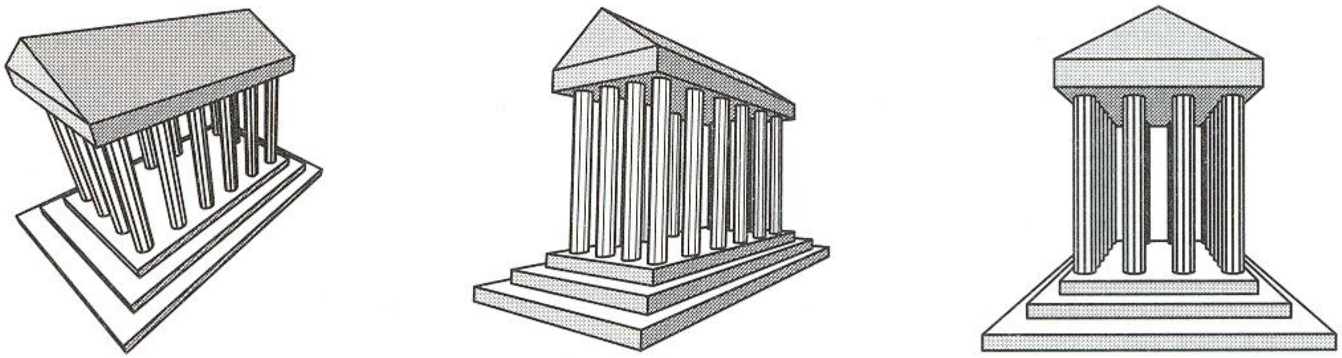
2-point perspective



1-point perspective

Perspective projection

- Two vanishing point projections are often used in architecture, engineering, industrial design and advertising.
- Three vanishing point drawings add little additional realism over two vanishing point drawings.



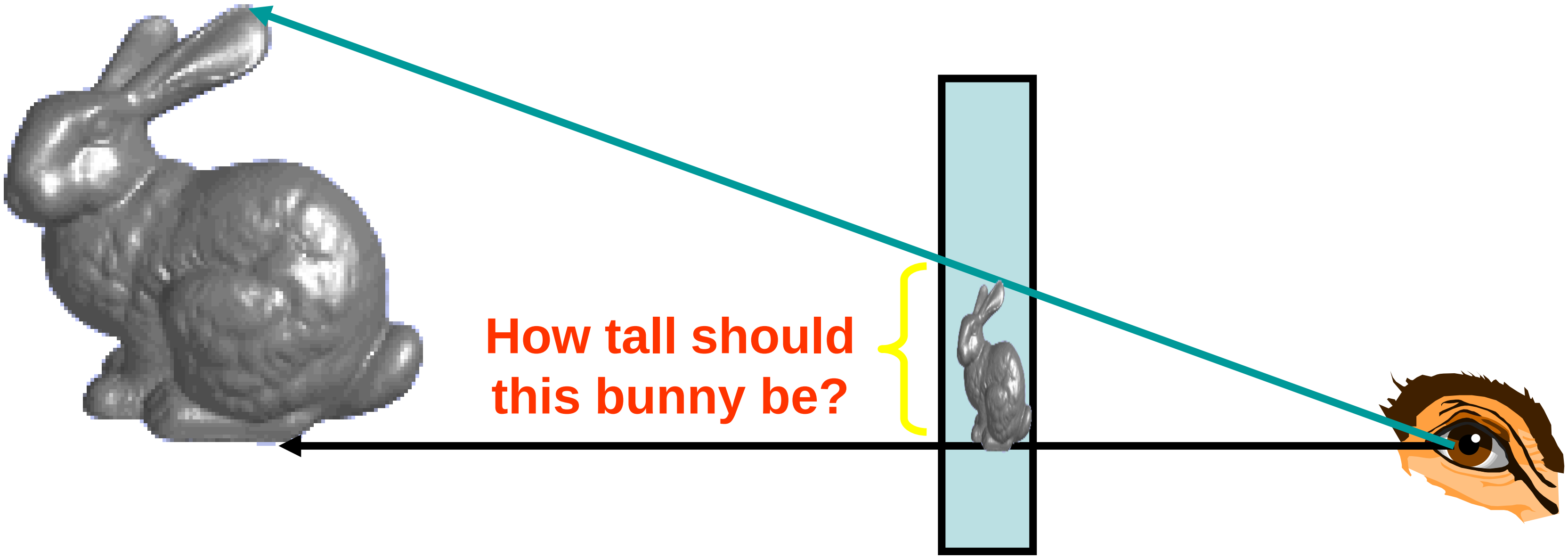
3-point perspective

2-point perspective

1-point perspective

Perspective projection

- 3-D graphics → think of the screen as a 2-D window onto the 3-D world



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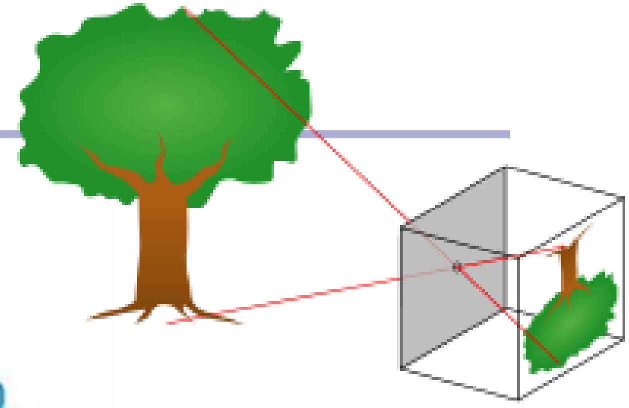
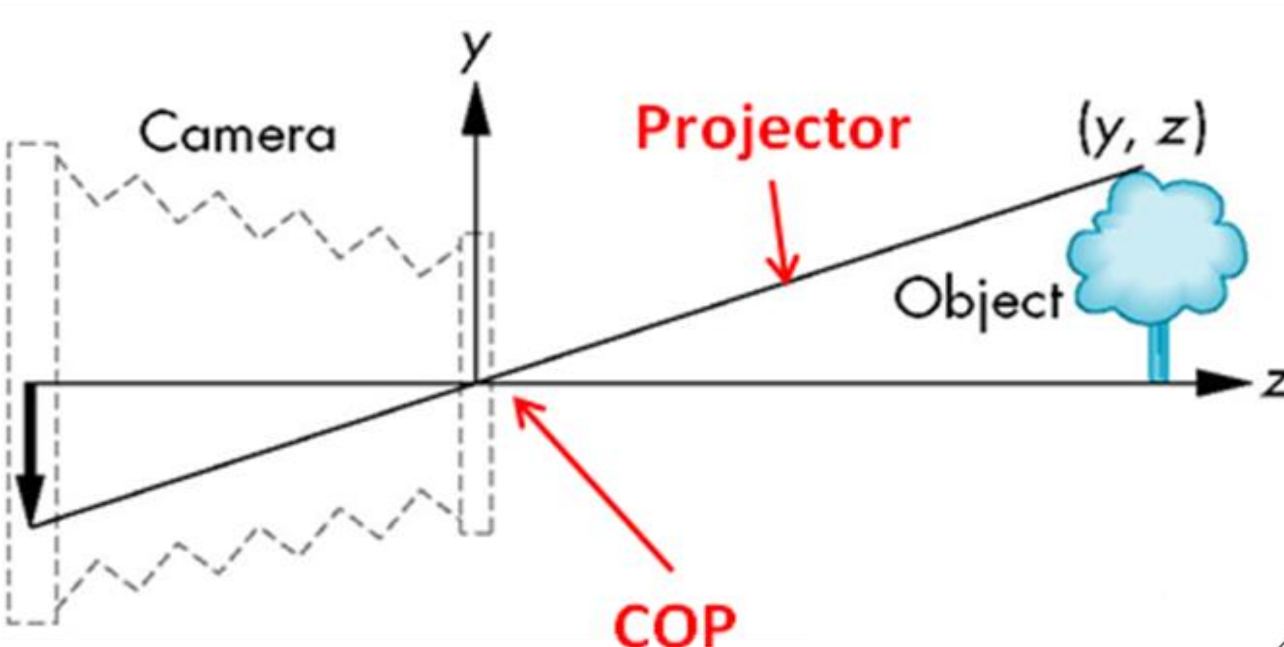
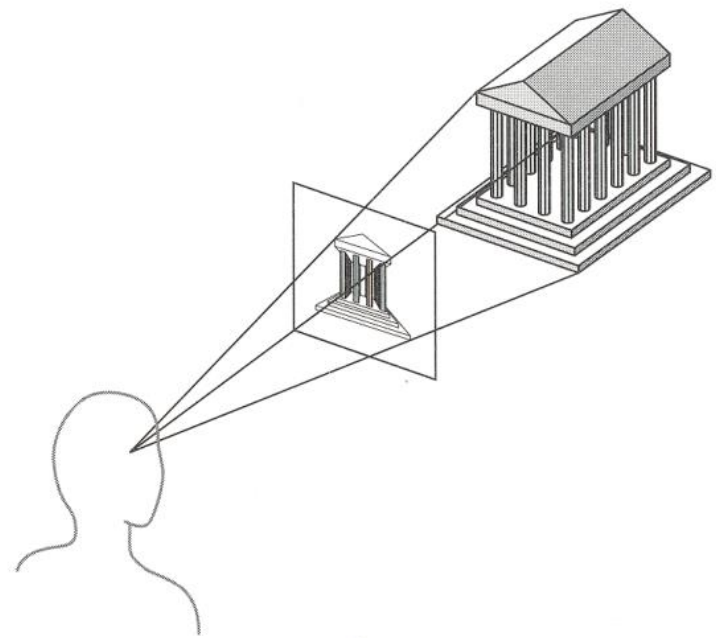
Pinhole Camera

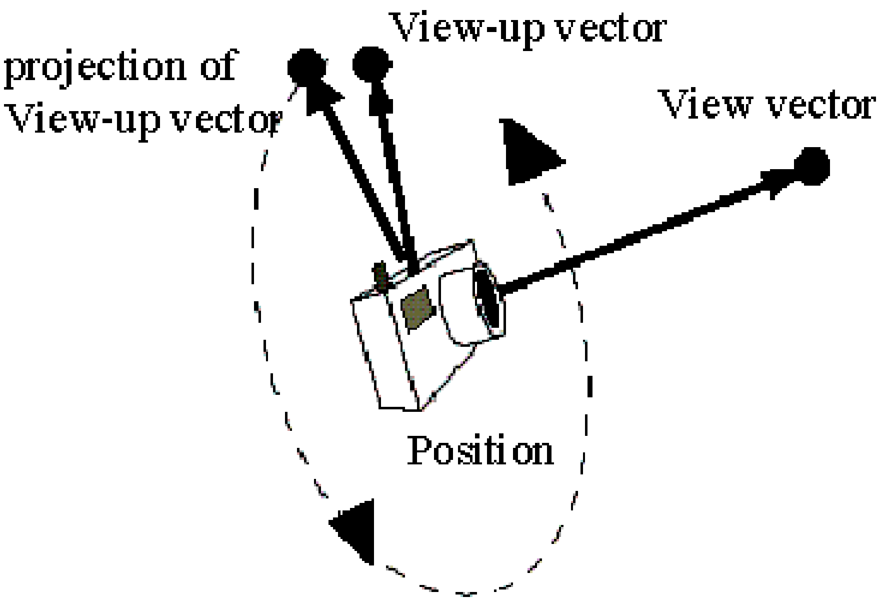


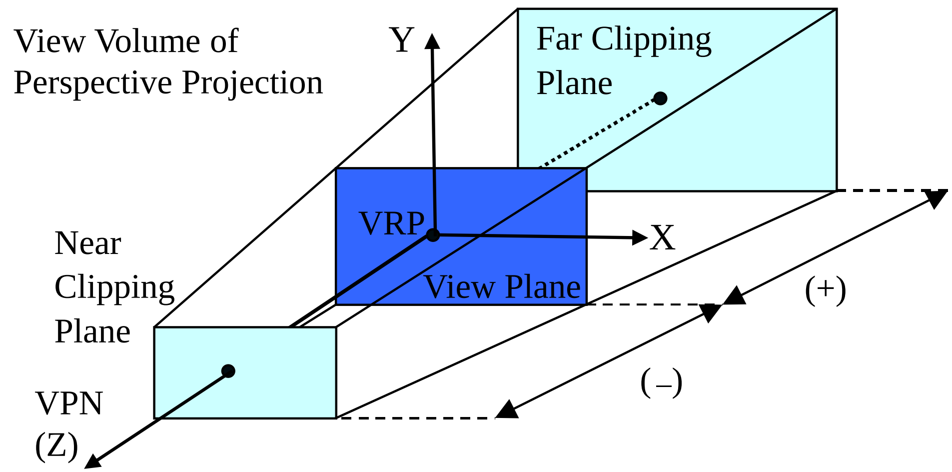




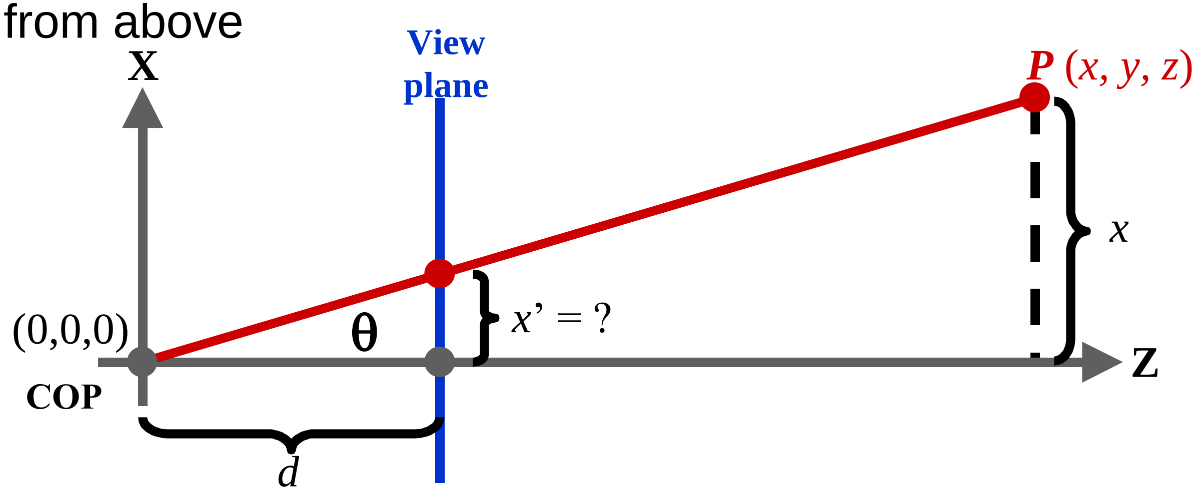
Synthetic Camera

- The geometry of the situation is that of similar triangles





View from above



$$\frac{x'}{d} = \frac{x}{z}$$

What is x' ?

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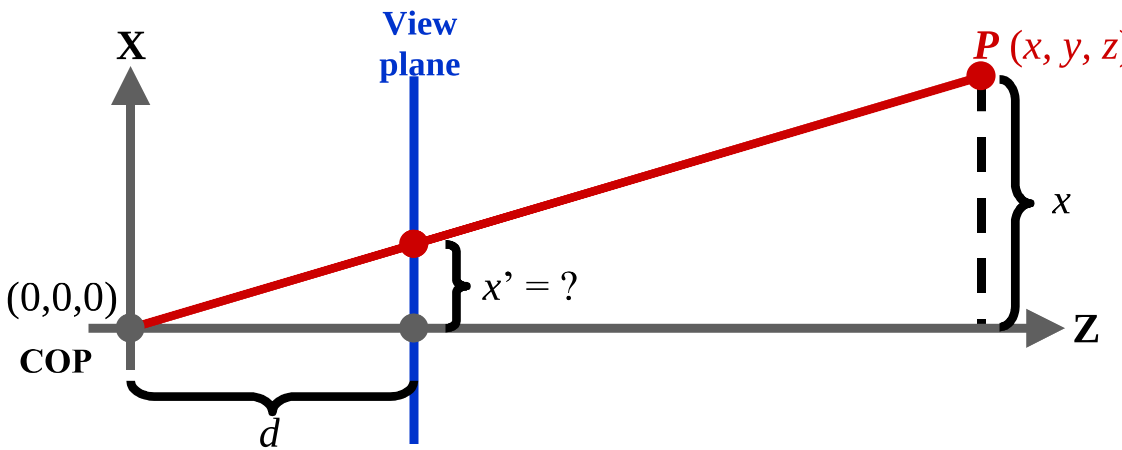
Perspective projection

- Desired result for a point $[x, y, z, 1]^T$ projected onto the view plane:



$$\frac{x'}{d} = \frac{x}{z}, \quad \frac{y'}{d} = \frac{y}{z}$$

$$x' = \frac{d \cdot x}{z} = \frac{x}{z/d}, \quad y' = \frac{d \cdot y}{z} = \frac{y}{z/d}, \quad z' = d = \frac{z}{z/d}$$

What could a matrix look like to do this?

Perspective projection matrix

$$x' = \frac{d \cdot x}{z} = \frac{x}{z/d}, \quad y' = \frac{d \cdot y}{z} = \frac{y}{z/d}, \quad z' = d = \frac{z}{z/d}$$

$$P_{perspective} = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

Perspective projection matrix

$$x' = \frac{d \cdot x}{z} = \frac{x}{z/d}, \quad y' = \frac{d \cdot y}{z} = \frac{y}{z/d}, \quad z' = d = \frac{z}{z/d}$$

We use: $w = z/d$

We have: $x' = \frac{d \cdot x}{z} = \frac{x}{z/d} = \frac{x}{w},$

$$y' = \frac{d \cdot y}{z} = \frac{y}{z/d} = \frac{y}{w},$$

$$z' = d = \frac{z}{z/d} = \frac{z}{w}$$

Perspective projection matrix

$$x' = \frac{d \cdot x}{z} = \frac{x}{z/d}, \quad y' = \frac{d \cdot y}{z} = \frac{y}{z/d}, \quad z' = d = \frac{z}{z/d}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

- in 3-D coordinates: $\left(\frac{x}{z/d}, \quad \frac{y}{z/d}, \quad d \right)$

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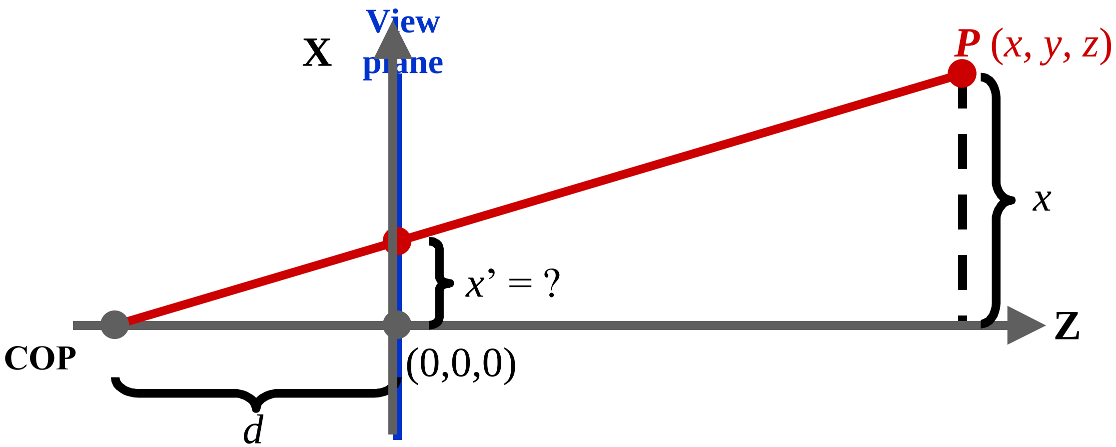
Perspective projection matrix

$$x' = \frac{d \cdot x}{z} = \frac{x}{z/d}, \quad y' = \frac{d \cdot y}{z} = \frac{y}{z/d}, \quad z' = d = \frac{z}{z/d}$$

$$P_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

Perspective projection: origin in view plane

- Desired result for a point $[x, y, z, 1]^T$ projected onto the view plane:



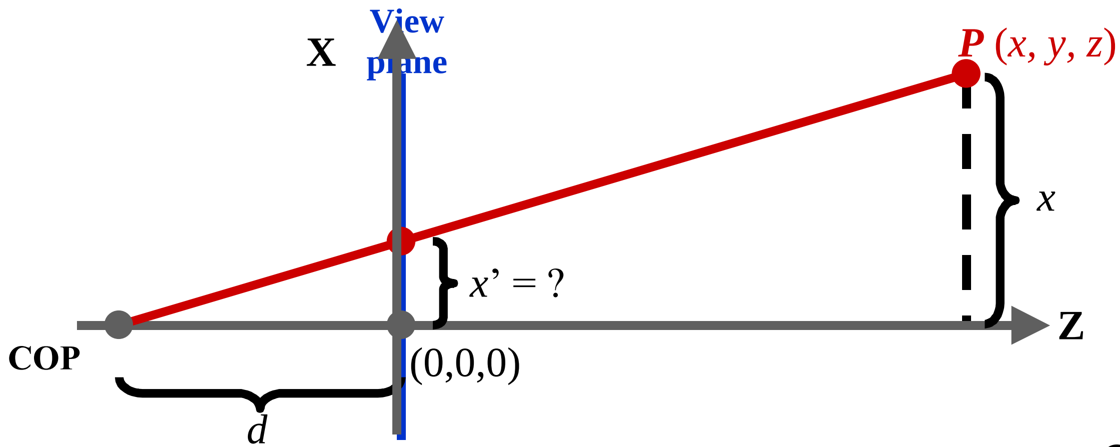
$$\frac{x'}{d} = \frac{x}{z+d}, \quad \frac{y'}{d} = \frac{y}{z+d}, \quad z' = 0$$

$$x' = \frac{d \cdot x}{z+d} = \frac{x}{z/d+1}, \quad y' = \frac{d \cdot y}{z+d} = \frac{y}{z/d+1}, \quad z' = 0 = \frac{0}{z/d+1}$$

What could a matrix look like to do this?

Perspective projection: origin in view plane

- Desired result for a point $[x, y, z, 1]^T$ projected onto the view plane:



$$w = z/d + 1: \quad x' = \frac{x}{w}, \quad y' = \frac{y}{w}, \quad z' = \frac{0}{w}$$

$$P'_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}$$

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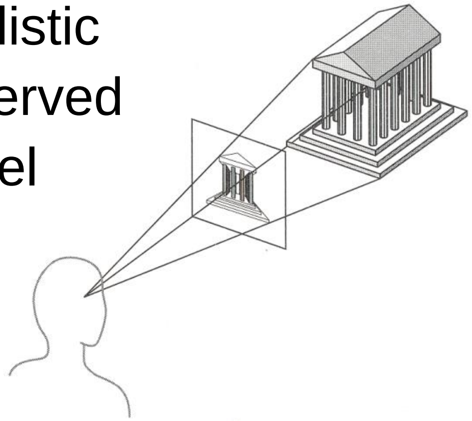
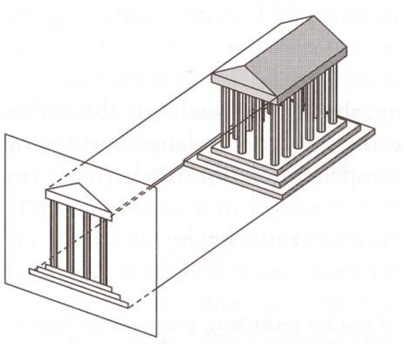
Perspective vs. Parallel

- Perspective projection
  - \+ Size varies inversely with distance - looks realistic
  - – Distance and angles are not (in general) preserved
  - – Parallel lines do not (in general) remain parallel



- Parallel projection
  - \+ Good for exact measurements
  - \+ Parallel lines remain parallel
  - – Angles are not (in general) preserved
  - – Less realistic looking



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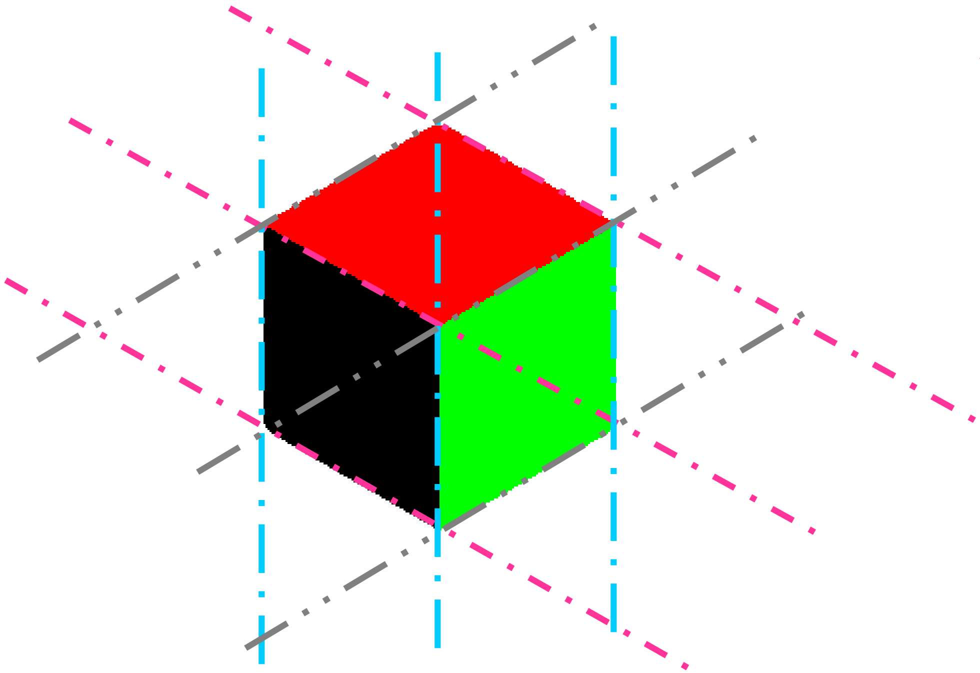
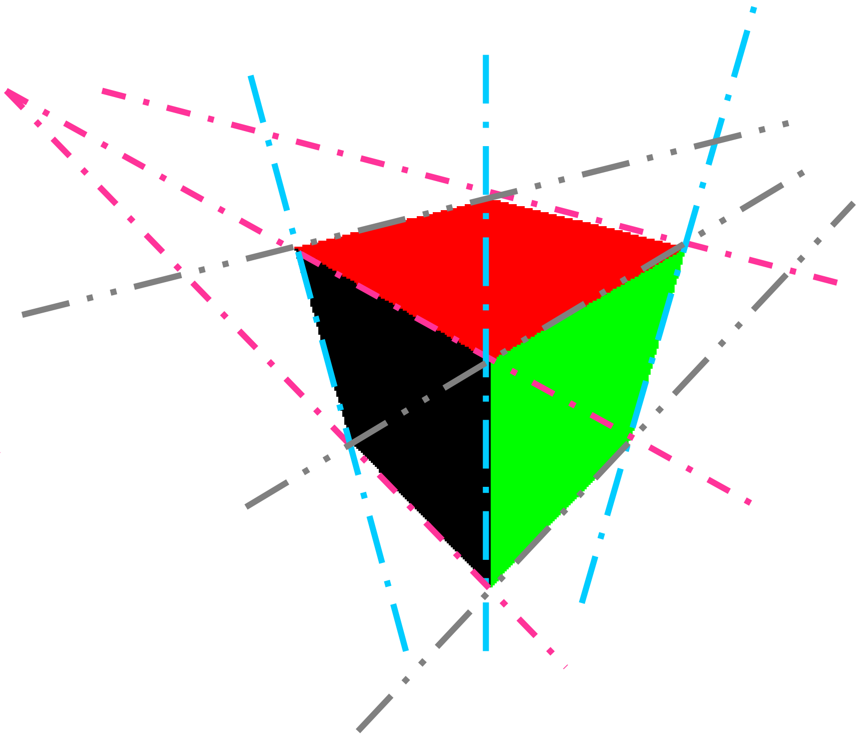
“Isometric” view

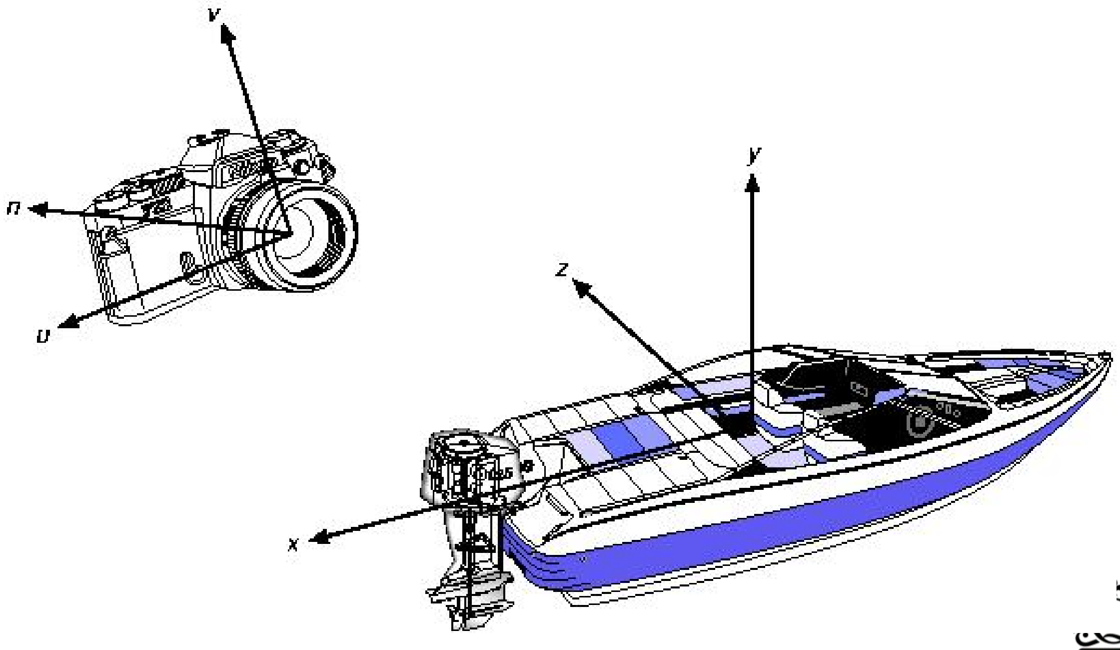




Viewing with a Camera: Coordinate Systems

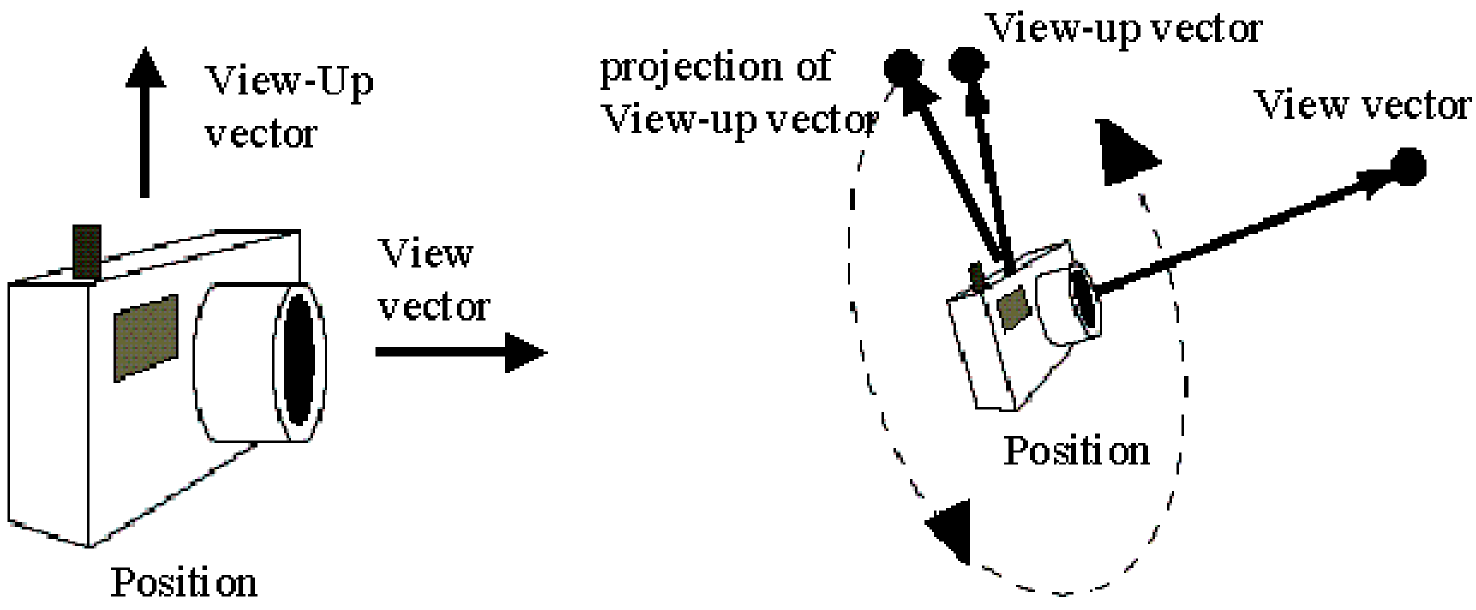
The process of viewing an object in 3D is conceptually similar to viewing it through a camera viewfinder:

- **Moving the camera around the scene - changes the camera location.**
- **Zooming the zoom lens - changes the part of the scene in the viewfinder; in some sense it changes the viewing plane.**
- **Pointing the lens in different directions - changes the coordinate system of the camera.**



Specification of 3D view projection parameters

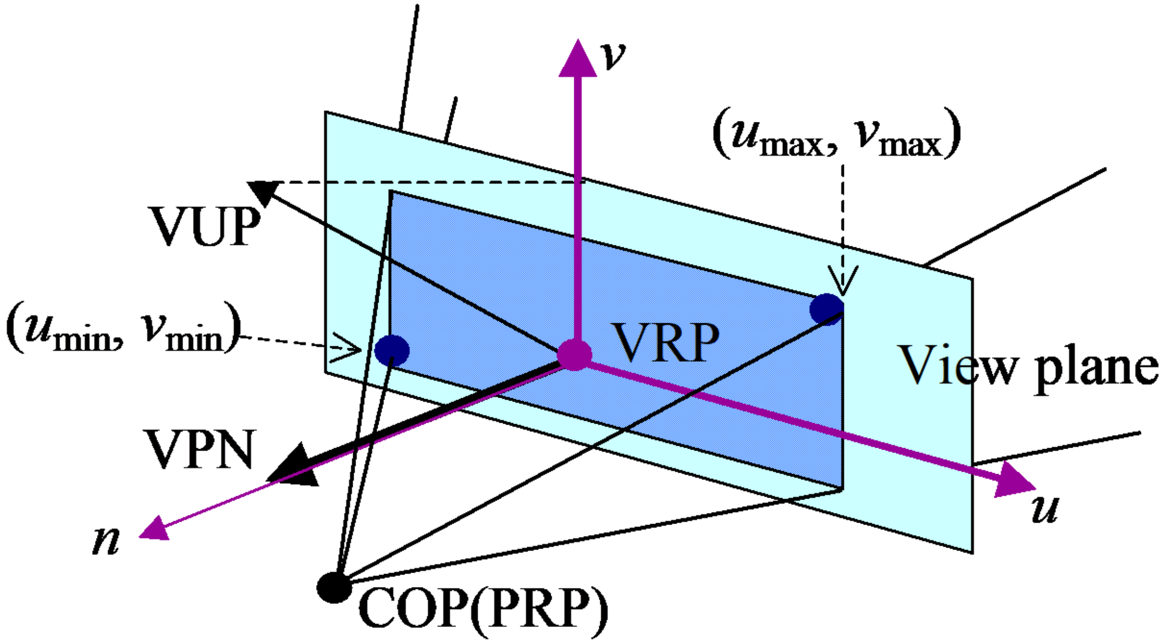
- *Position* of the camera (from where it's looking)
- The *view vector* specifies in what direction the camera is pointing
- The camera's *orientation* is determined by the view direction vector and the angle through which the camera is rotated (rolled) about that vector, i.e., the direction of the view-up vector (VUP). The view-up vector is not necessarily perpendicular to the view vector.



View Reference Coordinate System

View Reference Coordinate (VRC) System

- **View plane (projection plane) is specified by a normal and a point:**
  - **The view reference point (VRP) is where the camera or eye is, and it is taken as a point on the view plane.**
  - **The view plane normal (VPN) is the normal to the view plane.**

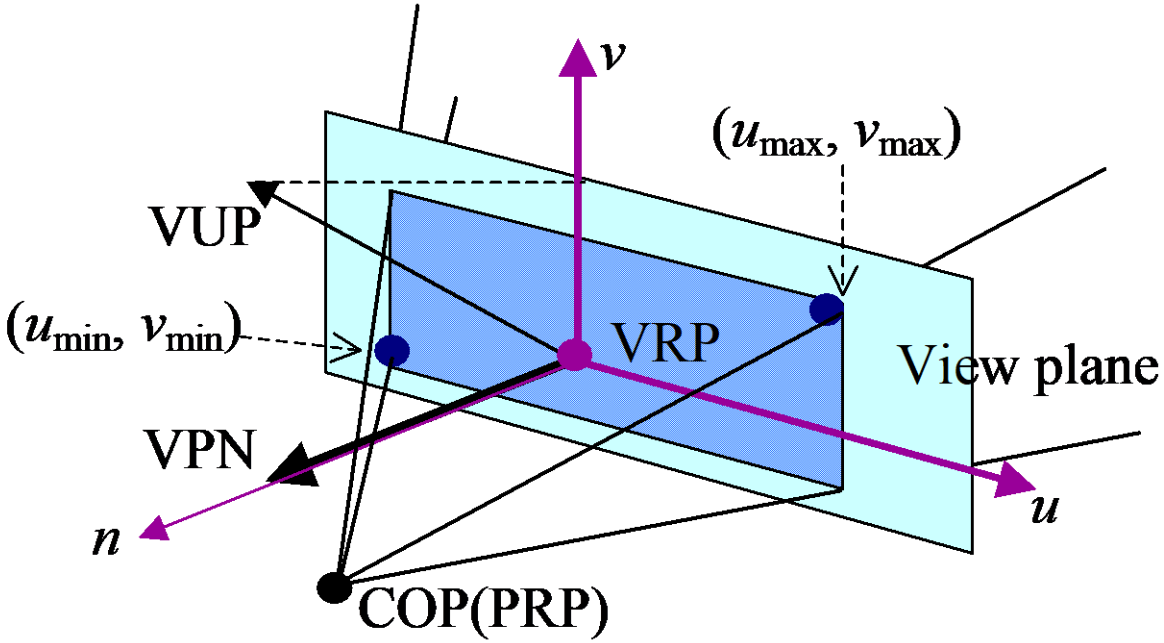


View Reference Coordinate System

View Reference Coordinate (VRC) System

- **View window can be specified by:**
  - A width, a height and a point: the centre of the window (CW).
  - Two points: min and max window coordinates for a rectangle.

The window's role is similar to that of a 2D window: its contents are mapped into the viewport, and any part of the 3D world that projects onto the view plane outside of the window is not displayed.



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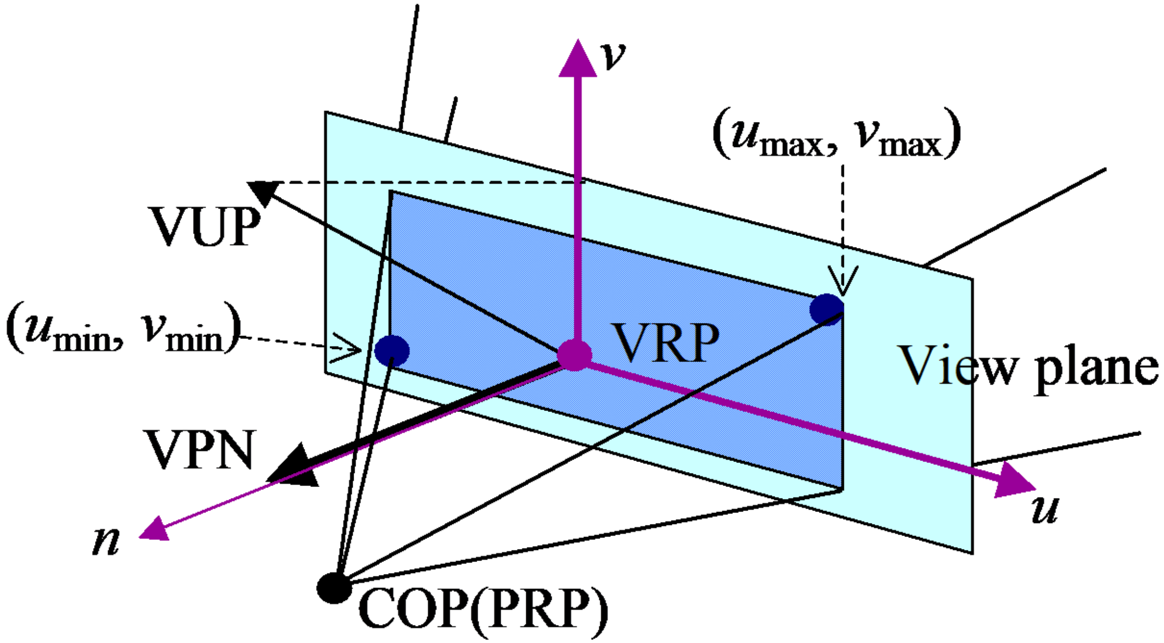
View Reference Coordinate System

View Reference Coordinate (VRC) System

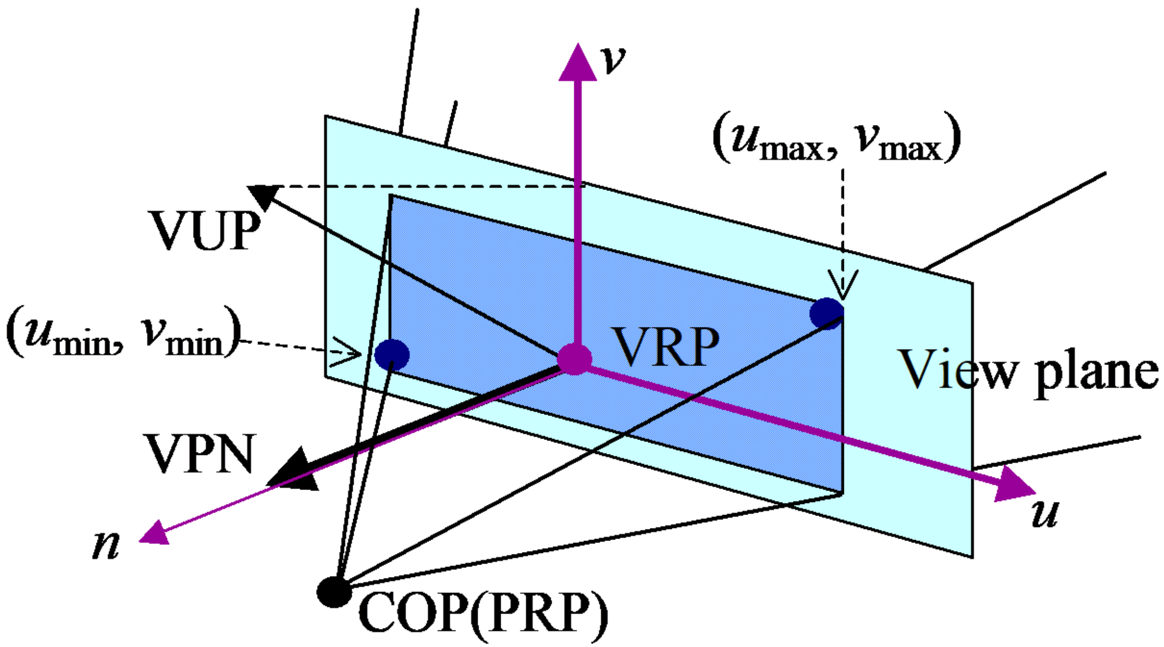
- **VRP is taken as the origin**
- **Three orthogonal axes in the view plane to define the orientation of the window**



View Reference Coordinate System

View Reference Coordinate (VRC) System

- **Three orthogonal axes:**
  - **- VPN is one axis (*n*-axis).**
  - **- The second axis (*v*-axis): projection of *view-up vector* (VUP) onto the view plane.**
  - **- The third axis (*u*-axis) can be easily found in the right-handed coordinate system.**



$$n = \frac{\mathrm{VPN}}{|\mathrm{VPN}|}$$

$$u = \frac{\mathrm{VUP} \times \mathrm{VPN}}{|\mathrm{VUP} \times \mathrm{VPN}|}$$

$$v = n \times u$$

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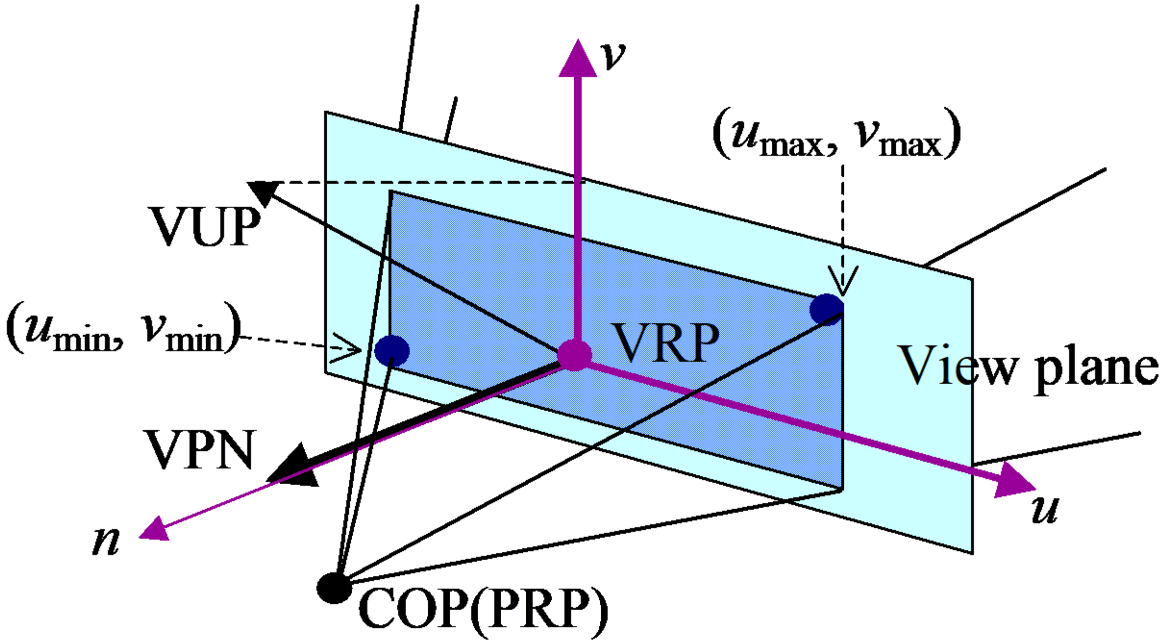
View Reference Coordinate System

View Reference Coordinate (VRC) System

- **Finally, the centre of projection (COP) is defined by a *projection reference point* (PRP).**



Transform world coordinate into VRC

1. **Translation of the coordinate system to the origin in homogeneous matrix form is:**

$$T = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. **Using the unit vectors of the coordinate axes, the resulting rotation matrix is:**

$$R = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. **Combination for the single transformation matrix (parallel):**

$$M = R \cdot T = \begin{bmatrix} u_x & u_y & u_z & -(u_x \cdot VRP_x + u_y \cdot VRP_y + u_z \cdot VRP_z) \\ v_x & v_y & v_z & -(v_x \cdot VRP_x + v_y \cdot VRP_y + v_z \cdot VRP_z) \\ n_x & n_y & n_z & -(n_x \cdot VRP_x + n_y \cdot VRP_y + n_z \cdot VRP_z) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Transform world coordinate into VRC

4. Parallel or Perspective projection:

$$P_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \text{ or } P'_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix} \text{ or } P''_{parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Combination for the single transformation matrix :

$$M = P \cdot R \cdot T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix} \cdot \begin{bmatrix} u_x & u_y & u_z & -(u_x \cdot VRP_x + u_y \cdot VRP_y + u_z \cdot VRP_z) \\ v_x & v_y & v_z & -(v_x \cdot VRP_x + v_y \cdot VRP_y + v_z \cdot VRP_z) \\ n_x & n_y & n_z & -(n_x \cdot VRP_x + n_y \cdot VRP_y + n_z \cdot VRP_z) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} u_x & u_y & u_z & -(u_x \cdot VRP_x + u_y \cdot VRP_y + u_z \cdot VRP_z) \\ v_x & v_y & v_z & -(v_x \cdot VRP_x + v_y \cdot VRP_y + v_z \cdot VRP_z) \\ n_x & n_y & n_z & -(n_x \cdot VRP_x + n_y \cdot VRP_y + n_z \cdot VRP_z) \\ \frac{n_x}{d} & \frac{n_y}{d} & \frac{n_z}{d} & -\frac{n_x \cdot VRP_x + n_y \cdot VRP_y + n_z \cdot VRP_z}{d} \end{bmatrix}$$

(perspective projection with origin at d distance from in the view plane)

$or\ M' = P' \cdot R \cdot T$ (perspective projection with origin in the view plane)

$or\ M'' = P'' \cdot R \cdot T$ (parallel projection)

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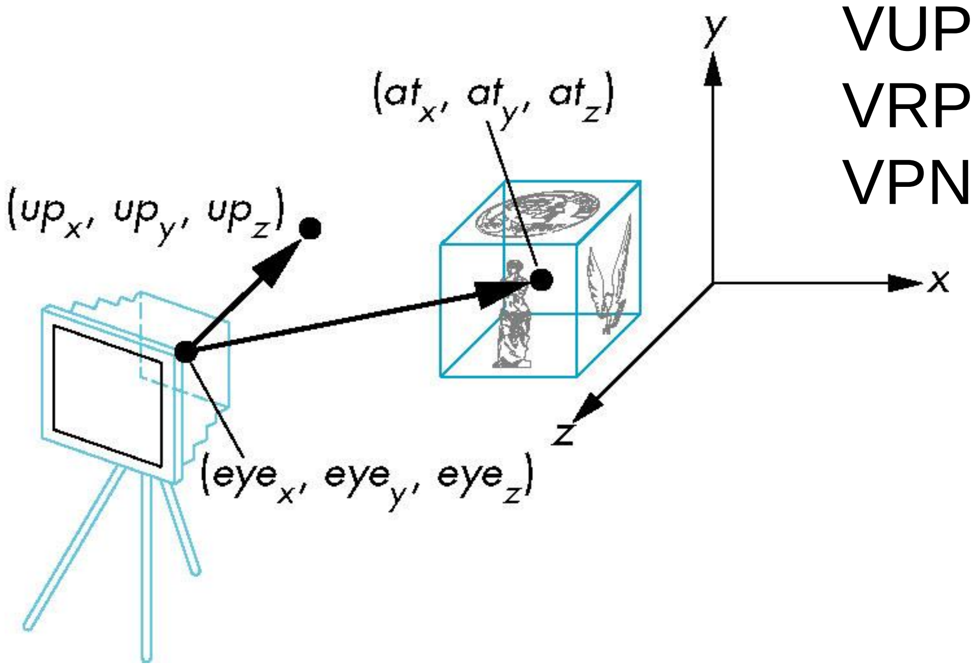
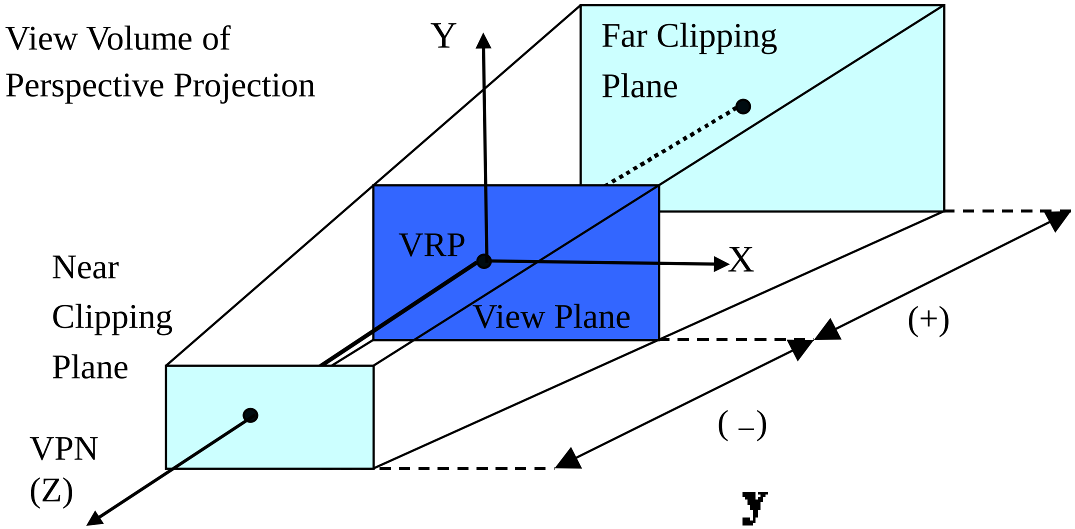
More Direct Method: Eye to Look-at

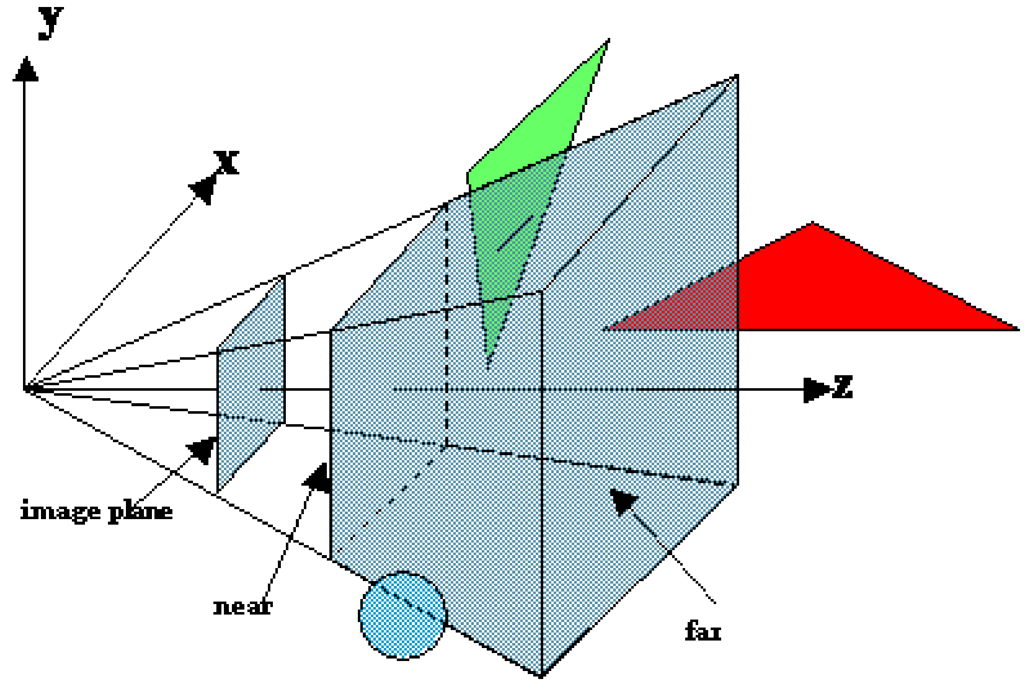
- Specify
  - the eye point → a point the camera is located in world space
  - the lookat point → a point in world space that we wish to become the center of view
  - the up vector → a vector in world space that we wish to point up in camera image



VUP = up vector
VRP = eye point
VPN = at point – eye point

View Volume and 3D Clipping





Exercise

What kind of projection is represented in the matrix below? Justify your answer.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

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Question

What happens to a projected object when the Centre of Projection is pushed further away from the projection plane?

Question

Is "isometric view" a special case of perspective projection? Justify your answer.

What did we learn?

- Taxonomy of projections
- Parallel projection
- Perspective projection