To Ponder

- Pick 5 cards
- Prepare to be astonished…

Security: Cryptography

Lecture 26

Some High-Level Goals

- Confidentiality
  - Non-authorized users have limited access
- Integrity
  - Accuracy/correctness/validity of data
- Availability
  - No down-time or disruptions
- Authenticity
  - Agents are who they claim to be
- Non-repudiation
  - A party to a transaction can not later deny their participation

Methods of Attack

- Target people ("social engineering")
  - Phishing: email, phone, surveys, …
  - Baiting: click & install, physical media, …
- Target software ("exploits")
  - Unpatched OS, browser, programs
  - Buffer overflow
  - Code injection and cross-site scripting
- Target channel ("man-in-the-middle")
  - Eavesdropping
  - Masquerading, tampering, replay

Cryptography

- Etymology (Greek)
  - *kryptos*: hidden or secret
  - *grapho*: write
- Basic problem:
  - 2 agents (traditionally "Alice" and "Bob")
  - A & B want to exchange private messages
  - Channel between A & B is not secure ("Eve" is eavesdropping)
- Solution has other applications too
  - Protect *stored* data (*e.g.* on disk, or in cloud)
  - Digital *signatures* for non-repudiation
  - Secure *passwords* for authentication

Core Idea: The Secret

- Alice & Bob share some *secret*
  - Secret can not be the message itself
  - Secret used to protect arbitrary messages
- Crude analogy: a padlock
  - Copies of the physical key are the secret
  - Alice puts message in box and locks it
  - Bob unlocks box and reads message
- But real channels are bit streams
  - Eve can see the bits!
  - Message must be garbled in some way
  - Secret is strategy for garbling/degarbling

Protecting Messages

- Alice garbles (encrypts) the message
- Sends the encrypted cipher-text
- Bob knows how to degarble (decrypt) cipher-text back into plain-text

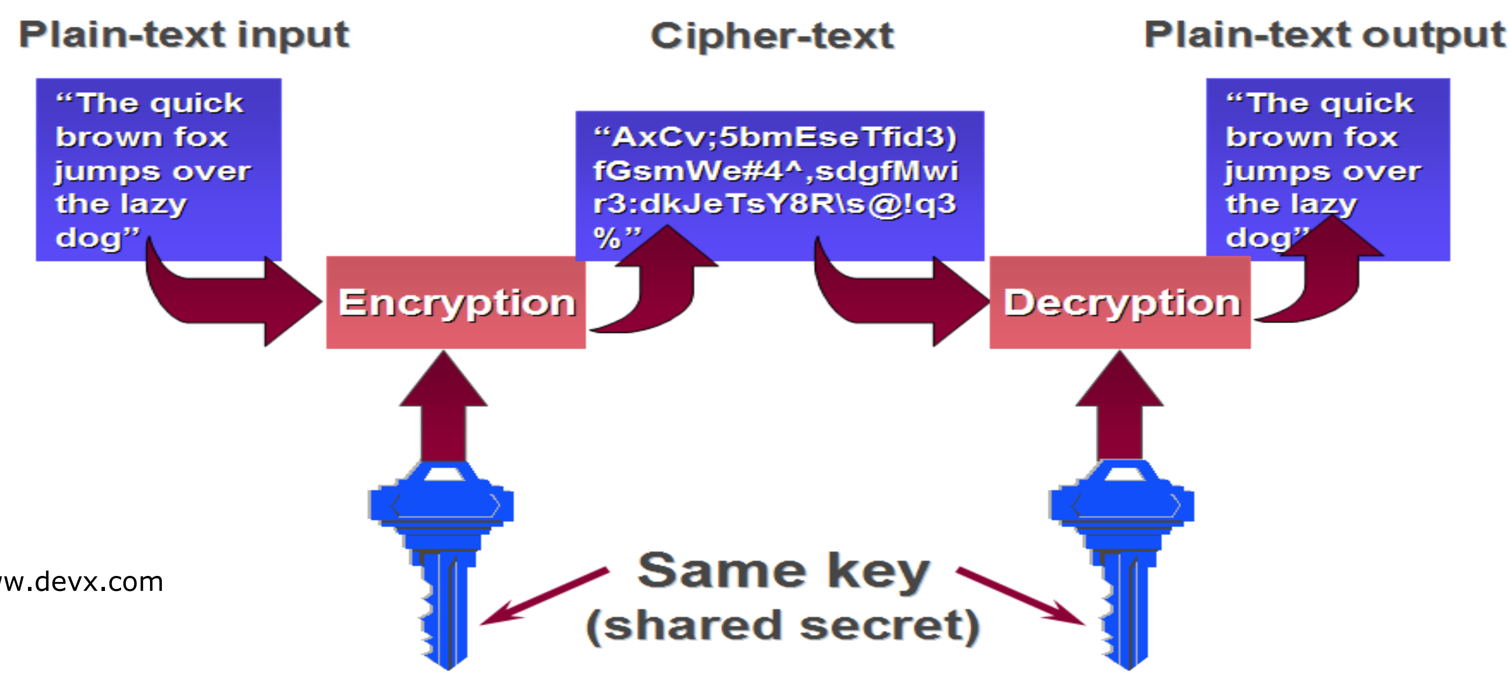
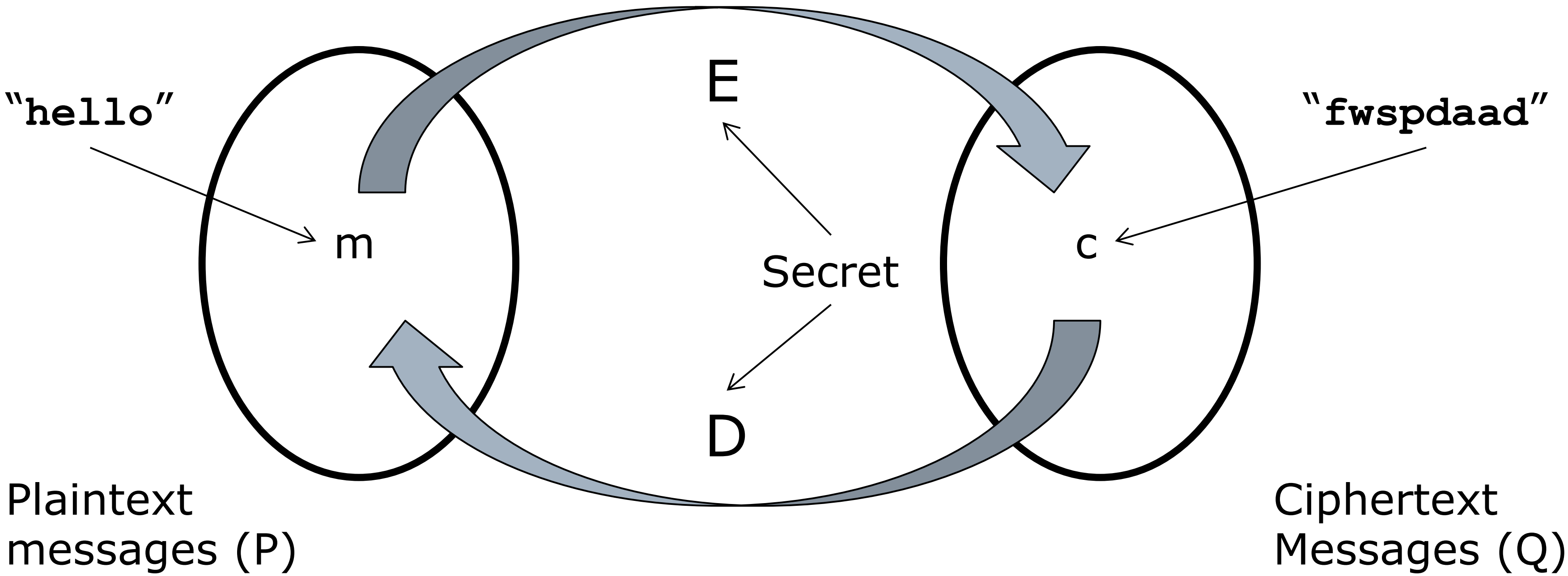


Image: www.devx.com

Encryption/Decryption Function



E: P → Q
D: Q → P

E(m) = c
D(c) = m
i.e. D = E^{-1}

Note: often P = Q
So E is a *permutation*

Families of Encryption Functions

- Each pair of agents needs their own E
  - Many E's (& corresponding D's) needed
- But good E's are hard to invent
- Solution: design one (good) E, which is parameterized by a number
  - That is, have a huge *family* of E's: E_0, E_1, E_2, … E_K
  - Secret: which E_i is used (i is the key)

Classic Example: Caesar Cipher

- Shift each letter by *x* positions in alphabet
  - *E.g.* $x = 3$
    `a` → **`d`**, **`b`** → **`e`**, **`c`** → **`f`**, **`d`** → **`g`**, **`e`** → **`h`**, …
  - The key is *x*
- Encode a string character-by-character
  - For m = "**`hello world`**", E_3(m) = "**`khoor zruog`**"
- Questions:
  - What is P (set of plaintext messages)?
  - What is Q (set of ciphertext messages)?
  - How many different ciphers?
  - Is this a strong or weak cipher?

Generalized Caesar Cipher

- Generalization: arbitrary mapping
  - *E.g.* The qwerty shift
    `a` → **`s`**, **`b`** → **`n`**, **`c`** → **`v`**, **`d`** → **`f`**, **`e`** → **`r`**, …
  - For m = "**`hello world`**",
    E(m) = "**`jraap eptaf`**"
  - 26! possible ciphers… that's a lot!
    - Approximately 4 x 10^{26}
    - There are ~10^{18} nanoseconds/century
- Weakness?

Generalized Caesar Cipher

- Generalization: arbitrary mapping
  - *E.g.* The qwerty shift
    a → **s**, **b** → **n**, **c** → **v**, **d** → **f**, **e** → **r**, …
  - For m = "**hello world**",
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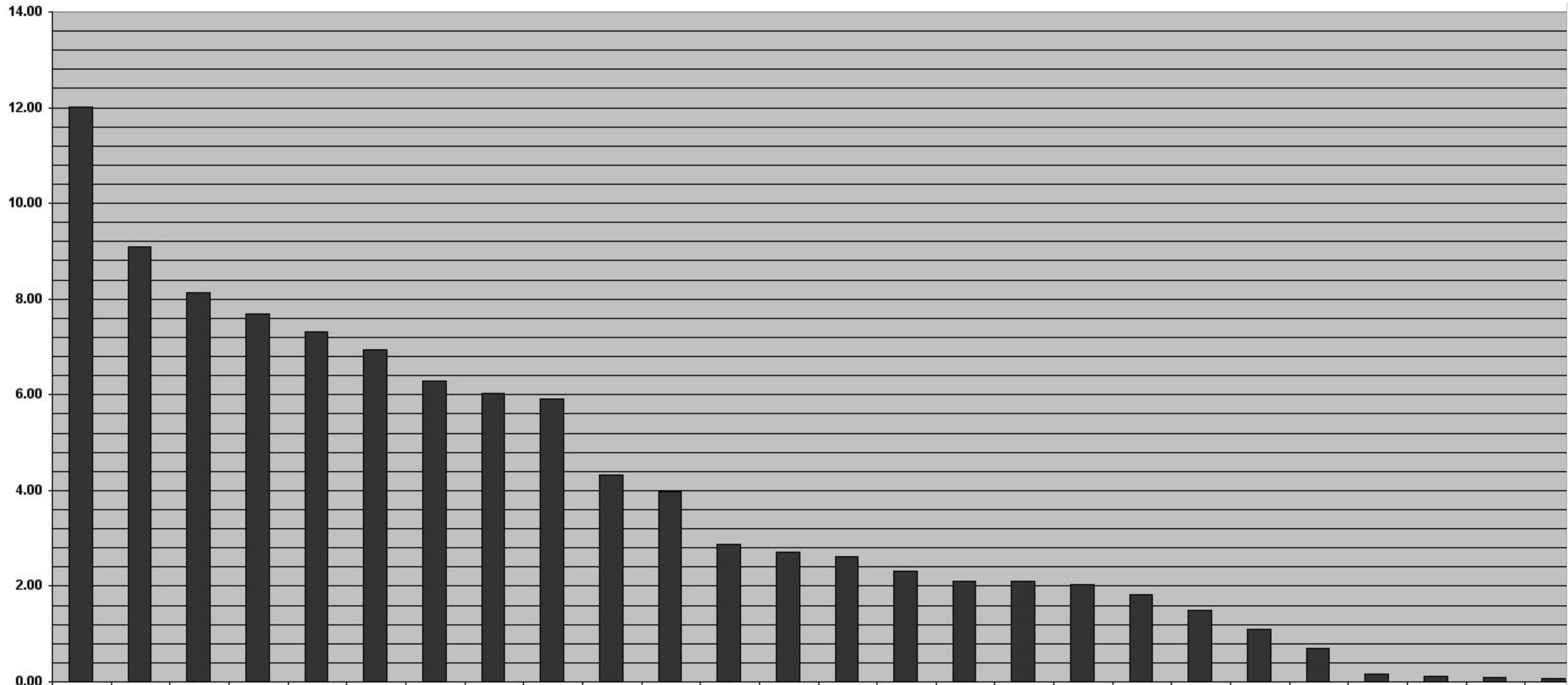
  - 26! possible ciphers… that's a lot!
    - Approximately 4 x 10^{26}
    - There are ~10^{18} nanoseconds/century
- Weakness?
  - In English text, letters appear in predictable ratios
  - From enough ciphertext, can infer E

Frequency Analysis



	E	T	A	O	I	N	S	R	H	D	L	U	C	M	F	Y	W	G	P	B	V	K	X	Q	J	Z
Frequency	12.02	9.10	8.12	7.68	7.31	6.95	6.28	6.02	5.92	4.32	3.98	2.88	2.71	2.61	2.30	2.11	2.09	2.03	1.82	1.49	1.11	0.69	0.17	0.11	0.10	0.07

Leon Battista Alberti

WW II: Enigma Machine

Polyalphabetic Cipher

- Alberti's idea: Use different E_i's within the same message
  - E("**`hello world`**") = E_a("**`h`**")E_b("**`e`**")E_c("**`l`**")E_d("**`l`**")E_e("**`o`**")…
- Alice & Bob need to agree on the *sequence* of E's to use
- Claude Shannon proved that this method is perfectly secure (1949)
  - Precise information-theoretic meaning
  - Known as a *one-time pad*

One-Time Pad

- Message is a sequence of bits

  $\boldsymbol{m}_0\ \boldsymbol{m}_1\ \boldsymbol{m}_2\ \boldsymbol{m}_3\ \boldsymbol{m}_4\ \boldsymbol{m}_5\ \boldsymbol{m}_{6\dots}$

- One-time pad is *random* bit sequence

  $\boldsymbol{x}_0\ \boldsymbol{x}_1\ \boldsymbol{x}_2\ \boldsymbol{x}_3\ \boldsymbol{x}_4\ \boldsymbol{x}_5\ \boldsymbol{x}_{6\dots}$

- E is bit-wise XOR operation, $\oplus$
- Cipher text is

  $\boldsymbol{m}_0\oplus\boldsymbol{x}_0\ \boldsymbol{m}_1\oplus\boldsymbol{x}_1\ \boldsymbol{m}_2\oplus\boldsymbol{x}_2\ \boldsymbol{m}_3\oplus\boldsymbol{x}_3\ \boldsymbol{m}_4\oplus\boldsymbol{x}_4\ \boldsymbol{m}_5\oplus\boldsymbol{x}_5\ \boldsymbol{m}_6\oplus\boldsymbol{x}_{6\dots}$

- Problem: Pad is long and cannot be re-used (hence cumbersome to share)
- "Solution": pseudo-random sequence, generated from a seed (the key)

Comparison: Stream *vs* Block

Stream Cipher

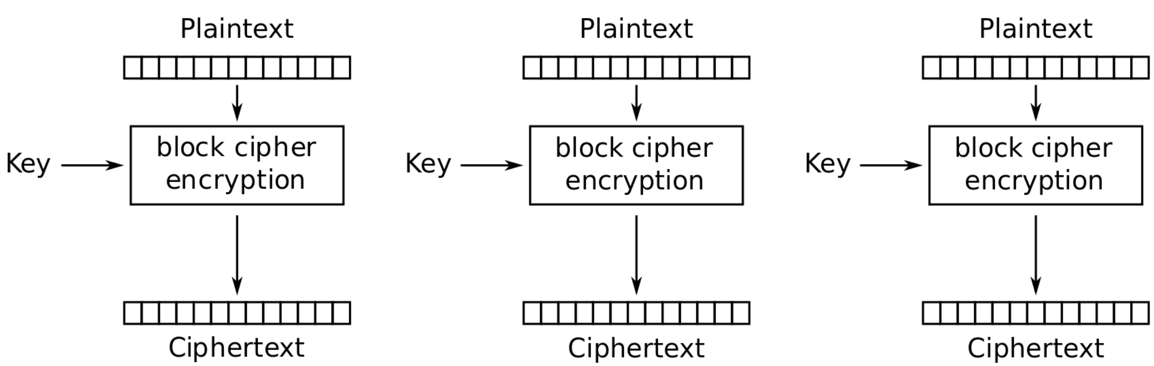
- Encrypts bit-by-bit
- $|P| = |Q| = 2$
- Few choices for E (roughly 2)
- Message can have any length

Block Cipher

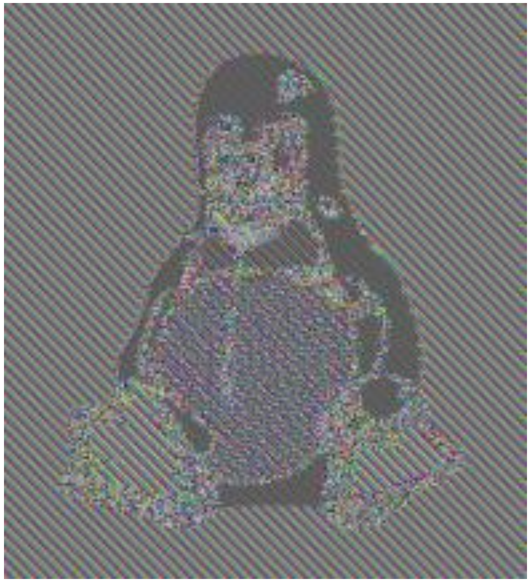
- Encrypts a fixed-length (k-bit) sequence
- $|P| = |Q| = 2^k$
- Many choices for E (roughly $2^k!$)
- Padding added s.t. $|m|$ mod $k = 0$

Limitation of Fixed Block Size

- Message can be longer than block size
- Reuse same E for each block?
  - Danger: Frequency analysis vulnerability
  - Don't do this (for multiblock messages)!



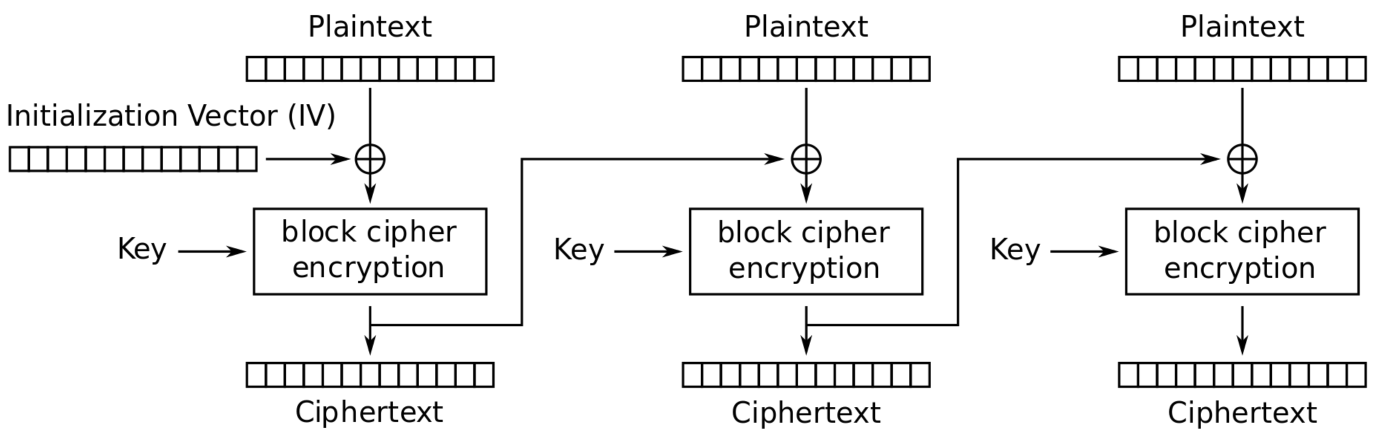
Electronic Codebook (ECB) mode encryption



https://en.wikipedia.org/wiki/Image:Tux_ecb.jpg

Solution: Initialization Vector

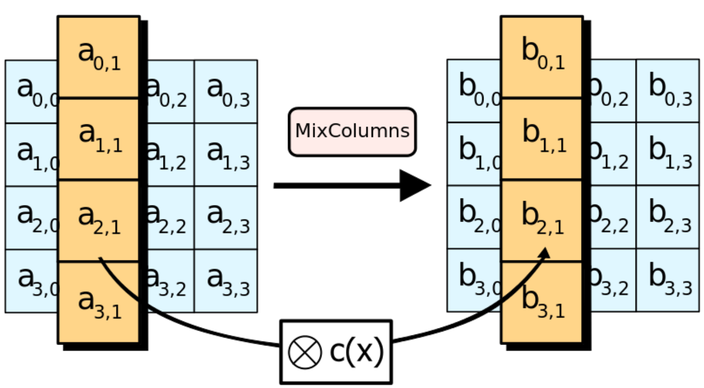
- Add a random block to start
- Combine adjacent blocks to make ciphertext block
  - Many combination strategies

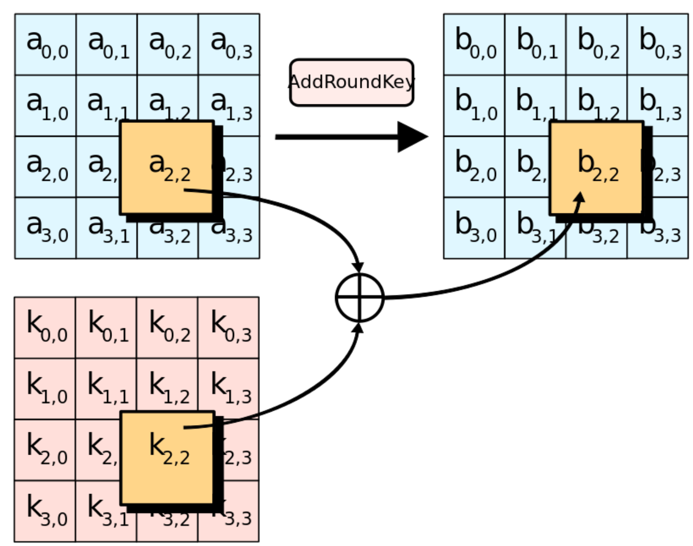


Cipher Block Chaining (CBC) mode encryption

AES

- Advanced Encryption Standard (2001)
  - Replaced DES (1977)
- Block size always 128 bits (4x4 bytes)
- Key size is 128, 192, or 256 bits
- Multi-step algorithm, many rounds





Security: Cryptography

Symmetric Key

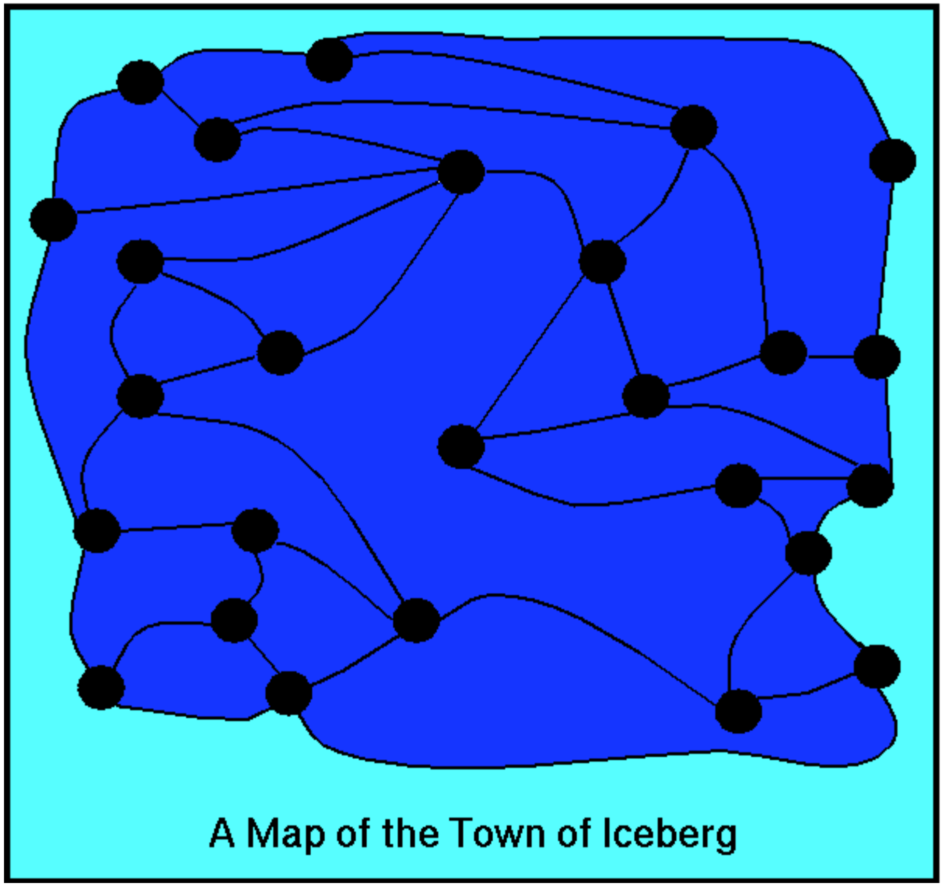
- For ciphers (so far): Knowing E is enough to figure out D (its inverse)
  - If you know how to encrypt, you can decrypt too
  - Known as a *symmetric key* cipher
- Example: Caesar cipher
  - If $E(m) = m + 3$, $D(m) = m - 3$
- Example: One-time pad
  - Use same pad and same operation (xor)
- Example: AES
  - Use same key, reverse rounds and steps

One-Way Functions

- For some functions, the inverse is hard to calculate
  - One direction (P→Q) is easy, but opposite direction (Q→P) is hard/expensive/slow
- Intuition:
  - Given a puzzle *solution*, easy to design a puzzle with that solution (the "forward" direction)
  - Given the puzzle, hard to come up with the solution (the "inverse" direction)

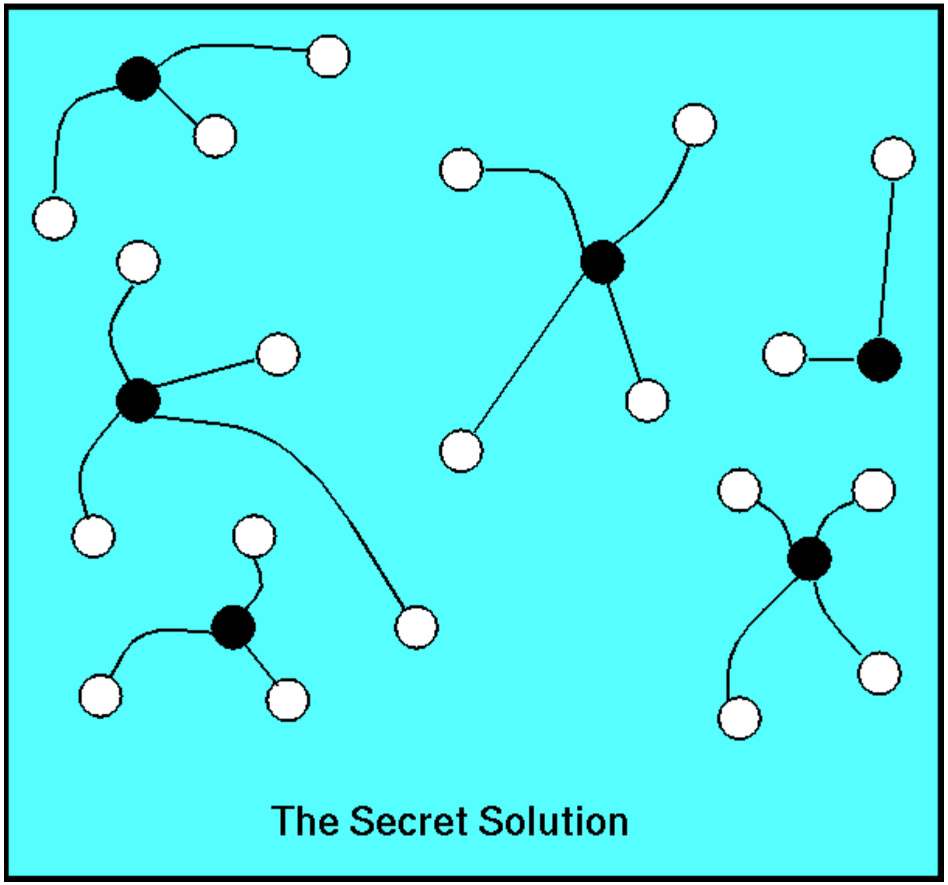
Example: Dominating Set

- Hard direction: Find a dominating set of size at most 6 in the following graph…



Example: Dominating Set

- Easy direction: Create a graph with a dominating set of size 6 from this forest…



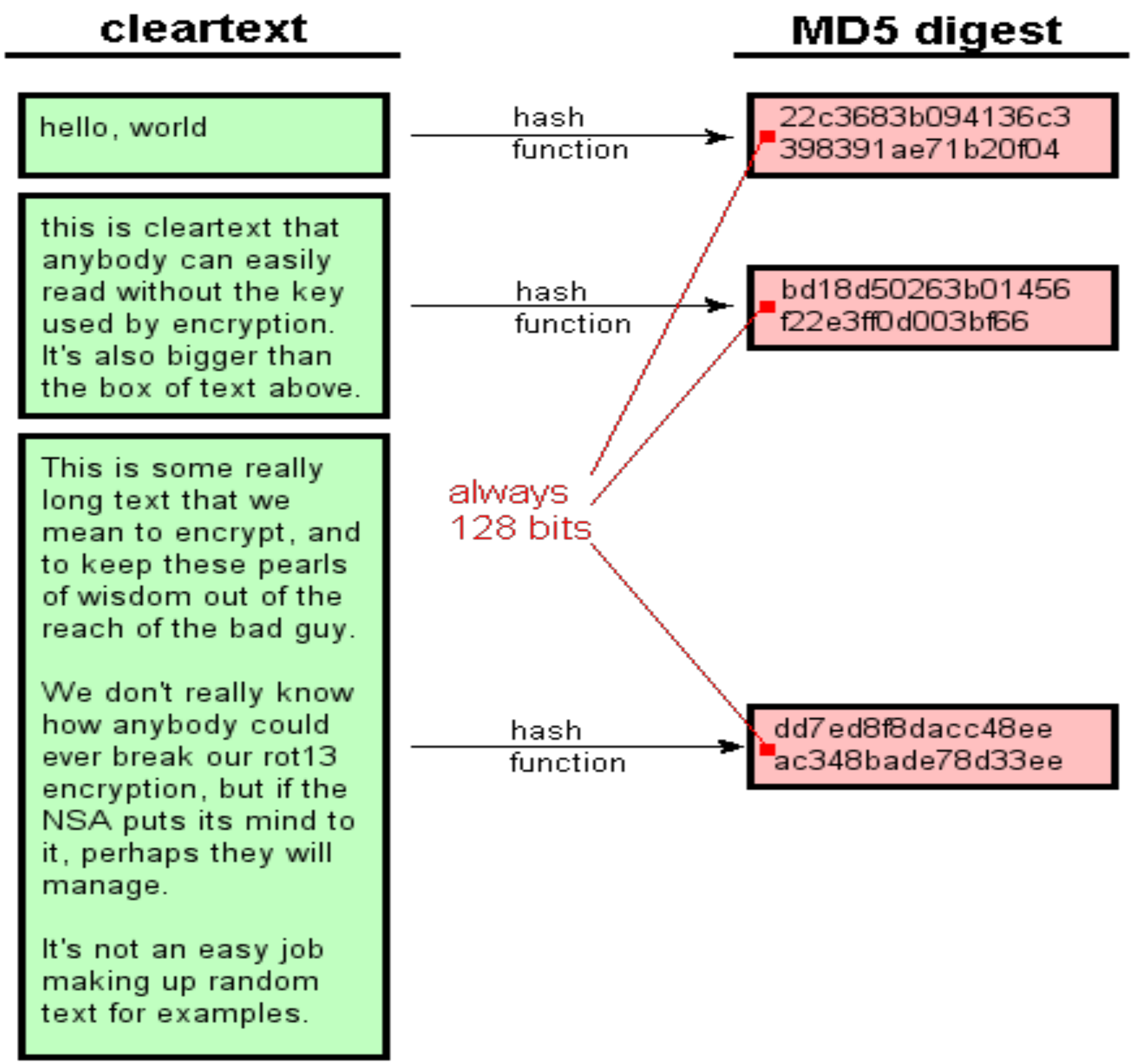
Example: Factoring

- Multiplying numbers is easy (*i.e.* fast)
  - Can multiply 2 n-bit numbers in n^2 steps
- Factoring a number is hard (*i.e.* slow)
  - To factor an n-bit number, need 2^n steps (approximately the number's value)
- Aside:
  - Primality testing is fast (recall lab activity in Software I and Fermat's Little Theorem)
  - But this fast test doesn't reveal the *factors* of a composite number

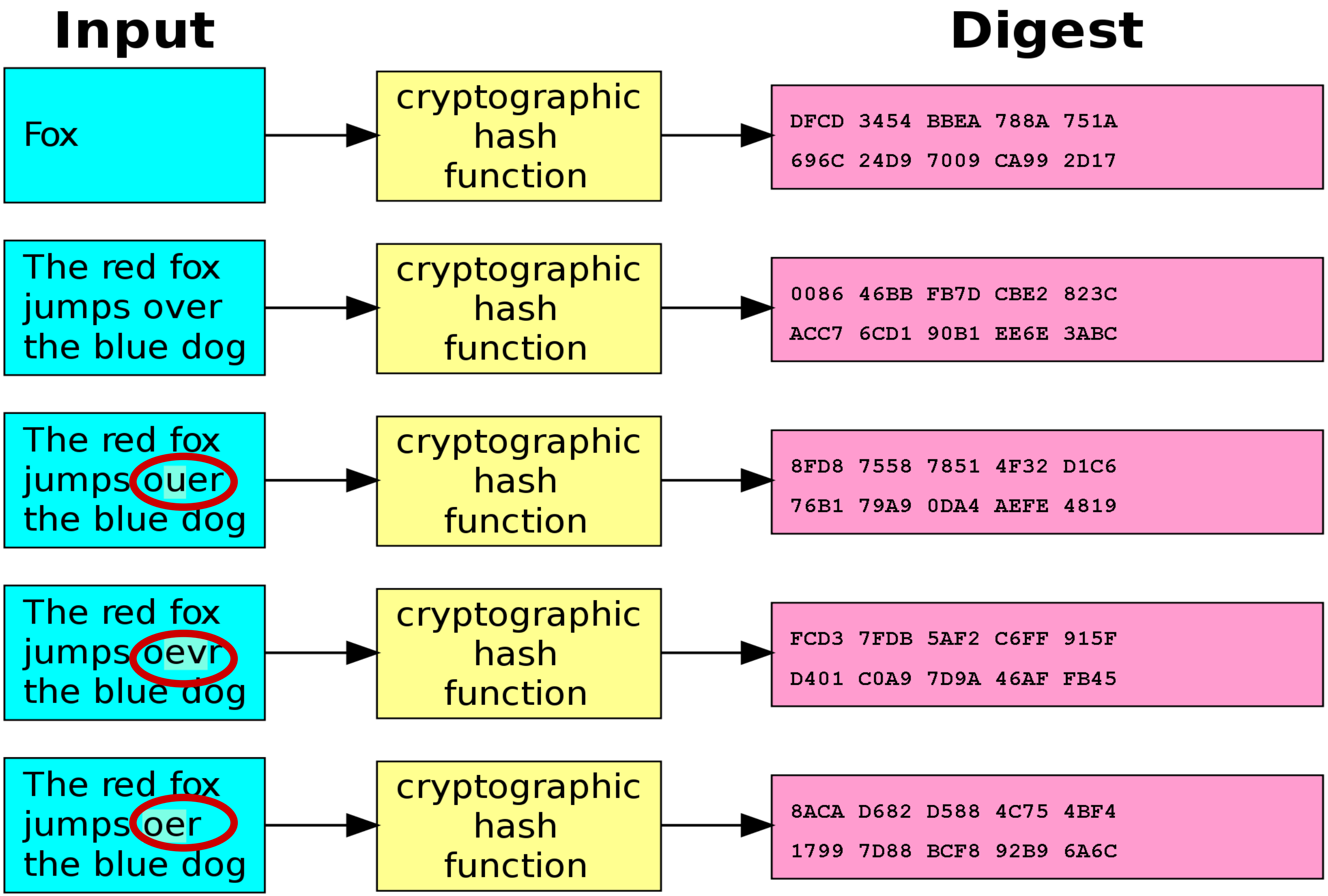
Cryptographic Hash Functions

- A hash function: $\mathbb{Z} \rightarrow \mathbb{Z}_B$
  - *Every* message, regardless of its length, maps to a number in the range $0..B-1$
  - Result called a *digest* (constant-length, $\lg B$)
  - Good hashes give uniform distribution: small diff in message → big diff in digest
- *Cryptographic* hash func's are one-way
  - Given a digest, computationally infeasible to find *any* m that hashes to it
  - Collisions must still exist ($B \ll |\text{messages}|$), but are infeasible to find for large enough B
  - Digest = a *fingerprint* of m (small, fixed-size)

Fixed-Length Digests



Crypto. Hash as Fingerprint



Common Cryptographic Hashes

- MD5
  - Flaws discovered: “cryptographically broken”
  - Do not use!
- SHA-1: deprecated
  - Windows, Chrome, Firefox reject (2017)
  - 160-bit digests (*i.e.* 40 hex digits)
- Replaced by SHA-2 (still common)
  - A family of 6 different hash functions
  - Digest sizes: 224, 256, 384, or 512 bits
  - Names: SHA-224, SHA-256, SHA-512, etc
- Currently SHA-3
  - Entirely different algorithm
  - Names: SHA3-224, SHA3-256, SHA3-512, etc

Utility of Crypto. Hashes

- Integrity verification (super-checksum)
  - File download, check digest matches
- Password protection
  - Server stores the *hash* of user's password
  - Check entered password by computing its hash and comparing hash to the stored value
  - Benefit: Passwords are not stored (directly) in the database! If server is compromised, intruder finds hashes but not passwords
- Problem:
  - See md5decrypt.net/en/Sha256/

  c023d5796452ad1d80263a05d11dc2a42b8c19c5d7c88c0e84ae3731b73a3d34

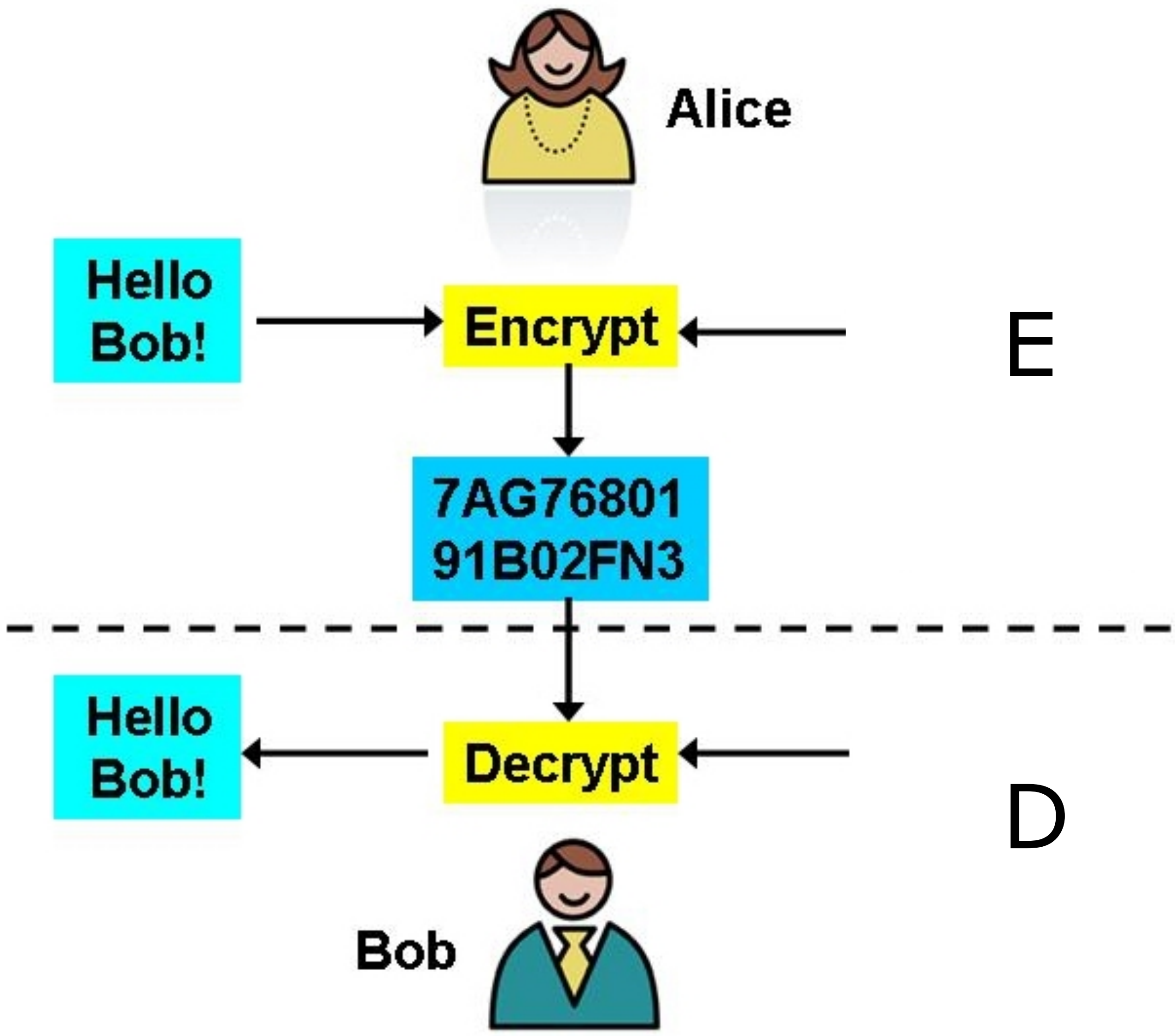
Role of Salt

- Danger:
  - Intruder pre-computes hashes for many (common) passwords: aka a *rainbow table*
  - Scan stolen hashes for matches
- Solution: *salt*
  - Server prepends text to password before hashing
  - Text must be *unique* to user
  - Text does not *need* to be secret
    - Ok: Deterministic value based on user name
    - Better: Random value, stored in the table
- Protects the fingerprint, by making it not mass pre-computable

One-Way Function with Trapdoor

- Function *appears* to be one-way
  - But, in reality, the inverse is easy if one knows a secret (the “trapdoor”)
- There are two very different functions:
  - The one-way-seeming function, E
  - The trapdoor for its inverse, D
- Knowing E is *not enough* to infer D
- Creates an asymmetry:
  - Alice knows E
  - Bob (and only Bob) knows D

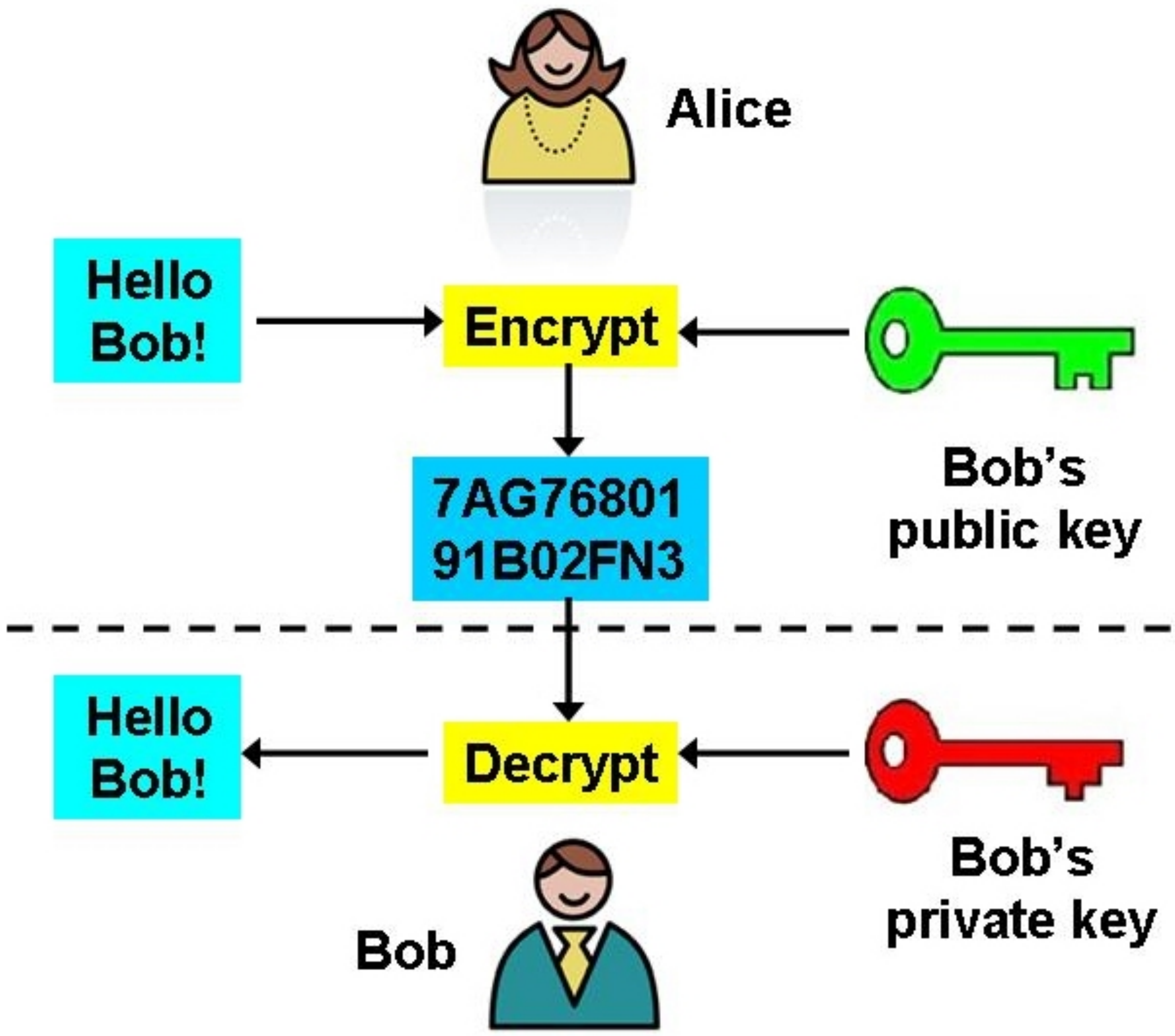
Asymmetry: Alice vs Bob



Public-Key Encryption

- Algorithms for E and D known by all
  - But parameterized by matched keys
- Asymmetry
  - Key for Bob’s E is *public*
  - Key for Bob’s D is *private*
- Anyone can encrypt messages for Bob
- Only Bob can decrypt these messages
- Important consequences
  - Each agent needs only 1 public key
  - No pre-existing shared secret needed

Public and Private Keys



RSA

- E and D are actually the same function
  $$m^k \bmod n$$
  - Parameterized by pair (k, n), *i.e.* the key
- Private key: (d, n)
  - $D(m) = m^d \bmod n$
- Public key: (e, n)
  - $E(m) = m^e \bmod n$
- Choice of *e* & *d* is based on factoring
  - Choose 2 large **prime** numbers, *p* and *q*
  - Calculate their product, *n = pq*
  - Pick any *d* relatively prime with *(p-1)(q-1)*
  - Find an *e* s.t. *ed = 1 mod (p-1)(q-1)*

Digital Signature

- Usual direction for encryption:

  $D(E(m)) = (m^e)^d = m^{ed} = m, \bmod n$
- One-to-one, so backwards works too!

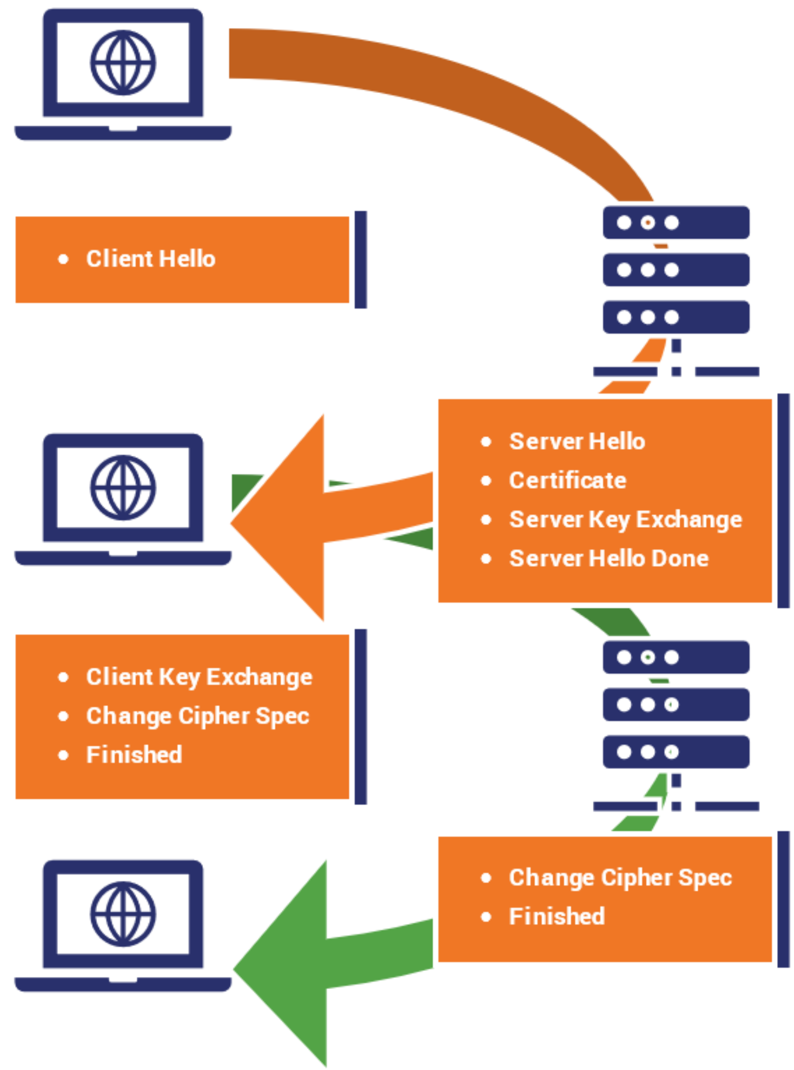
  $E(D(m)) = (m^d)^e = m^{de} = m, \bmod n$
- Consider:
  - Bob "encrypts" *m* using his **private** key, *d*
  - Bob sends **both** *m* and *D(m)*
  - Anyone can undo the "encrypted" part using Bob's **public** key, *e*
  - Result will be *m*
- D(m) serves as a digital **signature** of *m*
  - Only Bob could have created this signature
  - Use: non-repudiation

Performance Considerations

- Symmetric key algorithms are faster than public key algorithms
- Optimization for encryption (RSA)
  - Create a fresh symmetric key, *k*
  - Use symmetric algorithm to encrypt *m*
  - Use recipient's public key to encrypt *k*
- Optimization for digital signatures
  - Calculate the digest for *m* (always short)
  - Use sender's private key to encrypt digest

TLS 1.3: Handshake

- Certificate authority
  - Connects public key to identity
- Client:
  - Get server's public key
  - Make new (symmetric) session key
  - Sends this key to server (encrypted with public key)



Take Home Message

- Don’t try to roll your own crypto/security implementation
- Use (trusted) libraries
- Recognize role and importance of (eg):
  - Initialization vector
  - Cryptographic hash/digest
  - Salt
  - Private key vs public key

Summary

- Symmetric-key encryption
  - Sender and receiver share (same) secret key
  - Stream ciphers work one bit at a time (*e.g.*, one-time pad)
  - Block ciphers work on larger blocks of bits (*e.g.*, SHA-2)
- One-way functions: Hard to invert
  - Cryptographic hash produces fixed-size digest
  - Digest serves as a fingerprint
- Public key encryption
  - Matching keys: $k_{private}$, k_{public}
  - Anyone can use public key to encrypt
  - Only holder of private key can decrypt
  - Use private key to create a digital signature