IDSC 6490
Finals
Work Book

Part (1)

Finding min/max for multivariable Calculus.

Suppose $f(x,y) = 2 \cdot x^2 - xy - y^2$

We will need f_x, f_{xx}, f_y, f_{yy}, and f_{xy}.

The critical points or Stationary points are where $f_x = f_y = 0$ or is undefined.

Note this is the same as saying

$\nabla f(x,y) = 0$.

Here $f_x = -2x - y$

$f_y = -x - 2y$

$\nabla f(x,y) = f_x \vec{i} + f_y \vec{j} = 0$ when

$f_x = 0$ and $f_y = 0$ (just being pedantic *wink*)

$f_x = f_y = 0$ when

$$-2x - y = -x - 2y$$
$$+x \qquad +x$$
$$-x - y = -2y$$
$$+y \quad +y$$
$$-x = -y \quad \text{or} \quad x = y$$

If $x = y$ then

$f_x = -2x - y = 0$ when

$-2x - x = 0$

$-3x = 0$ so $x = 0$, that is

$x = y = 0$ or $(0,0)$ is the critical point. (Also, Wolfram will do this ☺).

So $(0,0)$ is critical

$f_{xx} = -2$

$f_{yy} = -2$

$f_{xy} = -1$

$$D(x,y) = f_{xx} f_{yy} - f_{xy}^2$$
$$= (-2)(-2) - (-1)^2 = 3 > 0$$

Since it is a min or max

Recall $D > 0$ just tells you that your critical number is a min or a max — not which.

look at f_{xx} to find out.

Here $f_{xx} = -2 < 0$ So $(0,0)$ is a local max (☺) Hurray!

$f(0,0) = 0$ so

That's pretty much what you need for #(1), or any problem like it (☺).

OH, $D < 0 \Rightarrow$ saddle point

$D = 0$ means ???