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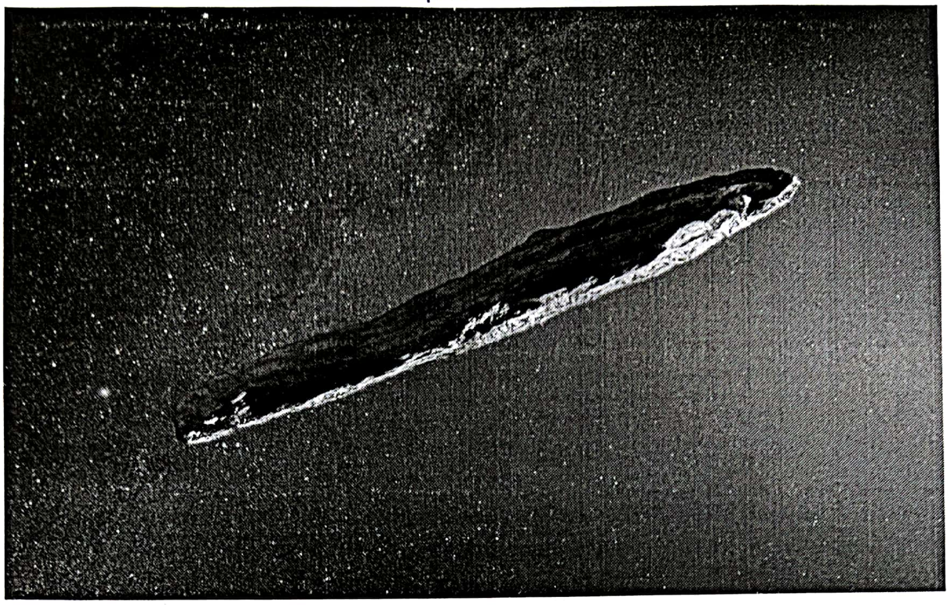
IDSC 6490 Alien Space Probe Linear Edition



This is real ☺

I mean, it's an artist's rendering, but the science is real. It just happened. Did you hear about it?

Here is a link to the paper. Maybe just read the summary at the end ☺.
https://arxiv.org/pdf/1810.11490.pdf

1. OK, some equally as interesting stuff.

Please sole the system of linear equations by Gauss-Jordan elimination. I'll be looking for every detail please ☺. I'll leave lots of room.

$$x + 2y + 3z = 1$$

$$x + 3y + 6z = 3$$

$$2x + 6y + 13z = 5$$

① $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 3 & 6 & 3 \\ 2 & 6 & 13 & 5 \end{array}\right)$

Step 1:
ⓐ Subtract $2R_2$ from R_3 $\quad 2R_2 = [2 \;\; 6 \;\; 12 \;\; 6]$
ⓑ Subtract $R_2 - R_1$

ⓐ
$$\begin{array}{rrrr} 2 & 6 & 13 & 5 \\ -2 & 6 & 12 & 6 \\ \hline z\ \boxed{0} & \boxed{0} & \boxed{1} & \boxed{-1} \end{array}$$

ⓑ
$$\begin{array}{rrrr} 1 & 3 & 6 & 3 \\ -1 & 2 & 3 & 1 \\ \hline 0 & 1 & 3 & 2 \end{array}$$

Some more space for you

② $\begin{array}{c} \\ \\ z \end{array}\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)$

Step 2:
Subtract $3R_3$ from R_2 $\quad 3R_3 = [0\ 0\ 3\ -3]$

$$\begin{array}{rcccc} & 0 & 1 & 3 & 2 \\ - & 0 & 0 & 3 & -3 \\ \hline y & \boxed{0} & \boxed{1} & \boxed{0} & \boxed{5} \end{array}$$

③ $\begin{array}{c} \\ y \\ z \end{array}\left(\begin{array}{cccc} 1 & 2 & 3 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right)$

Step 3:
Subtract $2R_2$ from R_1 $\quad 2R_2 = [0\ 2\ 0\ 10]$

$$\begin{array}{rcccc} & 1 & 2 & 3 & 1 \\ - & 0 & 2 & 0 & 10 \\ \hline & 1 & 0 & 3 & -9 \end{array}$$

④ $\begin{array}{c} \\ y \\ z \end{array}\left(\begin{array}{cccc} 1 & 0 & 3 & -9 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right)$

Step 4:
Subtract $3R_3$ from R_1 $\quad 3R_3 = [0\ 0\ 3\ -3]$

$$\begin{array}{rcccc} & 1 & 0 & 3 & -9 \\ - & 0 & 0 & 3 & -3 \\ \hline x & \boxed{1} & \boxed{0} & \boxed{0} & \boxed{-6} \end{array}$$

done!

⑤ $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right)$ $\quad x = -6$ ★ $\quad y = 5 \quad z = -1$

① $-6 + 2(5) + 3(-1) = 1$
$-6 + 10 - 3 = 1$
$1 = 1$ ✓

② $-6 + 3(5) + 6(-1) = 3$
$-6 + 15 - 6 = 3$
$3 = 3$ ✓

③ $2(-6) + 6(5) + 13(-1) = 5$
$-12 + 30 - 13 = 5$
$5 = 5$ ✓

2. Determine if the following matrix s symmetric or skew symmetric.

$$A = A^T$$

$$\begin{bmatrix} 5 & -7 & 1 \\ -7 & -8 & 2 \\ 1 & 2 & -4 \end{bmatrix}$$

Symmetric: $A = A^T$

Skew symmetric: $-A = A^T$

$$A = \begin{bmatrix} 5 & -7 & 1 \\ -7 & -8 & 2 \\ 1 & 2 & -4 \end{bmatrix}$$

Row 1 = $[5 \ -7 \ 1]$

Row 2 = $[-7 \ -8 \ 2]$

Row 3 = $[1 \ 2 \ -4]$

$$A^T = \begin{bmatrix} 5 & -7 & 1 \\ -7 & -8 & 2 \\ 1 & 2 & -4 \end{bmatrix}$$

This matrix A is symmetric.

3. Please determine whether or not the vectors form a basis for $\mathbf{R}^4$. Show you methodology please ☺.

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} \quad v_3 = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 4 \end{bmatrix} \quad v_4 = \begin{bmatrix} 2 \\ 6 \\ 8 \\ 5 \end{bmatrix}$$

If not, which of these vectors are linearly dependent and what is the relationship of the dependent vectors to them? Numerically, that is. I'll leave some space.

Gauss Jordan Elimination

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 5 & 6 \\ 1 & 3 & 6 & 8 \\ 1 & 2 & 4 & 5 \end{bmatrix}$$

① Swap R_2 and R_4

Subtract R_2 from R_4

$$\begin{array}{r} \\ \text{new } R_2 \rightarrow \\ \\ R_4 \text{ (done)} \rightarrow \end{array} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 4 & 5 \\ 1 & 3 & 6 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

② Subtract R_1 from R_2

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

③ $R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 1 & 1 \end{bmatrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{dependence}$$

④ $R_3 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

⑤ $R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

⑥ $R_2 - 2R_4$

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

⑦ Swap R_3 and R_4

$$\left.\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{row of zeroes signifies dependence.}}\right\} \text{Row reduced echelon form}$$

Using my Gauss Jordan elimination, I noticed this:

$$\boxed{R_3 - R_1 = 2[R_2 - R_1]}$$ ← well, my "new" R_2

$R_1 = [1\ 1\ 2\ 2]$

$R_2 = [1\ 2\ 4\ 5]$

$R_3 = [1\ 3\ 6\ 8]$

Proof.

$$[1\ 3\ 6\ 8] - [1\ 1\ 2\ 2] = 2\cdot([1\ 2\ 4\ 5] - [1\ 1\ 2\ 2])$$

$$[0\ 2\ 4\ 6] = 2([0\ 1\ 2\ 3]$$

Weird that the possible space probe barely cracked the news huh? Or is it ☺ Maniacal laugh!!!!!!

$$[0\ 2\ 4\ 6] = [0\ 2\ 4\ 6]$$

$$R_3 = 2[R_2 - R_1] - R_1$$

4. Consider the exciting matrix $A = \begin{bmatrix} 2 & 4 \\ 3 & 13 \end{bmatrix}$.

By hand 100% please find the eigenvalues and the eigenvectors and decompose A into PDP^{-1} where D is the diagonal matrix composed of the eigenvalues and P is constructed with the eigenvectors. I'll leave LOT's of space ☺.

(a) Find λ or eigenvalues

$$A = \begin{pmatrix} 2 & 4 \\ 3 & 13 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 2-\lambda & 4 \\ 3 & 13-\lambda \end{pmatrix}$$

$$\det \begin{vmatrix} 2-\lambda & 4 \\ 3 & 13-\lambda \end{vmatrix} = (2-\lambda)(13-\lambda) - 12 = 0$$

$$26 - 13\lambda - 2\lambda + \lambda^2 - 12 = 0$$

$$14 - 15\lambda + \lambda^2 = 0$$

$$\lambda^2 - 15\lambda + 14 = 0$$

$$(\lambda - 1)(\lambda - 14) = 0 \quad \boxed{\lambda = 1 \text{ or } \lambda = 14}$$ Eigenvalues

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(b) Find eigenvectors

$(A - \lambda I)(\vec{v}) = 0$ Start w/ $\lambda = 1$ (easier ☺)

$$\left(\begin{array}{cc:c} 2-\lambda & 4 & 0 \\ 3 & 13-\lambda & 0 \end{array}\right) = \begin{pmatrix} 2-1 & 4 \\ 3 & 13-1 \end{pmatrix} = \left(\begin{array}{cc:c} 1 & 4 & 0 \\ 3 & 12 & 0 \end{array}\right)$$

Use Gauss Jordan and row reduce

(1) $R_2 - 3R_1$

$$\begin{array}{cc} v_1 & v_2 \end{array}$$
$$\left(\begin{array}{cc:c} 1 & 4 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

Done

This says $V_1 + 4V_2 = 0$

$V_1 = -4V_2$

$V_2 = t$ (any value)

I will use 1

$\vec{V}_1 = \begin{pmatrix} V_1 \\ -V_2 \end{pmatrix}$

$\vec{V}_1 = \begin{pmatrix} -4t \\ t \end{pmatrix}$

$\boxed{\vec{V}_1 = \begin{pmatrix} -4 \\ 1 \end{pmatrix}}$ First Eigenvector

Now use $\lambda = 14$

$$\left(\begin{array}{cc:c} 2-\lambda & 4 & 0 \\ 3 & 13-\lambda & 0 \end{array}\right) = \left(\begin{array}{cc:c} -12 & 4 & 0 \\ 3 & -1 & 0 \end{array}\right)$$

Use Gauss Jordan and row reduce
Swap $R_1 + R_2$

$$\left(\begin{array}{cc:c} 3 & -1 & 0 \\ -12 & 4 & 0 \end{array}\right)$$

Then $R_2 + 4R_1$

$$= \left(\begin{array}{cc:c} 3 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

Done

Then $\frac{R_1}{3}$ to get 1 in Pivot

$$= \begin{array}{cc} v_1 & v_2 \end{array}$$
$$\left(\begin{array}{cc:c} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \end{array}\right)$$

any value

$v_1 - \frac{1}{3}v_2 = 0$

$v_1 = \frac{1}{3}v_2$

$v_2 = t$ (again, we'll use 1)

$$\vec{v_2} = \begin{pmatrix} \frac{1}{3}t \\ t \end{pmatrix} \quad \boxed{\vec{v_2} = \begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix}}$$

Second Eigenvector

Ⓒ Decompose A into PDP^{-1}

Eigenvalues	Eigenvectors
$\lambda = 1$	$\vec{v_1} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$
$\lambda = 14$	$\vec{v_2} = \begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix}$

①

$$A = \underbrace{\begin{array}{cc} v_1 & v_2 \\ \hline \end{array}}_{}\!\!\!\!\!\! \boxed{\overset{P}{\begin{pmatrix} -4 & \frac{1}{3} \\ 1 & 1 \end{pmatrix}}} \boxed{\overset{D}{\begin{pmatrix} 1 & 0 \\ 0 & 14 \end{pmatrix}}} \overset{P^{-1}*}{-\frac{3}{13}\begin{pmatrix} 1 & -\frac{1}{3} \\ -1 & -4 \end{pmatrix}}$$

* Find P^{-1}

$$P^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$P^{-1} = \frac{1}{(-4)(1) - (\frac{1}{3})(1)}\begin{pmatrix} 1 & -\frac{1}{3} \\ -1 & -4 \end{pmatrix}$$

$$P^{-1} = \frac{1}{-4-\frac{1}{3}}\begin{pmatrix} 1 & -\frac{1}{3} \\ -1 & -4 \end{pmatrix}$$

$$\boxed{P^{-1} = -\frac{3}{13}\begin{pmatrix} 1 & -\frac{1}{3} \\ -1 & -4 \end{pmatrix}}$$

②

$$A = -\frac{3}{13}\overset{P}{\begin{pmatrix} -4 & \frac{1}{3} \\ 1 & 1 \end{pmatrix}}\overset{D}{\begin{pmatrix} 1 & 0 \\ 0 & 14 \end{pmatrix}}\begin{pmatrix} 1 & -\frac{1}{3} \\ -1 & -4 \end{pmatrix}$$

③ Multiply P and D

$$P \cdot D = \begin{pmatrix} -4 & \frac{1}{3} \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 14 \end{pmatrix} = \begin{pmatrix} -4 & \frac{14}{3} \\ 1 & 14 \end{pmatrix}$$

$$A = -\frac{3}{13}\begin{pmatrix} -4 & \frac{14}{3} \\ 1 & 14 \end{pmatrix}\begin{pmatrix} 1 & -\frac{1}{3} \\ -1 & -4 \end{pmatrix}$$

$$PD \cdot P^{-1} = \begin{pmatrix} -4 & \frac{14}{3} \\ 1 & 14 \end{pmatrix}\begin{pmatrix} 1 & -\frac{1}{3} \\ -1 & -4 \end{pmatrix} = \begin{pmatrix} -\frac{26}{3} & -\frac{52}{3} \\ -13 & -\frac{169}{3} \end{pmatrix}$$

④ Multiply $PD \cdot P^{-1}$

$$A = -\frac{3}{13}\begin{pmatrix} -\frac{26}{3} & -\frac{52}{3} \\ -13 & -\frac{169}{3} \end{pmatrix} = \begin{pmatrix} (-\frac{3}{13})(-\frac{26}{3}) & (-\frac{3}{13})(-\frac{52}{3}) \\ (-\frac{3}{13})(-13) & (-\frac{3}{13})(-\frac{169}{3}) \end{pmatrix} = \begin{pmatrix} \frac{26}{13} & \frac{52}{13} \\ \frac{3 \cdot \cancel{13}}{\cancel{13}} & \frac{169}{13} \end{pmatrix} = \boxed{\begin{pmatrix} 2 & 4 \\ 3 & 13 \end{pmatrix}}$$

$P = \begin{pmatrix} -4 & \frac{1}{3} \\ 1 & 1 \end{pmatrix}$ $D = \begin{pmatrix} 1 & 0 \\ 0 & 14 \end{pmatrix}$ $P^{-1} = -\frac{3}{13}\begin{pmatrix} 1 & -\frac{1}{3} \\ -1 & -4 \end{pmatrix}$

$\lambda = 1$ $\vec{v}_1 = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

$\lambda = 14$

$\vec{v}_2 = \begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix}$

5. Following in my footsteps is uncovering the matrix U from the svd of A = $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ please be good enough to find the matrix V where A = U∑V. Hint: Use R *LOL*. Cut and paste your code as well as clearly stating the answer ☺.

The matrix V = [-0.4082 -0.8165 0.4082]

[-0.7071 0 -07.071]

[-0.5774 0.5774 0.5774]

```
In [17]: import numpy as np
         import scipy as sp
```

```
In [22]: a = [1, 1, 0]
         b = [0, 1, -1]

         A = np.array([a, b])
         M, N = A.shape

         u, s, v = np.linalg.svd(A)
         Sigma = sp.linalg.diagsvd(s, M, N)
```

Here is U

```
In [19]: print(np.round(u, 4))
```

```
[[-0.7071 -0.7071]
 [-0.7071  0.7071]]
```

This is Σ

```
In [20]: print(Sigma)
```

```
[[1.73205081 0.         0.        ]
 [0.         1.         0.        ]]
```

This is V

```
In [21]: print(np.round(v, 4))
```

```
[[-0.4082 -0.8165  0.4082]
 [-0.7071  0.     -0.7071]
 [-0.5774  0.5774  0.5774]]
```

This is the reconstructred matrix

```
In [26]: print(np.round(u @ (Sigma @ v)))
```

```
[[ 1.  1.  0.]
 [ 0.  1. -1.]]
```