IDSC 6490 Semi Evil Homework V

Graph Theory

Solve the horrible problems listed below and remember to show all your work.

Please read chapter 4, 8, and 9 in you Schaum's Discrete textbook. It will read fast ☺

Problem 1.

Investigate the number of Hamilton Circuits for any particular graph. That is, do some research on what is known at this time. Wikipedia is fine. Being able to "figure it out" in the middle of the night, so to speak – is a useful and necessary skill set. Some of the information you will find will be non-trivially difficult/impossible to understand at a single reading.

When I was solving a problem in Quantum Mechanics I had to read a particular paper about 30 times before I really understood it ☺. I'm not asking you for that (now anyway *LOL*)

Talk to me about Dirac's Theorem and create two graphs, each with 5 vertices (draw a picture please). One graph which has at least one Hamilton Circuit and then use Dirac's work to tell me how many Hamilton Circuits it has. Reference this Theorem directly. I'm looking to see if you can apply novel information to immediate problems which need to be solved. The other graph should have NO Hamilton circuits. Explain exactly why it does not please ☺.

I should guess that this will be a page or two of paper done correctly. Actually, make that three of four pages. While you are looking into Hamilton circuits you will run in to the terms NP, NP hard, and NP-complete. I want you to write an intelligible description of what these terms mean. Write it in a way that shows not only that you read these descriptions, but that you have absorbed a bit of it. Please don't just spit back the definition ☺.

I get that I just asked a LOT from you ☺.

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One of the greatest challenges in mathematics is the problem known as the Hamiltonian circuit problem. This is the challenge of determining whether a Hamiltonian cycle exists in any given graph G. The reason this is such a challenge in computer science and mathematics is that there is no easy way to determine, in general, whether a graph has a Hamilton circuit, and no general method for building one. This is known as an NP-complete problem (NP standing for "non-deterministic polynomial time."). Any given solution to an NP-complete problem can be determined to be right once it has been found, but there's no efficient way to actually find the solution in the first place. The time required to solve the problem using known algorithms (in the case of the Hamiltonian circuit problem, that would be things like brute force search, which is essentially traversing every edge in the graph to determine the circuit, and or if you are looking for a Hamilton circuit of minimum weight, then you'd use the nearest neighbor algorithm) will increase exponentially as you increase the complexity of the problem.

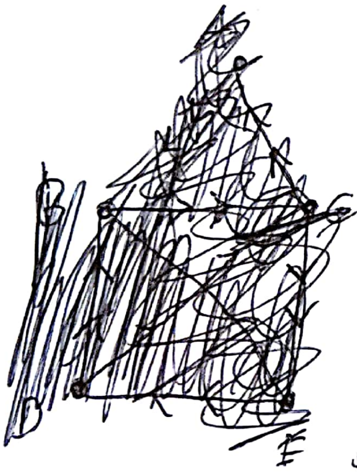
An NP-Complete problem belongs in the class of decision problems known as NP. (As a side note, I think it's absolutely fascinating that there are problems ~~as~~ out there in the world that we know there is a solution to, but we don't know how or where the solution lies, and that we've been studying them for so many year!)

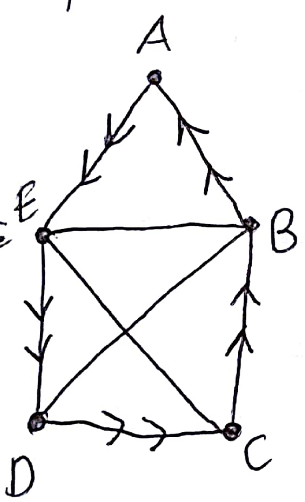
NP, or nondeterministic polynomial time, describes a set of decision problems. When I relate this to graph theory, the simplest example I can think of, and why I think you emphasized this in our homework, is the "traveling salesman" example; determining a route visiting all cities in a certain distance matrix that is the cheapest or most efficient. We actually had to build an algorithm in our Python class to do this, and I wish I had known about this sooner before building my program! A way to do this is simply add all the corresponding paths in the matrix and then calculate to shortest distance (which is what my program does), or you can build something that goes from one city, it guesses the next city to visit, until it's exhausted all cities, and then returns the shortest route. But again, this gets more and more complex as you add more and more cities and/or vertices.

Finally, NP Hardness relates to the class of decision problems that are at least as hard as the hardest problems in NP. What really cooks my noodle is that NP Hard problems don't necessarily belong in NP; they may not be able to be solved in polynomial time (the time required for a computer to solve a problem). The video you linked gave a great explanation, with this class of problems being something like checking the best move in a game of chess based on the positions of the board, and one by ~~one~~ checking every single move to determine the right answer. Again, I'm going to relate this to graph theory by stating that there's no way, or intelligible way, to determine if a graph has a Hamilton circuit until you physically go through the iterations and come up with an answer.

ⓐ Design a graph that has at least one Hamilton Circuit and then use Dirac's theorem to show how many it has.

A graph G has a Hamilton circuit if you can visit each vertex exactly once, and return to the first vertex.





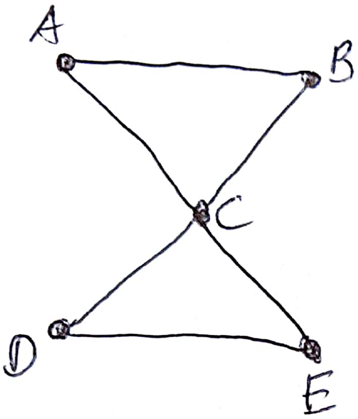
HP= {C, B, A, E, D, C}

Dirac's Theorem states that any simple graph with n vertices greater than/= 3 ($n \geq 3$) is Hamiltonian if every vertex degree (k) = n/2 or greater.

deg(A)= 2
deg(B)= 4
deg(C)= 3
deg(D)= 3
deg(E)= 4

For any complete, undirected graphs there are (n-1)! distinct Hamilton circuits. For a graph with 5 vertices, there are (5-1)! = 4! = 24 distinct Hamilton circuits.

(5) Design a graph with 5 vertices that has no Hamilton circuits. and explain why.



There is no Hamilton circuit for this graph because no matter what starting point is used, you must pass through C at least twice to close the circuit.

A Hamilton circuit must travel through each vertex once and only once, and start and end at the same vertex.