OK, three pages is probably enough *LOL*

Problem 2. Determine or define a set/class of graphs which have both Euler Circuits and Hamilton Circuits. Convince me that this "kind" of graph has both of these properties. There is an obvious and pretty simple answer – if you already know it *LOL*. Don't get mad, you're getting wicked smart!

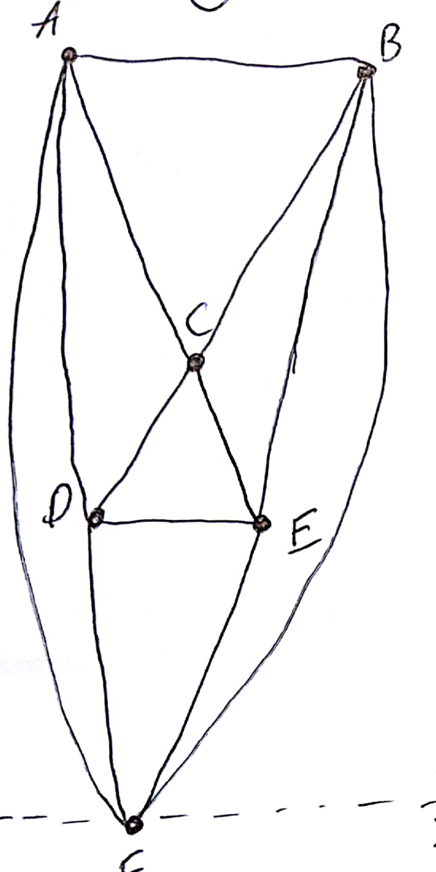
A graph that contains both Euler and Hamilton circuits, must statisfy this criteria: ① A circuit that visits every edge of the graph once, that begins and ends on the same vertex

Euler

② A circuit that visits every vertex in the graph exactly once.

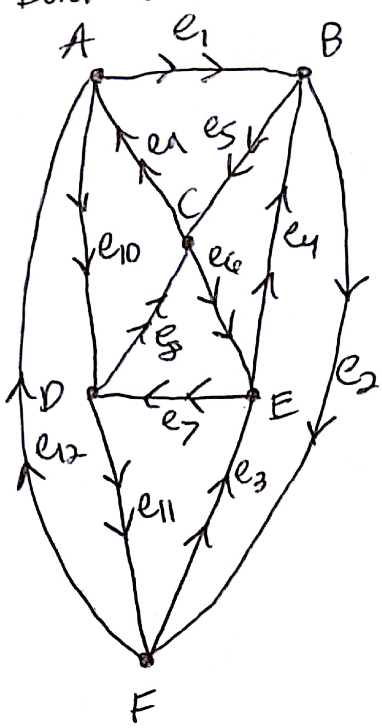
Hamiltonian

G



Graph with both Euler/Hamilton Circuits.

Euler Circuit.



A graph G has a Euler circuit if every vertice is even.

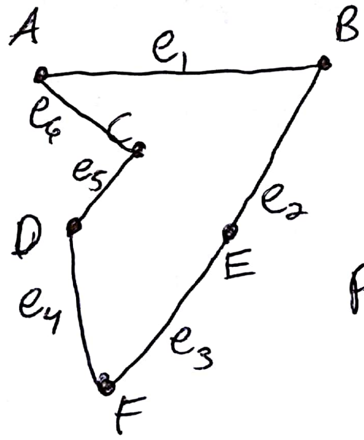
$\deg(A) = \overline{AB}, \overline{AC}, \overline{AD}, \overline{AF} = 4$
$\deg(B) = \overline{BA}, \overline{BC}, \overline{BE}, \overline{BF} = 4$
$\deg(C) = \overline{CA}, \overline{CB}, \overline{CD}, \overline{CE} = 4$
$\deg(D) = \overline{DA}, \overline{DC}, \overline{DE}, \overline{DF} = 4$
$\deg(E) = \overline{EB}, \overline{EC}, \overline{ED}, \overline{EF} = 4$
$\deg(F) = \overline{FA}, \overline{FB}, \overline{FD}, \overline{FE}, 4$

$P = \{\overline{AB}, \overline{BF}, \overline{FE}, \overline{EB}, \overline{BC}, \overline{CE}, \overline{ED}, \overline{DC}, \overline{CA}, \overline{AD}, \overline{DF}, \overline{FA}\}$

$P = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}\}$

Hamilton Circuit

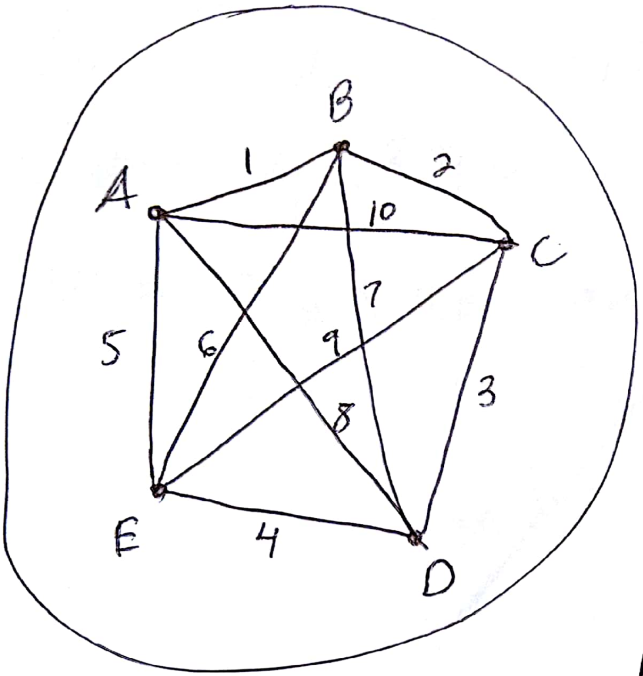
A graph has a Hamilton circuit if you travel to each vertex once.



$P = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

Problem 3. Can a simple graph have 5 vertices and 13 edges? If so, draw it; if not, explain why it is not possible to have such a graph.

A simple graph is an undirected graph that has no loops (edges connected at both ends to the same vertex), and no more than one edge between any two different vertices. A simple graph can have at most $\binom{n}{2} = \frac{n(n-1)}{2}$ edges, see the next page for demonstration.



$n = 5$

$$\text{Max}(\text{edge}) = \frac{n(n-1)}{2}$$

$$\text{max}(\text{edge}) = \frac{5(4)}{2}$$

$$\text{max}(\text{edge}) = \frac{20}{2} = \boxed{10}$$

$$P = \{ \overline{AB}, \overline{AC}, \overline{AD}, \overline{AE}, \overline{BC}, \overline{BD}, \overline{BE}, \overline{CD}, \overline{CE}, \overline{DE} \}$$

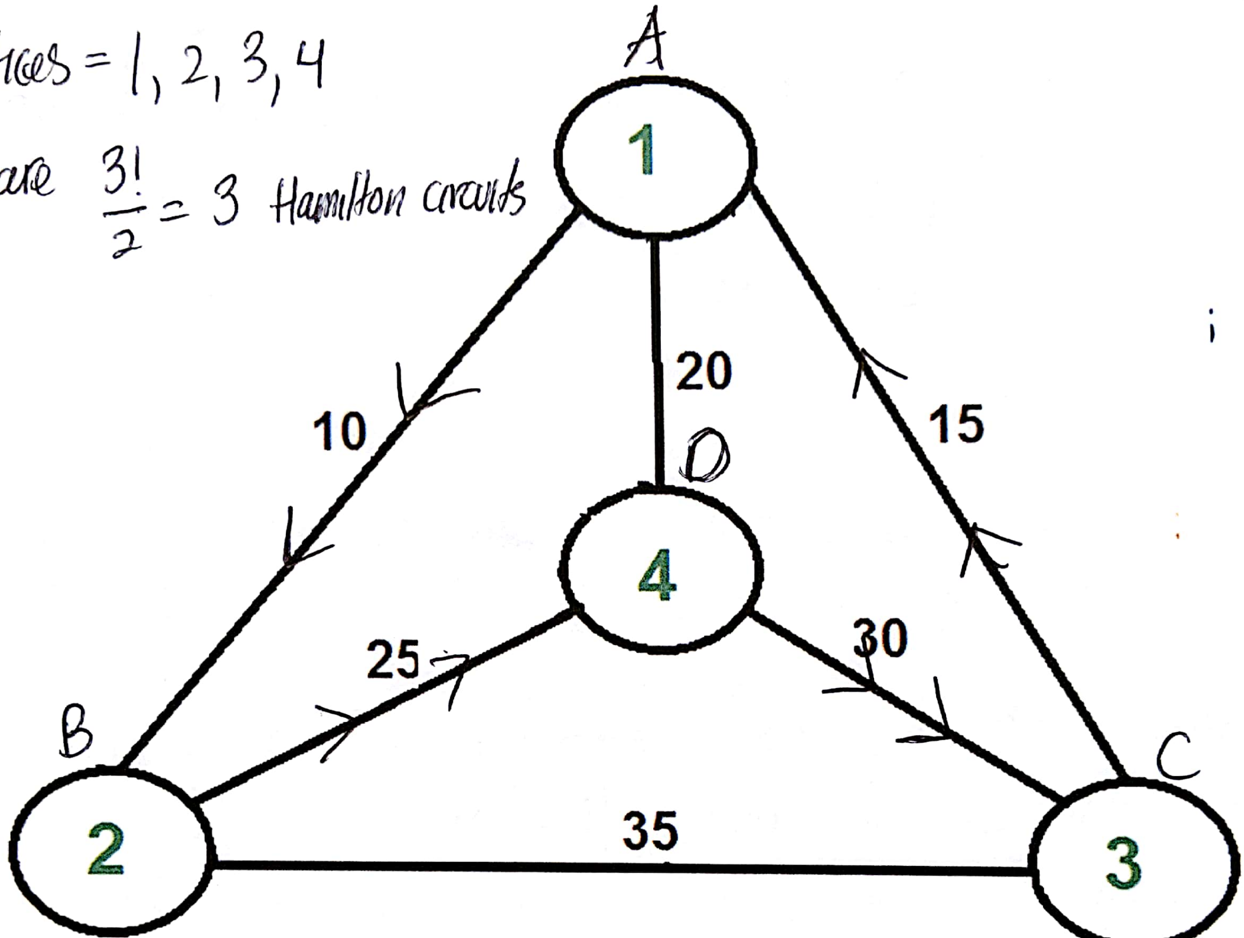
$$P = \{ e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10} \}$$

Problem 4. The Travelling Salesperson Problem.

Given a set of cities and distance between every pair of cities, the problem is to find the shortest possible route that visits every city exactly once and returns to the starting point.
Note the difference between Hamiltonian Cycle and TSP. The Hamiltonian cycle problem is to find if there exist a tour that visits every city exactly once. Here we know that Hamiltonian Tour exists (because the graph is complete) and in fact many such tours exist, the problem is to find a minimum weight Hamiltonian Cycle. Please start at city 1 and use the Nearest Neighbor Algorithm to find a "reasonably priced" tour. Is it optimal? I'll leave an extra page here ☺.

Vertices = 1, 2, 3, 4

There are $\frac{3!}{2} = 3$ Hamilton circuits



The nearest neighbor algorithm states, given a starting vertex, choose the edge with the least weight to the next vertex, the closest vertex, and that vertex becomes the starting vertex until the circuit is complete.

In this case, it would be $|1, 2, 4, 3, 1| = 10 + 25 + 30 + 15$

$= \boxed{80}$

You're ALMOST done yea!!!

A reasonably priced tour is 80.

Is it optimal? Next page.....

Here are the possible Hamilton circuits:

$H_1 = |1, 2, 3, 4, 1| = 10 + 35 + 30 + 20 = 95$

$H_2 = |1, 3, 4, 2, 1| = 15 + 30 + 25 + 10 = \boxed{80}$

$H_3 = |1, 3, 2, 4, 1| = 15 + 35 + 25 + 20 = 95$

Keep going….

H_3 is the minimum weight Hamilton circuit, since it equals 80.

The nearest neighbor algorithm gives you the inverse of H_2, or $|1, 2, 4, 3, 1|$, which gives you the same weight, which is the least expensive tour. So yes, this is optimal.

FIN