IDSC 6490 Super Easy Homework II

Counting Problems and Recursion

This is due on Saturday September 22 ☺

1. **Read Chapter 5 in your Discrete Mathematics Textbook.**

2. **Watch Week II lectures at least once. ☺**

3. **Solve these fun and easy problems listed below and remember to show all your work.**

Name: Danny Moncada 5445381

Problem 1.

Your company is creating product code for its inventory control. They need you to figure out some stuff ☺ for LOTS of cash ☺. If the code consists of 3 small letters from our English alphabet followed by a dash, and then 4 numbers followed by a dash and then followed by 2 letters (again small English alphabet), please answer the following important cooperate questions ☺.

A-Z A-Z A-Z - 0-9 0-9 0-9 0-9 - A-Z A-Z

(a) How many different product codes are there if there are no restrictions?

Codes = $26 \cdot 26 \cdot 26 - 10 \cdot 10 \cdot 10 \cdot 10 - 26 \cdot 26$

Codes = $(26)^3 \cdot (10)^4 \cdot (26)^2 = \boxed{118{,}813{,}760{,}000 \text{ product codes}}$

(b) How many product codes are there if the initial 3 letter section must all distinct vowels and the first number in the number part must be a 3 or a 7 and the final 2 letters must be different.

$P(5,3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \cdot 4 \cdot 3 = \boxed{60}$

$5\ 4\ 3 - 2^{*}\ 10\ 10\ 10 - 26\ 25$

5 vowels = 5

3 or 7 = 2^{*}

$P(26,2) = \frac{26!}{(26-2)!} = \frac{26!}{24!} = 26 \cdot 25 =$

$60 \cdot 2000 \cdot 650$

product codes = $60 \cdot 2000 \cdot 650$

$\boxed{\text{product codes} = 78{,}000{,}000}$

Problem 2. You have 3 different math books, 4 different biology book, and 2 different sociology books. In how many ways can you arrange them on a shelf if:

(a) There are no restrictions.

Arrangements = (3 math + 4 biology + 2 sociology)!

$= (3+4+2)! = 9! = \boxed{362{,}880}$

(b) They must be grouped by subject matter. That is, all the math books must be together, all the biology books must be together and all the sociology books must be together.

Arrangements = 3! (types of book) $\cdot$ 3! (math) 4! (biology) 2! (sociology) $= 3!3!4!2! = \boxed{1728}$

3. In how many ways can 5 people (different and not clones from Gattaca *LOL*) be arranged in a straight line? How about around a circular table with 5 seats? Explain what these numbers are different in a conceptual way ☺.

Order matters with a permutation, such as standing in a line. In this example you have $\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$. $P(5,5) = 5!$.
Therefore you have $\boxed{120}$ ways for 5 people to be arranged in a straight line.

For circular permutations, in the case of seating around a circular table, objects (or people) can be arranged in $(n-1)$ ways. One person can sit at any place, so the remaining people can sit $(5-1)$ or $4! = \boxed{24}$ ways.

P.S. I LOVE GATTACA!!

4. Consider the recurrence relation

$$f_n = f_{n-1} + f_{n-2} \quad \text{for } n \geq 3.$$

$f_1 = 1$ and $f_2 = 3$

f^1	f^2	f^3	f^4	f^5	f^6	f^7
1	3	4	7	11	18	29

(a) Please compute the first seven numbers in this sequence.

$f_3 = f_{3-1} + f_{3-2}$ $\quad f_4 = f_{4-1} + f_{4-2}$ $\quad f_5 = f_{5-1} + f_{5-2}$ $\quad f_6 = f_{6-1} + f_{6-2}$ $\quad f_7 = f_{7-1} + f_{7-2}$

$f_3 = f_2 + f_1$ $\quad f_4 = f_3 + f_2$ $\quad f_5 = f_4 + f_3$ $\quad f_6 = f_5 + f_4$ $\quad f_7 = f_6 + f_5$

$f_3 = 3 + 1$ $\quad f_4 = 4 + 3$ $\quad f_5 = 7 + 4$ $\quad f_6 = 11 + 7$ $\quad f_7 = 18 + 11$

$\boxed{1, 3, 4, 7, 11, 18, 29}$

(b) Find the closed form for this recurrence relation. I'll be looking for every detail please. Solving the characteristic equation, and solving for constants ☺. I'm leaving plenty of space.

$f_n = f_{n-1} + f_{n-2}$

$r^n = r^{n-1} + r^{n-2}$

$\frac{r^n}{r^{n-2}} = \frac{r^{n-1}}{r^{n-2}} + \frac{r^{n-2}}{r^{n-2}}$

$r^{n-(n-2)} = r^{n-(n-1)} + 1$

$r^2 = r + 1$

$r^2 - r - 1 = 0$

$a = 1$
$b = -1$
$c = 1$

$r = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{1+4}}{2}$

$r_1 = \frac{1+\sqrt{5}}{2}, \quad r_2 = \frac{1-\sqrt{5}}{2}$

$f(n) = C_1 r_1^n + C_2 r_2^n$

$f(1) = 1 = C_1\left(\frac{1+\sqrt{5}}{2}\right)^1 + C_2\left(\frac{1-\sqrt{5}}{2}\right)^1$

$f(2) = 3 = C_1\left(\frac{1+\sqrt{5}}{2}\right)^2 + C_2\left(\frac{1-\sqrt{5}}{2}\right)^2$

Solving for C_2 using $f(2) = 3$ $\qquad (a+b)^2 = a^2 + 2ab + c$

① $C_1\left(\frac{1+\sqrt{5}}{2}\right)^2 + C_2\left(\frac{1-\sqrt{5}}{2}\right)^2 = 3$

② $\frac{(1+\sqrt{5})^2}{4} C_1 + \frac{(\sqrt{5}-1)^2}{4} C_2 = 3$

③ $\cancel{\frac{(1+\sqrt{5})^2}{4} C_1} + \frac{(\sqrt{5}-1)^2}{4} C_2 - \cancel{\frac{(1+\sqrt{5})^2}{4} C_1} = 3 - \frac{(1+\sqrt{5})^2}{4} C_1$

④ $\frac{(\sqrt{5}-1)^2}{4} C_2 = 3 - \frac{(1+\sqrt{5})^2}{4} C_1$; simplify $(1+\sqrt{5})^2 = (1+\sqrt{5})(1+\sqrt{5})$

$= 1^2 + 2 \cdot 1 \cdot \sqrt{5} + \sqrt{5}^2$

$= 1 + 2\sqrt{5} + 5$

$= 6 + 2\sqrt{5}$

$= 2(3+\sqrt{5})$

⑤ $\frac{(\sqrt{5}-1)^2}{4} C_2 = 3 - \frac{\cancel{2}(3+\sqrt{5})}{\cancel{4}\,2} C_1$

⑥ $\frac{(\sqrt{5}-1)^2}{4}C_2 = 3 - \frac{3+\sqrt{5}}{2}C_1$; simplify $(\sqrt{5}-1)^2 = (\sqrt{5}-1)(\sqrt{5}-1)$

$= \sqrt{5}^2 - 2\cdot\sqrt{5}\cdot -1 + (-1)^2$

⑦ $\frac{2(3-\sqrt{5})}{\cancel{4}\,2}C_2 = 3 - \frac{3+\sqrt{5}}{2}C_1$

$= 5 - 2\sqrt{5} + 1$

$= 6 - 2\sqrt{5}$

$= 2(3-\sqrt{5})$

⑧ $\cancel{2}\left(\frac{3-\sqrt{5}}{\cancel{2}}\right)C_2 = \left(3 - \frac{3+\sqrt{5}}{\cancel{2}}\right)\cdot \cancel{2}\cdot C_1$

⑨ $(3-\sqrt{5})C_2 = 6-(3+\sqrt{5})C_1$; factor out $(3-\sqrt{5})$ to isolate C_2

⑩ $\frac{3-\sqrt{5}}{3-\sqrt{5}}C_2 = \frac{6}{3-\sqrt{5}} - \frac{(3+\sqrt{5})C_1}{3-\sqrt{5}}$

⑪ $C_2 = \frac{6}{3-\sqrt{5}} - \frac{3+\sqrt{5}}{3-\sqrt{5}}C_1$; rationalize denominator by multiplying by $(3+\sqrt{5})$

⑫ $C_2 = \frac{(6-(3+\sqrt{5})C_1)(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$; simplify $(3-\sqrt{5})(3+\sqrt{5})$ $(a-b)(a+b) = a^2 - b^2$

$= 3^2 - \sqrt{5}^2 = 9 - 5 = 4$ $a=3$, $b=\sqrt{5}$

⑬ $\boxed{C_2 = \frac{(3+\sqrt{5})(6-(3+\sqrt{5})C_1}{4}}$

Solve for C_1 by plugging C_2 back into original equation (I will use x in place of C_1) $C_1 = X$

① $X\left(\frac{1+\sqrt{5}}{2}\right)^2 + \frac{(3+\sqrt{5})(6-(3+\sqrt{5})X}{4}\left(\frac{1-\sqrt{5}}{2}\right)^2 = 3$ (original equation)

② $\frac{(1+\sqrt{5})^2}{4}X + \frac{(3+\sqrt{5})(6-(3+\sqrt{5})X}{4}\left(\frac{\sqrt{5}-1}{2}\right)^2 = 3$;

③ $\frac{(1+\sqrt{5})^2}{4}x + \frac{(3+\sqrt{5})(6-(3+\sqrt{5})x}{4} \cdot \frac{(\sqrt{5}-1)^2}{4} = 3$; multiply fractions $\frac{a}{b}\cdot\frac{c}{d} = \frac{a\cdot c}{b\cdot d}$

④ $\frac{(1+\sqrt{5})^2 x}{4} + \frac{(3+\sqrt{5})(6-(3+\sqrt{5})x)(\sqrt{5}-1)^2}{16} = 3$; multiply by 16 to remove fraction

⑤ $4\frac{(1+\sqrt{5})^2 x}{\cancel{4}}\cdot\cancel{16} + \frac{(3+\sqrt{5})(6-(3+\sqrt{5})x(\sqrt{5}-1)^2}{\cancel{16}}\cdot\cancel{16} = 3\cdot 16 = 48$

$\rightarrow$

⑥ $4(1+\sqrt{5})^2x + (3+\sqrt{5})(\sqrt{5}-1)^2(6-(3+\sqrt{5})x = 48$; simplify $(1+\sqrt{5})^2 = (1)^2 + 2\cdot\sqrt{5}\cdot 1\cdot\sqrt{5}^2$

⑦ $4(6+2\sqrt{5})x + (3+\sqrt{5})(\sqrt{5}-1)^2(6-(3+\sqrt{5})x = 48$; simplify $(\sqrt{5}-1)^2$ $\quad 1+2\sqrt{5}+5 = 6+2\sqrt{5}$

$= \sqrt{5}^2 + 2\cdot\sqrt{5}\cdot(-1) + (-1)^2 = 6-2\sqrt{5}$

⑧ $4(6+2\sqrt{5})x + (3+\sqrt{5})(6-2\sqrt{5})(6-(3+\sqrt{5})x = 48$; distribute x

Thanks ☺

⑨ $4(6+2\sqrt{5})x + (3+\sqrt{5})(6-2\sqrt{5})(6-3x-\sqrt{5x}) = 48$; distribute $4x$ on first part of equation

⑩ $24x + 8\sqrt{5}x + \underbrace{(3+\sqrt{5})(6-2\sqrt{5})}_{8}(6-3x-\sqrt{5x}) = 48$; solve $(3+\sqrt{5})(6-2\sqrt{5})$*

$ac+ad+bc+bd$

⑪ $24x + 8\sqrt{5}x + 8(6-3x-\sqrt{5x}) = 48$; expand 8 $\quad = (3\cdot 6)+(3\cdot -2\sqrt{5})+(\sqrt{5}\cdot 6) + (-2\sqrt{5}\cdot\sqrt{5})$

⑫ $24x + 8\sqrt{5}x + 48 - 24x - 8\sqrt{5}x = 48$; group like terms

$= 18\cancel{0} - \cancel{6\sqrt{5}} + \cancel{6\sqrt{5}} + 2\sqrt{5}\sqrt{5}$

⑬ $24x + 8\sqrt{5}x + 48 - 24x - 8\sqrt{5}x = 48$ $\quad = 18 - 0 + 2\cdot 5 = \boxed{8}$*

⑭ $\cancel{24x} - \cancel{24x} + \cancel{8\sqrt{5}x} - \cancel{8\sqrt{5}x} = 48$

⑮ $\boxed{x = 48}$ or $\boxed{C_1 = 48}$

Test $C_1 = 48$ and $C_2 = \frac{(3+\sqrt{5})(6-(3+\sqrt{5})(48)}{4}$ in original function

① $48\left(\frac{1+\sqrt{5}}{2}\right)^2 + \frac{(3+\sqrt{5})(6-(3+\sqrt{5})(48)}{4}\left(\frac{1-\sqrt{5}}{2}\right)^2 = 3$

② $48\left(\frac{(1+\sqrt{5})(1+\sqrt{5})}{2^2}\right) + \frac{(3+\sqrt{5})(6-(3+\sqrt{5})(48)}{4}\left(\frac{1-\sqrt{5}}{2}\right)^2 = 3$; simplify $(1+\sqrt{5})$ $= 1^2 + 2\cdot 1\cdot\sqrt{5}\cdot\sqrt{5}^2 = 6+2\sqrt{5}$

③ $48\left(\frac{6+2\sqrt{5}}{2^2}\right) + \frac{(3+\sqrt{5})(6-(3+\sqrt{5})(48)}{4}\left(\frac{1-\sqrt{5}}{2}\right)^2 = 3$; factor out 2 from $6+2\sqrt{5}$ $\quad \frac{\cancel{2}(3+\sqrt{5})}{\cancel{2^2}}$

④ $\cancel{48}\cdot\frac{3+\sqrt{5}}{\cancel{2}} + \frac{(3+\sqrt{5})(6-(3+\sqrt{5})(48)}{4}\left(\frac{1-\sqrt{5}}{2}\right)^2 = 3$; factor out 2

⑤ $24(3+\sqrt{5}) + \frac{(3+\sqrt{5})(6-(3+\sqrt{5})(48)}{4}\left(\frac{1-\sqrt{5}}{2}\right)^2 = 3$; expand $= (6-(3+\sqrt{5})48$ $= 6 - 48(3+\sqrt{5})$ $= 6 - 144 - 48\sqrt{5}$ $= -138 - 48\sqrt{5}$

⑥ $24(3+\sqrt{5}) + \frac{(3+\sqrt{5})(-138-48\sqrt{5})}{4}\left(\frac{1-\sqrt{5}}{2}\right)^2 = 3$; factor $-138-48\sqrt{5}$

⑦ $24(3+\sqrt{5}) + \dfrac{(3+\sqrt{5})(-\cancel{6}^{3}(23+8\sqrt{5}))}{\cancel{4}\,2}\left(\dfrac{1-\sqrt{5}}{2}\right)^2 = 3$; factor out 2 and move -6 out

⑧ $24(3+\sqrt{5}) - \dfrac{3(3+\sqrt{5})(23+8\sqrt{5})}{2}\left(\dfrac{1-\sqrt{5}}{2}\right)^2 = 3$; move 3 over and solve

$=(3+\sqrt{5})(23+8\sqrt{5})$

$=(3\cdot 23)+(\sqrt{5}\cdot 23)+(3\cdot 8\sqrt{5})+\sqrt{5}\cdot 8\sqrt{5}$

$= 69+23\sqrt{5}+24\sqrt{5}+8\sqrt{5}^2$

$=69+23\sqrt{5}+24\sqrt{5}+40$

$=109+47\sqrt{5}$

⑨ $24(3+\sqrt{5}) - \dfrac{3(109+47\sqrt{5})}{2}\left(\dfrac{1-\sqrt{5}}{2}\right)^2 = 3$

⑩ $24(3+\sqrt{5}) - \dfrac{327+141\sqrt{5}}{2}\left(\dfrac{1-\sqrt{5}}{2}\right)^2 = 3$; simplify $(1-\sqrt{5})(1-\sqrt{5}) = 1^2 + 2\cdot\sqrt{5}\cdot(-1) + (\sqrt{5})^2$

$=1-2\sqrt{5}+5$

$=6-2\sqrt{5} = 2(3-\sqrt{5})$

⑪ $24(3+\sqrt{5}) - \dfrac{327+141\sqrt{5}}{2}\left(\dfrac{\cancel{2}(3-\sqrt{5})}{\cancel{2}^2}\right) = 3$; cancel 2

⑫ $24(3+\sqrt{5}) - \dfrac{327+141\sqrt{5}}{2}\cdot\dfrac{3-\sqrt{5}}{2} = 3$; multiply fractions - $\dfrac{(327+141\sqrt{5})(3-\sqrt{5})}{2\cdot 2}$

⑬ $24(3+\sqrt{5}) - \dfrac{(327+141\sqrt{5})(3-\sqrt{5})}{4} = 3$; expand $(327+141\sqrt{5})\cdot(3-\sqrt{5})$

$=(327\cdot 3)+(327\cdot(-\sqrt{5}))+(141\sqrt{5}\cdot 3)+141\sqrt{5}(-\sqrt{5})$

⑭ $24(3+\sqrt{5}) - \dfrac{276+96\sqrt{5}}{4} = 3$; factor $276+96\sqrt{5}$ $= 981 - 327\sqrt{5} + 423\sqrt{5} - 705$ $(141\cdot -5)$

$= 981-705+423\sqrt{5}-327\sqrt{5}$

$= 276+96\sqrt{5}$

⑮ $24(3+\sqrt{5}) - \dfrac{\cancel{12}^{3}(23+8\sqrt{5})}{\cancel{4}} = 3$

⑯ $24(3+\sqrt{5}) - 3(23+8\sqrt{5}) = 3$; expand all numbers

⑰ $72+24\sqrt{5}-69-24\sqrt{5} = 3$; simplify! OMG It WORKS!

⑱ $72-69+\cancel{24\sqrt{5}}-\cancel{24\sqrt{5}}=3$

⑲ $72-69 = 3$

⑳ $\boxed{3=3}$ ✓