IDSC 6490 Reasonably Famous Monsters Homework

Week 9 The Multivariate Calculus

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1. Suppose $f(x, y, z) = 2xy^2 + \sin(xz^2)$ please find the following:

① $f(x, y, z) = 2xy^2 + \sin(xz^2)$

$\sin'\square = \cos\square + \square'$
$\cos'\square = -\sin\square + \square'$
$(f\cdot g)' = fg' + gf'$

ⓐ $f_x = 2y^2 + \sin'(xz^2)$

$f_x = 2y^2 + \cos(xz^2)\cdot z^2$

$$\boxed{f_x = 2y^2 + z^2\cos(xz^2)}$$

ⓑ $f_{yx} = \frac{d}{dx\,dy}$ $\quad f_y = 2xy^2 + \cancel{C}$

$f_y = 4xy$

$f_x = 4xy$

$$\boxed{f_{yx} = 4y}$$

ⓒ $f_{zz} = \frac{d}{dz\,dz}$ $\quad f_z = \cancel{C} + \sin'(xz^2)$

$f_z = \cos(xz^2)\cdot(xz^2)'$

$f_z = 2xz\cos(xz^2)$

$$f_{zz} = 2x\cdot\overset{f}{[z]}\cdot\overset{g}{\cos(xz^2)}$$

$$= 2x\left(\frac{d}{dz}[z]\cdot\cos(xz^2) + z\cdot\frac{d}{dz}[\cos(xz^2)]\right]$$

$$= 2x\left[(1)\cdot\cos(xz^2) + z\cdot -\sin(xz^2)\cdot(xz^2)'\right]$$

$$= 2x(\cos(xz^2) + z\cdot -\sin(xz^2)\cdot 2xz)$$

$$\boxed{f_{zz} = 2x(\cos(xz^2) - 2xz^2\sin(xz^2))}$$

$f(x,y) = xy - 2x^2 - y^2 + 110x + 74y.$

$f(x,y) = xy - 2x^2 - y^2 + 110x + 74y$

$\boxed{\begin{array}{l} f_x = y - 4x + 110 \\ f_y = x - 2y + 74 \end{array}}$

$y = 4x - 110$

$x - 2(4x - 110) = -74$

$x - 8x + 220 = -74$

$\frac{-7x}{7} = \frac{-294}{7}$ $\boxed{x = 42}$

$y = 4(42) - 110$

$y = 268 - 110$

$\boxed{y = 58}$

The critical point is $(42, 58)$.

$f_x = y - 4x + 110$ $\quad f_y = x - 2y + 74$ $\quad \boxed{f_{xy} = 1}$

$\boxed{f_{xx} = -4}$ $\quad \boxed{f_{yy} = -2}$

$\boxed{\begin{array}{l} D(a,b) < 0 \text{ saddle point} \\ D(a,b) = 0 \text{ inclusive} \\ D(a,b) > 0 \text{ evaluate } f_{xx} \end{array}}$

$\boxed{\begin{array}{l} f_{xx} < 0 = \text{maximum} \\ f_{xx} > 0 = \text{minimum} \end{array}}$

$D(x,y) = (f_{xx})(f_{yy}) - (f_{xy})^2$

$D(x,y) = (-4)(-2) - (1)^2$

$\boxed{D(x,y) = 7}$ This is greater than 0, so we evaluate f_{xx}.

$f_{xx} = -4$; $-4 < 0$, therefore maximum.

$f(42,58) = (42)(58) - 2(42^2) - 58^2 + 110(42) + 74(58)$

$f(42,58) = 2436 - 3528 - 3364 + 4620 + 4292$

$\boxed{\text{The critical point } (42, 58) \text{ is a maximum.}}$

$\boxed{f(42,58) = 4456}$ $\boxed{\text{The maximum point is } 4456}$.

3. Please find the derivative of $f(x,y) = (x + 4y)^3$ in the direction of $\vec{v} = <1, -1>$ at the point (2,3). That is please find $\nabla f(x,y) \cdot \vec{u}$ at the point (2,3) where $\vec{u}$ is the [redacted] vector in the direction of $\vec{v}$.

unit

found another typo

$$f(x,y) = (x+4y)^3$$

$$\vec{v} = \langle 1, -1 \rangle \qquad \boxed{\vec{u} = \frac{1i}{\sqrt{2}} - \frac{1j}{\sqrt{2}}}$$

$$\vec{u} = \frac{1i - 1j}{\sqrt{(1)^2 + (-1)^2}}$$

$$\nabla f(x,y) = f_x \vec{i} + f_y \vec{j}$$

$$f_x = (3)\left(4y + \frac{d}{dx}[x]\right)(x+4y)^2$$

$$f_x = (3)(1)(x+4y)^2$$

$$\boxed{f_x = 3(x+4y)^2}$$

$$f_y = (3)\left(4 \cdot \frac{d}{dy}[y] + \frac{d}{dy}[x]\right)(x+4y)^2$$

$$\boxed{f_y = 12(x+4y)^2}$$

$$\nabla f(x,y) = 3(x+4y)^2 \vec{i} + 12(x+4y)^2 \vec{j} \Big|_{2,3}$$

$$= 3(2+4(3))^2 \vec{i} + 12(2+4(3))^2 \vec{j}$$

$$= 588i + 2352j$$

$$\nabla f(2,3) = \langle 588, 2352 \rangle$$

$$\nabla f(2,3) = \langle 588, 2352 \rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right\rangle$$

$$\nabla f(2,3) = \frac{588}{\sqrt{2}}, \frac{-2352}{\sqrt{2}}$$

$$\nabla f(2,3) = \frac{-1764}{\sqrt{2}} = \boxed{-1247.34}$$

4. Please evaluate the following multiple integrals ☺.

I know I only did a "two-er" on the vid but I'm making sure that you can extrapolate.

$$\left(\int_{-1}^{1}\left(\int_{0}^{1}\left(\int_{0}^{1} xyz\, dx\right)^{1} dy\right)^{2} dz\right)^{3}$$

① $\int_0^1 xyz\, dx = \frac{1}{2}x^2yz \Big|_0^1 \frac{1}{2}(1^2)(yz) - \frac{1}{2}(0^2)(yz)$

$= \frac{1}{2}(1)(yz) - 0$

$= \frac{1}{2}yz$

② $\int_0^1 \frac{1}{2}yz\, dy = \frac{1}{4}y^2z \Big|_0^1 \frac{1}{4}(1^2)(z) - \frac{1}{4}(0^2)(z)$

$= \frac{1}{4}(1)(z) - 0$

$= \frac{1}{4}z$

③ $\int_{-1}^1 \frac{1}{4}z = \frac{1}{8}z^2 \Big|_{-1}^1 \frac{1}{8}(1)^2 - \frac{1}{8}(-1)^2$

$= \frac{1}{8} - \frac{1}{8}$

$= \boxed{0}$

5. Please integrate the function $f(x,y) = x + 2y$ where $0 \leq x \leq 1$ and $1 \leq y \leq \sqrt{x}$. You will definitely want to draw a picture here ☺.

$$f(x,y) = x + 2y$$

$$\int_0^1 \int_1^{\sqrt{x}} x + 2y \, dy dx$$

① $$\int_1^{\sqrt{x}} x + 2y \, dy = \int_1^{\sqrt{x}} xy + \frac{y^2}{4} \Big|_1^{\sqrt{x}}$$

$$= x\sqrt{x} + \frac{(\sqrt{x})^2}{4} - \left(x(1) + \frac{(1)^2}{4}\right)$$

$$= x^{3/2} + \frac{x}{4} - \left(x + \frac{1}{4}\right)$$

$$= \boxed{x^{3/2} + \frac{x}{4} - x + \frac{1}{4}}$$

$$\int_0^1 x^{3/2} + \frac{x}{4} - x + \frac{1}{4} \, dx = \frac{1}{40} x\left(16x^{3/2} - 15x + 10\right) \Big|_0^1 \quad - 0$$

$$= \frac{1}{40}\left(16(1)^{3/2} - 15(1) + 10\right) - 0$$

$$= \frac{1}{40}(16 - 15 + 10)$$

$$= \frac{1}{40}(11) = \boxed{\frac{11}{40}}$$