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IDSC 6490 Probability Week III Homework

Happy Business People Edition

Please be sure to read Chapter 1.1 – 1.5 in your Probability and Statistical Inference textbook.

This homework is due on **September 29st** ☺

Name

1. You are in a frequentist probability class and the Professor boldly claims that the probability that there exists life in outer-space is greater than 50%. Concisely explain why your Professor (the other guy *LOL*) is full of beans using your knowledge of the frequentist model.

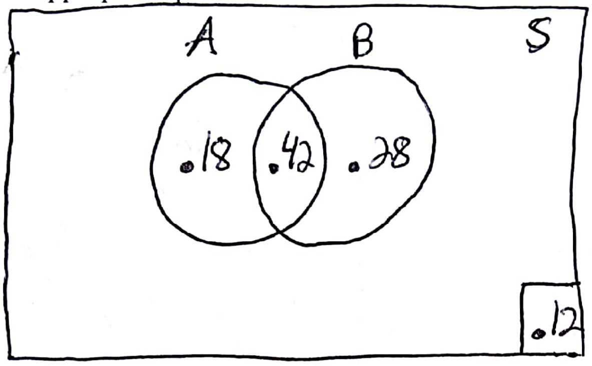
The professor would be wrong in this case because the frequentist model is based on empirical evidence/observed trials. If you were flipping a coin 50 times, the frequentist probability theory state you should expect about 25 heads, because this is already observed behavior. This model works with well-defined random experiments, because it tells you the number of ways you can get what you are looking for/your expected results. The existence of life in outer space is not well defined, nor has it been observed. Therefore his claim doesn't make any sense when applying it to the frequentist model.

2. Given a probability space such that P(A) = .6, P(A∩B) = .42, and P(B) = .7. Please answer the following questions.

(a) Please draw the appropriate picture.

$P(A)=.6$ $\quad .6-.42=.18$

$P(A\cap B)=.42$



$P(B)=.7$ $\quad .7-.42=.28$

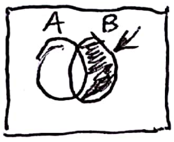
$P(B\cap A)=.42$

$.18+.42+.28=.88$ $P(A\cup B)$

$1-.88=.12$

(b) What is P(B|A^c)? The probability of B given the compliment of A $\quad P(B)=.7 \quad P(A^c)=1-.6=.4$

$$P(B|A^c)=\frac{P(B\cap A^c)}{P(A^c)} \qquad P(B\cap A^c)=.28$$



$$P(B|A^c)=\frac{P(B\cap A^c)}{P(A^c)}=\frac{.28}{.4}=\boxed{.7}$$

(c) What is P[(A∪B)^c]?

$P(A\cup B)=.18+.42+.28=.88$

$$P[(A\cup B)^c]=1-P(A\cup B)$$

$$P[(A\cup B)^c]=1-.88=\boxed{.12}$$

(d) Are A and B independent? Yes or no is of no value ☺ Explanation please.

In mathematical terms, the notion of independence is $P(B|A)=P(B)$.
That is, the probability of B, given A, is equal to the probability of B.
In the case above, $P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{.42}{.7}=\boxed{.7}$ or $P(B)$.
Going one step forther, if $\boxed{P(B\cap A)=P(B)P(A)}$, or if the intersection of $P(B\cap A)$ is equal to P(B) times P(A), they are independent.
Again, in case above $\quad P(B\cap A)=\boxed{.42}$

$P(B)(P(A))=(.7)(.6)=\boxed{.42}$ $\quad$ They are independent.

P.S. When I saw this in the video and then only noticed that .6 times .7 was .42, I had the eureka moment!

3. Suppose that a particular now extinct dinosaur hatched its children with the probability of a boy at 2/5 and the probability of a girl at 3/5. What is the probability that if the dinosaur had four offspring that there were 3 boys and 1 girl? You may assume independence ☺.

$n = 4$ offspring

$p = \frac{2}{5}$ (boy) $\quad q = \frac{3}{5}$ (girl)

$P(k) = P(k \text{ successes}) = \binom{n}{k} p^k q^{n-k}$

Find $P = p(3 \text{ boys})$

$$P = \binom{4}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^{4-3}$$

$$P = (4)\left(\frac{5}{78}\right)\left(\frac{3}{5}\right)$$

$$P = \left(\frac{21}{82}\right)\left(\frac{3}{5}\right)$$

$$\boxed{P = \frac{2}{13} \text{ or } .153846}$$

4. To make a seven-hundred and fifty – piece jigsaw puzzle more challenging, a puzzle company includes five extra pieces in the box along with the other seven-hundred and fifty pieces, and those five extra pieces do not fit anywhere in the puzzle (nice guys *LOL*).

If you buy such a puzzle box, and immediately select one piece at random, what is the probability that it will be one of the extra pieces? Make some kind of argument please - not just a number.

My logic/reasoning is as follows: you had an original box of 750 pieces (good) where there were 5 "bad" pieces added, giving you a new total of 755 pieces to choose from. Since you are only selecting one piece at random, you would think your probability is $\frac{1}{755}$ <u>but</u> you have five bad pieces to choose from. In this case, the probability is $\frac{5}{755}$ or .006623 chance of getting a "bad" piece.

X	Good	Bad
	$\frac{756}{755}$	$\frac{5}{755}$