IDSC 6490 Week 1

Please solve these problems and send back to me by Saturday, September 15th.

All solution MUST be accompanied by the necessary work to receive ANY credit.

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1. GameDude, the manufacturer of the popular video game Crush, Kill, Destroy® has determined that the profits from the sales of this video game can be modeled by the function $P(t) = \frac{e^t + e^{-t}}{2}$ where t is time in months and P(t) is millions of dollars.

$P(t) = \frac{e^t + e^{-t}}{2}$

$P(-t) = \frac{e^{-t} + e^{-(-t)}}{2} = P(t) = \frac{e^{-t} + e^{t}}{2}$

$P(4) = \frac{e^4 + e^{-4}}{2} = \frac{2.71^4 + 2.71^{-4}}{2} = 27.30816$

$P(-4) = \frac{e^{-4} + e^{-(-4)}}{2} = \frac{2.71^{-4} + 2.71^4}{2} = 27.30816$

(a) Is this function even or odd or neither? Please use the definitions of even, or odd and show that you Grok. One of the things that you are learning to do in this class is "how" to get it done on the fly. If necessary, use Google "How do I…whatever." This is a perfectly legitimate tool as long as you really learn the general principles. Probably also want to Google "Grok." ☺.

The function $P(t) = \frac{e^t + e^{-t}}{2}$ is even because substituting $-x$ (or $-t$) into the equation produces the original starting function. In my example, plugging in 4 and -4 into the equation produce the same result, which makes this function even.

(b) If this function is not one-to-one? If not, make a rational redaction to make the function one-to-one and tell me your reasoning.

This function is not one-to-one because not every value that gets plugged into the function will have a distinct or unique result. For example, 4 and -4, produce the same exact result. A one-to-one function must have distinct outputs for each input.

To make this function one-to-one, simply change the function to $P(t) = \frac{e^t - e^{-t}}{2}$. When you plug in (4, -4) you get

$P(4) = \frac{e^4 - e^{-4}}{2} = 26.95863$ (4, 26.95863)

$P(-4) = \frac{e^{-4} - e^4}{2} = -26.95863$ (-4, -26.95863)

$P(t) = \frac{e^t - e^{-t}}{2}$

(c) Now that you have P(t) as a one-to-one function, please derive the inverse function $P^{-1}(t)$. P(t) has another name, Cosh(t) and the inverse is easily found on the internet – I want you to show me how, the actual answer is irrelevant – I'll be grading the process.

Hint: Use some algebra to try to make this look like a quadratic equation, then you will need you logs.

① $P(t) = \frac{e^t - e^{-t}}{2}$ or $y = \frac{e^t - e^{-t}}{2}$. Solve for t.

② $t = \frac{e^y - e^{-y}}{2}$. Factor out 2

③ $2 \cdot t = \left(\frac{e^y - e^{-y}}{\cancel{2}}\right)(\cancel{2})$.

④ $2t = e^y - \frac{1}{e^y}$ (apply exponent)

⑤ $2t \cdot e^y = \left(e^y - \frac{1}{e^y}\right)(e^y)$

⑥ $2te^y = e^{y^2} - 1$

⑦ $0 = e^{y^2} - 2te^y - 1$ Quadratic formula

$\boxed{a = 1, \; b = 2t, \; c = -1}$

⑧ Use quadratic equation to solve for y

$\frac{-b \pm \sqrt{(b^2) - 4(a)(c)}}{2a} = x(e^y)$

⑨ $= \frac{-(-2t) \pm \sqrt{(-2t)^2 - 4(1)(-1)}}{2(1)}$

⑩ $= \frac{2t \pm \sqrt{4t^2 + 4}}{2}$

⑪ $= \frac{2t \pm \sqrt{4(t^2 + 1)}}{2}$

⑫ $= \frac{2t \pm (\sqrt{4})(\sqrt{t^2 + 1})}{2}$

I'm leaving you another page for your derivation ☺

⑬ $= \frac{2t \pm 2\sqrt{t^2 + 1}}{2}$

Keep going, keep going ☺

(14) Factor out common 2

$$\frac{2 \cdot \left(t \pm \sqrt{t^2+1}\right)}{2}$$

(15) $t \pm \sqrt{t^2+1} = e^y$

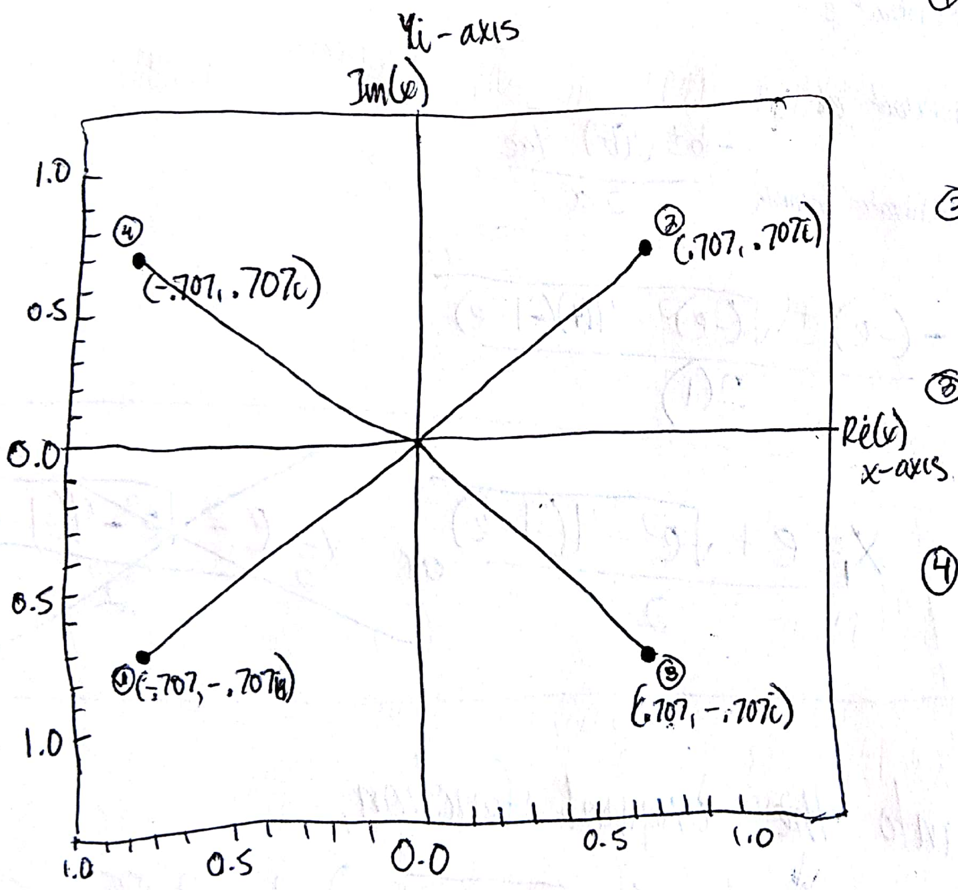
(16) $\boxed{y = \ln\left(t+\sqrt{t^2+1}\right), \ln\left(t-\sqrt{t^2+1}\right)}$

Here is the inverse function.

2. Got to www.wolframalpha.com and type in the following equation:

x^4 + 1= 0.

Please graph the solutions in the complex plane. I just want you to get used to wolfram a bit and you will be delving in to the Complex Plane in you coursework – this class and others.



(1) $X = -\sqrt[4]{-1}$

$(-.707, -.707i)$

(2) $X = \sqrt[4]{-1}$

$(.707, .707i)$

(3) $X = -(-1)^{3/4}$

$(.707, -.707i)$

(4) $X = (-1)^{3/4}$

$(-.707, .707i)$

3. Solve for x if possible. If necessary go to your favorite search engine and look up "how do I solve logarithmic equations" or some such thing. Again – bootstrapping is at the

heart of research and discovery. I cannot emphasis enough the necessity or your being able to figure it out on the fly (or fake it well *LOL*) More about this later *LOL*.

$\ln e^x = x$ so $\ln e^1 = 1$

(a) $\ln(x^2 - 1) = \ln(x + 1) + 1$

① $\ln(x^2-1) = \ln(x+1) + \ln(e^1)$

② $\cancel{\ln}(x^2-1) = \cancel{\ln}((x+1)e)$ $\quad \ln(f(x)) = \ln(g(x))$ or $f(x) = g(x)$

③ $x^2 - 1 = (x+1)e$ $\quad$ simplify

④ $x^2 - 1 = ex + e$ $\quad$ subtract e

⑤ $x^2 - 1 - e = ex$ $\quad$ subtract ex

⑥ $x^2 - ex - 1 - e = 0$ $\quad$ Quadratic formula $\dfrac{-b \pm \sqrt{(b^2) - 4ac}}{2a}$

⑦ $a = 1$, $b = -e$, $c = -1-e$ $\Big\}$ plug into quadratic form. $\quad = \dfrac{-(-e) \pm \sqrt{(-e)^2 - 4(1)(-1-e)}}{2(1)}$

⑧ $= \dfrac{e \pm \sqrt{e^2 - 4(-1-e)}}{2}$

$$\boxed{x_1 = \frac{e + \sqrt{e^2 - 4(-1-e)}}{2} \text{ or } \cancel{x_2 = \frac{e - \sqrt{e^2 - 4(-1-e)}}{2}}}$$

When you plug in x_1 into the orginal function,

$\ln\left(\left(\frac{e + \sqrt{e^2 - 4(-1-e)}}{2}\right)^2 - 1\right) \approx 2.55$ and $\ln\left(\frac{e + \sqrt{e^2 - 4(-1-e)}}{2} + 1\right) + 1 \approx 2.55$ ✓

However, when you plug in x_2,

$\ln\left(\left(\frac{e - \sqrt{e^2 - 4(-1-e)}}{2}\right)^2 - 1\right) \approx -35.35$ and $\ln\left(\frac{e - \sqrt{e^2 - 4(-1-e)}}{2} + 1\right) + 1 \approx$ undefined

So x_2 does not work.