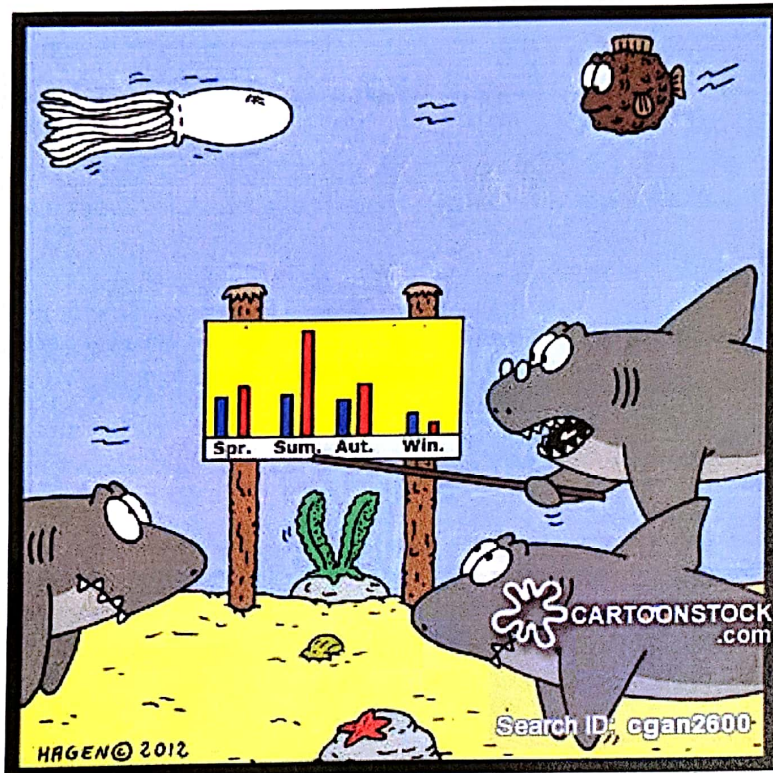IDSC 6490 Week 4 Distributions Homework

Please read the appropriate chapters in your Statistics textbook.

This Homework is due **October 6th**

Research shows that while the number of surfers is somewhat constant over the year, there is a sudden increase in casual bathers over Summer...

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1. Let X be the Random Variable which maps tossing a fair quarter 100 times and recording the number of heads that land face up. Please answer the following questions.

$$X \sim Binomial\ (n=100,\ p=.5)$$

(a) What is the expected value of this random variable? E(X) = ?

$$E(x) = np \qquad E(x) = (100)(.5)$$

$$\boxed{E(x) = 50}$$

The expected value is 50 heads.

(b) What is Standard Deviation of this Random Variable?

$$\sigma = \sqrt{npq} \qquad \sigma = \sqrt{100 \cdot .5 \cdot .5} \qquad \text{The standard deviation}$$

$$p=.5$$
$$q=.5$$

$$\sigma = \sqrt{25} = \boxed{5} \qquad \text{is } 5.$$

(c) What is the probability (or approximately the probability) of obtaining between 40 and 60 heads?

$$P(40 \le X \le 60) \qquad \mu = 50$$
$$\sigma = 5$$

$$P\left(\frac{40-50}{5} \le \frac{x-50}{5} \le \frac{60-50}{5}\right)$$

$$P(-2 \le 2 \le 2)$$

I used a look-up table (which I'll attach) to get

$$0.982 - 0.028 = \boxed{0.954}$$

The probability of obtaining between 40 and 60 heads (approx) is 0.954 or 95.4%

2. Suppose the heights of students at the Carlson School of Management has been measured this year and found to follow a normal distribution with mean $\mu = 75$ inches and standard deviation $\sigma = 3$ inches. Please answer the following questions.

(a) What is the probability that any given C.S.O.M student is taller than 78 inches?

$$\boxed{P(x > 78) = 0.1587}$$

$$1 - P\left(\frac{x-75}{3} \leq \frac{78-75}{3}\right)$$

$$1 - P(z \leq \tfrac{3}{3})$$

$$1 - P(z \leq 1) = 1 - .8413 = 0.1587$$

(b) What is the probability that a student from C.S.O.M is between 69 inches and 78 inches?

$$\boxed{P(69 \leq x \leq 78) = 0.8185}$$

$$P\left(\frac{69-75}{3} \leq \frac{x-75}{3} \leq \frac{78-75}{3}\right) = P(-2 \leq z \leq 1) = P(z \leq 1) - P(z \leq -2)$$

$$.8413 - .0228 = 0.8185$$

(c) What is the probability that a student from C.S.O.M is less than 55 inches tall?

$$P(x \leq 55) = 0.00000 \text{ (essentially zero)}$$

$$P\left(\frac{x-75}{3} \leq \frac{55-75}{3}\right) = P\left(z \leq \frac{-20}{3}\right) = P(z \leq -6.6667)$$

Not on any charts, from your video it is because anything past -3.4 is pratically zero.

3. Consider a bag of well mixed marbles. The bag contains 8 black marbles and 7 white marbles. You are to draw out 4 marbles without replacement. What is the probability of getting 3 black marbles?

$S = 15$

$A = 1$ black marble
$B = 2^{nd}$ black marble
$C = 3^{rd}$ black marble

$P(A + B + C)$

$P(A) = \frac{8}{15}$

$P(A+B+C) = P(A) \cdot P(B/A) \cdot P(C/B)$

$P(B/A) = \frac{7}{14}$

$P(A+B+C) = \left(\frac{8}{15}\right) \cdot \left(\frac{7}{14}\right) \cdot \left(\frac{6}{13}\right)$

$P(C/B) = \frac{6}{13}$

$P(A+B+C) = 0.5333 \cdot 0.5 \cdot 0.4615385$

$P(A+B+C) = 0.1230769$

The probability of 3 marbles (black) is 0.123 or 12.3%.

4. Evil Mike, a professor at C.S.O.M give an exam for which the results are approximately normally distributed with a mean $\mu = 65$ and standard deviation $\sigma = 5$. To get an A you need to be in the top 10th percentile, the is the top 10% of the class. What is the lowest score on Evil Mike's test will get you an A?

$X \in N(60, 5^2)$

0.9015

A

90th

Z $Z = 1.29$
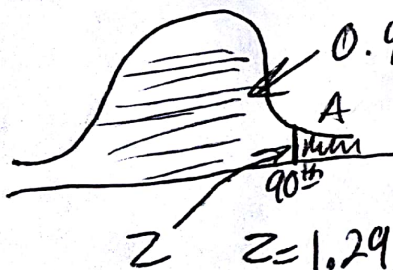
$Z = 1.29 = 0.9015$
to make sure you are in the 90th percentile.

$5(1.29) = \left(\frac{X - 65}{5}\right) 5$

$6.45 = X - 65$

$X = 65 + 6.45$

$X = 71.45$

The lowest score to get you in the top 10th percentile is 71.45.