Differential Privacy

Matt Kusner

This Talk

A Web Search Example

Privacy via Randomization

Differential Privacy

Global Sensitivity

Privacy Mechanisms

- The Laplace Mechanism
- The Gaussian Mechanism
- The Exponential Mechanism

Local Sensitivity

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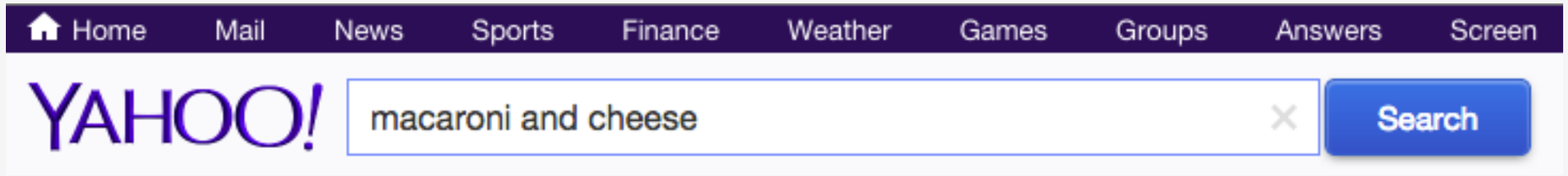
Local Sensitivity

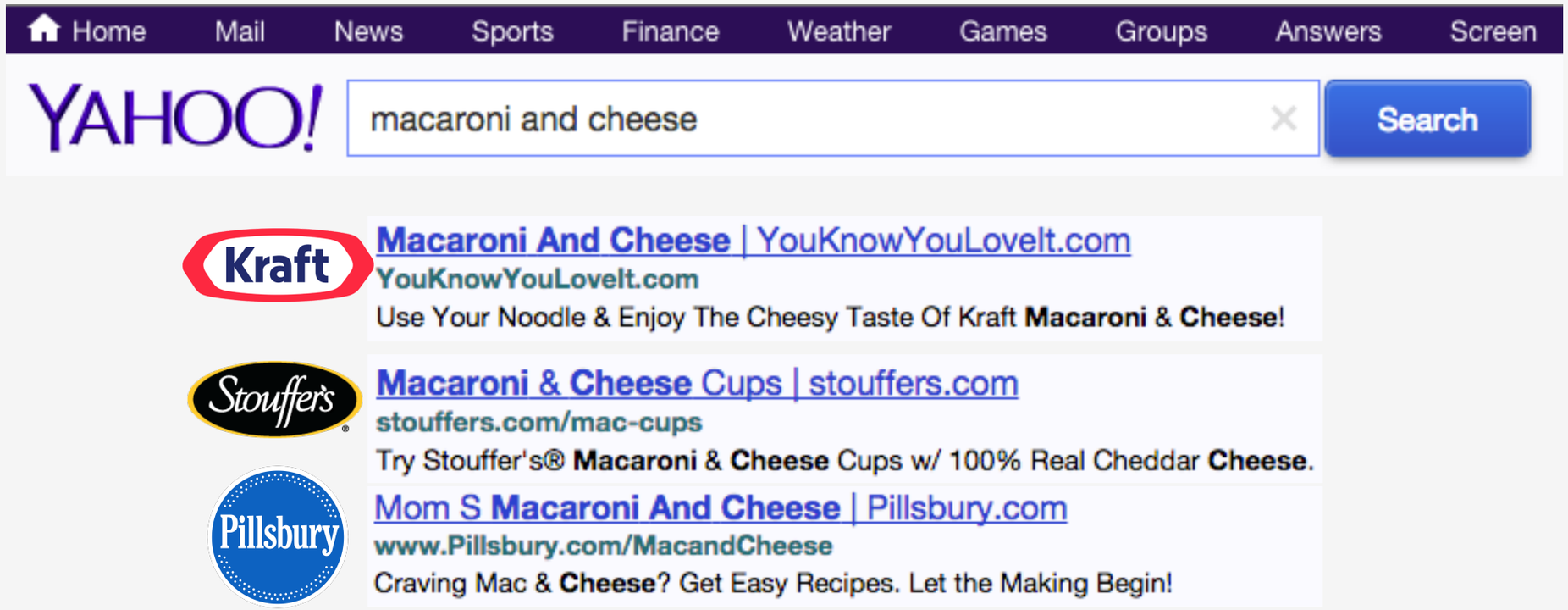
Just a Web Search?

somePerson@yahoo.com **searches**



Just a Web Search?

somePerson@yahoo.com **searches**



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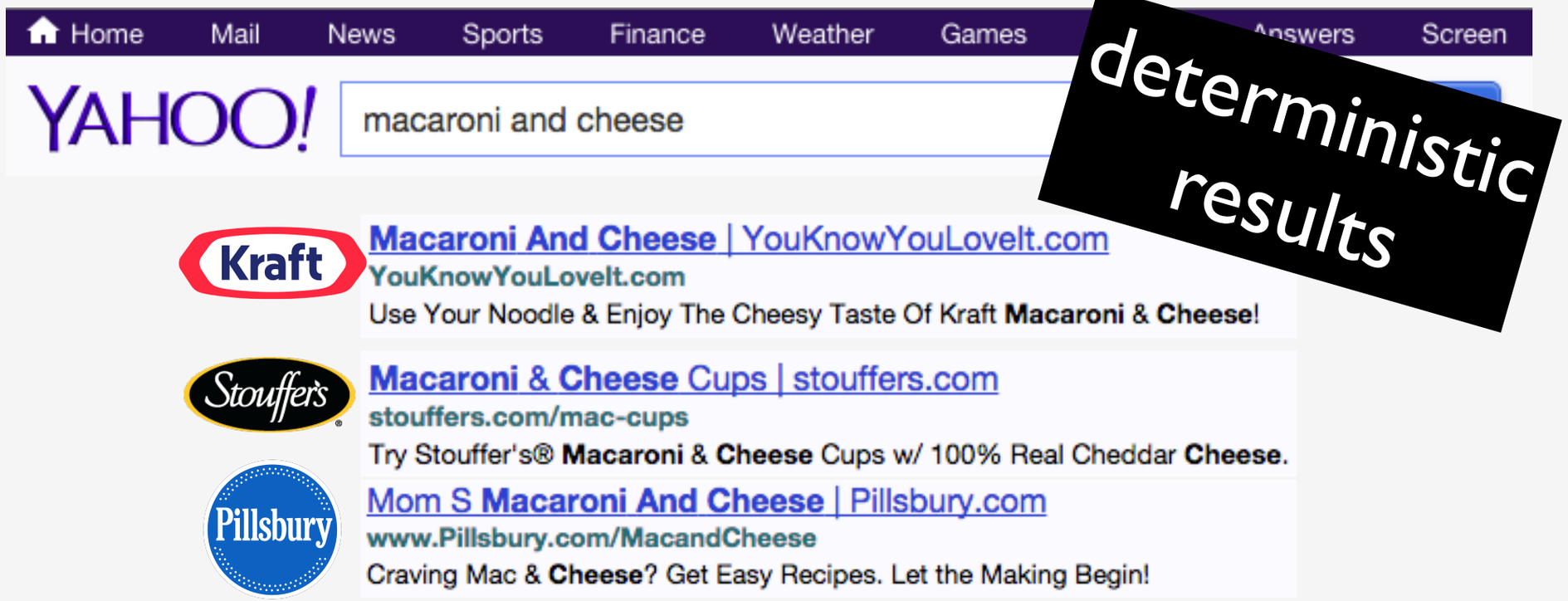

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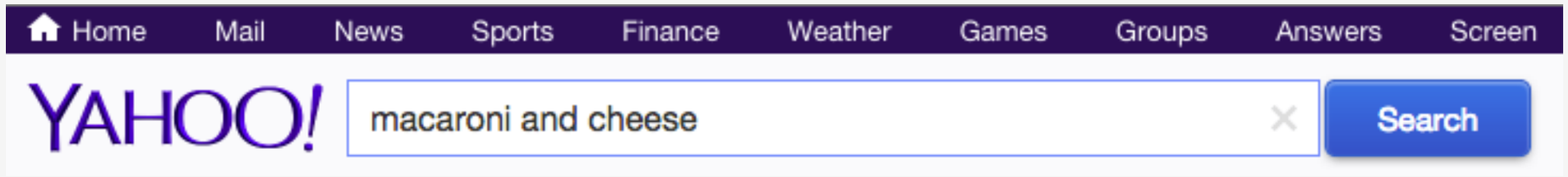

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Use Your Noodle & Enjoy The Cheesy Taste Of Kraft **Macaroni** & **Cheese**!

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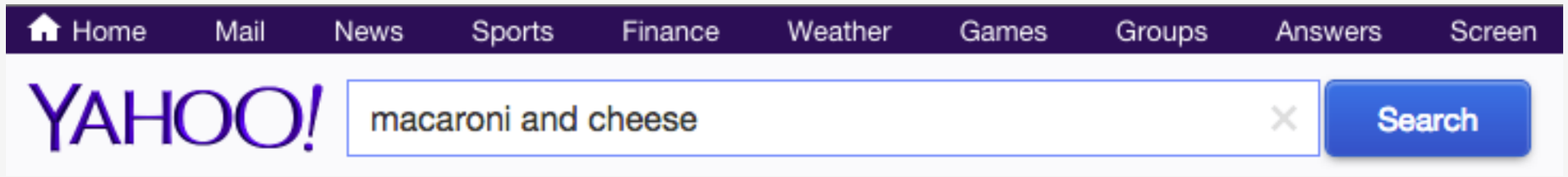
somePerson@yahoo.com **searches**



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somePerson@yahoo.com **clicks!**

Just a Web Search?

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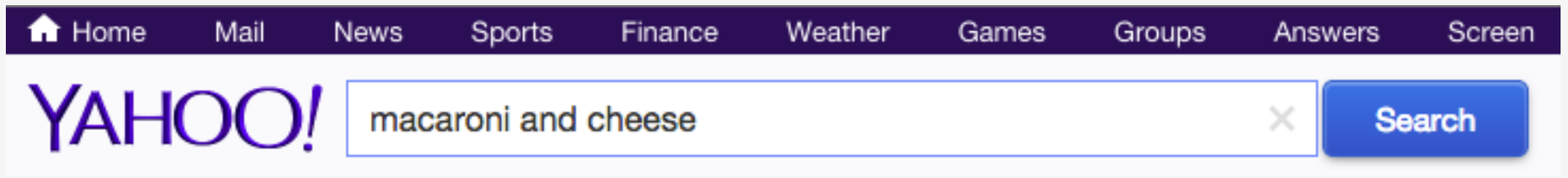

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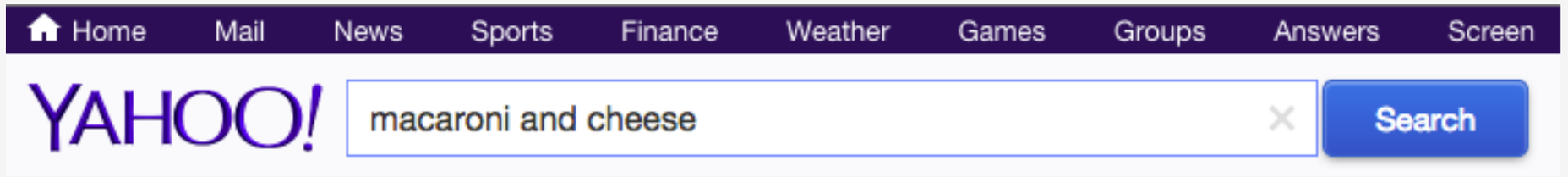
somePerson@yahoo.com **clicks!**

YAHOO!

update ad-recommendation algorithm using profile data on somePerson@yahoo.com

Just a Web Search?

somePerson@yahoo.com **searches**



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somePerson@yahoo.com **clicks!**

YAHOO!

somePerson@yahoo.com

age: 55 | gender: M | SSN: 83...1

Just a Web Search?

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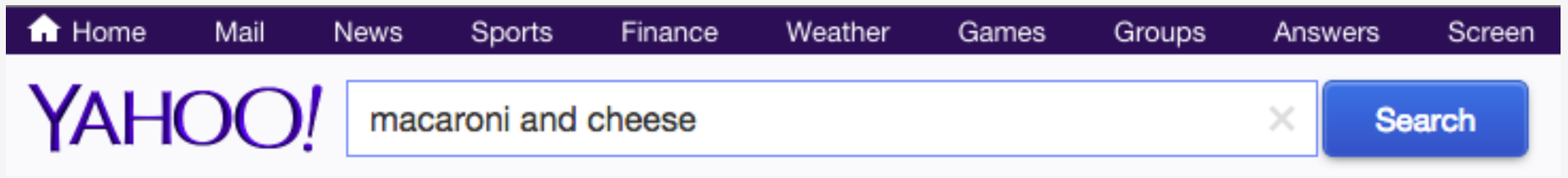
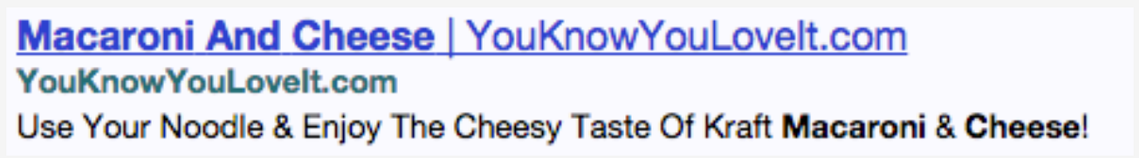
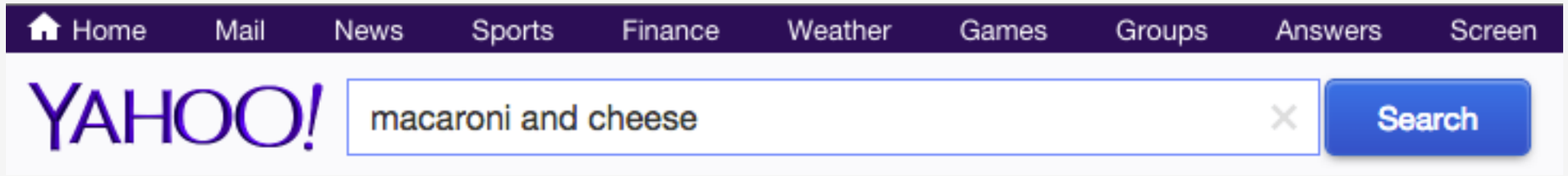


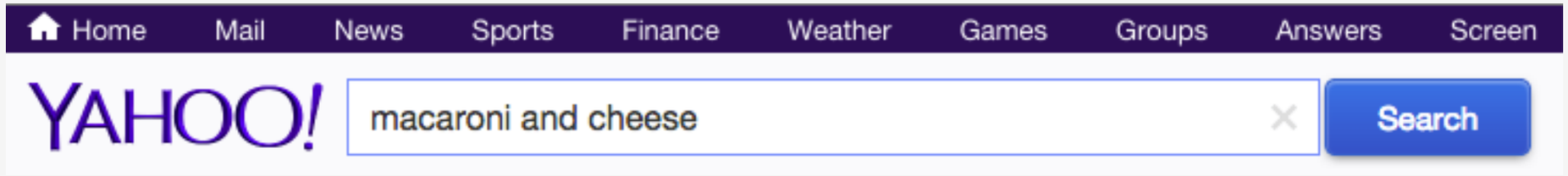


somePerson@yahoo.com **clicks!**

The Attack

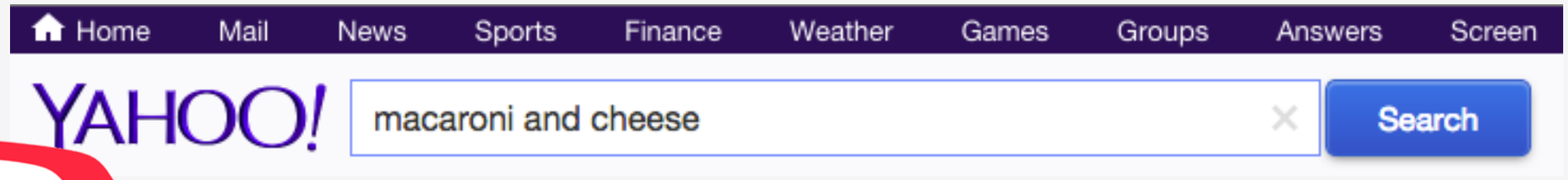

Home
Mail
News
Sports
Finance
Weather
Games
Groups
Answers
Screen
YAHOO!
macaroni and cheese
Search

The Attack



Is `somePerson@yahoo.com` >49?

The Attack



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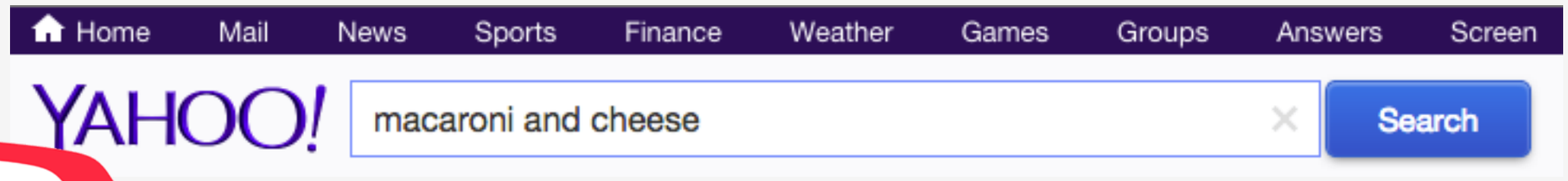


Is `somePerson@yahoo.com` >49?

account 1

`old@yahoo.com`

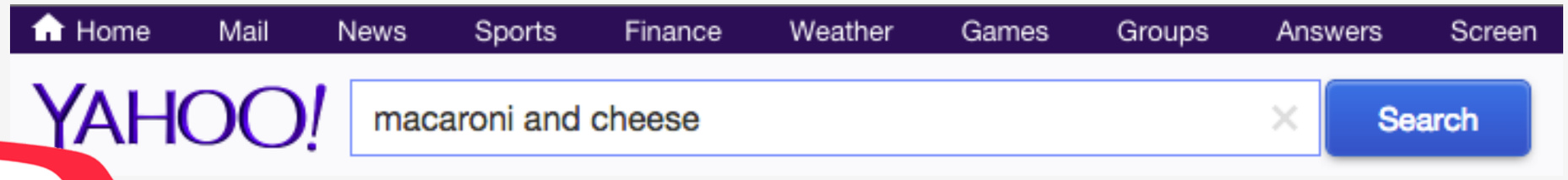
age: 50

account 2

`young@yahoo.com`

age: 49

The Attack



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account 2

`young@yahoo.com`

age: 49

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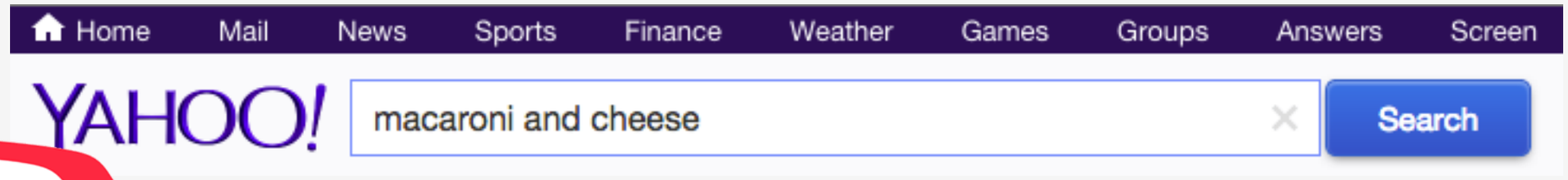

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`young@yahoo.com`

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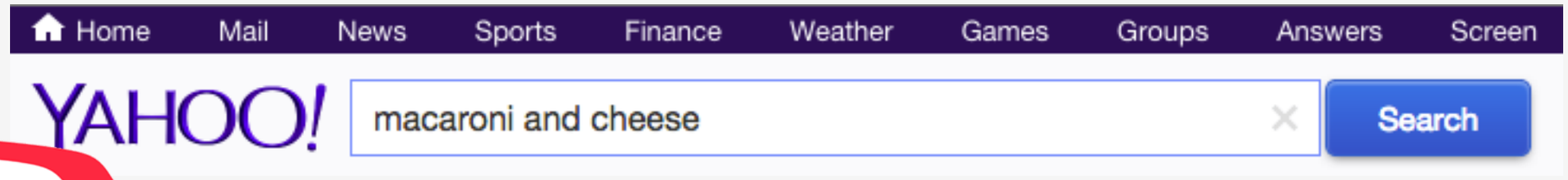

Is `somePerson@yahoo.com` >49?

account 1	account 2
`old@yahoo.com` age: 50	`young@yahoo.com` age: 49

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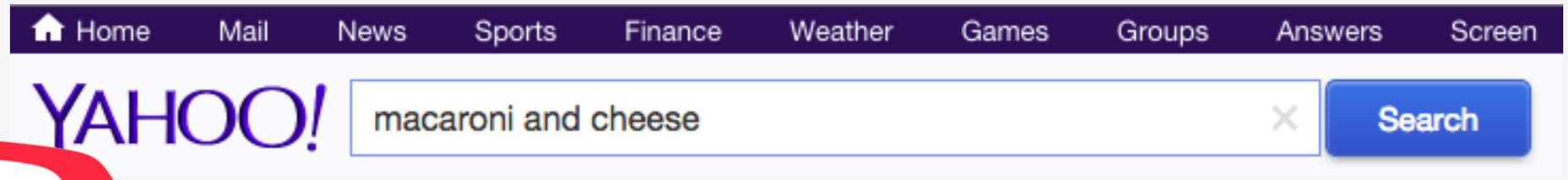

Is `somePerson@yahoo.com` >49?

Yes!

Data Leaks

Aol.

private search logs

650,000 users

NETFLIX

private movie ratings

33 million users

Data Leaks

Aol.

private search logs

650,000 users

NETFLIX

private movie ratings

33 million users

How to keep data private?

This Talk

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Privacy via Randomization

Differential Privacy

Global Sensitivity

Privacy Mechanisms

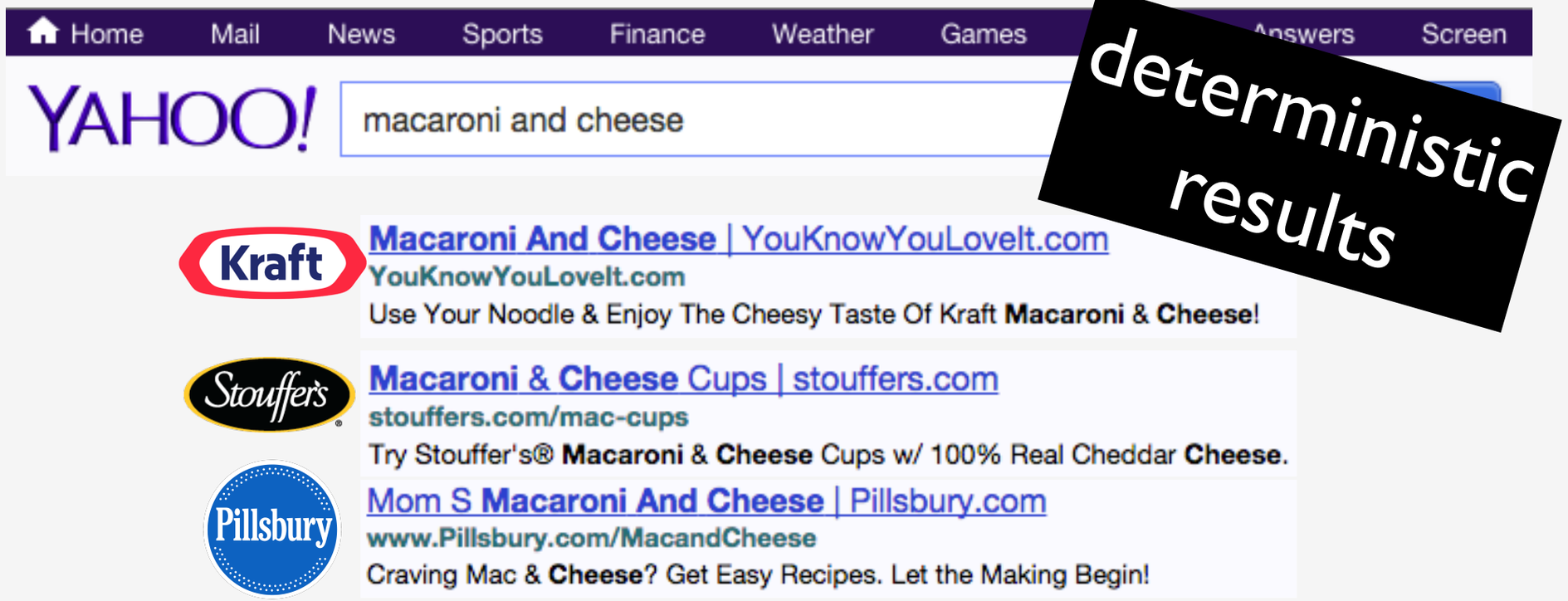
- The Laplace Mechanism
- The Gaussian Mechanism
- The Exponential Mechanism

Local Sensitivity

Privacy via Randomization

Privacy via Randomization

somePerson@yahoo.com **searches**



Privacy via Randomization

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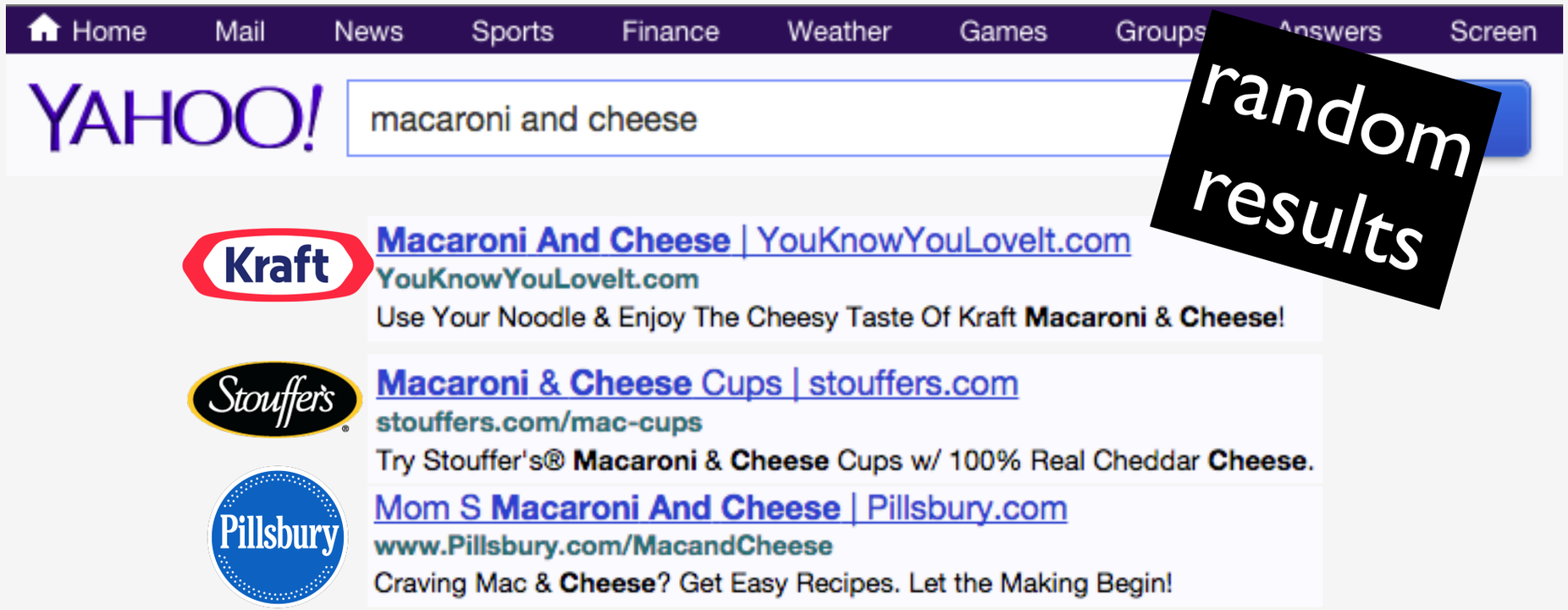

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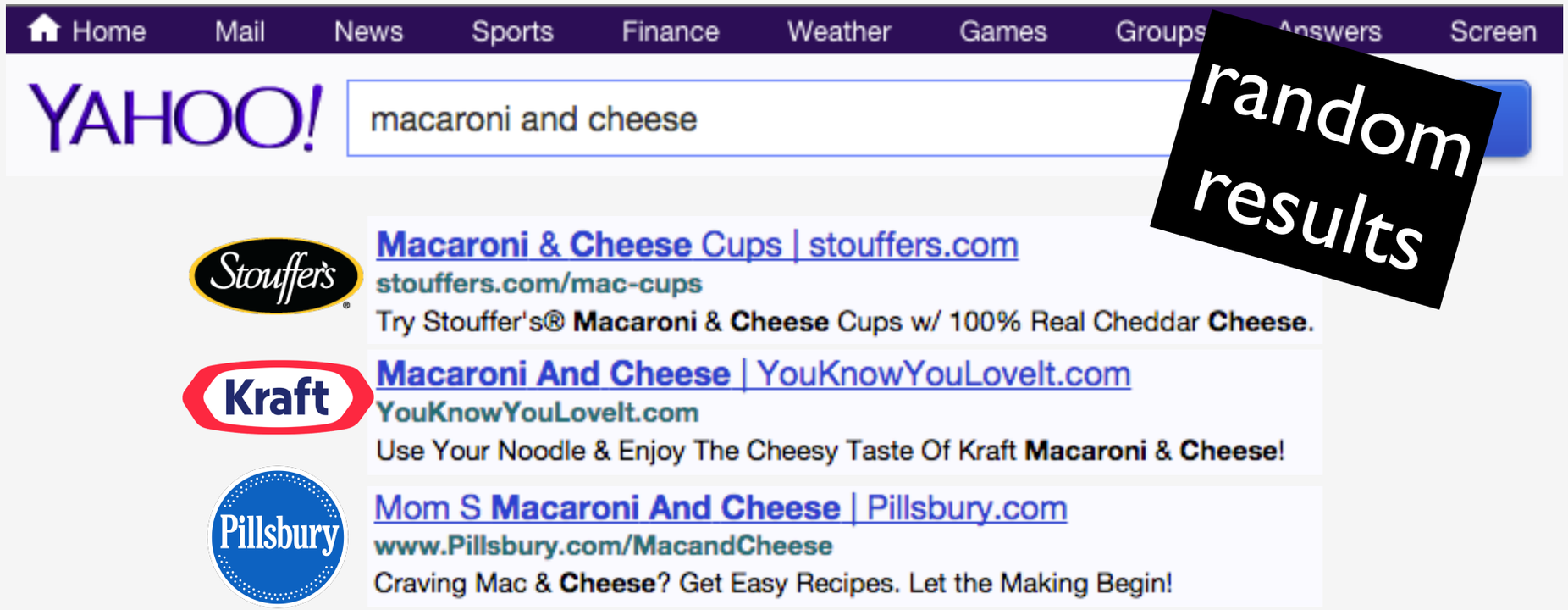

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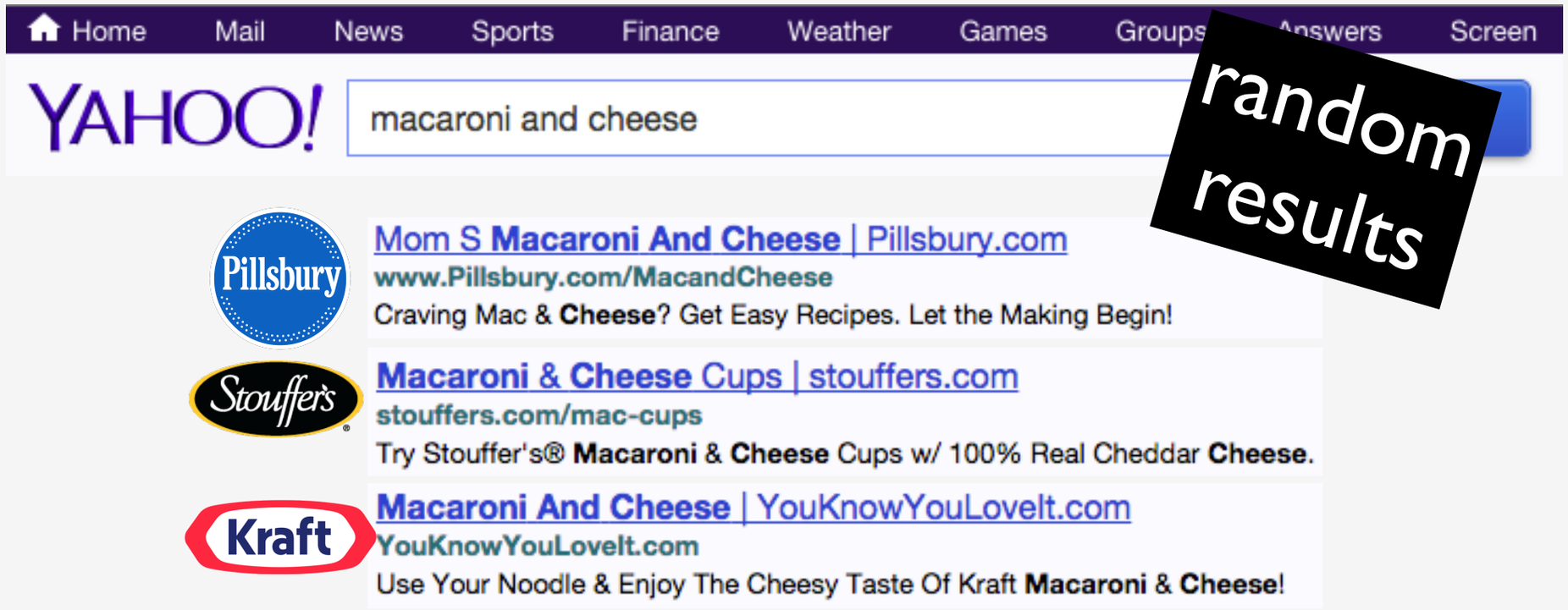

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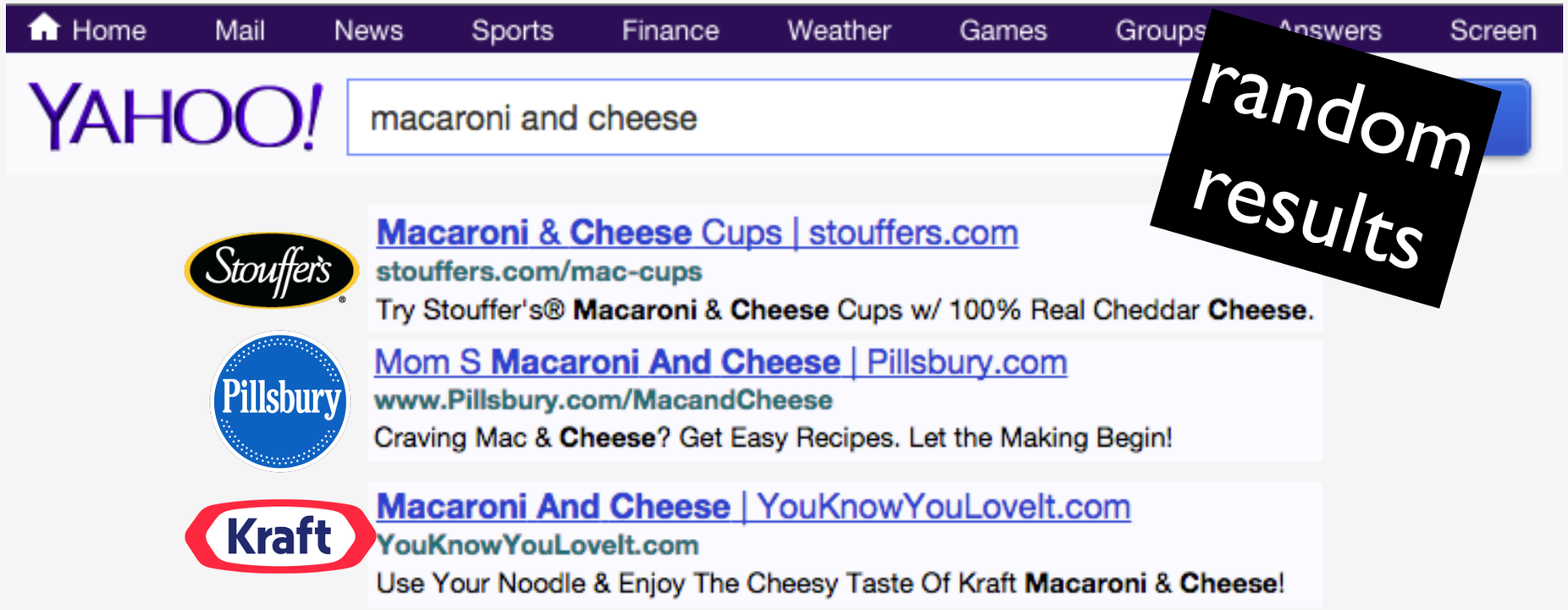

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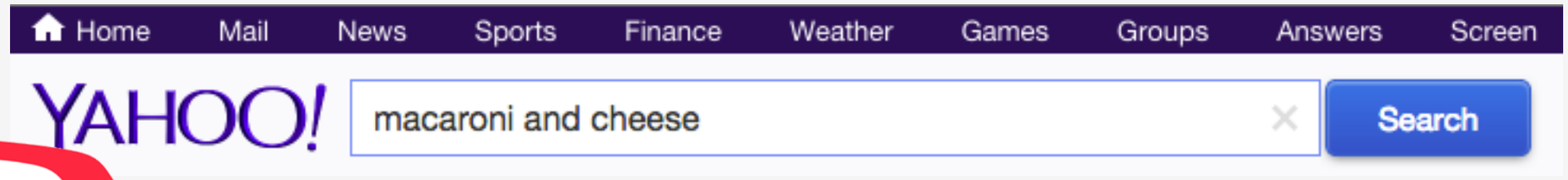

Is `somePerson@yahoo.com` >49?

account 1

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age: 50

account 2

`young@yahoo.com`

age: 49

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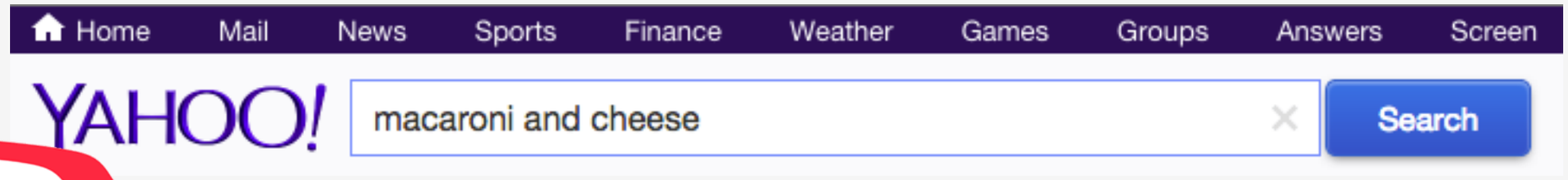

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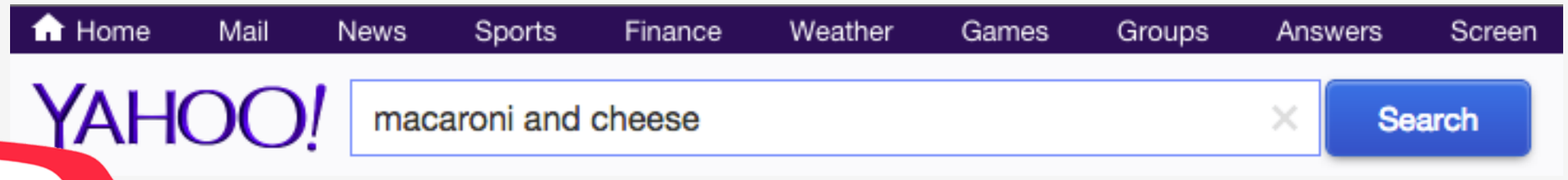

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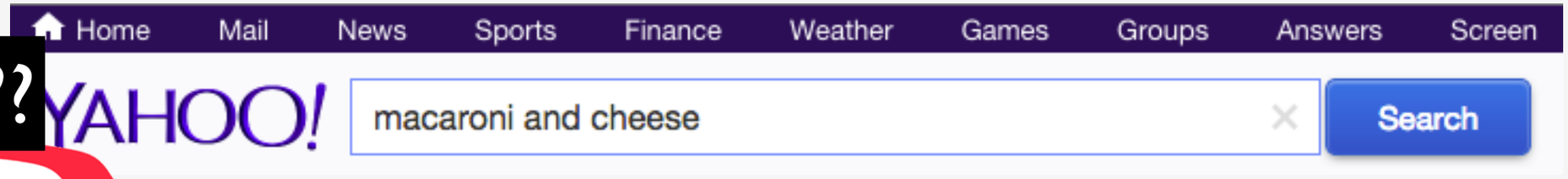

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Privacy via Randomization

Implies a **trade-off**

Privacy  Accuracy

increased randomness

decreased randomness

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Local Sensitivity

Differential Privacy [Dwork et al., 2006]

Differential Privacy [Dwork et al., 2006]

A formalization of "privacy through randomness"

Differential Privacy

[Dwork et al., 2006]

A formalization of "privacy through randomness"

data

age: 20	gender: M	SSN: 42...2
age: 91	gender: M	SSN: 21...0
age: 82	gender: F	SSN: 46...7
age: 39	gender: F	SSN: 91...3
⋮	⋮	⋮

Differential Privacy

[Dwork et al., 2006]

A formalization of "privacy through randomness"

algorithm

$\mathcal{A}$

data

age: 20	gender: M	SSN: 42...2
age: 91	gender: M	SSN: 21...0
age: 82	gender: F	SSN: 46...7
age: 39	gender: F	SSN: 91...3
•	•	•
•	•	•
•	•	•

[Dwork et al., 2006]

Differential Privacy

A formalization of "privacy through randomness"

algorithm data output

$$\mathcal{A}\left(\begin{array}{|c|c|c|}\hline \text{age: 20} & \text{gender: M} & \text{SSN: 42...2} \\ \hline \text{age: 91} & \text{gender: M} & \text{SSN: 21...0} \\ \hline \text{age: 82} & \text{gender: F} & \text{SSN: 46...7} \\ \hline \text{age: 39} & \text{gender: F} & \text{SSN: 91...3} \\ \hline \vdots & \vdots & \vdots \\ \hline \end{array}\right) = \mathbf{x}$$

[Dwork et al., 2006]

Differential Privacy

A formalization of "privacy through randomness"

algorithm data $\mathcal{D}$ output

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Differential Privacy

A formalization of "privacy through randomness"

algorithm $\mathcal{A}$ data $\mathcal{D}'$

age: 20	gender: M	SSN: 42...2
age: 91	gender: M	SSN: 21...0
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⋮	⋮	⋮
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informally: $\mathbf{x} \approx \mathbf{x}'$

[Dwork et al., 2006]

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$$\frac{\Pr[\mathcal{A}(\mathcal{D}) = \mathbf{x}]}{\Pr[\mathcal{A}(\mathcal{D}') = \mathbf{x}]} \leq e^{\epsilon}$$

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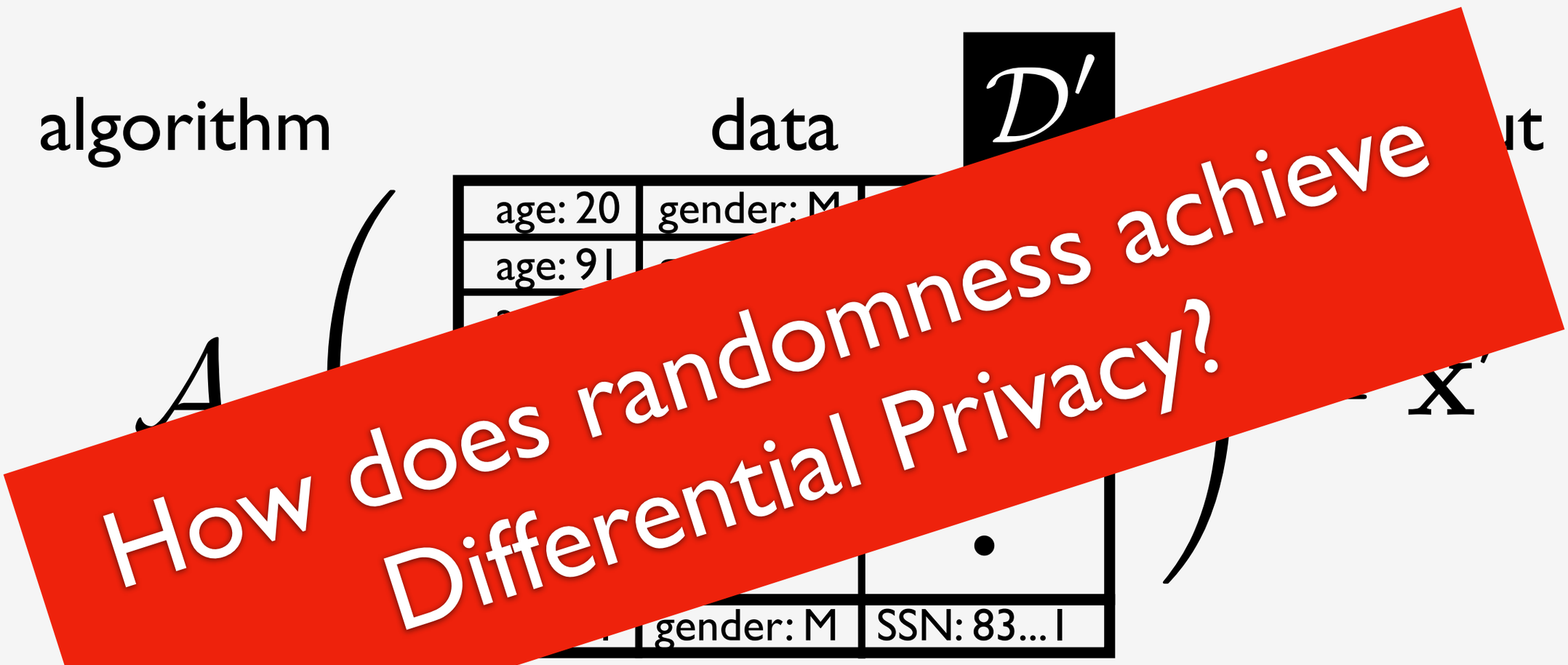
$\mathcal{D}, \mathcal{D}'$ differ in one row

Differential Privacy

[Dwork et al., 2006]

A formalization of "privacy through randomness"



How does randomness achieve Differential Privacy?

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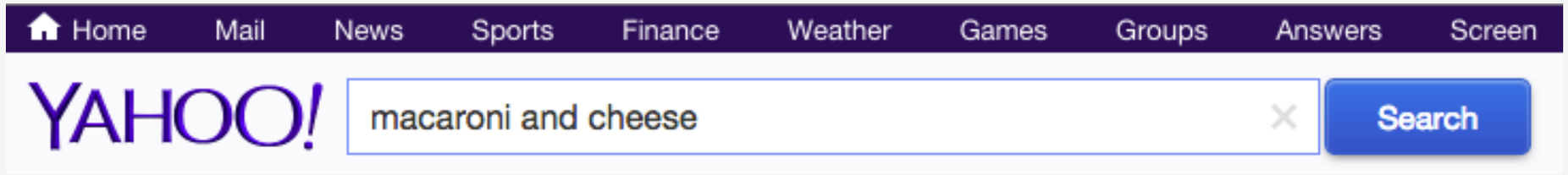
Randomized Response

[Warner, 1965]

somePerson@yahoo.com **searches**



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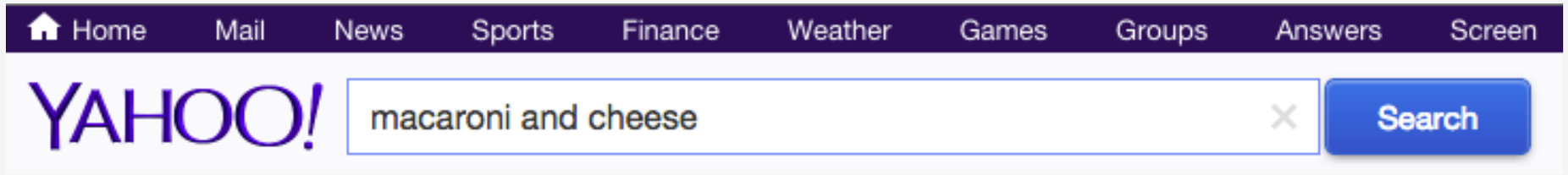

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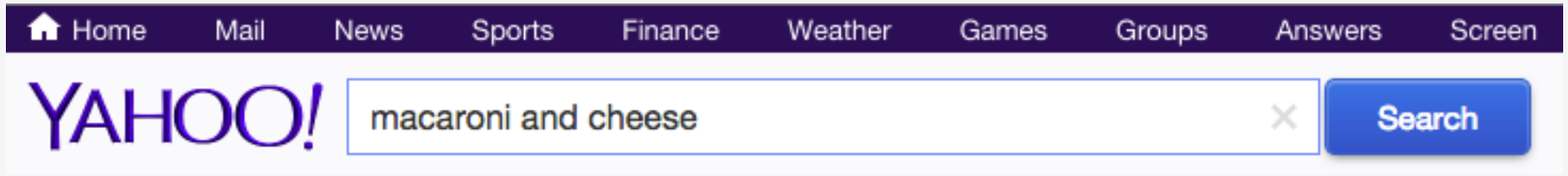

tails

true prediction

old@yahoo.com

age: 50	gender: M	SSN: 83...1

Randomized Response

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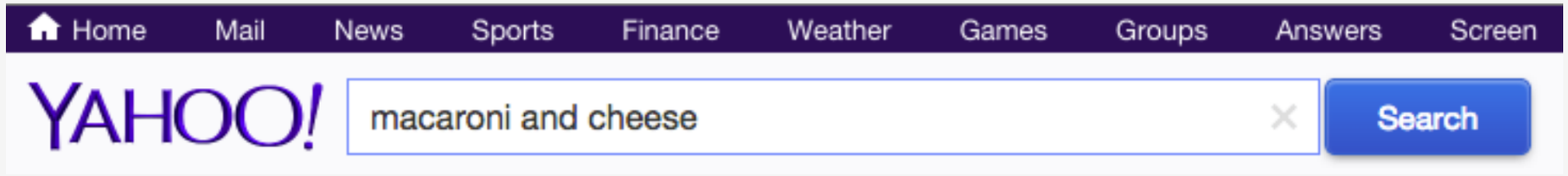

tails 

heads 

true prediction

old@yahoo.com

age: 50	gender: M	SSN: 83...1

Kraft

randomised prediction

tails 

Kraft

heads 

Stouffer's

Randomized Response [Warner, 1965]

Claim:

Randomized response is (log 3)-differentially private

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$\mathcal{A}$ is (ϵ)-differentially private if:

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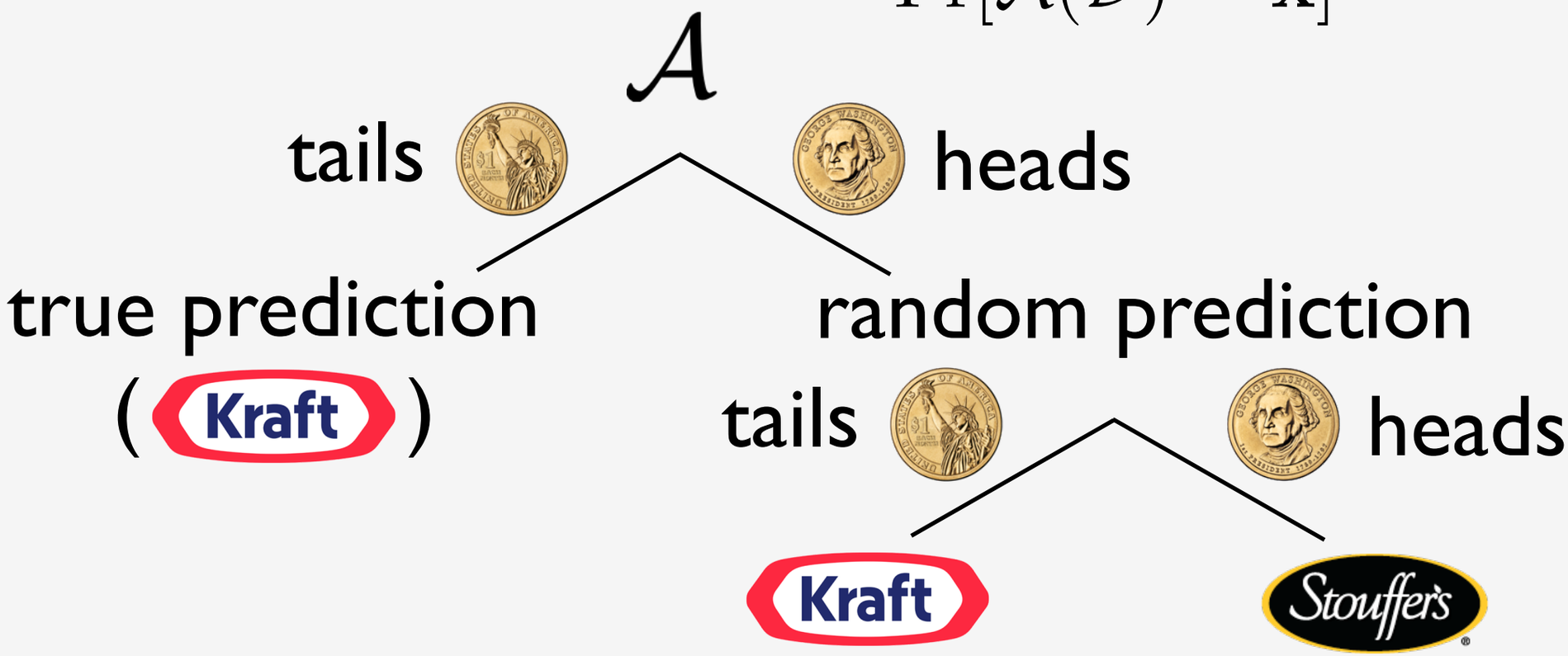

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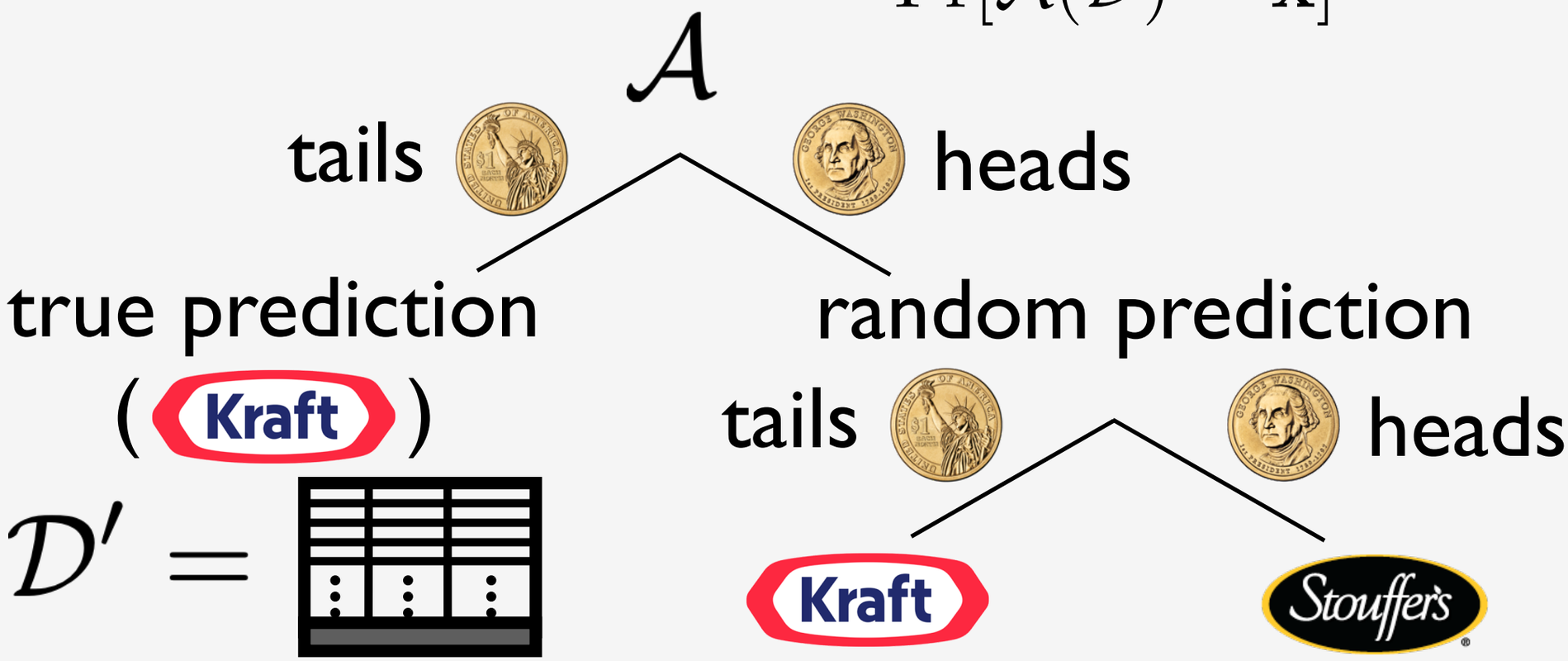

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[Warner, 1965]

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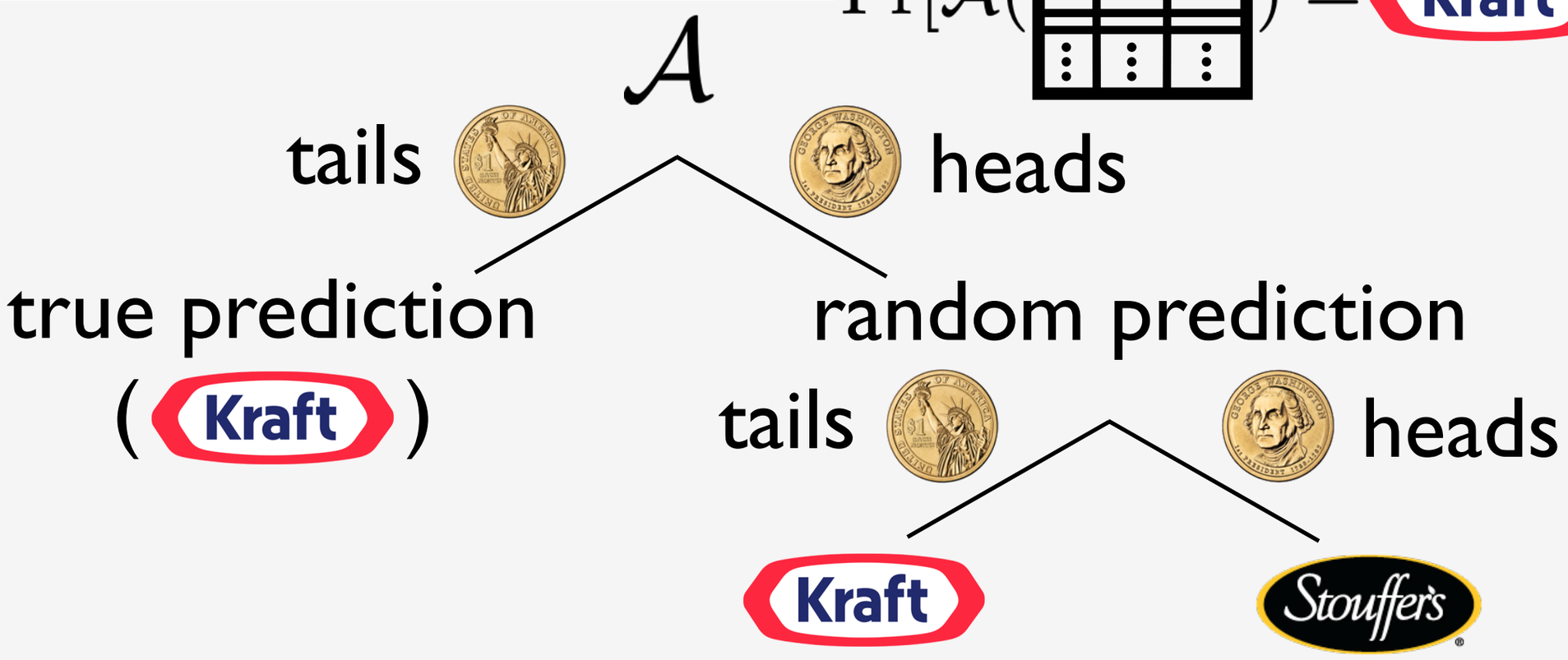
Proof:

$\mathcal{A}$ is (ϵ)-differentially private if:

$$\frac{\Pr[\mathcal{A}(\text{[table]}) = \text{Kraft}]}{\Pr[\mathcal{A}(\text{[table]}) = \text{Kraft}]} \leq e^{\epsilon}$$



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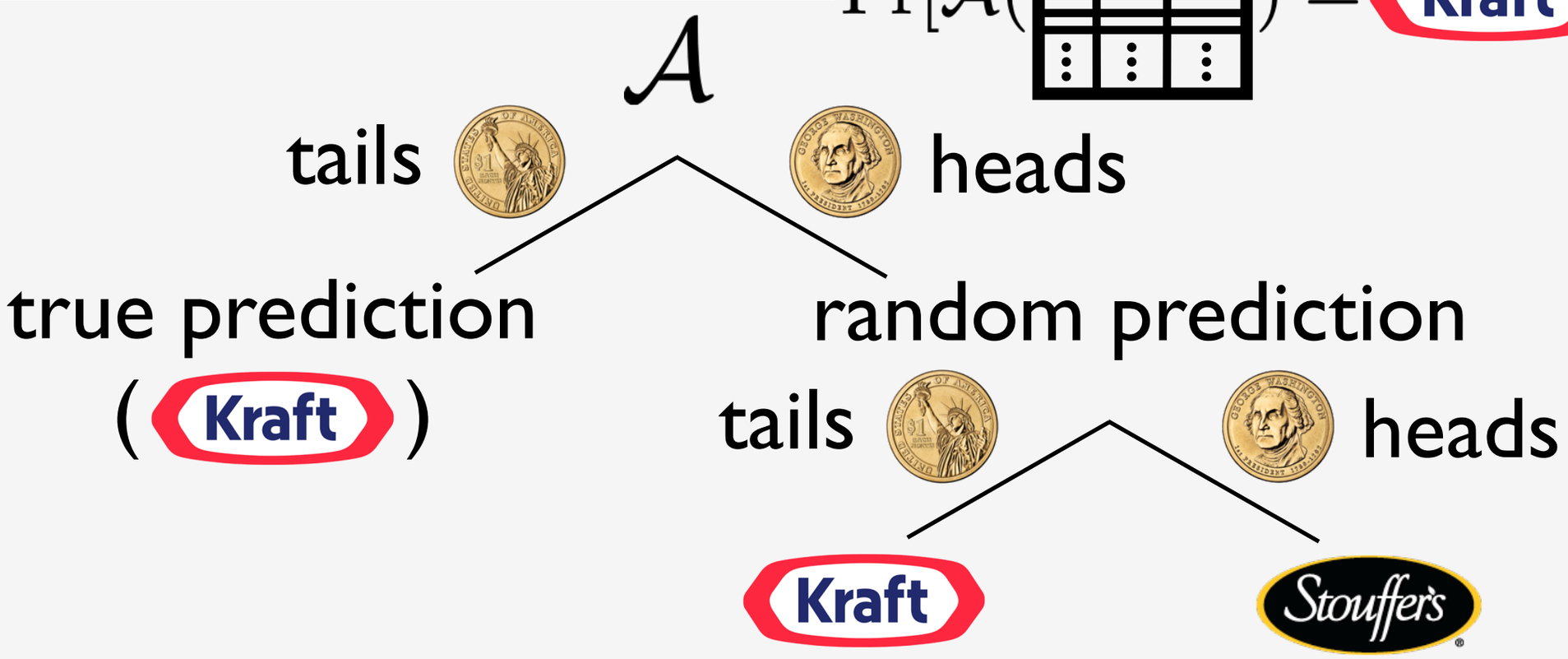

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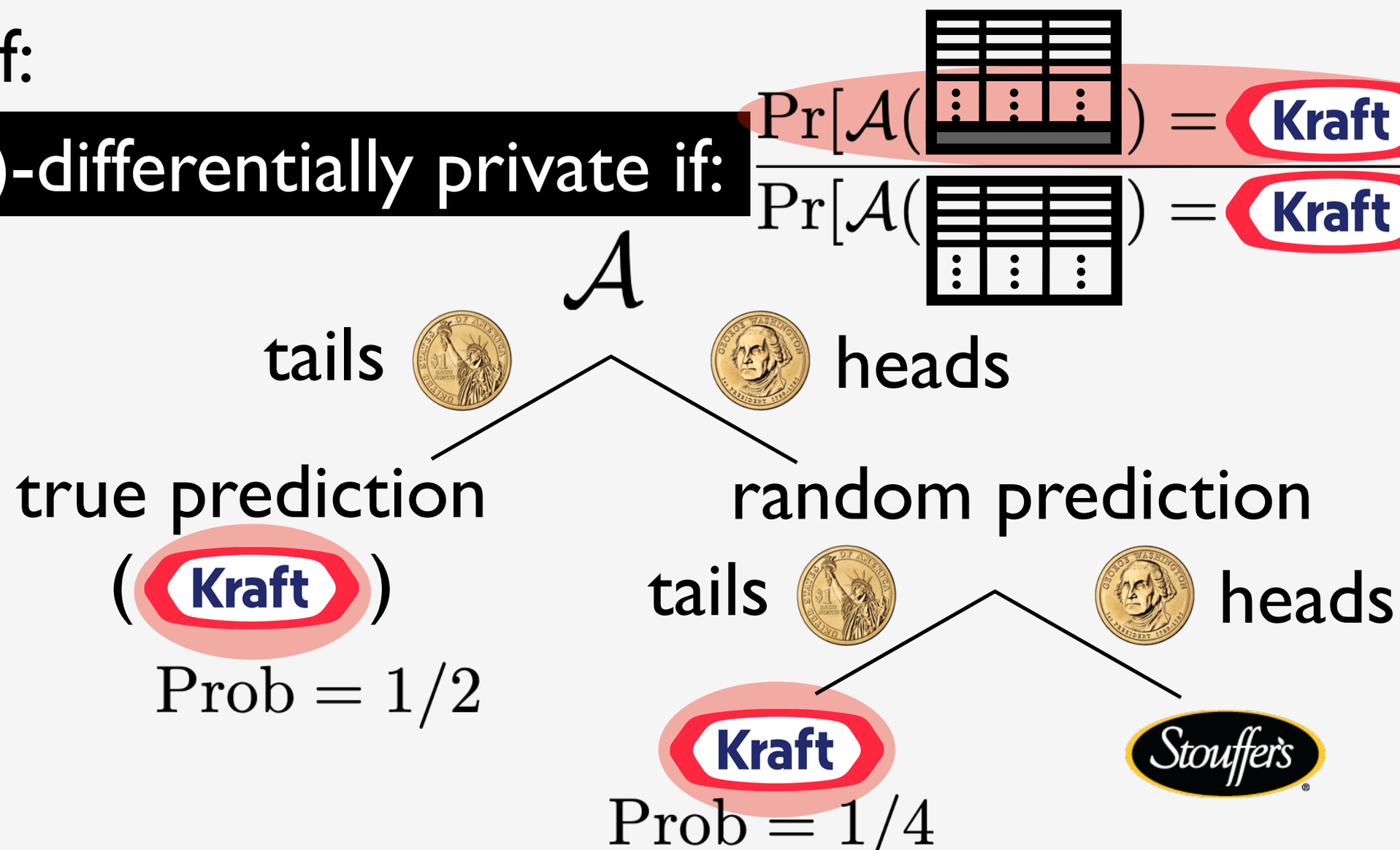


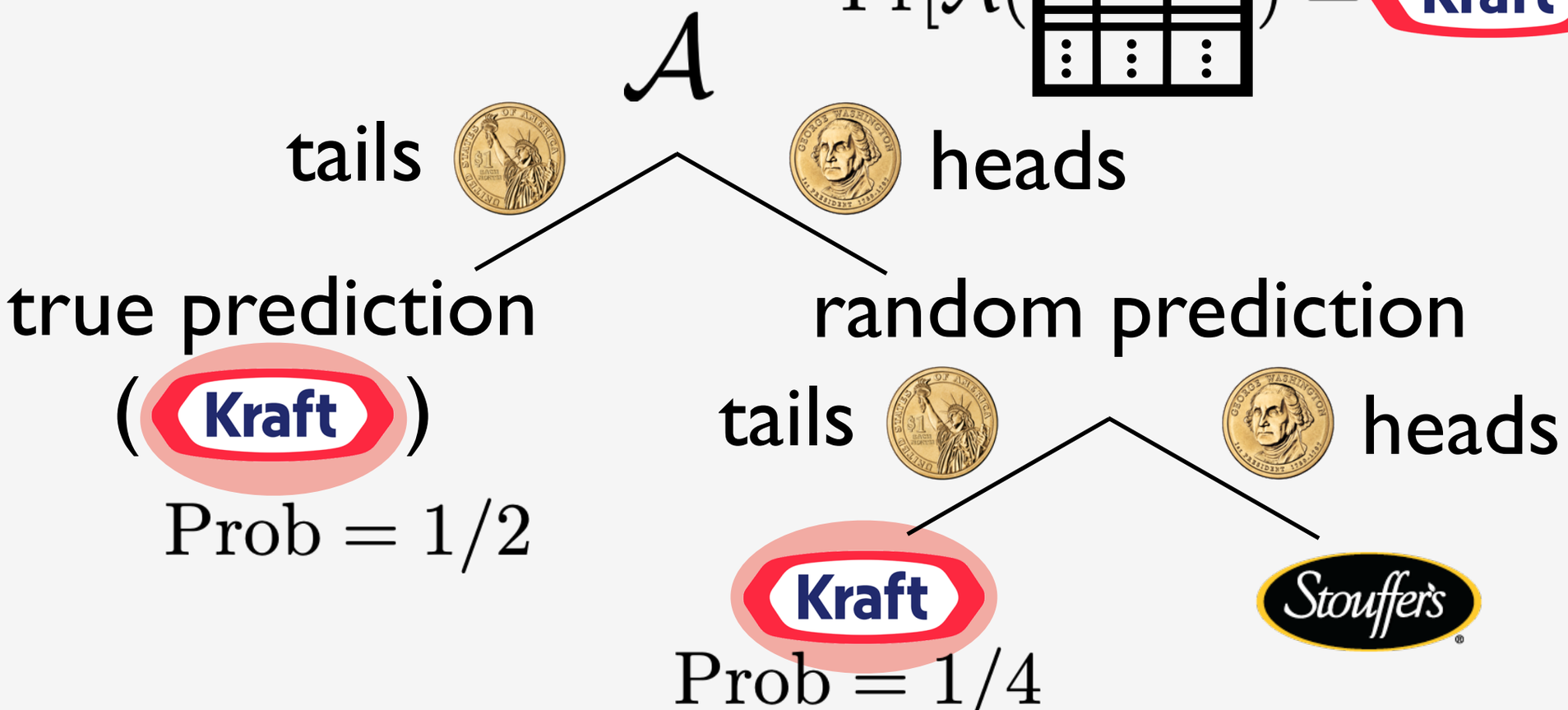
Randomized Response [Warner, 1965]

Claim:

Randomized response is (log 3)-differentially private

Proof:

$\mathcal{A}$ is (ϵ)-differentially private if: $\dfrac{3/4}{\Pr[\mathcal{A}(\text{table}) = \text{Kraft}]} \leq e^{\epsilon}$



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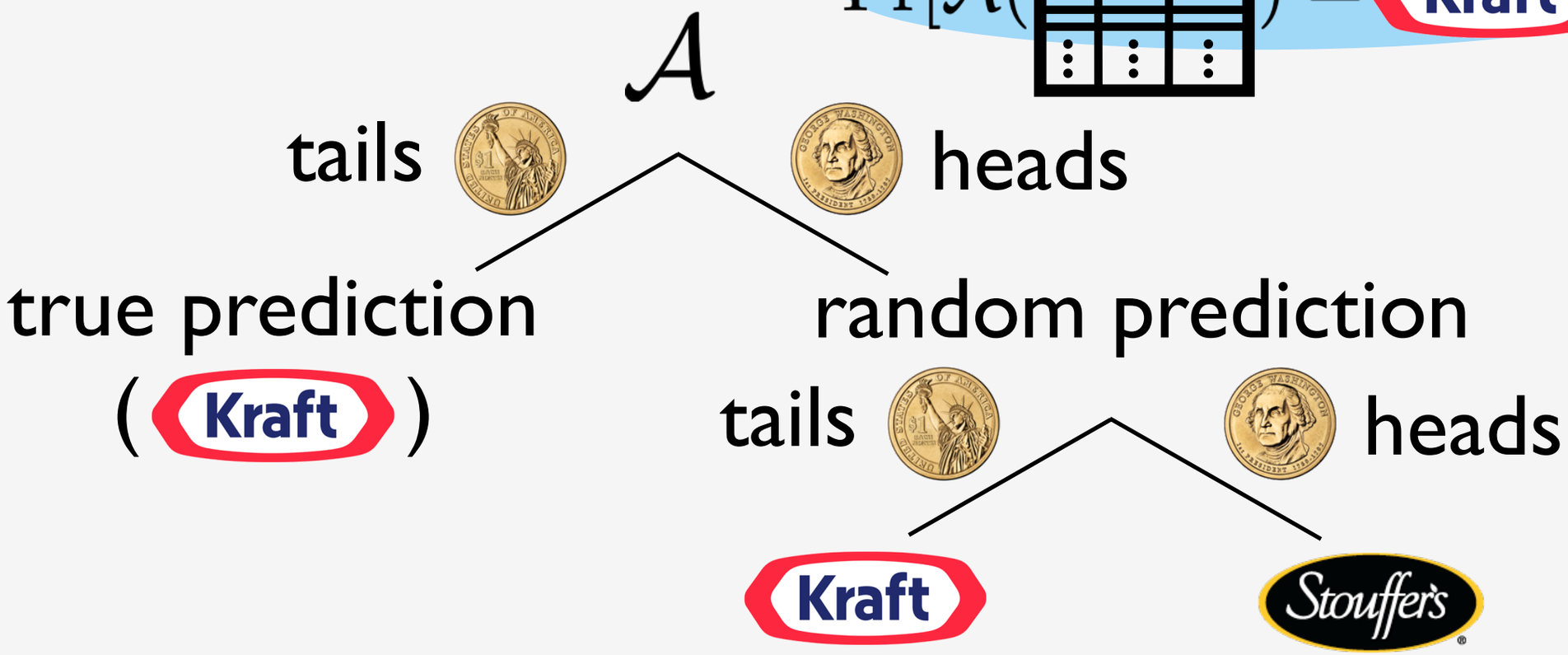

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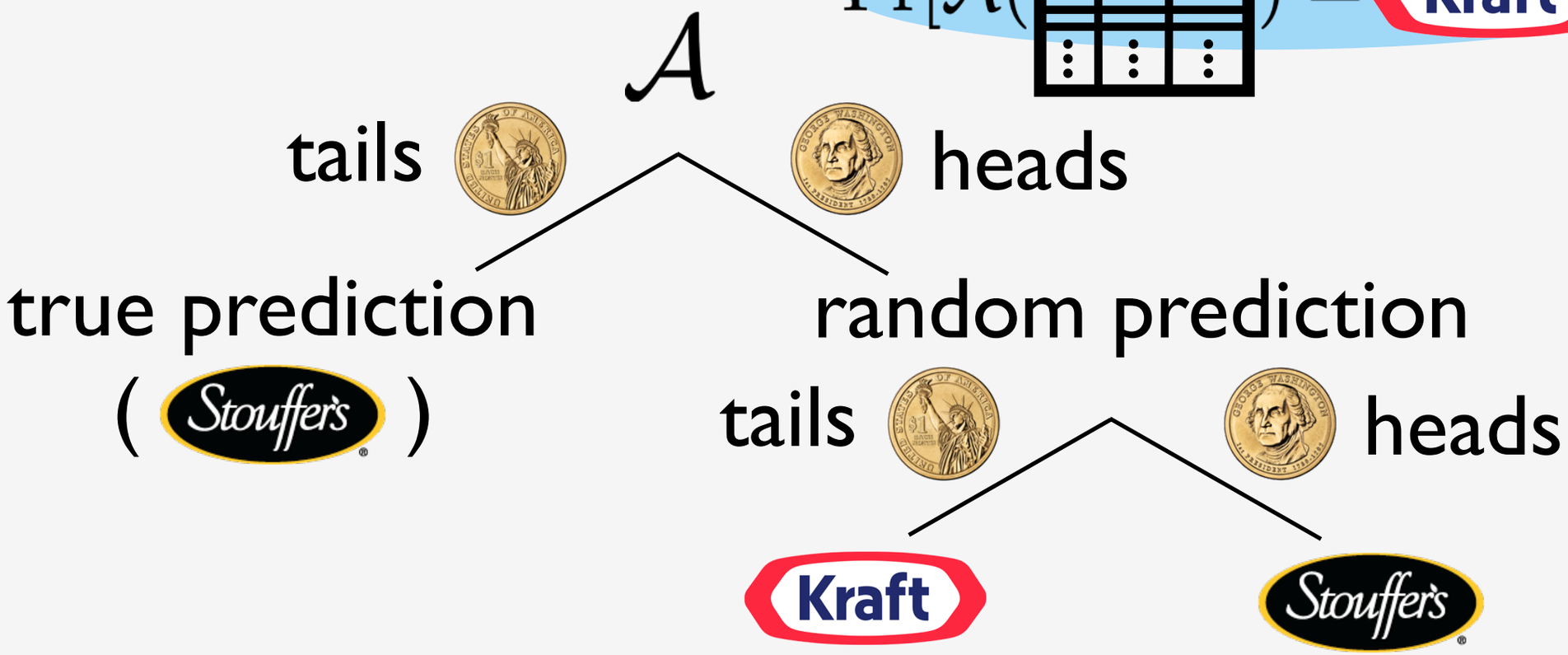

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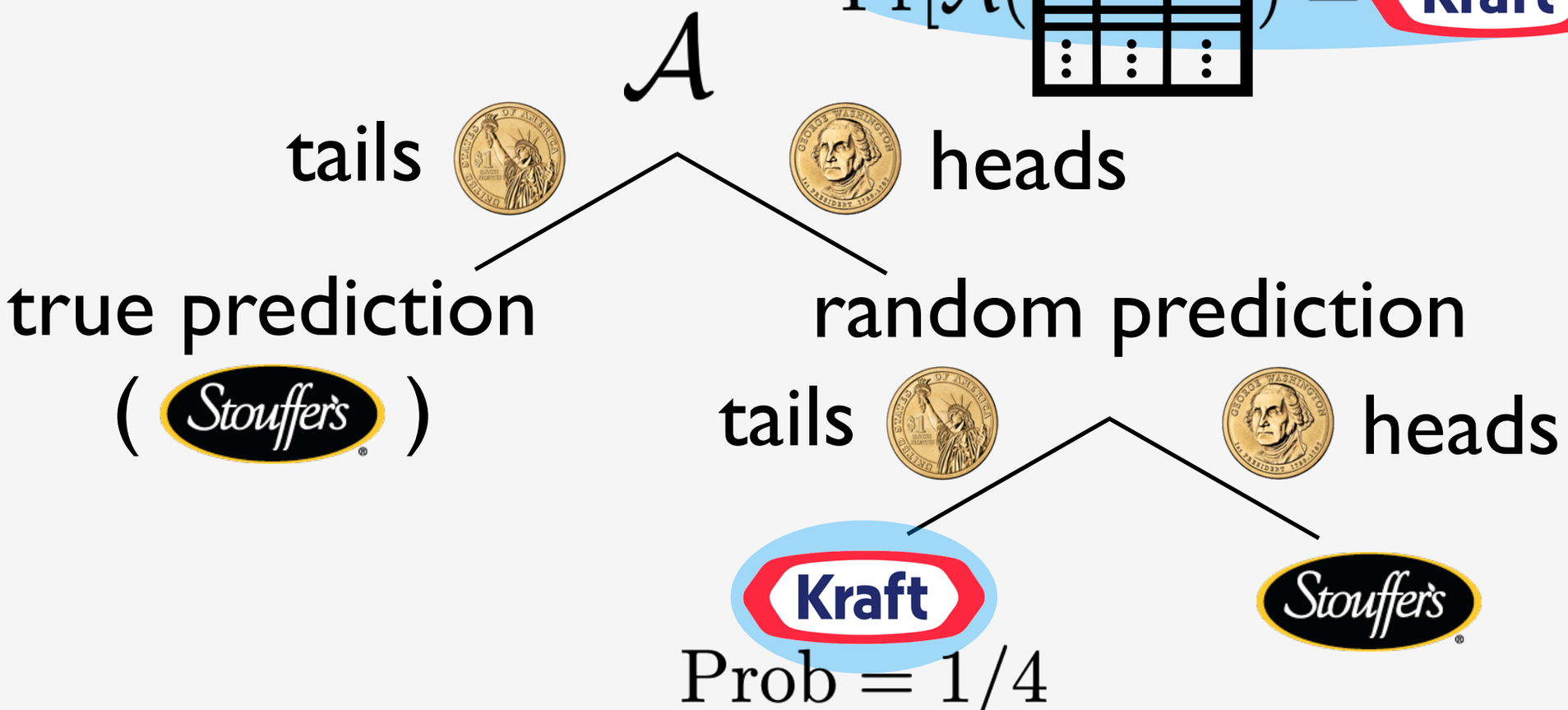

Randomized Response [Warner, 1965]

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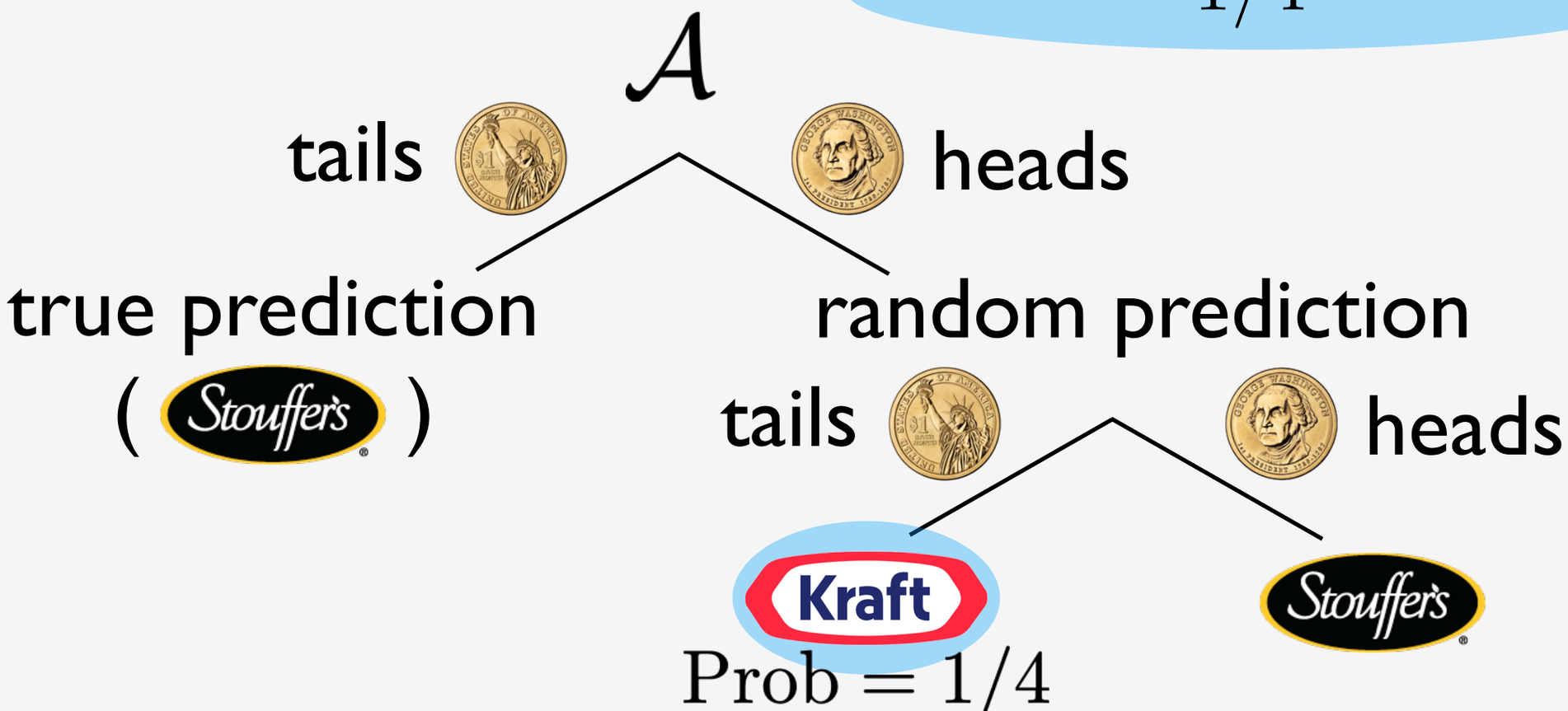
Proof:

$\mathcal{A}$ is (ϵ)-differentially private if:

$$\frac{3/4}{1/4} \leq e^{\epsilon}$$



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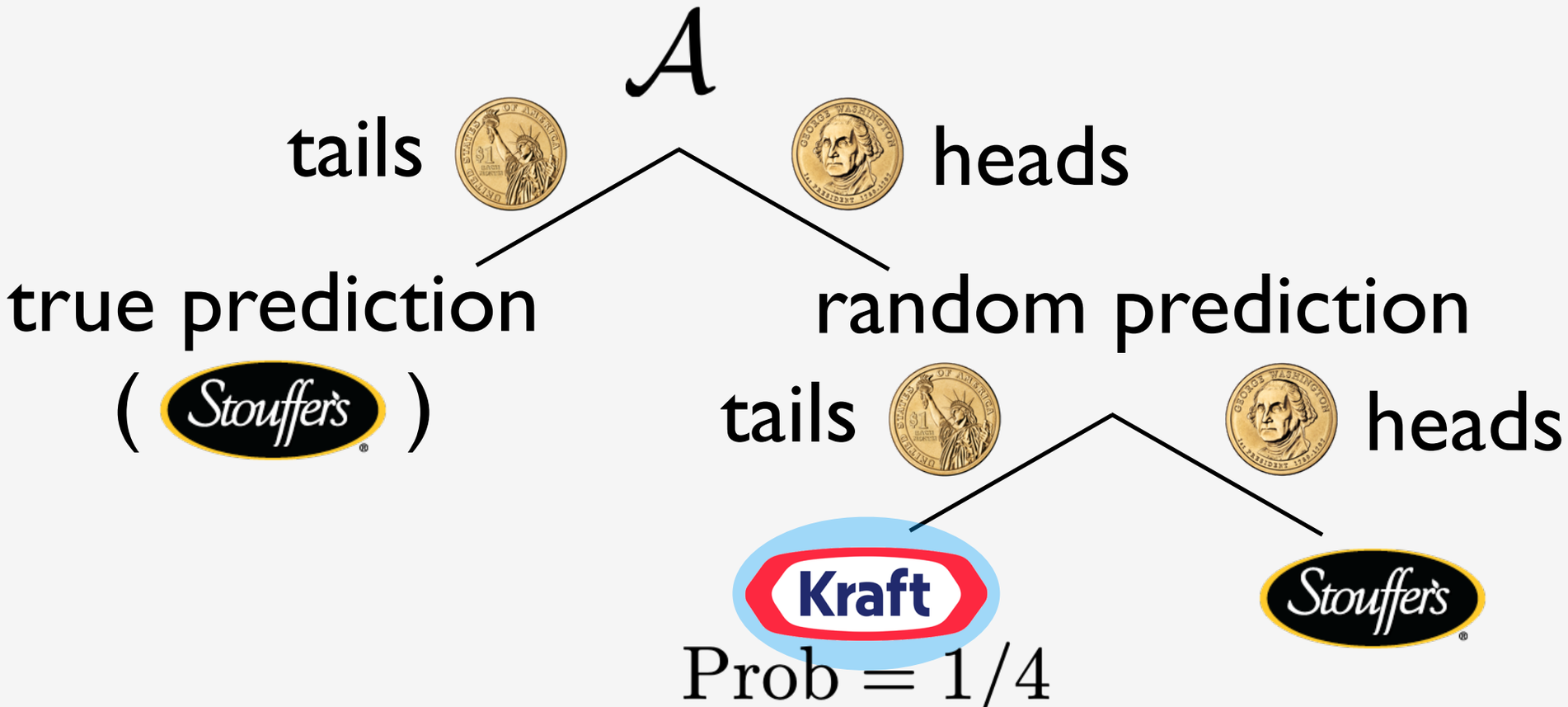
$\mathcal{A}$ is (ϵ)-differentially private if: $\log(3) \leq \epsilon$



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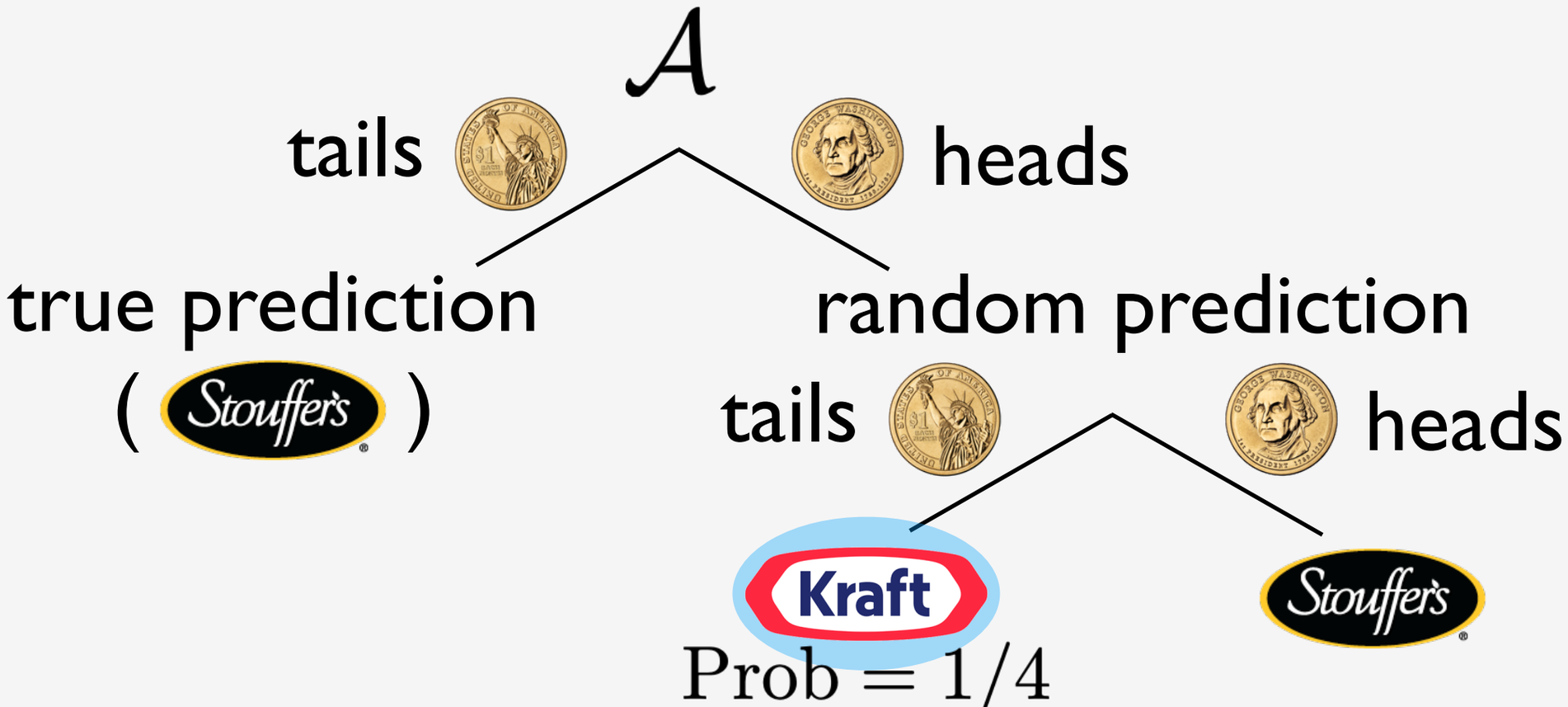
□



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[Dwork et al., 2006]

A formalization of "privacy through randomness"

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Differential Privacy [Dwork et al., 2006]

A formalization of "privacy through randomness"

properties

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3. Immune to common attacks:
   - linkage attacks
   - differencing attacks

4. Doesn't compromise privacy of few for sake of all

This Talk

A Web Search Example

Privacy via Randomization

Differential Privacy

Global Sensitivity

Privacy Mechanisms

- The Laplace Mechanism
- The Gaussian Mechanism
- The Exponential Mechanism

Local Sensitivity

Global Sensitivity

[Dwork et al., 2006]

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How much could $\mathcal{A}$ change?

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Global Sensitivity of $\mathcal{A}$

$$\Delta_{\mathcal{A}} := \max_{D,D' \text{ s.t. } D,D' \text{ diff by one row}} \|\mathcal{A}(\mathcal{D}) - \mathcal{A}(\mathcal{D}')\|$$

Global Sensitivity

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$$\Delta^2_{\mathcal{A}} := \max_{D,D' \text{ s.t. } D,D' \text{ diff by one row}} \|\mathcal{A}(\mathcal{D}) - \mathcal{A}(\mathcal{D}')\|_2$$

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The Laplace Mechanism

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$$\Delta_{\mathcal{A}}^{1} := \max_{D,D' \text{ s.t. } D,D' \text{ diff by one row}} \|\mathcal{A}(\mathcal{D}) - \mathcal{A}(\mathcal{D}')\|_1$$

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1. Compute $\mathcal{A}(\mathcal{D})$

The Laplace Mechanism

[Dwork et al., 2006]

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The Laplace Mechanism

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$$\mathrm{Lap}(r \mid \theta) = \frac{1}{2\theta} \exp\left(-\frac{|r|}{\theta}\right)$$

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[Dwork et al., 2006]

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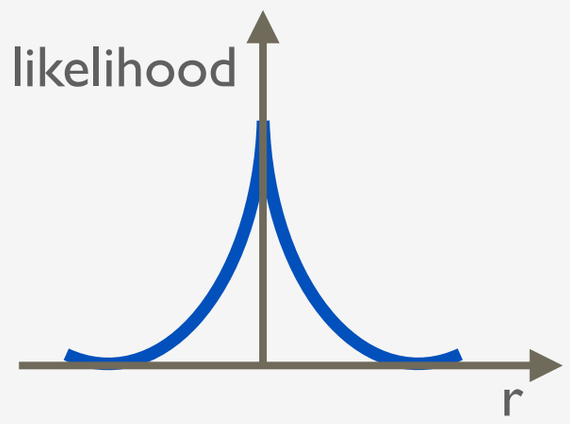

The Laplace Mechanism

[Dwork et al., 2006]

$$\Delta^1_{\mathcal{A}} := \max_{D,D' \text{ s.t. } D,D' \text{ diff by one row}} \|\mathcal{A}(\mathcal{D}) - \mathcal{A}(\mathcal{D}')\|_1$$

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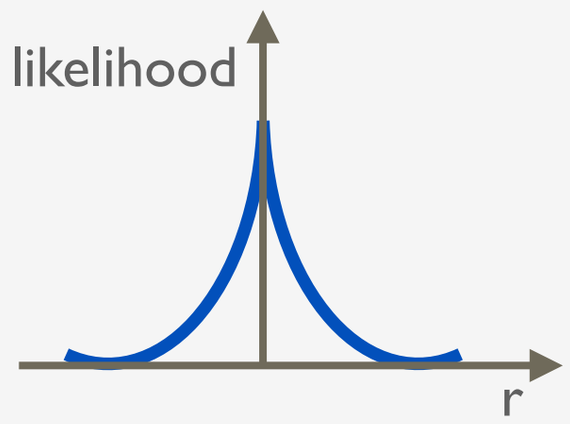

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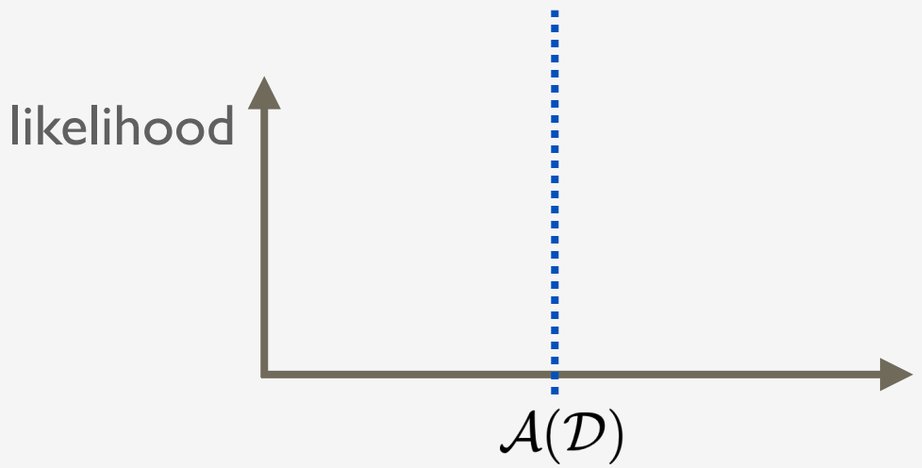

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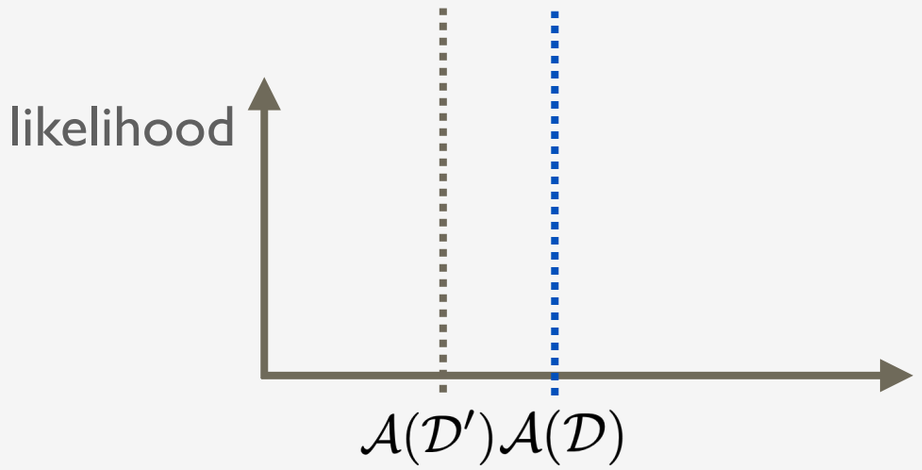

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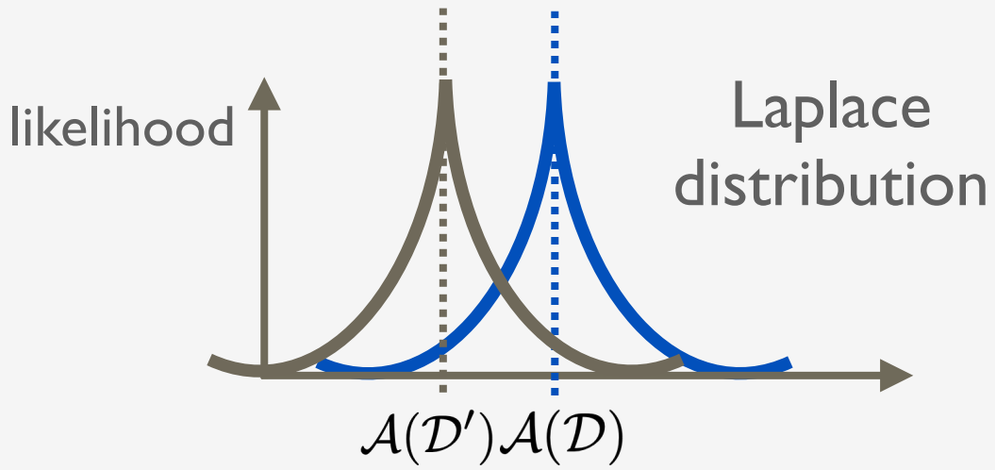

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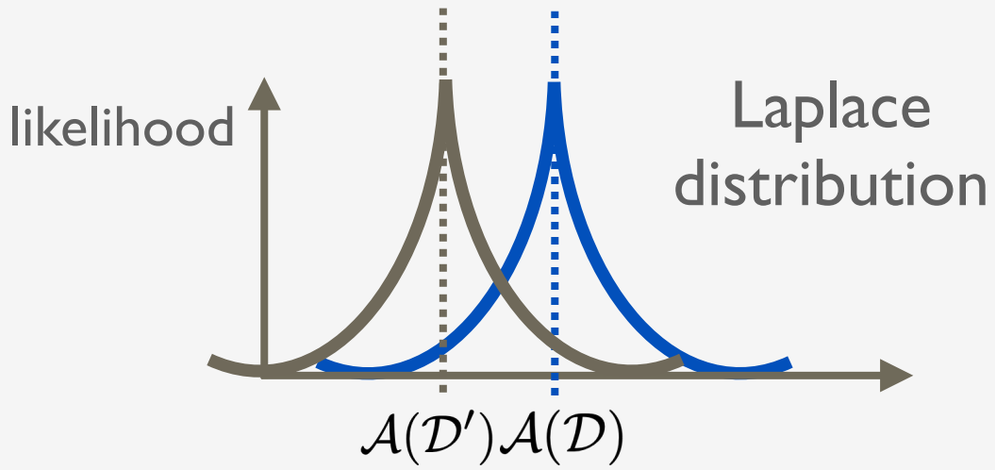

The Laplace Mechanism guarantees $(\epsilon, 0)$ differential privacy

The Gaussian Mechanism

[Dwork & Roth, 2014]

$$\Delta_{\mathcal{A}}^2 := \max_{D,D' \text{ s.t. } D,D' \text{ diff by one row}} \|\mathcal{A}(\mathcal{D}) - \mathcal{A}(\mathcal{D}')\|_2$$

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1. Compute $\mathcal{A}(\mathcal{D})$ — scalar (for simplicity)
2. Draw $r \sim \mathcal{N}(r \mid \Delta_{\mathcal{A}}^2 \sqrt{2 \log(1.25/\delta)}/\epsilon)$
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Gaussian distribution (mean 0)

$$\mathcal{N}(r \mid \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

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This Talk

A Web Search Example

Privacy via Randomization

Differential Privacy

Global Sensitivity

Privacy Mechanisms

The Laplace Mechanism
The Gaussian Mechanism
— Continuous data

The Exponential Mechanism — Discrete data

Local Sensitivity

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[McSherry and Talwar, 2007]

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$$q(\mathcal{D}, \mathbf{x}) \to \mathbb{R}$$

The Exponential Mechanism

[McSherry and Talwar, 2007]

a general mechanism for ϵ-differential privacy

$q(\mathcal{D}, \mathbf{x}) \rightarrow \mathbb{R}$

$\mathbf{x} \in Range(\mathcal{A})$

discrete

The Exponential Mechanism

[McSherry and Talwar, 2007]

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$q(\mathcal{D}, \mathbf{x}) \to \mathbb{R}$ "quality function"

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$q(\mathcal{D}, \mathbf{x}) \to \mathbb{R}$ "quality function"

e.g., score of ranking $\mathbf{x}$ for dataset $\mathcal{D}$

The Exponential Mechanism

[McSherry and Talwar, 2007]

a general mechanism for ϵ-differential privacy

$$q(\mathcal{D}, \mathbf{x}) \to \mathbb{R}$$

"quality function"

ideally...

$$\mathcal{A}(\mathcal{D}) = \arg\max_{\mathbf{x} \in \mathcal{X}} q(\mathcal{D}, \mathbf{x})$$

The Exponential Mechanism

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deterministic!

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[McSherry and Talwar, 2007]

a general mechanism for ϵ-differential privacy

$q(\mathcal{D}, \mathbf{x}) \to \mathbb{R}$ "quality function"

changes in $\mathcal{D}$ may cause changes in q!

Definition. *The* ***global sensitivity*** *of q is defined as,*

$$\Delta_q \triangleq \max_{\mathcal{D},\mathcal{D}',\mathbf{x}} |q(\mathcal{D}, \mathbf{x}) - q(\mathcal{D}', \mathbf{x})|$$

where $\mathcal{D}, \mathcal{D}'$ are neighboring datasets.

The Exponential Mechanism

[McSherry and Talwar, 2007]

a general mechanism for ϵ-differential privacy

$q(\mathcal{D}, \mathbf{x}) \to \mathbb{R}$ "quality function"

Definition. *Given a dataset $\mathcal{D}$ the* ***exponential mechanism*** *$\mathcal{A}$ selects an element $\mathbf{x} \in \mathcal{X}$ with probability,*

$$\Pr[\mathbf{x}] = \exp\left(\frac{\epsilon q(\mathcal{D}, \mathbf{x})}{2\Delta_q}\right)\frac{1}{\mathbf{Z}}$$

The Exponential Mechanism

[McSherry and Talwar, 2007]

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normalizing constant over all $\mathbf{x} \in \mathcal{X}$

The Exponential Mechanism

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$$\mathcal{A}(\mathcal{D}) \approx \arg\max_{\mathbf{x}\in\mathcal{X}} q(\mathcal{D}, \mathbf{x})$$

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Theorem. *The exponential mechanism $\mathcal{A}$ is ϵ-differentially private.*

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Theorem. *Given that $\hat{\mathbf{x}}$ is output by the exponential mechanism, then*

$$q(\mathcal{D}, \hat{\mathbf{x}}) \geq \max_{\mathbf{x} \in \mathcal{X}} q(\mathcal{D}, \mathbf{x}) - \frac{2\Delta_q}{\epsilon}(\log|\mathcal{X}| + t)$$

with probability at least $1 - e^{-t}$.

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Local Sensitivity [Nissim et al., 2007]

Global Sensitivity of $\mathcal{A}$

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Local Sensitivity of $\mathcal{A}$, for specific dataset $\mathcal{D}$

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Multiple DP algorithms use local sensitivity

[Nissim et al., 2007] [Dwork & Lei, 2009] [Smith & Thakurta, 2013] [Dwork & Roth, 2014]

Takeaways

Losing privacy is easier than it seems

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account

old@yahoo.com

age: 50

Differential privacy quantifies

privacy via randomisation

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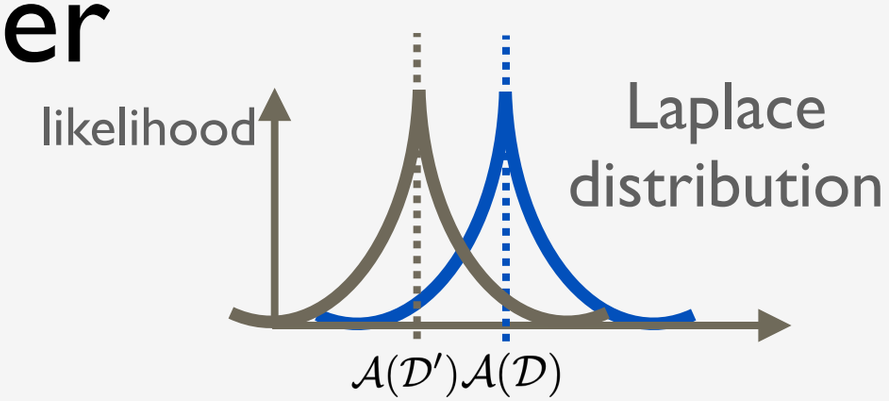
privacy via randomisation

Main idea: add noise on the order of the **global sensitivity**



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Differential privacy quantifies **privacy via randomisation**

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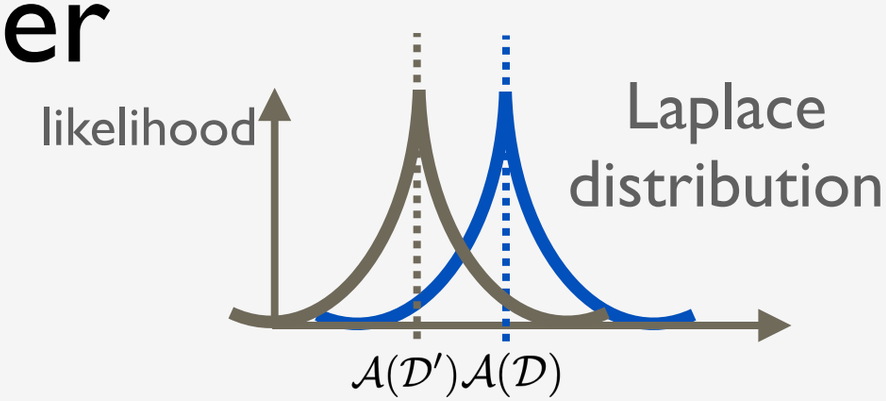

If too much noise, can sometimes use **local sensitivity** instead!

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