

## 5.6

a) We started by counting the occurrence of the letters in the sentence. Using frequency analysis we came to the conclusion that B must represent E. Meaning the key must equal 3. Meaning

EBOBFPXKBKDIFPEPBKQBKZBQEXQZLKQXFKPJLOBLCQEBIBQQBOBQEXKLCXKVL  
QEBOIBQQBO would translate into:  
HEREISANGLISHSENTENCETHATCONTAINSMOREOFTHELETTERETHANOFANYOT  
HERLETTER.

b)

AJTSHJIVFYWXJVRSHOLVKQZWFSKZXUWRTSHZEQIFMGXIFKJPPYQTVWFWHOLRX  
MMWGHIVVYSWNMTFQZRG

AJTSH JIVFY WXJVR SHOLV KQZWF SKZXU WRTSH ZEQIF MGXIF KJPPY QTVWF  
WHOLR XMMWG HIVVY SWNMT FQZRG

We noticed that HIVVY and PEARL give the key SEVEN in the table. Using a hint we knew that we had to look for a word with length 5, so we started comparing the word PEARL with every 5 letter combination from our cipher. Meaning this sentence would translate into:  
IFYOUREABLETOREADTHISMESSAGETHENYOUHAVESUCCESSFULLYPASSEDTHEFI  
RSTPEARLASSIGNMENT

5.7)

14-7-27-27-9-25

letters -> decimals

01110- 00111-11011-11011-01001-11001

decimals -> binary

011-100-011-111-011-110-110-100-111-001

"shift" r=3

3-bits	IV	IV decimal	$EK(IV) = 3 * IV + 2 * 13 \text{ mod } 32$	XOR 3 bits $EK(IV)$
011	10110	22	11100	100
100	11100	28	01110	111
011	01110	14	00100	010
111	00100	4	00110	110
011	00110	6	01100	000
110	01100	12	11110	001
110	11110	30	10100	011
100	10100	20	10110	001
111	10110	22	11100	000
001	11100	28	01110	010

100-111-010-110-000-001-011-001-000-010

10011-10101-10000-00101-10010-00010

19 21 16 5 18 2 -> SUPERB

5.8 )

$$\Phi(n) = (p-1)(q-1)$$

$$N = p \cdot q$$

$$q = N/p$$

$$\Phi(n) = (p-1)(q-1)$$

$$\Phi(n) = pq - p - q + 1$$

$$\Phi(n) = N - p - N/p + 1$$

$$p + N/p = N+1 - \Phi(n) \quad | \cdot p$$

$$p^2 + N = p(N+1 - \Phi(n))$$

$$p^2 - p(N+1 - \Phi(n)) + N = 0$$

$$a = 1, b = N+1 - \Phi(n), c = N$$

$$X = \frac{-b \pm \sqrt{b^2 - 4a}}{2a} \text{ where } d = b^2 - 4a$$

$$p, q = \frac{(N+1 - \Phi(n)) \pm \sqrt{(N+1 - \Phi(n))^2 - 4N}}{2}$$

5.9)

$$a) N = pq = 47 \cdot 61 = 2867$$

$$\Phi(n) = (p-1)(q-1) = (46 \cdot 60) = 2760$$

$$e=7. \text{ Because } \gcd(7, 2760) = 1$$

$$pk = (N, e) = (2867, 7)$$

$$b) e \cdot d \bmod \Phi(n) = 1$$

Knowing that:  $\Phi(n)x + ey = \gcd(\Phi(n), e) \Rightarrow 2760x + 7y = 1$  we should find  $y$  using

Extended Euclidean Algorithm:

q	r	x	y	a	b	x2	x1	y2	y1
0	0	0	0	2760	7	1	0	0	1
394	2	1	-394	7	2	0	1	1	-394

3	1	-3	1183	1	1	1	-3	-394	1183
2	0	-5	-2760	1	0	-3	-5	1183	-2760

So the secret key  $D = 1183$

c)

The ciphertext =  $E(m, e) = m^e \bmod N = 5^7 \bmod 2867 = 716$

d)

$s = 9$ ,  $e = 7$ ,  $d = 395$ ,  $N = 2867$ , and  $m = 813$

$m' = E(s, e) = s^e \bmod N = 9^7 \bmod 2867 = 4\,782\,969 \bmod 2867 = 813 = m \Rightarrow$  the signature  $s=9$  is a valid RSA signature