

Homework 1

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Problems

4.15.5

(a)

$$Performance = ClockRate / CPI$$

$$P1 = 3.0 * 10^9 / 1.5 = 2.0 * 10^9 \text{ IPS}$$

$$P2 = 2.5 * 10^9 / 1.0 = 2.5 * 10^9 \text{ IPS}$$

$$P3 = 4.0 * 10^9 / 2.2 = 1.8 * 10^9 \text{ IPS}$$

Processor P2 has the highest performance expressed in giga instructions per second

(b)

$$Number of cycles = (Clock \text{ Rate}) * (Execution \text{ Time in seconds})$$

$$Number of instructions = (IPS) * (Execution \text{ Time in seconds})$$

For P1

$$\text{-Number of cycles} = (3.0 * 10^9) * (10) = 3.0 * 10^{10} \text{ cycles}$$

$$\text{-Number of instructions} = (2.0 * 10^9) * (10) = 2.0 * 10^{10} \text{ instructions}$$

For P2

$$\text{-Number of cycles} = (2.5 * 10^9) * (10) = 2.5 * 10^{10} \text{ cycles}$$

$$\text{-Number of instructions} = (2.5 * 10^9) * (10) = 2.5 * 10^{10} \text{ instructions}$$

For P3

$$\text{-Number of cycles} = 4.0 * 10^9 * (10) = 4.0 * 10^{10}$$

$$\text{-Number of instructions} = (1.8 * 10^9) * (10) = 1.8 * 10^{10}$$

(c)

$$Clock \text{ Rate} = ((Total \text{ number of instructions}) * (CPI)) / (Execution \text{ Time})$$

For P1

$$\text{-Total number of instructions} = 2.0 * 10^{10}$$

$$\text{-CPI} = 1.2 * 1.5 = 1.8$$

$$\text{-Execution Time} = 0.7 * 10 = 7 \text{ seconds}$$

$$Clock \text{ Rate} = ((2.0 * 10^{10}) * 1.8) / 7 = 5.14 \text{ GHZ}$$

For P2

$$\text{-Total number of instructions} = 2.5 * 10^{10}$$

$$\text{-CPI} = 1.2 * 1 = 1.2$$

-Execution Time = $0.7 * 10 = 7$ seconds
 Clock Rate = $((2.5 * 10^{10}) * 1.2) / 7 = 4.28$ GHZ
 For P3
 -Total number of instructions = $1.8 * 10^{10}$
 -CPI = $1.2 * 2.2 = 2.64$
 -Execution Time = $0.7 * 10 = 7$ seconds
 Clock Rate = $((1.8 * 10^{10}) * 2.64) / 7 = 6.79$ GHZ

4.15.8

(a)

Execution Time = Instruction Count * CPI * Clock Cycle Time
 Average CPI = (Execution Time) / (Instruction Count * Clock Cycle Time)
 Compiler A - $1.1 / 1.0e^9 * 1 = 1.1$
 Compiler B - $1.5 / 1.2e^9 * 1 = 1.25$

(b)

Execution Time = Instruction Count * CPI * Clock Cycle Time
 Execution Time / Clock Cycle Time = Instruction Count * CPI
 $1 / \text{Clock Cycle Time} = (\text{Instruction Count} * \text{CPI}) / \text{Execution Time}$
 $1 / \text{Clock Cycle Time} = \text{Clock Rate}$
 Clock Rate = (Instruction Count * CPI) / Execution Time
 Same CPU time so allow Execution Time to be 1
 Clock Rate = CR
 $CR_B / CR_A = (\text{Instruction Count}_B * CPI_B) / (\text{Instruction Count}_A * CPI_A)$
 $CR_B / CR_A = (1.2e^9 * 1.25) / (1.0e^9 * 1.1) \approx 1.37$
 B is 1.37 times faster, so A is .73 times slower

(c)

Speedup = $\frac{\text{Execution Time}_A}{\text{Execution Time}_{new}}$
 $\frac{\text{Execution Time}_A}{\text{Execution Time}_{new}} = \frac{(\text{Instruction Count}_A * CPI_A)}{(\text{Instruction Count}_{new} * CPI_{new})} = (1.0e^9 * 1.1) / (6.0e^8) \approx 1.67$ times faster
 $\frac{\text{Execution Time}_B}{\text{Execution Time}_{new}} = \frac{(\text{Instruction Count}_B * CPI_B)}{(\text{Instruction Count}_{new} * CPI_{new})} = (1.2e^9 * 1.25) / (6.0e^8) \approx 2.27$ times faster

4.5.10

(a)

Processors - 1

Execution Time = Clock Cycle / Clock Rate

Clock Rate = 2GHz = $2e9$

Clock Cycles = $(CPI_{arithmetic} * Instructions_{arithmetic}) + (CPI_{load/store} * Instructions_{load/store}) + (CPI_{branch} * Instructions_{branch})$

$$\text{Clock Cycles} = \frac{(2.56e9*1)}{0.7*1} + \frac{(1.28e9*12)}{0.7*1} + (2.56e8 * 5)$$

$$\text{Clock Cycles} = (3.66e9) + (2.194e10) + (1.28e9) = 26.88e9 \text{ cycles}$$

$$\text{Execution Time} = \frac{26.88e9 \text{ cycles}}{2e9 \text{ cycles/sec}} = 13.44 \text{ seconds}$$

$$\text{Relative Speedup} = \frac{Processors-1}{Processors-p} = \frac{13.44}{13.44} = 1$$

(Processors - 2)

$$\text{Clock cycles} = \frac{(2.56e9*1)}{0.7*2} + \frac{(1.28e9*12)}{0.7*2} + (2.56e8 * 5)$$

$$\text{Clock cycles} = \frac{2.56e9}{1.4} + \frac{1.536e10}{1.4} + (1.28e9)$$

$$\text{Clock cycles} = 1.404e10 \text{ cycles}$$

$$\text{Execution Time} = \frac{1.404e10 \text{ cycles}}{2e9 \text{ cycles/sec}} = 7.02 \text{ seconds}$$

$$\text{Relative Speedup} = \frac{Processors-1}{Processors-p} = \frac{13.44}{7.02} = 1.91$$

Processors - 4

$$\text{Clock cycles} = \frac{(2.56e9*1)}{0.7*4} + \frac{(1.28e9*12)}{0.7*4} + (2.56e8 * 5)$$

$$\text{Clock cycles} = \frac{2.56e9}{2.8} + \frac{1.536e10}{2.8} + (1.28e9)$$

$$\text{Clock cycles} = 7.22e9 \text{ cycles}$$

$$\text{Execution Time} = \frac{7.22e9 \text{ cycles}}{2e9 \text{ cycles/sec}} = 3.86 \text{ seconds}$$

$$\text{Relative Speedup} = \frac{Processors-1}{Processors-p} = \frac{13.44}{3.86} = 3.48$$

Processors - 8

$$\text{Clock cycles} = \frac{(2.56e9*1)}{0.7*8} + \frac{(1.28e9*12)}{0.7*8} + (2.56e8 * 5)$$

$$\text{Clock cycles} = \frac{2.56e9}{5.6} + \frac{1.536e10}{5.6} + (1.28e9)$$

$$\text{Clock cycles} = 4.5e9 \text{ cycles}$$

$$\text{Execution Time} = \frac{4.5e9 \text{ cycles}}{2e9 \text{ cycles/sec}} = 2.25 \text{ seconds}$$

$$\text{Relative Speedup} = \frac{Processors-1}{Processors-p} = \frac{13.44}{2.25} = 4.27$$

(b)

Processors - 1

Execution Time = Clock Cycle / Clock Rate

Clock Rate = 2 Ghz = $2e9$

$$\text{Clock Cycle} = (2.56e9 * 2) + (1.28e9 * 12) + (2.56e8 * 5)$$

$$\text{Clock Cycle} = (5.12e9) + (1.536e10) + (1.28e9) = 21.76e9$$

$$\text{Execution Time} = \frac{21.76e9 \text{ cycles}}{2e9 \text{ cycles/sec}} = 10.88 \text{ seconds}$$

Processors - 2

$$\text{Execution Time} = \text{Clock Cycle} / \text{Clock Rate}$$

$$\text{Clock Rate} = 2 \text{ Ghz} = 2e9$$

$$\text{Clock Cycle} = \frac{(2.56e9*2)}{0.7*2} + \frac{(1.28e9*12)}{0.7*2} + (2.56e8 * 5)$$

$$\text{Clock Cycle} = (3.657e9) + (1.097e10) + (1.280e9) = 15.94e9$$

$$\text{Execution Time} = \frac{15.907e9 \text{ cycles}}{2e9 \text{ cycles/sec}} = 7.954 \text{ seconds}$$

Processors - 4

$$\text{Execution Time} = \text{Clock Cycle} / \text{Clock Rate}$$

$$\text{Clock Rate} = 2 \text{ Ghz} = 2e9$$

$$\text{Clock Cycle} = \frac{(2.56e9*2)}{0.7*4} + \frac{(1.28e9*12)}{0.7*4} + (2.56e8 * 5)$$

$$\text{Clock Cycle} = (1.829e9) + (5.486e9) + (1.280e9) = 8.595e9$$

$$\text{Execution Time} = \frac{8.595e9 \text{ cycles}}{2e9 \text{ cycles/sec}} = 4.298 \text{ seconds}$$

Processors - 8

$$\text{Execution Time} = \text{Clock Cycle} / \text{Clock Rate}$$

$$\text{Clock Rate} = 2 \text{ Ghz} = 2e9$$

$$\text{Clock Cycle} = \frac{(2.56e9*2)}{0.7*8} + \frac{(1.28e9*12)}{0.7*8} + (2.56e8 * 5)$$

$$\text{Clock Cycle} = (9.142e8) + (2.743e9) + (1.280e9) = 4.937e9$$

$$\text{Execution Time} = \frac{4.937e9 \text{ cycles}}{2e9 \text{ cycles/sec}} = 2.469 \text{ seconds}$$

(c)

Execution Time for 4 Processors

$$\text{Clock cycles} = \frac{(2.56e9*1)}{0.7*4} + \frac{(1.28e9*12)}{0.7*4} + (2.56e8 * 5)$$

$$\text{Clock cycles} = \frac{2.56e9}{2.8} + \frac{1.536e10}{2.8} + (1.28e9)$$

$$\text{Clock cycles} = 7.22e9 \text{ cycles}$$

$$\text{Execution Time} = \frac{7.22e9 \text{ cycles}}{2e9 \text{ cycles/sec}} = 3.86 \text{ seconds}$$

Find new CPI for single processor

x will represent the new CPI for load/store instructions that we will find

$$\text{Clock Cycles} = (2.56e9 * 1) + (1.28e9 * x) + (2.56e8 * 5)$$

$$\text{Clock Cycles} = (2.56e9) + (1.28e9 * x) + (1.28e9)$$

$$\text{Clock Cycles} = (3.84e9) + (1.28e9 * x)$$

$$\text{Execution Time} = \frac{3.84e9 + (1.28e9 * x)}{2e9}$$

$$3.86 = \frac{3.84e9}{2e9} + \frac{1.28e9 * x}{2e9}$$

$$3.86 = 1.92 + .64 * x$$

$$x = 3.03$$

$$\frac{3.03}{12} = .25 = 25\%$$

The CPI of load/store instructions should be reduced by 25% in order for a single processors to match the performance of four processors using its original CPI value

4.15.13

(a)

$$\text{Execution Time} = \text{Clock Cycles} / \text{Clock Rate}$$

$$\text{Clock Cycles} = (\text{CPI} * \text{Instructions})$$

$$\text{P1 Clock Rate} = 4\text{GHz} = 4.0e9$$

$$\text{P1 CPI} = 0.9$$

$$\text{P1 Instructions} = 5.0e9$$

$$\text{P2 Clock Rate} = 3\text{GHz} = 3.0e9$$

$$\text{P2 CPI} = 0.75$$

$$\text{P2 Instructions} = 1.0e9$$

$$\text{Execution Time}_{P1} = \frac{0.9 * 5.0e9}{4.0e9} = \frac{4.5e9}{4.0e9} = 1.125 \text{ seconds}$$

$$\text{Execution Time}_{P2} = \frac{0.75 * 1.0e9}{3.0e9} = \frac{0.75e9}{3.0e9} = 0.25 \text{ seconds}$$

We can see that this fallacy is not true for P1 and P2, although P1 has a higher clock rate, it is slower than P2 as shown above

(b)

Find P1 execution time

$$\text{Execution Time} = \text{Clock Cycles} / \text{Clock Rate}$$

$$\text{Clock Cycles} = (\text{CPI} * \text{Instructions})$$

$$\text{P1 Clock Rate} = 4\text{GHz} = 4.0e9$$

$$\text{P1 CPI} = 0.9$$

$$\text{P1 Instructions} = 1.0e9$$

$$\text{Execution Time}_{P1} = \frac{0.9 * 1.0e9}{4.0e9} = .225 \text{ seconds}$$

Find the numbers of instructions P2 can execute

Let x represent the numbers of instructions P2 can execute

$$\text{P2 Clock Rate} = 3\text{GHz} = 3.0e9$$

$$\text{P2 CPI} = 0.75$$

$$\text{P2 Instructions} = 1.0e9$$

$$\text{Execution Time}_{P1} = \frac{(0.75 * x)}{3.0e9}$$

$$.225 = \frac{.75 * x}{3.0e9}$$

$$0.675e9 = .75 * x$$

$$0.9e9 = x$$

$$x = 9.0e8$$

P2 can execute 9.0e8 instructions in the same time that P1 executes 1.0e9 instructions

(c)

$$\text{MIPS} = \text{Number of Instructions} / \text{Execution Time} * 10^6$$

$$\text{MIPS}_{P1} = \frac{5.0e9}{1.125e6} = 4.44e3$$

$$\text{MIPS}_{P2} = \frac{1.0e9}{0.25e6} = 4.0e3$$

This fallacy is not true for P1 and P2, although P1 has a bigger MIPS, it is slower than P2 regarding performance

(d)

$$\text{MFLOPS} = \# \text{ FP Operations} / \text{Execution Time} * 1.0e6$$

$$\# \text{ FP Operations}_{P1} = 5.0e9 * .40 = 2.0e9$$

$$\# \text{ FP Operations}_{P2} = 1.0e9 * .40 = 4.0e8$$

$$\text{MLOPS}_{P1} = \frac{2.0e9}{1.125*1.0e6} = 1.77e3$$

$$\text{MLOPS}_{P2} = \frac{4.0e8}{.25*1.0e6} = 1.6e3$$

4.15.14

Total Time = 250 seconds

FP Instructions Time = 70 seconds

Load/Store Instructions Time = 85 seconds

Branch Instruction Time = 40 seconds

(a)

Reduced time for FP instructions = 70 * .80 = 56 seconds

Total Time = 56 + (250 - 70) = 236 seconds

Total Time Reduced = 250 - 236 = 14 seconds

$$\frac{14}{250} * 100 = 5.6\%$$

Reducing FP operations by 20% will cause a 5.6% reduction in total time

(b)

Original Int Time = 250 - 70 - 85 - 40 = 55

Total Time reduced by 20% = 250 * (1 - .20) = 200 seconds

Total Time = FP time + l/s time + branch time + new int time

$$200 = 70 + 85 + 40 + \text{new int time}$$

new int time = 5 seconds

$$\frac{5}{55} * 100 = 9.09\%$$

Int operation time is reduced by 90.9% when total time is reduced by 20%

(c)

Total Time reduced by 20% = 250 * (1 - .20) = 200 seconds

Let x represent branch time

$$200 = 70 + 85 + x + 55$$

$$200 = 210 + x$$

$$x = -10$$

We see here that in order for Total time to be reduced by 20%, branch time will have to go into the negatives, which is not possible. Thus the total time cannot be reduced by 20% by reducing only the time for branch instructions.

4.15.15

(a)

$$\text{Execution Time} = \text{Clock Cycles} / \text{Clock Rate}$$

$$\text{Clock Rate} = 2\text{Ghz} = 2e9$$

$$\text{Clock Cycles} = (CPI_{FP} * Instructions_{FP}) + (CPI_{INT} * Instructions_{INT}) + (CPI_{L/S} * Instructions_{L/S}) + (CPI_{branch} * Instructions_{branch})$$

$$\text{Clock Cycles} = (50e6 * 1) + (110e6 * 1) + (80e6 * 4) + (16e6 * 2)$$

$$\text{Clock Cycles} = 50e6 + 110e6 + 320e6 + 32e6$$

$$\text{Clock Cycles} = 512e6$$

$$\text{Execution Time} = \frac{512e6}{2e9} = 256e_{-3} = 0.256 \text{ seconds}$$

$$\text{Execution Time two times faster} = \frac{.256}{2} = 0.128 \text{ seconds}$$

Let x represent what FP instructions average CPI has to be to make execution time 2 times faster

$$0.128 = \frac{(50e6*x) + (110e6*1) + (80e6*4) + (16e6*2)}{2e9}$$

$$0.128 = \frac{(50e6*x) + 110e6 + 320e6 + 32e6}{2e9}$$

$$0.128 = \frac{(50e6*x) + 462e6}{2e9}$$

$$0.128 = \frac{50e6*x}{2e9} + \frac{462e6}{2e9}$$

$$0.128 = \frac{50e6*x}{2e9} + .231$$

$$-0.103 = \frac{50e6*x}{2e9}$$

$$-0.103 = 25e_{-3} * x$$

$$-0.103 = 0.025 * x$$

$$x = -4.12$$

Since x comes out to be negative, it is impossible to make the program run two times faster by only improving CPI of FP instructions

(b)

$$\text{Execution Time} = \text{Clock Cycles} / \text{Clock Rate}$$

$$\text{Clock Rate} = 2\text{Ghz} = 2e9$$

$$\text{Clock Cycles} = (CPI_{FP} * Instructions_{FP}) + (CPI_{INT} * Instructions_{INT}) + (CPI_{L/S} * Instructions_{L/S}) + (CPI_{branch} * Instructions_{branch})$$

$$\text{Clock Cycles} = (50e6 * 1) + (110e6 * 1) + (80e6 * 4) + (16e6 * 2)$$

$$\text{Clock Cycles} = 50e6 + 110e6 + 320e6 + 32e6$$

$$\text{Clock Cycles} = 512e6$$

$$\text{Execution Time} = \frac{512e6}{2e9} = 256e_{-3} = 0.256 \text{ seconds}$$

$$\text{Execution Time two times faster} = \frac{.256}{2} = 0.128 \text{ seconds}$$

Let x represent what L/S instructions average CPI has to be to make execution time 2 times faster

$$0.128 = \frac{(50e6*1) + (110e6*1) + (80e6*x) + (16e6*2)}{2e9}$$

$$0.128 = \frac{50e6 + 110e6 + 32e6 + (80e6*x)}{2e9}$$

$$0.128 = \frac{(80e6*x) + 192e6}{2e9}$$

$$0.128 = \frac{(80e6*x)}{2e9} + \frac{192e6}{2e9}$$

$$0.128 = \frac{(80e6*x)}{2e9} + .096$$

$$0.032 = .040 * x$$

$$x = 0.8$$

$$\frac{4}{8} * 100 = 500\%$$

We would have to improve CPI of L/S Instructions by 500% if we want the program to run two times faster

(c)

$$\text{New } CPI_{INT} = 1 * (1 - .40) = .6$$

$$\text{New } CPI_{FP} = 1 * (1 - .40) = .6$$

$$\text{New } CPI_{L/S} = 4 * (1 - .30) = 2.8$$

$$\text{New } CPI_{Branch} = 2 * (1 - .30) = 1.4$$

$$\text{Execution Time} = \text{Clock Cycles} / \text{Clock Rate}$$

$$\text{Clock Rate} = 2\text{Ghz} = 2e9$$

$$\text{Clock Cycles} = (CPI_{FP} * Instructions_{FP}) + (CPI_{INT} * Instructions_{INT}) + (CPI_{L/S} * Instructions_{L/S}) + (CPI_{branch} * Instructions_{branch})$$

$$\text{Clock Cycles} = (50e6 * .6) + (110e6 * .6) + (80e6 * 2.8) + (16e6 * 1.4)$$

$$\text{Clock Cycles} = 30e6 + 66e6 + 224e6 + 22.4e6$$

$$\text{Clock Cycles} = 342.4e6$$

$$\text{Execution Time} = \frac{342.4e6}{2e9} = 171.2e-3 = 0.1712 \text{ seconds}$$

$$\frac{0.256}{.1712} = 1.495$$

Execution Time improved by 1.495 times.

4.15.16

(a)

$$\text{Time} = 100 \text{ seconds}$$

$$\text{Per-Processor Tim}(T(p)) = \text{Time}(T) / \text{Processors}(p) + 4$$

$$T(2) = \frac{100}{2} + 4 = 54 \text{ seconds}$$

$$\text{Speedup} = \frac{100}{54} = 1.85$$

$$\text{Ideal Speedup} = \frac{100}{\frac{100}{2}} = \frac{100}{50} = 2$$

$$\text{Ratio} = \frac{1.85}{2} = 0.925$$

$$T(4) = \frac{100}{4} + 4 = 29 \text{ seconds}$$

$$\text{Speedup} = \frac{100}{29} = 3.44$$

$$\text{Ideal Speedup} = \frac{100}{\frac{100}{4}} = \frac{100}{25} = 4$$

$$\text{Ratio} = \frac{3.44}{4} = 0.86$$

$$T(8) = \frac{100}{8} + 4 = 16.5 \text{ seconds}$$

$$\text{Speedup} = \frac{100}{16.5} = 6.06$$

$$\text{Ideal Speedup} = \frac{100}{\frac{100}{8}} = \frac{100}{12.5} = 8$$

$$\text{Ratio} = \frac{6.06}{8} = 0.7575$$

$$T(16) = \frac{100}{16} + 4 = 10.25 \text{ seconds}$$

$$\text{Speedup} = \frac{100}{10.25} = 9.75$$

$$\text{Ideal Speedup} = \frac{100}{\frac{100}{16}} = \frac{100}{6.25} = 16$$

$$\text{Ratio} = \frac{9.75}{16} = 0.609$$

$$T(32) = \frac{100}{32} + 4 = 7.125 \text{ seconds}$$

$$\text{Speedup} = \frac{100}{7.125} = 14.035$$

$$\text{Ideal Speedup} = \frac{100}{\frac{100}{32}} = \frac{100}{3.125} = 32$$

$$\text{Ratio} = \frac{14.035}{32} = 0.439$$

$$T(64) = \frac{100}{64} + 4 = 5.5625 \text{ seconds}$$

$$\text{Speedup} = \frac{100}{5.5625} = 17.97$$

$$\text{Ideal Speedup} = \frac{100}{\frac{100}{64}} = \frac{100}{1.5625} = 64$$

$$\text{Ratio} = \frac{17.97}{64} = 0.28$$

$$T(128) = \frac{100}{128} + 4 = 4.781 \text{ seconds}$$

$$\text{Speedup} = \frac{100}{4.781} = 20.91$$

$$\text{Ideal Speedup} = \frac{100}{\frac{100}{128}} = \frac{100}{0.78125} = 128$$

$$\text{Ratio} = \frac{20.91}{128} = 0.163$$