Homework 1

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Problems

4.15.5

(a)

$Performance = ClockRate/CPI$

P1 = $3.0 * 10^9/1.5 = 2.0 * 10^9$ IPS
P2 = $2.5 * 10^9/1.0 = 2.5 * 10^9$ IPS
P3 = $4.0 * 10^9/2.2 = 1.8 * 10^9$ IPS

Processor P2 has the highest performance expressed in giga instructions per second

(b)

$Numberofcycles = (Clock\ Rate) * (Execution\ Time\ in\ seconds)$
$Numberofinstructions = (IPS) * (Execution\ Time\ in\ seconds)$

For P1
-Number of cycles = $(3.0 * 10^9) * (10) = 3.0 * 10^{10}$ cycles
-Number of instructions = $(2.0 * 10^9) * (10) = 2.0 * 10^{10}$ instructions

For P2
-Number of cycles = $(2.5 * 10^9) * (10) = 2.5 * 10^{10}$ cycles
-Number of instructions = $(2.5 * 10^9) * (10) = 2.5 * 10^{10}$ instructions

For P3
-Number of cycles = $4.0 * 10^9 * (10) = 4.0 * 10^{10}$
-Number of instructions = $(1.8 * 10^9) * (10) = 1.8 * 10^{10}$

(c)

Clock Rate = $((Total\ number\ of\ instructions) * (CPI))/(Execution\ Time)$
For P1
-Total number of instructions = $2.0 * 10^{10}$
-CPI = $1.2 * 1.5 = 1.8$
-Execution Time = $0.7 * 10 = 7$ seconds
Clock Rate = $((2.0 * 10^{10}) * 1.8)/7 = 5.14$ GHZ
For P2
-Total number of instructions = $2.5 * 10^{10}$
-CPI = $1.2 * 1 = 1.2$

-Execution Time $= 0.7 * 10 = 7$ seconds

Clock Rate $= ((2.5 * 10^{10}) * 1.2)/7 = 4.28$ GHZ

For P3

-Total number of instructions $= 1.8 * 10^{10}$

-CPI $= 1.2 * 2.2 = 2.64$

-Execution Time $= 0.7 * 10 = 7$ seconds

Clock Rate $= ((1.8 * 10^{10}) * 2.64)/7 = 6.79$ GHZ

4.15.8

(a)

Execution Time = Instruction Count * CPI * Clock Cycle Time

Average CPI = (Execution Time) / (Instruction Count * Clock Cycle Time)

Compiler A - $1.1/1.0e^9 * 1 = 1.1$

Compiler B - $1.5/1.2e^9 * 1 = 1.25$

(b)

Execution Time = Instruction Count * CPI * Clock Cycle Time

Execution Time/Clock Cycle Time = Instruction Count * CPI

1 / Clock Cycle Time = (Instruction Count * CPI) / Execution Time

1 / Clock Cycle Time = Clock Rate

Clock Rate = (Instruction Count * CPI) / Execution Time

Same CPU time so allow Execution Time to be 1

Clock Rate = CR

$CR_A/CR_B = (Instruction\ Count_A * CPI_A)/(Instruction\ Count_B * CPI_B)$

$CR_B/CR_A = (1.0e^9 * 1.1)/(1.2e^9 * 1.25) \approx .733$

Compiler A is .733 times faster

(c)

Speedup $= \frac{Execution\ Time_A}{Execution\ Time_{new}}$

$\frac{Execution\ Time_A}{Execution\ Time_{new}} = \frac{(Instruction\ Count_A * CPI_A)}{(Instruction\ Count_{new} * CPI_{new})} = (1.0e^9 * 1.1)/(6.0e^8) \approx 1.67$ times faster

$\frac{Execution\ Time_B}{Execution\ Time_{new}} = \frac{(Instruction\ Count_B * CPI_B)}{(Instruction\ Count_{new} * CPI_{new})} = (1.2e^9 * 1.25)/(6.0e^8) \approx 2.27$ times faster

4.5.10

(a)

Processors - 1

Execution Time = Clock Cycle / Clock Rate
Clock Rate = 2GHz = $2e9$

Clock Cycles = $(CPI_{arithmetic} * Instructions_{arithmetic}) + (CPI_{load/store} * Instructions_{load/store}) + (CPI_{branch} * Instructions_{branch})$

Clock Cycles = $\frac{(2.56e9*1)}{0.7*1} + \frac{(1.28e9*12)}{0.7*1} + (2.56e8*5)$
Clock Cycles = $(3.66e9) + (2.194e10) + (1.28e9) = 26.88e9$ cycles
Execution Time = $\frac{26.88e9\ cycles}{2e9\ cycles/sec} = 13.44$ seconds

Relative Speedup = $\frac{Processors-1}{Processors-p} = \frac{13.44}{13.44} = 1$

(Processors - 2)

Clock cycles = $\frac{(2.56e9*1)}{0.7*2} + \frac{(1.28e9*12)}{0.7*2} + (2.56e8*5)$

Clock cycles = $\frac{2.56e9}{1.4} + \frac{1.536e10}{1.4} + (1.28e9)$

Clock cycles = $1.404e10$ cycles

Execution Time = $\frac{1.404e10\ cycles}{2e9\ cycles/sec} = 7.02$ seconds

Relative Speedup = $\frac{Processors-1}{Processors-p} = \frac{13.44}{7.02} = 1.91$ times faster

Processors - 4

Clock cycles = $\frac{(2.56e9*1)}{0.7*4} + \frac{(1.28e9*12)}{0.7*4} + (2.56e8*5)$

Clock cycles = $\frac{2.56e9}{2.8} + \frac{1.536e10}{2.8} + (1.28e9)$

Clock cycles = $7.22e9$ cycles

Execution Time = $\frac{7.22e9\ cycles}{2e9\ cycles/sec} = 3.86$ seconds

Relative Speedup = $\frac{Processors-1}{Processors-p} = \frac{13.44}{3.86} = 3.48$ times faster

Processors - 8

Clock cycles = $\frac{(2.56e9*1)}{0.7*8} + \frac{(1.28e9*12)}{0.7*8} + (2.56e8*5)$

Clock cycles = $\frac{2.56e9}{5.6} + \frac{1.536e10}{5.6} + (1.28e9)$

Clock cycles = $4.5e9$ cycles

Execution Time = $\frac{4.5e9\ cycles}{2e9\ cycles/sec} = 2.25$ seconds

Relative Speedup = $\frac{Processors-1}{Processors-p} = \frac{13.44}{2.25} = 5.97$ times faster

(b)

Processors - 1

Execution Time = Clock Cycle / Clock Rate
Clock Rate = 2 Ghz = $2e9$
Clock Cycle = $(2.56e9*2) + (1.28e9*12) + (2.56e8*5)$

Clock Cycle $= (5.12e9) + (1.536e10) + (1.28e9) = 21.76e9$

Execution Time $= \frac{21.76e9\ cycles}{2e9\ cycles/secs} = 10.88$ seconds

Processors - 2

Execution Time = Clock Cycle / Clock Rate
Clock Rate = 2 Ghz = $2e9$

Clock Cycle $= \frac{(2.56e9*2)}{0.7*2} + \frac{(1.28e9*12)}{0.7*2} + (2.56e8*5)$

Clock Cycle $= (3.657e9) + (1.097e10) + (1.280e9) = 15.94e9$

Execution Time $= \frac{15.907e9\ cycles}{2e9\ cycles/secs} = 7.954$ seconds

Processors - 4

Execution Time = Clock Cycle / Clock Rate
Clock Rate = 2 Ghz = $2e9$

Clock Cycle $= \frac{(2.56e9*2)}{0.7*4} + \frac{(1.28e9*12)}{0.7*4} + (2.56e8*5)$

Clock Cycle $= (1.829e9) + (5.486e9) + (1.280e9) = 8.595e9$

Execution Time $= \frac{8.595e9\ cycles}{2e9\ cycles/secs} = 4.298$ seconds

Processors - 8

Execution Time = Clock Cycle / Clock Rate
Clock Rate = 2 Ghz = $2e9$

Clock Cycle $= \frac{(2.56e9*2)}{0.7*8} + \frac{(1.28e9*12)}{0.7*8} + (2.56e8*5)$

Clock Cycle $= (9.142e8) + (2.743e9) + (1.280e9) = 4.937e9$

Execution Time $= \frac{4.937e9\ cycles}{2e9\ cycles/secs} = 2.469$ seconds

(c)

Execution Time for 4 Processors

Clock cycles $= \frac{(2.56e9*1)}{0.7*4} + \frac{(1.28e9*12)}{0.7*4} + (2.56e8*5)$

Clock cycles $= \frac{2.56e9}{2.8} + \frac{1.536e10}{2.8} + (1.28e9)$

Clock cycles $= 7.22e9$ cycles

Execution Time $= \frac{7.22e9\ cycles}{2e9\ cycles/sec} = 3.86$ seconds

Find new CPI for single processor

x will represent the new CPI for load/store instructions that we will find
Clock Cycles $= (2.56e9*1) + (1.28e9*x) + (2.56e8*5)$
Clock Cycles $= (2.56e9) + (1.28e9*x) + (1.28e9)$

Clock Cycles = $(3.84e9) + (1.28e9 * x)$

Execution Time = $\frac{3.84e9+(1.28e9*x)}{2e9}$

$3.86 = \frac{3.84e9}{2e9} + \frac{1.28e9*x}{2e9}$

$3.86 = 1.92 + .64 * x$

$x = 3.03$
$\frac{3.03}{12} = .25 = 25\%$

The CPI of load/store instructions should be reduced by 25% in order for a single processors to match the performance of four processors using its original CPI value

4.15.13

(a)

Execution Time = Clock Cycles / Clock Rate
Clock Cycles = (CPI * Instructions)
P1 Clock Rate = 4GHz = 4.0e9
P1 CPI = 0.9
P1 Instructions = 5.0e9

P2 Clock Rate = 3GHz = 3.0e9
P2 CPI = 0.75
P2 Instructions = 1.0e9

$Execution\ Time_{P1} = \frac{0.9*5.0e9}{4.0e9} = \frac{4.5e9}{4.0e9} = 1.125$ seconds
$Execution\ Time_{P2} = \frac{0.75*1.0e9}{3.0e9} = \frac{0.75e9}{3.0e9} = 0.25$ seconds
We can see that this fallacy is not true for P1 and P2, although P1 has a higher clock rate, it is slower than P2 as shown above

(b)

Find P1 execution time

Execution Time = Clock Cycles / Clock Rate
Clock Cycles = (CPI * Instructions)
P1 Clock Rate = 4GHz = 4.0e9
P1 CPI = 0.9
P1 Instructions = 1.0e9

$Execution\ Time_{P1} = \frac{0.9*1.0e9}{4.0e9} = .225\ seconds$

Find the numbers of instructions P2 can execute

Let x represent the numbers of instructions P2 can execute
P2 Clock Rate = 3GHz = 3.0e9
P2 CPI = 0.75
P2 Instructions = 1.0e9

$Execution\ Time_{P1} = \frac{(0.75*x)}{3.0e9}$
$.225 = \frac{.75*x}{3.0e9}$
$0.675e9 = .75 * x$
$0.9e9 = x$

$x = 9.0e8$

P2 can execute 9.0e8 instructions in the same time that P1 executes 1.0e9 instructions

(c)

MIPS = Number of Instructions / Execution Time * 10^6

$MIPS_{P1} = \frac{5.0e9}{1.125e6} = 4.44e3$

$MIPS_{P2} = \frac{1.0e9}{0.25e6} = 4.0e3$

This fallacy is not true for P1 and P2, although P1 has a bigger MIPS, it is slower than P2 regarding performance

(d)

MFLOPS = # FP Operations / Execution Time * 1.0e6

$\# \ FP \ Operations_{P1} = 5.0e9 * .40 = 2.0e9$

$\# \ FP \ Operations_{P2} = 1.0e9 * .40 = 4.0e8$

$MLOPS_{P1} = \frac{2.0e9}{1.125 * 1.0e6} = 1.77e3$

$MLOPS_{P2} = \frac{4.0e8}{.25 * 1.0e6} = 1.6e3$

4.15.14

Total Time = 250 seconds

FP Instructions Time = 70 seconds

Load/Store Instructions Time = 85 seconds

Branch Instruction Time = 40 seconds

(a)

Reduced time for FP instructions = 70 * .80 = 56 seconds

Total Time = 56 + (250 - 70) = 236 seconds

Total Time Reduced = 250 - 236 = 14 seconds

$\frac{14}{250} * 100 = 5.6\%$

Reducing FP operations by 20% will cause a 5.6% reduction in total time

(b)

Original Int Time = 250 - 70 - 85 - 40 = 55

Total Time reduced by 20% = $250 * (1 - .20) = 200$ seconds

Total Time = FP time + l/s time + branch time + new int time

200 = 70 + 85 + 40 + new int time

new int time = 5 seconds

$\frac{5}{55} * 100 = 90.9\%$

Int operation time is reduced by 90.9% when total time is reduced by 20%

(c)

Total Time reduced by 20% = $250 * (1 - .20) = 200$ seconds

Let x represent branch time

200 = 70 + 85 + x + 55

200 = 210 + x

x = -10

We see here that in order for Total time to be reduced by 20%, branch time will have to go into the negatives, which is not possible. Thus the total time cannot be reduced by 20% by reducing only the time for branch instructions.

4.15.15

(a)

Execution Time = Clock Cycles / Clock Rate

Clock Rate = 2Ghz = 2e9

Clock Cycles = $(CPI_{FP} * Instructions_{FP}) + (CPI_{INT} * Instructions_{INT}) + (CPI_{L/S} * Instructions_{L/S}) + (CPI_{branch} * Instructions_{branch})$

Clock Cycles = $(50e6 * 1) + (110e6 * 1) + (80e6 * 4) + (16e6 * 2)$

Clock Cycles = $50e6 + 110e6 + 320e6 + 32e6$

Clock Cycles = $512e6$

Execution Time = $\frac{512e6}{2e9} = 256e_{-3} = 0.256$ seconds

Execution Time two times faster = $\frac{.256}{2} = 0.128$ seconds

Let x represent what FP instructions average CPI has to be to make execution time 2 times faster

$0.128 = \frac{(50e6*x)+(110e6*1)+(80e6*4)+(16e6*2)}{2e9}$

$0.128 = \frac{(50e6*x)+110e6+320e6+32e6}{2e9}$

$0.128 = \frac{(50e6*x)+462e6}{2e9}$

$0.128 = \frac{50e6*x}{2e9} + \frac{462e6}{2e9}$

$0.128 = \frac{50e6*x}{2e9} + .231$

-0.103 = $\frac{50e6*x}{2e9}$

-0.103 = $25e_{-3} * x$

-0.103 = $0.025 * x$

x = -4.12

Since x comes out to be negative, it is impossible to make the program run two times faster by only improving CPI of FP instructions

(b)

Execution Time = Clock Cycles / Clock Rate

Clock Rate = 2Ghz = 2e9

Clock Cycles = $(CPI_{FP} * Instructions_{FP}) + (CPI_{INT} * Instructions_{INT}) + (CPI_{L/S} * Instructions_{L/S}) + (CPI_{branch} * Instructions_{branch})$

Clock Cycles = $(50e6 * 1) + (110e6 * 1) + (80e6 * 4) + (16e6 * 2)$

Clock Cycles = $50e6 + 110e6 + 320e6 + 32e6$

Clock Cycles = $512e6$

Execution Time = $\frac{512e6}{2e9} = 256e_{-3} = 0.256$ seconds

Execution Time two times faster = $\frac{.256}{2} = 0.128$ seconds

Let x represent what L/S instructions average CPI has to be to make execution time 2 times faster

$0.128 = \frac{(50e6*1)+(110e6*1)+(80e6*x)+(16e6*2)}{2e9}$

$0.128 = \frac{50e6+110e6+32e6+(80e6*x)}{2e9}$

$0.128 = \frac{(80e6*x)+192e6}{2e9}$

$0.128 = \frac{(80e6*x)}{2e9} + \frac{192e6}{2e9}$

$0.128 = \frac{(80e6*x)}{2e9} + .096$

$0.032 = .040 * x$

x = 0.8

$\frac{4}{.8} * 100 = 500\%$

We would have to improve CPI of L/S Instructions by 500% if we want the program to run two times faster

(c)

New CPI_{INT} = 1 * (1 - .40) = .6
New CPI_{FP} = 1 * (1 - .40) = .6
New $CPI_{L/S}$ = 4 * (1 - .30) = 2.8
New CPI_{Branch} = 2 * (1 - .30) = 1.4
Execution Time = Clock Cycles / Clock Rate
Clock Rate = 2Ghz = 2e9
Clock Cycles = $(CPI_{FP} * Instructions_{FP}) + (CPI_{INT} * Instructions_{INT}) + (CPI_{L/S} * Instructions_{L/S}) + (CPI_{branch} * Instructions_{branch})$
Clock Cycles = $(50e6 * .6) + (110e6 * .6) + (80e6 * 2.8) + (16e6 * 1.4)$
Clock Cycles = $30e6 + 66e6 + 224e6 + 22.4e6$
Clock Cycles = $342.4e6$
Execution Time = $\frac{342.4e6}{2e9} = 171.2e_{-3} = 0.1712$ seconds
$\frac{0.256}{.1712} = 1.495$
Execution Time improved by 1.495 times.

4.15.16

(a)

Time = 100 seconds
Per-Processor Tim(T(p)) = Time(T) / Processors(p) + 4
T(2) = $\frac{100}{2} + 4 = 54$ seconds
Speedup = $\frac{100}{54} = 1.85$ times faster
Ideal Speedup = $\frac{100}{\frac{100}{2}} = \frac{100}{50} = 2$ times faster
Ratio = $\frac{1.85}{2} = 0.925$

T(4) = $\frac{100}{4} + 4 = 29$ seconds
Speedup = $\frac{100}{29} = 3.44$ times faster
Ideal Speedup = $\frac{100}{\frac{100}{4}} = \frac{100}{25} = 4$ times faster
Ratio = $\frac{3.44}{4} = 0.86$

T(8) = $\frac{100}{8} + 4 = 16.5$ seconds
Speedup = $\frac{100}{16.5} = 6.06$ times faster
Ideal Speedup = $\frac{100}{\frac{100}{8}} = \frac{100}{12.5} = 8$ times faster
Ratio = $\frac{6.06}{8} = 0.7575$

T(16) = $\frac{100}{16} + 4 = 10.25$ seconds
Speedup = $\frac{100}{10.25} = 9.75$ times faster
Ideal Speedup = $\frac{100}{\frac{100}{16}} = \frac{100}{6.25} = 16$ times faster
Ratio = $\frac{9.75}{16} = 0.609$

T(32) = $\frac{100}{32} + 4 = 7.125$ seconds
Speedup = $\frac{100}{7.125} = 14.035$ times faster
Ideal Speedup = $\frac{100}{\frac{100}{32}} = \frac{100}{3.125} = 32$ times faster
Ratio = $\frac{14.035}{32} = 0.439$

T(64) = $\frac{100}{64} + 4 = 5.5625$ seconds
Speedup = $\frac{100}{5.5625} = 17.97$ times faster

Ideal Speedup $= \frac{100}{\frac{100}{64}} = \frac{100}{1.5625} = 64$ times faster
Ratio $= \frac{17.97}{64} = 0.28$

T(128) $= \frac{100}{128} + 4 = 4.781$ seconds
Speedup $= \frac{100}{4.781} = 20.91$ times faster
Ideal Speedup $= \frac{100}{\frac{100}{128}} = \frac{100}{0.78125} = 128$ times faster
Ratio $= \frac{20.91}{128} = 0.163$