

Homework 3

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Boolean Algebra

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(a) $A * (\bar{A} + B * B) + \overline{(B + A)} * (\bar{A} + B)$

$$\begin{aligned} & A * (\bar{A} + B * B) + (\bar{B} * \bar{A}) * (\bar{A} + B) \quad \text{DeMorgan's} \\ & A * (\bar{A} + B) + (\bar{B} * \bar{A}) * (\bar{A} + B) \quad \text{Idempotent Law } (B * B = B) \\ & (A + (\bar{B} * \bar{A}))(\bar{A} + B) \quad \text{Distributive Law (Inversed)} \\ & (A + (\bar{A} * \bar{B}))(\bar{A} + B) \quad \text{Commutative Law} \\ & (A + \bar{B})(\bar{A} + B) \quad \text{Redundancy Law } (A + (\bar{A} * \bar{B}) = A + \bar{B}) \\ & A(\bar{A} + B) + \bar{B}(\bar{A} + B) \quad \text{Distributive Law} \\ & (A * \bar{A} + A * B) + \bar{B}(\bar{A} + B) \quad \text{Distributive Law} \\ & (0 + A * B) + \bar{B}(\bar{A} + B) \quad \text{Inverse Law } (A * \bar{A} = 0) \\ & (A * B) + \bar{B}(\bar{A} + B) \quad \text{Identity Law } (0 + A * B = A * B) \\ & (A * B) + (\bar{B} * \bar{A} + \bar{B} * B) \quad \text{Distributive Law} \\ & (A * B) + (\bar{B} * \bar{A} + 0) \quad \text{Inverse Law } (\bar{B} * B = 0) \\ & (A * B) + (\bar{B} * \bar{A}) \quad \text{Identity Law } (\bar{B} * \bar{A} + 0 = \bar{B} * \bar{A}) \end{aligned}$$

$$\boxed{A * B + \bar{B} * \bar{A}}$$

(b) $\overline{C * B} + (A * B * C) + \overline{A + C + \bar{B}}$

$$\begin{aligned} & \overline{C} + \bar{B} + (A * B * C) + \overline{A + C + \bar{B}} \quad \text{DeMorgan's} \\ & \overline{C} + (A * B * C) + \bar{B} + \overline{A + C + \bar{B}} \quad \text{Commutative Law} \\ & \overline{C} + (C * A * B) + \bar{B} + \overline{A + C + \bar{B}} \quad \text{Commutative Law} \\ & \overline{C} + (A * B) + \bar{B} + \overline{A + C + \bar{B}} \quad \text{Redundancy Law } (\overline{C} + (C * A * B) = \overline{C} + (A * B)) \\ & \overline{C} + \bar{B} + (A * B) + \overline{A + C + \bar{B}} \quad \text{Commutative Law} \\ & \overline{C} + \bar{B} + (B * A) + \overline{A + C + \bar{B}} \quad \text{Commutative Law} \\ & \overline{C} + (\bar{B} + A) + \overline{A + C + \bar{B}} \quad \text{Redundancy Law } (\bar{B} + (B * A) = \bar{B} + A) \\ & \overline{C} + (\bar{B} + A) + \bar{A} * \bar{C} * \bar{B} \quad \text{DeMorgan's} \\ & \overline{C} + (\bar{B} + A) + \bar{A} * \bar{C} * B \quad \text{Double Negation Law} \\ & \overline{C} + (\bar{A} * \bar{C} * B) + (\bar{B} + A) \quad \text{Commutative Law} \\ & \overline{C} + (\bar{C} * \bar{A} * B) + (\bar{B} + A) \quad \text{Commutative Law} \\ & \overline{C} + (\bar{B} + A) \quad \text{Absorption Law } (\overline{C} + (\bar{C} * \bar{A} * B) = \overline{C}) \end{aligned}$$

$$\boxed{\overline{C} + \bar{B} + A}$$

$$(c) (A + B) * (\bar{A} + C) * (\bar{C} + B)$$

$$\begin{aligned}
& (A(\bar{A} + C) + B(\bar{A} + C)) * (\bar{C} + B) \quad \text{Distributive Law} \\
& ((A * \bar{A} + A * C) + B(\bar{A} + C)) * (\bar{C} + B) \quad \text{Distributive Law} \\
& (A * \bar{A} + A * C + B * \bar{A} + B * C) * (\bar{C} + B) \quad \text{Distributive Law} \\
& ((A * \bar{A} * \bar{C}) + (A * C * \bar{C}) + (B * \bar{A} * \bar{C}) + (B * C * \bar{C})) + \\
& ((A * \bar{A} * B) + (A * C * B) + (B * \bar{A} * B) + (B * C * B)) \quad \text{Distributive Law} \\
& ((A * \bar{A} * \bar{C}) + (A * C * \bar{C}) + (B * \bar{A} * \bar{C}) + (B * C * \bar{C})) + \\
& ((A * \bar{A} * B) + (A * C * B) + (B * \bar{A} * B) + (B * B * C)) \quad \text{Commutative Law} \\
& ((A * \bar{A} * \bar{C}) + (A * C * \bar{C}) + (B * \bar{A} * \bar{C}) + (B * C * \bar{C})) + \\
& ((A * \bar{A} * B) + (A * C * B) + (B * \bar{A} * B) + (B * C)) \quad \text{Idempotent Law} \\
& ((A * \bar{A} * \bar{C}) + (A * C * \bar{C}) + (B * \bar{A} * \bar{C}) + (B * C * \bar{C})) + \\
& ((A * \bar{A} * B) + (A * C * B) + (B * B * \bar{A}) + (B * C)) \quad \text{Commutative Law} \\
& ((A * \bar{A} * \bar{C}) + (A * C * \bar{C}) + (B * \bar{A} * \bar{C}) + (B * C * \bar{C})) + \\
& ((A * \bar{A} * B) + (A * C * B) + (B * \bar{A}) + (B * C)) \quad \text{Idempotent Law} \\
& ((0 * \bar{C}) + (A * C * \bar{C}) + (B * \bar{A} * \bar{C}) + (B * C * \bar{C})) + \\
& ((A * \bar{A} * B) + (A * C * B) + (B * \bar{A}) + (B * C)) \quad \text{Inverse Law} \\
& ((0 * \bar{C}) + (A * 0) + (B * \bar{A} * \bar{C}) + (B * C * \bar{C})) + \\
& ((A * \bar{A} * B) + (A * C * B) + (B * \bar{A}) + (B * C)) \quad \text{Inverse Law} \\
& ((0 * \bar{C}) + (A * 0) + (B * \bar{A} * \bar{C}) + (B * 0)) + \\
& ((A * \bar{A} * B) + (A * C * B) + (B * \bar{A}) + (B * C)) \quad \text{Inverse Law} \\
& ((0 * \bar{C}) + (A * 0) + (B * \bar{A} * \bar{C}) + (B * 0)) + \\
& ((0 * B) + (A * C * B) + (B * \bar{A}) + (B * C)) \quad \text{Inverse Law} \\
& ((0) + (0) + (B * \bar{A} * \bar{C}) + (0)) + \\
& ((0) + (A * C * B) + (B * \bar{A}) + (B * C)) \quad \text{Law of Zeros (Too much to do them one by one ...)} \\
& (B * \bar{A} * \bar{C}) + (A * C * B) + (B * \bar{A}) + (B * C) \quad \text{Identity Law (Too much to do them one by one...)} \\
& (B * \bar{A}) + (B * \bar{A} * \bar{C}) + (A * C * B) + (B * C) \quad \text{Commutative Law} \\
& (B * \bar{A}) + (A * C * B) + (B * C) \quad \text{Absorption Law } ((B * \bar{A}) + (B * \bar{A} * \bar{C}) = (B * \bar{A})) \\
& (B * \bar{A}) + (B * C) + (A * C * B) \quad \text{Commutative Law} \\
& (B * \bar{A}) + (B * C) + (B * C * A) \quad \text{Commutative Law} \\
& (B * \bar{A}) + (B * C) \quad \text{Absorption Law } ((B * C) + (B * C * A) = (B * C)) \\
& B * (\bar{A} + C) \quad \text{Distributive Law (Inversed)}
\end{aligned}$$

$$\boxed{B * (\bar{A} + C)}$$

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(a)