5.2 base case: $1 = 2^0$; 1 is a distinct sum of powers of 2

Induction Hypothesis: $n = 2^k$, $n \geq 1$

Case 1: $k+1$ is even

1. $\frac{(k+1)}{2}$ must be a natural number less then or equal to k
2. $\frac{(k+1)}{2} = 2^{x_1} + 2^{x_2} + \ldots + 2^{x_y}$ for $0 \leq x_1 < \ldots < x_y$
3. $k+1 = 2(2^{x_1} + 2^{x_2} + \ldots + 2^{x_y})$; multiply both sides by 2
4. $k+1 = 2^{x_1+1} + \ldots + 2^{x_y+1}$; can be represented as a distinct sum of 2^{x_y}

Case 2: $k+1$ is odd

1. k must be even, thus each power must be atleast 1 so k is divisible by 2
2. $k = 2^{x_1} + \ldots + 2^{x_y}$ for $0 < x_1 < x_y$
3. $k+1 = 2^{x_1} + \ldots + 2^{x_y} + 1 = 2^{x_1} + \ldots + 2^{x_y} + 2^0$; can be represented

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Base cases

5.39

12	13	14	15
4,4,4	4,4,5	4,5,5	5,5,5

Induction step: $P(n) \Rightarrow P(n+4)$

$P(n) = 4k + 5j$

$P(n+4) = (4k + 5j) + 4$

$P(n+4) = 4(k+1) + 5j$

since we have shown that every four cents you can add a four cent stamp, and that there are four base cases, this induction proof shows that there is a combination of 4 & 5 cent stamps that can make of every value ≥ 12

6.8 a. $n^7 < 2^n$ using induction $n \geq 37$

1. base case: $37^7 < 2^{37}$

   94, 931, 877, 133 < 137, 438, 953, 472 ; true

2. Induction step: $P(n) \rightarrow P(n+1)$ $7 < n$

3. $P(n) = n^7 < 2^n$ $21 < n^2$

4. $P(n+1) = (n+1)^7 < 2^{n+1}$ $35 < n^3$

5. $(n+1)^7 = n^7 + 7n^6 + 21n^5 + 35n^4 + 35n^3 + 21n^2 + 7n + 1$

6. $(n+1)^7 < n^7 + (n)n^6 + (n^2)n^5 + (n^3)n^4 + (n^4)n^3 + (n^5)n^2 + (n^6)n + (n^7)$

7. $(n+1)^7 < 8n^7$

8. $8n^7 < 2^{n+1}$; since the left side only goes up by 8, but the right side doubles, value is true

b: Prove using leaping induction

1. base cases: 37: $9.4 \times 10^{10} < 1.3 \times 10^{11}$

   38: $1.1 \times 10^{11} < 2.7 \times 10^{11}$ $14 < n$

2. $P(n) = n^7 < 2^n$ $84 < n^2$

3. $P(n+2) = (n+2)^7 < 2^{n+2}$ $280 < n^3$

   $560 < n^4$

   $672 < n^5$

4. $(n+2)^7 = n^7 + 14n^6 + 84n^5 + 280n^4 + 560n^3 + 672n^2 + 448n + 128$

5. $(n+2)^7 < n^7 + n \cdot n^6 + n^2 \cdot n^5 + n^3 \cdot n^4 + n^4 \cdot n^3 + n^5 \cdot n^2 + n^6 \cdot n + n^7$

6. $(n+2)^7 < 8n^7$

7. $8n^7 < 2^{n+2}$