Statistics with Recitation — Quiz 4

November 11, 2025

Answer Key

1. (**12 points**) A recent Urban Safety Poll plans to estimate the proportion of adults nationally who think that cities should lower residential speed limits to 25 miles per hour. Based on prior data, they expect the true proportion to be about 61%. The research team would like to report results with a 95% confidence level and a margin of error of 3 percentage points.

   (a) (**6 points**) What sample size is required to achieve a 3% margin of error at the 95% confidence level?

   (b) (**6 points**) Suppose the poll actually surveys this many adults and finds that 61% of them think cities should lower residential speed limits to 25 mph. Construct a 95% confidence interval for the population proportion and determine whether the poll provides convincing evidence that more than 60% of the population hold this opinion.

Suggested Answers for Problem 1:

(a) Let the target margin of error be $\text{ME} = 0.03$ and the anticipated proportion be $\hat{p} = 0.61$. For a 95% confidence level, $z^\star \approx 1.96$.

We use the margin of error formula and solve for n:

$$\text{ME} = z^\star\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad\Rightarrow\quad n = \hat{p}(1-\hat{p})\left(\frac{z^\star}{\text{ME}}\right)^2.$$

$$n = 0.61(1-0.61)\left(\frac{1.96}{0.03}\right)^2 \approx 0.61(0.39)\cdot(65.33)^2 \approx 1015.4$$

We must round up to ensure the margin of error is at most 3%, so the poll should survey at least

$$n = 1016 \text{ adults}.$$

Grading Criterion

- (6 pts): Correct setup and formula, solves for n, and rounds up to $n \approx 1016$.
- (1–5 pts): Partially correct answer.
- (0 pts): Incorrect method.

(b) Using $n = 1016$ and $\hat{p} = 0.61$, the margin of error at 95% is

$$\text{ME} = 1.96\sqrt{\frac{0.61(1-0.61)}{1016}} \approx 0.03,$$

so the 95% confidence interval is

$$\hat{p} \pm \text{ME} = 0.61 \pm 0.03,$$

which is

$$(0.58,\ 0.64) \quad \text{or} \quad (58\%,\ 64\%).$$

Since 0.60 lies inside the 95% confidence interval, values of the population proportion both above and below 60% are plausible. Therefore the poll does **not** provide convincing evidence that more than 60% of the population support the policy.

Grading Criterion

- (6 pts): Correct CI computation, correct comparison with 0.60, and a clear conclusion.
- (1–5 pts): Partially correct answer.
- (0 pts): Incorrect answer.

2. (**18 points**) A large city's public-transit app occasionally experiments with design and link placement. Product analysts tested three different placements of a "Download Offline Map" button on the app's home screen to see which location, if any, led to the most taps. The number of app sessions included in the experiment was 1000 and is captured in one of the response combinations in the following table:

	Tap	**No Tap**
Position 1	14.9%	16.3%
Position 2	13.5%	20.0%
Position 3	11.2%	24.1%

   (a) (**6 points**) Calculate the actual number of sessions in each of the six response categories.

   (b) (**6 points**) Each session had an equal chance of being shown any of the three positions. However, we see slightly different totals for the groups. Is there any evidence that the groups were actually imbalanced? Clearly state hypotheses, check conditions, and calculate the appropriate test statistic. Then draw a conclusion in context. (You do not need to compute a precise p-value.)

   (c) (**6 points**) Conduct an appropriate hypothesis test to check whether there is evidence that the tap rate on the button differs among the three positions. Clearly state hypotheses, check conditions, and calculate the appropriate test statistic. Then draw a conclusion in context. (You do not need to compute a precise p-value.)

Suggested Answers for Problem 2:

(a) With $n = 1000$, multiply each percentage by 1000:

$$\begin{array}{lll} \text{Position 1:} & \text{Tap} = 149, & \text{No Tap} = 163 \\ \text{Position 2:} & \text{Tap} = 135, & \text{No Tap} = 200 \\ \text{Position 3:} & \text{Tap} = 112, & \text{No Tap} = 241 \end{array}$$

Grading Criterion

- (6 pts): All six integers correct.
- (1–5 pts): Partially correct answer.
- (0 pts): Incorrect answer.

(b) **Hypotheses:**

$$\begin{cases} H_0 : \text{sessions were equally assigned across positions (each group proportion} = 1/3); \\ H_A : \text{at least one group proportion differs from } 1/3. \end{cases}$$

Conditions:

- Independence: sessions are randomly assigned to positions, so observations are independent.
- Sample size: all expected counts are at least 5. Under H_0, each group has expected count $1000/3 \approx 333.33 > 5$.

Conditions are satisfied.

Test statistic: Observed group totals: P1 = 312, P2 = 335, P3 = 353 and we expect each = $1000/3 \approx 333.33$.

$$X^2 = \sum \frac{(O-E)^2}{E} = \frac{(312-333.33)^2}{333.33} + \frac{(335-333.33)^2}{333.33} + \frac{(353-333.33)^2}{333.33} \approx 2.53.$$

with df $= 3 - 1 = 2$. From a chi-square table, $X^2 = 2.53$ with df $= 2$ gives a p-value between 0.2 and 0.3, which is greater than $\alpha = 0.05$.

Conclusion: At the 5% significance level, we fail to reject H_0. There is no evidence that sessions were imbalanced across the three positions; the observed totals are consistent with equal assignment.

Grading Criterion

- (6 pts): Correct H_0/H_A, appropriate condition checking, correct $X^2 \approx 2.53$ with df$=2$, and correct conclusion.
- (1–5 pts): Partially correct answer (e.g., missing some conditions or minor arithmetic errors).
- (0 pt): Incorrect answer.

(c) We can calculate the row sums and column sums:

	Tap	No Tap	Total
Position 1	149	163	312
Position 2	135	200	335
Position 3	112	241	353
Total	396	604	1000

Row sums: $(312, 335, 353)$, column sums: $(396, 604)$.

Hypotheses:

$$\begin{cases} H_0 : \text{Tap rate is the same across positions (tap and no tap independent of position).} \\ H_A : \text{At least one position has a different tap rate.} \end{cases}$$

Conditions:

- Independence: sessions are randomly assigned to positions, so observations are independent.
- Sample size: all expected cell counts are at least 5,

$$E_{ij} = \frac{(\text{row}_i)(\text{col}_j)}{1000} > 5.$$

Conditions are satisfied.

Test statistic: The expected counts are $E_{ij} = (\text{row}_i \cdot \text{col}_j)/1000$. The chi-square test statistic is

$$X^2 = \sum \frac{(O-E)^2}{E} \approx 17.89$$

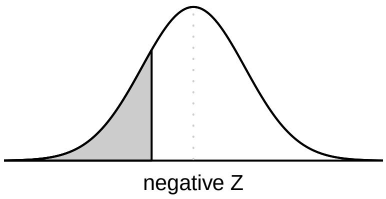
with df $= (3-1)(2-1) = 2$. From a chi-square table, this corresponds to a p-value < 0.001.

Conclusion: Since the p-value is much smaller than $\alpha = 0.05$, we reject H_0. There is strong evidence that tap rates differ by position.

Grading Criterion

- (6 pts): Correct hypotheses, condition checking, $X^2 \approx 17.89$ with df$= 2$, and correct conclusion.
- (1–5 pts): Partially correct answer.
- (0 pts): Incorrect answer.

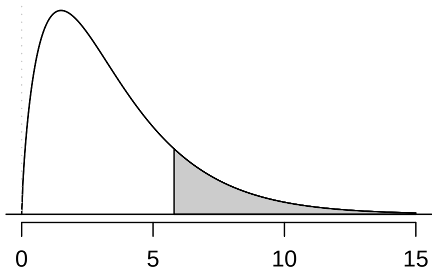
Table 1: **Standard normal probability table**



Second decimal place of Z										
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	Z
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	−3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	−3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	−3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	−3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	−3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	−2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	−2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	−2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	−2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	−2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	−2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	−2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	−2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	−2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	−2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	−1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	−1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	−1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	−1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	−1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	−1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	−1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	−1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	−1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	−1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	−0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	−0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	−0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	−0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	−0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	−0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	−0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	−0.0

*For $Z \leq -3.50$, the probability is less than or equal to 0.0002.

Table 4: **Chi-square probability table**



Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
	2	2.41	3.22	4.61	5.99	7.82	9.21	10.60	13.82
	3	3.66	4.64	6.25	7.81	9.84	11.34	12.84	16.27
	4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47
	5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52
	6	7.23	8.56	10.64	12.59	15.03	16.81	18.55	22.46
	7	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32
	8	9.52	11.03	13.36	15.51	18.17	20.09	21.95	26.12
	9	10.66	12.24	14.68	16.92	19.68	21.67	23.59	27.88
	10	11.78	13.44	15.99	18.31	21.16	23.21	25.19	29.59
	11	12.90	14.63	17.28	19.68	22.62	24.72	26.76	31.26
	12	14.01	15.81	18.55	21.03	24.05	26.22	28.30	32.91
	13	15.12	16.98	19.81	22.36	25.47	27.69	29.82	34.53
	14	16.22	18.15	21.06	23.68	26.87	29.14	31.32	36.12
	15	17.32	19.31	22.31	25.00	28.26	30.58	32.80	37.70
	16	18.42	20.47	23.54	26.30	29.63	32.00	34.27	39.25
	17	19.51	21.61	24.77	27.59	31.00	33.41	35.72	40.79
	18	20.60	22.76	25.99	28.87	32.35	34.81	37.16	42.31
	19	21.69	23.90	27.20	30.14	33.69	36.19	38.58	43.82
	20	22.77	25.04	28.41	31.41	35.02	37.57	40.00	45.31
	25	28.17	30.68	34.38	37.65	41.57	44.31	46.93	52.62
	30	33.53	36.25	40.26	43.77	47.96	50.89	53.67	59.70
	40	44.16	47.27	51.81	55.76	60.44	63.69	66.77	73.40
	50	54.72	58.16	63.17	67.50	72.61	76.15	79.49	86.66