Statistics with Recitation — Quiz 5

November 25, 2025

Answer Key

1. (**17 points**) An airport claims recent staffing changes have reduced average security screening waits to 8 minutes. To audit this claim, a simple random sample of 28 passengers' waits (in minutes) was recorded at a randomly chosen checkpoint during a typical weekday. Summary statistics are shown below. The point estimate suggests the mean wait may be *less than* 8 minutes. Is the result statistically significant?

n	$\bar{x}$	s	min	max
28	7.65	0.80	5.9	9.6

   (a) (**2 points**) Write the hypotheses in symbols and in words.

   (b) (**6 points**) Check conditions, then calculate the test statistic T and the associated degrees of freedom.

   (c) (**3 points**) According to the table, find a range for the p-value and interpret it.

   (d) (**2 points**) What is the conclusion of the (two-sided) hypothesis test at $\alpha = 0.05$?

   (e) (**4 points**) If you were to construct a 90% confidence interval corresponding to this test, would you expect 8 minutes to be in the interval? Explain briefly and show the interval.

Suggested Answers for Problem 1:

(a) Let μ be the true mean screening wait time (minutes) at this checkpoint.

$$H_0: \ \mu = 8 \qquad H_A: \ \mu < 8.$$

Grading Criterion

- **(2 pts)**: Correctly states both H_0 and H_A in symbols and words.
- **(1 pts)**: $H_A : \mu \neq 8$.
- **(0 pts)**: Incorrect answer.

(b) Conditions for one-sample t test:

- Independence: simple random sample of passengers; each wait time is independent.
- Approximately normal population or no strong skew/outliers in the sample.

Test statistic:

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{7.65 - 8}{0.80/\sqrt{28}} = \frac{-0.35}{0.1512} \approx -2.32, \quad \text{df} = n - 1 = 27.$$

Grading Criterion

- **(6 pts)**: Correct t-statistics and coniditions.
- **(1-5 pts)**: Partially correct answers.
- **(0 pts)**: Incorrect answer.

(c) For $T \approx -2.32$ with df $= 27$, the one-sided p-value

$$0.01 < p < 0.025$$

If the true mean wait were 8 minutes, a sample mean is lower than this for 1-2.5% of the time.

Alternatively, for the two sided p-value

$$0.02 < p < 0.05$$

If the true mean wait were 8 minutes, a sample mean is lower than this for 2-5% of the time.

Grading Criterion

- **(3 pts)**: Correct (one-sided or two-sided) p-value (range) and interpretations.
- **(1-2 pts)**: Partially correct answers.
- **(0 pts)**: Incorrect answer.

(d) Since $p < 0.025$, reject H_0. There is statistically significant evidence that the mean screening wait is less than 8 minutes.

Grading Criterion

- **(2 pts)**: Correct answer.
- **(1 pts)**: Incorrect answer.

(e) For a 90% two-sided interval, $t^\star \approx 1.703$ (df $= 27$).

$$\bar{x} \pm t^\star \frac{s}{\sqrt{n}} = 7.65 \pm 1.7 \left(\frac{0.80}{\sqrt{28}} \right) = 7.65 \pm 0.257 = (7.393,\ 7.907).$$

8 minutes is not in this interval.

Since 90% confidence interval is smaller than the 95% one, it H_0 is rejected at $\alpha = 0.05$, then it should also reject H_0 at $\alpha = 0.10$.

Grading Criterion

- **(4 pts)**: Correct confidence interval and interpretations.
- **(1-3 pts)**: Partially correct answers.
- **(0 pts)**: Incorrect answer.

2. (**13 points**) A telecom company operates six regional call centers and wants to know if average post-call satisfaction (0–100 scale) differs by center (each center has its own supervisor and training). The table below shows, for each center, the sample size n_i, sample mean $\bar{x}_i$, and sample standard deviation s_i from a random sample of recent calls.

	Ctr 1	Ctr 2	Ctr 3	Ctr 4	Ctr 5	Ctr 6
n_i	28	30	24	27	31	26
$\bar{x}_i$	78.4	81.0	76.9	79.2	83.1	77.5
s_i	10.5	9.7	10.9	10.1	9.3	11.2

The one-way ANOVA output below will be used to test differences between the average satisfaction scores across centers.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
center	5	1480.00	296.00	2.69	0.023
Residuals	160	17600.00	110.00		

(a) (**2 points**) Write the hypotheses in symbols and in words.

(b) (**6 points**) Check the ANOVA conditions.

(c) (**5 points**) Conduct a hypothesis test to determine if these data provide convincing evidence that the average satisfaction score varies across some (or all) centers. Explain the results.

Suggested Answers for Problem 2:

(a) Let μ_i be the true mean of the center i score.

$$H_0: \ \mu_1 = \mu_2 = \cdots = \mu_6$$

$$H_A: \ \text{At least one center's mean differs from the others.}$$

Grading Criterion

- **(2 pts)**: Correct null and alternative in words or symbols.
- **(1 pts)**: Only partially correct statement.
- **(0 pts)**: Incorrect or missing.

(b) ANOVA conditions:

- Independence: calls sampled randomly within centers; different calls are independent.
- Approximate normality: each group's distribution is roughly symmetric with no extreme outliers.
- Equal variances: sample standard deviations are similar.

Grading Criterion

- **(6 pts)**: Correct ANOVA coniditions.
- **(1-5 pts)**: Partially correct answers.
- **(0 pts)**: Incorrect answer.

(c) From the ANOVA table: $F = 2.69$, with $\text{df}_1 = 5$ and $\text{df}_2 = 160$. The reported p-value is $p = 0.023$. Since $p = 0.023 < \alpha = 0.05$, reject H_0. There is statistically significant evidence that average satisfaction scores differ across at least some call centers.

Grading Criterion

- **(5 pts)**: Correct $F = 2.69$, p-value ≈ 0.023, and conclusions.
- **(1-4 pts)**: Partially correct answers.
- **(0 pts)**: Incorrect answer.