# Statistics with Recitation — Quiz 5

November 25, 2025

Answer Key

1. (**17 points**) An airport claims recent staffing changes have reduced average security screening waits to 8 minutes. To audit this claim, a simple random sample of 28 passengers' waits (in minutes) was recorded at a randomly chosen checkpoint during a typical weekday. Summary statistics are shown below. The point estimate suggests the mean wait may be *less than* 8 minutes. Is the result statistically significant?

| $n$ | $\bar{x}$ | $s$ | min | max |
|---|---|---|---|---|
| 28 | 7.65 | 0.80 | 5.9 | 9.6 |

   (a) (**2 points**) Write the hypotheses in symbols and in words.

   (b) (**6 points**) Check conditions, then calculate the test statistic $T$ and the associated degrees of freedom.

   (c) (**3 points**) According to the table, find a range for the p-value and interpret it.

   (d) (**2 points**) What is the conclusion of the (two-sided) hypothesis test at $\alpha = 0.05$?

   (e) (**4 points**) If you were to construct a 90% confidence interval corresponding to this test, would you expect 8 minutes to be in the interval? Explain briefly and show the interval.

**Suggested Answers for Problem 1:**

(a) Let $\mu$ be the true mean screening wait time (minutes) at this checkpoint.

$$H_0: \ \mu = 8 \qquad H_A: \ \mu < 8.$$

Grading Criterion

- **(2 pts)**: Correctly states both $H_0$ and $H_A$ in symbols and words.
- **(0 pts)**: Incorrect answer.

(b) Conditions for one-sample $t$ test:

- Independence: simple random sample of passengers; each wait time is independent.
- Approximately normal population or no strong skew/outliers in the sample.

Test statistic:

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{7.65 - 8}{0.80/\sqrt{28}} = \frac{-0.35}{0.1512} \approx -2.32, \quad \text{df} = n - 1 = 27.$$

Grading Criterion

- **(6 pts)**: Correct t-statistics and coniditions.
- **(1-5 pts)**: Partially correct answers.
- **(0 pts)**: Incorrect answer.

(c) For $T \approx -2.32$ with df $= 27$, the two-sided p-value

$$0.01 < p < 0.025$$

If the true mean wait were 8 minutes, a sample mean is lower than this for 1-2.5% of the time.

Grading Criterion

- **(3 pts)**: Correct p-value (range) and interpretations.
- **(1-2 pts)**: Partially correct answers.
- **(0 pts)**: Incorrect answer.

(d) Since $p < 0.025$, reject $H_0$. There is statistically significant evidence that the mean screening wait is less than 8 minutes.

Grading Criterion

- **(2 pts)**: Correct answer.
- **(1 pts)**: Incorrect answer.

(e) For a 90% two-sided interval, $t^\star \approx 1.703$ (df $= 27$).

$$\bar{x} \pm t^\star \frac{s}{\sqrt{n}} = 7.65 \pm 1.7 \left(\frac{0.80}{\sqrt{28}}\right) = 7.65 \pm 0.257 = (7.393,\ 7.907).$$

8 minutes is not in this interval.

Since 90% confidence interval is smaller than the 95% one, it $H_0$ is rejected at $\alpha = 0.05$, then it should also reject $H_0$ at $\alpha = 0.10$.

Grading Criterion

- **(4 pts)**: Correct confidence interval and interpretations.
- **(1-3 pts)**: Partially correct answers.
- **(0 pts)**: Incorrect answer.

2. (**13 points**) A telecom company operates six regional call centers and wants to know if average post-call satisfaction (0–100 scale) differs by center (each center has its own supervisor and training). The table below shows, for each center, the sample size $n_i$, sample mean $\bar{x}_i$, and sample standard deviation $s_i$ from a random sample of recent calls.

| | Ctr 1 | Ctr 2 | Ctr 3 | Ctr 4 | Ctr 5 | Ctr 6 |
|---|---|---|---|---|---|---|
| $n_i$ | 28 | 30 | 24 | 27 | 31 | 26 |
| $\bar{x}_i$ | 78.4 | 81.0 | 76.9 | 79.2 | 83.1 | 77.5 |
| $s_i$ | 10.5 | 9.7 | 10.9 | 10.1 | 9.3 | 11.2 |

The one-way ANOVA output below will be used to test differences between the average satisfaction scores across centers.

| | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|---|---|---|---|---|---|
| center | 5 | 1480.00 | 296.00 | 2.69 | 0.023 |
| Residuals | 160 | 17600.00 | 110.00 | | |

(a) (**2 points**) Write the hypotheses in symbols and in words.

(b) (**6 points**) Check the ANOVA conditions.

(c) (**5 points**) Conduct a hypothesis test to determine if these data provide convincing evidence that the average satisfaction score varies across some (or all) centers. Explain the results.

**Suggested Answers for Problem 2:**

(a) Let $\mu_i$ be the true mean of the center $i$ score.

$$H_0: \ \mu_1 = \mu_2 = \cdots = \mu_6$$

$$H_A: \ \text{At least one center's mean differs from the others.}$$

> Grading Criterion
>
> - **(2 pts)**: Correct null and alternative in words or symbols.
> - **(1 pts)**: Only partially correct statement.
> - **(0 pts)**: Incorrect or missing.

(b) ANOVA conditions:

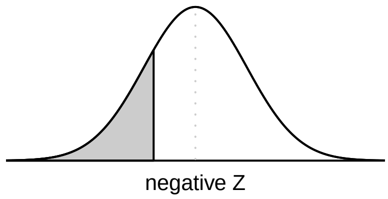
- Independence: calls sampled randomly within centers; different calls are independent.
- Approximate normality: each group's distribution is roughly symmetric with no extreme outliers.
- Equal variances: sample standard deviations are similar.

> Grading Criterion
>
> - **(6 pts)**: Correct ANOVA coniditions.
> - **(1-5 pts)**: Partially correct answers.
> - **(0 pts)**: Incorrect answer.

(c) From the ANOVA table: $F = 2.69$, with $\text{df}_1 = 5$ and $\text{df}_2 = 160$. The reported p-value is $p = 0.023$. Since $p = 0.023 < \alpha = 0.05$, reject $H_0$. There is statistically significant evidence that average satisfaction scores differ across at least some call centers.

> Grading Criterion
>
> - **(5 pts)**: Correct $F = 2.69$, p-value $\approx 0.023$, and conclusions.
> - **(1-4 pts)**: Partially correct answers.
> - **(0 pts)**: Incorrect answer.

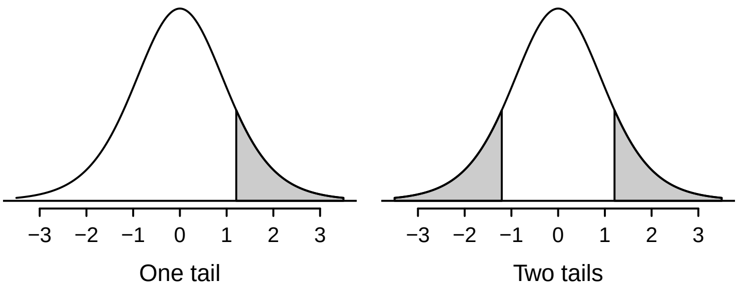
Table 1: **Standard normal probability table**




| Second decimal place of $Z$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.09 | 0.08 | 0.07 | 0.06 | 0.05 | 0.04 | 0.03 | 0.02 | 0.01 | 0.00 | $Z$ |
| 0.0002 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | −3.4 |
| 0.0003 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0005 | 0.0005 | 0.0005 | −3.3 |
| 0.0005 | 0.0005 | 0.0005 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0007 | 0.0007 | −3.2 |
| 0.0007 | 0.0007 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0009 | 0.0009 | 0.0009 | 0.0010 | −3.1 |
| 0.0010 | 0.0010 | 0.0011 | 0.0011 | 0.0011 | 0.0012 | 0.0012 | 0.0013 | 0.0013 | 0.0013 | −3.0 |
| 0.0014 | 0.0014 | 0.0015 | 0.0015 | 0.0016 | 0.0016 | 0.0017 | 0.0018 | 0.0018 | 0.0019 | −2.9 |
| 0.0019 | 0.0020 | 0.0021 | 0.0021 | 0.0022 | 0.0023 | 0.0023 | 0.0024 | 0.0025 | 0.0026 | −2.8 |
| 0.0026 | 0.0027 | 0.0028 | 0.0029 | 0.0030 | 0.0031 | 0.0032 | 0.0033 | 0.0034 | 0.0035 | −2.7 |
| 0.0036 | 0.0037 | 0.0038 | 0.0039 | 0.0040 | 0.0041 | 0.0043 | 0.0044 | 0.0045 | 0.0047 | −2.6 |
| 0.0048 | 0.0049 | 0.0051 | 0.0052 | 0.0054 | 0.0055 | 0.0057 | 0.0059 | 0.0060 | 0.0062 | −2.5 |
| 0.0064 | 0.0066 | 0.0068 | 0.0069 | 0.0071 | 0.0073 | 0.0075 | 0.0078 | 0.0080 | 0.0082 | −2.4 |
| 0.0084 | 0.0087 | 0.0089 | 0.0091 | 0.0094 | 0.0096 | 0.0099 | 0.0102 | 0.0104 | 0.0107 | −2.3 |
| 0.0110 | 0.0113 | 0.0116 | 0.0119 | 0.0122 | 0.0125 | 0.0129 | 0.0132 | 0.0136 | 0.0139 | −2.2 |
| 0.0143 | 0.0146 | 0.0150 | 0.0154 | 0.0158 | 0.0162 | 0.0166 | 0.0170 | 0.0174 | 0.0179 | −2.1 |
| 0.0183 | 0.0188 | 0.0192 | 0.0197 | 0.0202 | 0.0207 | 0.0212 | 0.0217 | 0.0222 | 0.0228 | −2.0 |
| 0.0233 | 0.0239 | 0.0244 | 0.0250 | 0.0256 | 0.0262 | 0.0268 | 0.0274 | 0.0281 | 0.0287 | −1.9 |
| 0.0294 | 0.0301 | 0.0307 | 0.0314 | 0.0322 | 0.0329 | 0.0336 | 0.0344 | 0.0351 | 0.0359 | −1.8 |
| 0.0367 | 0.0375 | 0.0384 | 0.0392 | 0.0401 | 0.0409 | 0.0418 | 0.0427 | 0.0436 | 0.0446 | −1.7 |
| 0.0455 | 0.0465 | 0.0475 | 0.0485 | 0.0495 | 0.0505 | 0.0516 | 0.0526 | 0.0537 | 0.0548 | −1.6 |
| 0.0559 | 0.0571 | 0.0582 | 0.0594 | 0.0606 | 0.0618 | 0.0630 | 0.0643 | 0.0655 | 0.0668 | −1.5 |
| 0.0681 | 0.0694 | 0.0708 | 0.0721 | 0.0735 | 0.0749 | 0.0764 | 0.0778 | 0.0793 | 0.0808 | −1.4 |
| 0.0823 | 0.0838 | 0.0853 | 0.0869 | 0.0885 | 0.0901 | 0.0918 | 0.0934 | 0.0951 | 0.0968 | −1.3 |
| 0.0985 | 0.1003 | 0.1020 | 0.1038 | 0.1056 | 0.1075 | 0.1093 | 0.1112 | 0.1131 | 0.1151 | −1.2 |
| 0.1170 | 0.1190 | 0.1210 | 0.1230 | 0.1251 | 0.1271 | 0.1292 | 0.1314 | 0.1335 | 0.1357 | −1.1 |
| 0.1379 | 0.1401 | 0.1423 | 0.1446 | 0.1469 | 0.1492 | 0.1515 | 0.1539 | 0.1562 | 0.1587 | −1.0 |
| 0.1611 | 0.1635 | 0.1660 | 0.1685 | 0.1711 | 0.1736 | 0.1762 | 0.1788 | 0.1814 | 0.1841 | −0.9 |
| 0.1867 | 0.1894 | 0.1922 | 0.1949 | 0.1977 | 0.2005 | 0.2033 | 0.2061 | 0.2090 | 0.2119 | −0.8 |
| 0.2148 | 0.2177 | 0.2206 | 0.2236 | 0.2266 | 0.2296 | 0.2327 | 0.2358 | 0.2389 | 0.2420 | −0.7 |
| 0.2451 | 0.2483 | 0.2514 | 0.2546 | 0.2578 | 0.2611 | 0.2643 | 0.2676 | 0.2709 | 0.2743 | −0.6 |
| 0.2776 | 0.2810 | 0.2843 | 0.2877 | 0.2912 | 0.2946 | 0.2981 | 0.3015 | 0.3050 | 0.3085 | −0.5 |
| 0.3121 | 0.3156 | 0.3192 | 0.3228 | 0.3264 | 0.3300 | 0.3336 | 0.3372 | 0.3409 | 0.3446 | −0.4 |
| 0.3483 | 0.3520 | 0.3557 | 0.3594 | 0.3632 | 0.3669 | 0.3707 | 0.3745 | 0.3783 | 0.3821 | −0.3 |
| 0.3859 | 0.3897 | 0.3936 | 0.3974 | 0.4013 | 0.4052 | 0.4090 | 0.4129 | 0.4168 | 0.4207 | −0.2 |
| 0.4247 | 0.4286 | 0.4325 | 0.4364 | 0.4404 | 0.4443 | 0.4483 | 0.4522 | 0.4562 | 0.4602 | −0.1 |
| 0.4641 | 0.4681 | 0.4721 | 0.4761 | 0.4801 | 0.4840 | 0.4880 | 0.4920 | 0.4960 | 0.5000 | −0.0 |

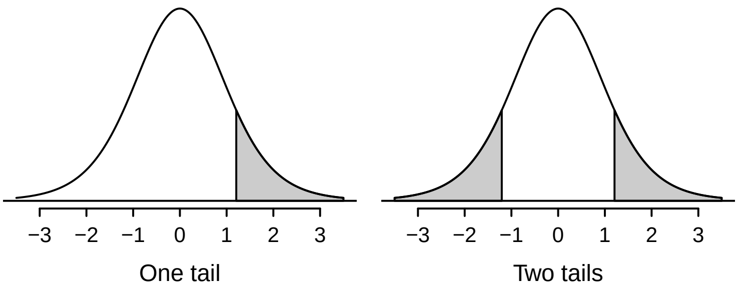
*For $Z \leq -3.50$, the probability is less than or equal to 0.0002.

Table 2: $t$ **distribution probability table** ($df \le 30$)




| one tail | | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 |
|---|---|---|---|---|---|---|
| two tails | | 0.200 | 0.100 | 0.050 | 0.020 | 0.010 |
| df | 1 | 3.08 | 6.31 | 12.71 | 31.82 | 63.66 |
| | 2 | 1.89 | 2.92 | 4.30 | 6.96 | 9.92 |
| | 3 | 1.64 | 2.35 | 3.18 | 4.54 | 5.84 |
| | 4 | 1.53 | 2.13 | 2.78 | 3.75 | 4.60 |
| | 5 | 1.48 | 2.02 | 2.57 | 3.36 | 4.03 |
| | 6 | 1.44 | 1.94 | 2.45 | 3.14 | 3.71 |
| | 7 | 1.41 | 1.89 | 2.36 | 3.00 | 3.50 |
| | 8 | 1.40 | 1.86 | 2.31 | 2.90 | 3.36 |
| | 9 | 1.38 | 1.83 | 2.26 | 2.82 | 3.25 |
| | 10 | 1.37 | 1.81 | 2.23 | 2.76 | 3.17 |
| | 11 | 1.36 | 1.80 | 2.20 | 2.72 | 3.11 |
| | 12 | 1.36 | 1.78 | 2.18 | 2.68 | 3.05 |
| | 13 | 1.35 | 1.77 | 2.16 | 2.65 | 3.01 |
| | 14 | 1.35 | 1.76 | 2.14 | 2.62 | 2.98 |
| | 15 | 1.34 | 1.75 | 2.13 | 2.60 | 2.95 |
| | 16 | 1.34 | 1.75 | 2.12 | 2.58 | 2.92 |
| | 17 | 1.33 | 1.74 | 2.11 | 2.57 | 2.90 |
| | 18 | 1.33 | 1.73 | 2.10 | 2.55 | 2.88 |
| | 19 | 1.33 | 1.73 | 2.09 | 2.54 | 2.86 |
| | 20 | 1.33 | 1.72 | 2.09 | 2.53 | 2.85 |
| | 21 | 1.32 | 1.72 | 2.08 | 2.52 | 2.83 |
| | 22 | 1.32 | 1.72 | 2.07 | 2.51 | 2.82 |
| | 23 | 1.32 | 1.71 | 2.07 | 2.50 | 2.81 |
| | 24 | 1.32 | 1.71 | 2.06 | 2.49 | 2.80 |
| | 25 | 1.32 | 1.71 | 2.06 | 2.49 | 2.79 |
| | 26 | 1.31 | 1.71 | 2.06 | 2.48 | 2.78 |
| | 27 | 1.31 | 1.70 | 2.05 | 2.47 | 2.77 |
| | 28 | 1.31 | 1.70 | 2.05 | 2.47 | 2.76 |
| | 29 | 1.31 | 1.70 | 2.05 | 2.46 | 2.76 |
| | 30 | 1.31 | 1.70 | 2.04 | 2.46 | 2.75 |

Table 3: $t$ **distribution probability table** ($df \geq 31$)




| one tail | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 |
|---|---|---|---|---|---|
| two tails | 0.200 | 0.100 | 0.050 | 0.020 | 0.010 |
| df 31 | 1.31 | 1.70 | 2.04 | 2.45 | 2.74 |
| 32 | 1.31 | 1.69 | 2.04 | 2.45 | 2.74 |
| 33 | 1.31 | 1.69 | 2.03 | 2.44 | 2.73 |
| 34 | 1.31 | 1.69 | 2.03 | 2.44 | 2.73 |
| 35 | 1.31 | 1.69 | 2.03 | 2.44 | 2.72 |
| 36 | 1.31 | 1.69 | 2.03 | 2.43 | 2.72 |
| 37 | 1.30 | 1.69 | 2.03 | 2.43 | 2.72 |
| 38 | 1.30 | 1.69 | 2.02 | 2.43 | 2.71 |
| 39 | 1.30 | 1.68 | 2.02 | 2.43 | 2.71 |
| 40 | 1.30 | 1.68 | 2.02 | 2.42 | 2.70 |
| 41 | 1.30 | 1.68 | 2.02 | 2.42 | 2.70 |
| 42 | 1.30 | 1.68 | 2.02 | 2.42 | 2.70 |
| 43 | 1.30 | 1.68 | 2.02 | 2.42 | 2.70 |
| 44 | 1.30 | 1.68 | 2.02 | 2.41 | 2.69 |
| 45 | 1.30 | 1.68 | 2.01 | 2.41 | 2.69 |
| 46 | 1.30 | 1.68 | 2.01 | 2.41 | 2.69 |
| 47 | 1.30 | 1.68 | 2.01 | 2.41 | 2.68 |
| 48 | 1.30 | 1.68 | 2.01 | 2.41 | 2.68 |
| 49 | 1.30 | 1.68 | 2.01 | 2.40 | 2.68 |
| 50 | 1.30 | 1.68 | 2.01 | 2.40 | 2.68 |
| 60 | 1.30 | 1.67 | 2.00 | 2.39 | 2.66 |
| 70 | 1.29 | 1.67 | 1.99 | 2.38 | 2.65 |
| 80 | 1.29 | 1.66 | 1.99 | 2.37 | 2.64 |
| 90 | 1.29 | 1.66 | 1.99 | 2.37 | 2.63 |
| 100 | 1.29 | 1.66 | 1.98 | 2.36 | 2.63 |
| 150 | 1.29 | 1.66 | 1.98 | 2.35 | 2.61 |
| 200 | 1.29 | 1.65 | 1.97 | 2.35 | 2.60 |
| 300 | 1.28 | 1.65 | 1.97 | 2.34 | 2.59 |
| 400 | 1.28 | 1.65 | 1.97 | 2.34 | 2.59 |
| 500 | 1.28 | 1.65 | 1.96 | 2.33 | 2.59 |
| $\infty$ | 1.28 | 1.65 | 1.96 | 2.33 | 2.58 |