Statistics with Recitation — Quiz 2

October 7, 2025

Answer Key

1. (**18 points**) A power bank's total available charge (in mAh) is denoted by X. A single phone charge uses a random amount of charge denoted by Y. Different power banks and phone charges vary slightly. Assume all quantities mentioned below are independent, and different phone charges are i.i.d. copies of Y.

	mean	SD	variance
X	10000	400	160000
Y	2000	300	90000

(a) (**6 points**) One full power bank, plus *four* single-phone charges from a second power bank, are used during a trip. What is the expected total charge delivered? What is its standard deviation?

(b) (**6 points**) How much charge would you expect to be left in a power bank after one phone charge? That is, find $\mathbb{E}[X - Y]$. What is the standard deviation of the remaining charge?

(c) (**6 points**) Using this context, explain why *variances* add even when we subtract one random variable from another (e.g., $\mathrm{Var}(X - Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$).

Suggested Answers for Problem 1:

(a) Total charge $= X + 4Y$.

$\mathbb{E}[X + Y_1 + Y_2 + Y_3 + Y_4] = \mathbb{E}[X] + 4\,\mathbb{E}[Y] = 10000 + 4(2000) = 18000$ mAh.

Independence $\Rightarrow$ Var$(X + Y_1 + Y_2 + Y_3 + Y_4) =$ Var$(X) + 4$ Var$(Y) = 160000 + 4(90000) = 520000$.

SD$(X + 4Y) = \sqrt{520000} \approx 721$ mAh.

> **Grading Criterion**
>
> - (6 pts): Correct 18000 mAh expectation and SD $\approx$ 721 (or $= 200\sqrt{13}$)
>
> - (2-4 pts): Partially correct answer
>
> - (0 pts): Incorrect

(b) Remaining charge $= X - Y$.

$\mathbb{E}[X - Y] = \mathbb{E}[X] - \mathbb{E}[Y] = 10000 - 2000 = 8000$ mAh.

Var$(X - Y) =$ Var$(X) +$ Var$(Y) = 160000 + 90000 = 250000$.

SD$(X - Y) = \sqrt{250000} = 500$ mAh.

> **Grading Criterion**
>
> - (6 pts): Correct 8000 mAh expectation and SD $= 500$ mAh
>
> - (2-4 pts): Partially correct answer
>
> - (0 pts): Incorrect

(c) In general, Var$(X \pm Y) =$ Var$(X) +$ Var$(Y) \pm 2$ Cov(X, Y). Under independence, Cov$(X, Y) = 0$, so even when we subtract, the covariance term vanishes and Var$(X - Y) =$ Var$(X) +$ Var(Y). Here, the variability in power-bank capacity and the variability in a phone's usage are independent sources of uncertainty, so their variances add.

> **Grading Criterion**
>
> - (6 pts): States variance-covariance identity and explains independence $\Rightarrow$ covariance $= 0$
>
> - (2-4 pts): Gives the correct rule with limited explanation
>
> - (0 pts): Incorrect

2. (**12 points**) Thalassemia is an autosomal recessive blood disorder. If both parents are carriers, each child has a 25% chance of having the disease, a 50% chance of being a carrier, and a 25% chance of being neither affected nor a carrier. Suppose two carrier parents have *four* children. Find the probability that

   (a) (**3 points**) none will have the disease;

   (b) (**3 points**) exactly one child will have the disease;

   (c) (**3 points**) at least one child will be neither affected nor a carrier;

   (d) (**3 points**) the first child with the disease will be the 4th child.

Suggested Answers for Problem 2:

(a) None diseased: $(0.75)^4 = \left(\frac{3}{4}\right)^4 = \frac{81}{256} \approx 0.3164$.

> **Grading Criterion**
>
> - (3 pts): Correct answer
> - (0 pts): Incorrect

(b) Let D denote "has disease" with $P(D) = 0.25$ and $P(\text{not } D) = 0.75$. Exactly one diseased among 4: $\binom{4}{1}(0.25)^1(0.75)^3 = 4 \cdot \frac{1}{4} \cdot \left(\frac{3}{4}\right)^3 = \frac{27}{64} \approx 0.4219$.

> **Grading Criterion**
>
> - (3 pts): Correct answer
> - (0 pts): Incorrect

(c) Let N denote "neither affected nor carrier" with $P(N) = 0.25$. At least one N: $1 - P(\text{no } N) = 1 - (0.75)^4 = 1 - \frac{81}{256} = \frac{175}{256} \approx 0.6836$.

> **Grading Criterion**
>
> - (3 pts): Correct answer
> - (0 pts): Incorrect

(d) First diseased is the 4th: first three not diseased, then diseased: $(0.75)^3(0.25) = \left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right) = \frac{27}{256} \approx 0.1055$.

> **Grading Criterion**
>
> - (3 pts): Correct answer
> - (0 pts): Incorrect