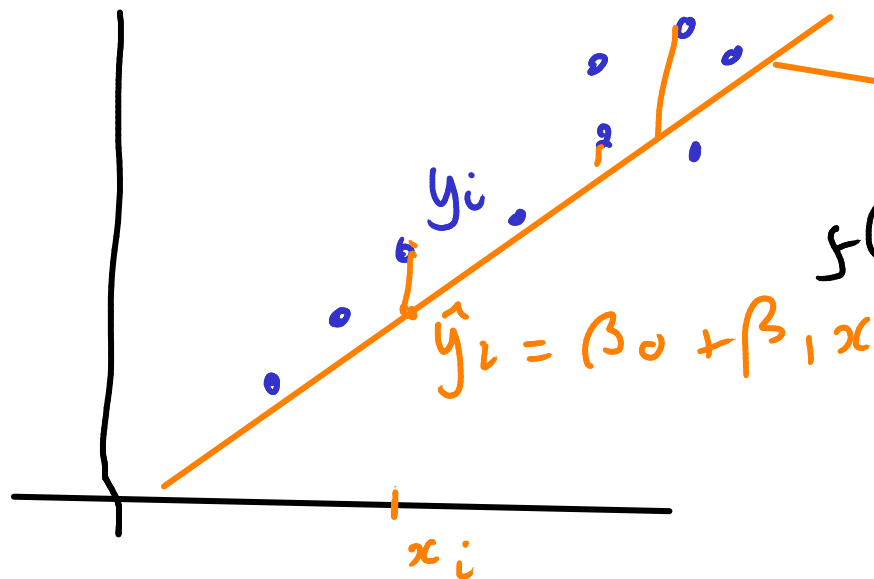
The background of the slide features a stylized globe on the left side, partially obscured by a dense pattern of binary code (0s and 1s) that recedes into the distance, creating a sense of depth and digital connectivity. The colors are primarily light blue and white.

# Foundations of Data Science:

## Logistic regression -

### Maximum likelihood estimation of logistic regression coefficients

# Principle of Maximum Likelihood



$$S(\beta_0, \beta_1) = \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2$$

Optimisation :

$$\beta_0 =$$

$$\beta_1 =$$

Adjust coefficients so as to maximise the likelihood of the data.

⇒ Expression for max. likelihood

Optimise w.r.t  $\beta_0, \beta_1, \dots$

## Maximum likelihood function

$y_i = 1 \Rightarrow \text{Success}$

$y_i = -1 \Rightarrow \text{Failure}$

$$P(Y = y_i | X = x_i) = \frac{y_i + 1}{2} f(\beta_0 + \beta_1 x_i)$$

$$+ \frac{-y_i + 1}{2} (1 - f(\beta_0 + \beta_1 x_i))$$
$$[1 - f(\beta_0 + \beta_1 x_i) = f(-\beta_0 - \beta_1 x_i)]$$

$$P(Y = y_i | X = x_i) = \frac{y_i + 1}{2} f(\beta_0 + \beta_1 x_i) + \left( \frac{-y_i + 1}{2} \right) f(-\beta_0 - \beta_1 x_i)$$

$$= \frac{y_i + 1}{2} f(y_i (\beta_0 + \beta_1 x_i)) + \left( \frac{-y_i + 1}{2} \right) f(y_i (\beta_0 + \beta_1 x_i))$$

$$= f(y_i (\beta_0 + \beta_1 x_i))$$

## Likelihood of dataset given model

$$\begin{aligned} P(Y=y \mid X=\underline{x}) &= P(Y=y_1 \mid X=x_1) P(Y=y_2 \mid X=x_2) \dots \\ &= \prod_{i=1}^n P(Y=y_i \mid X=x_i) \\ &= \prod_{i=1}^n f(y_i (\beta_0 + \beta_1 x_i)) \end{aligned}$$

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$$\ln ab = \ln a + \ln b$$

$$\ln \prod_{i=1}^n a_i = \sum_{i=1}^n \ln a_i$$

Log likelihood

$$\ln P(Y=y \mid X=\underline{x}) = \sum_{i=1}^n \ln f(y_i (\beta_0 + \beta_1 x_i))$$

Optimisation  $\Rightarrow \hat{\beta}_0$  and  $\hat{\beta}_1$ .



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UPDATE

Log your COVID vaccination in the app

# 39,014

Total number of new daily cases  
across the UK

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You can help fight COVID-19  
by aiding research

<https://covid.joinzoe.com/>