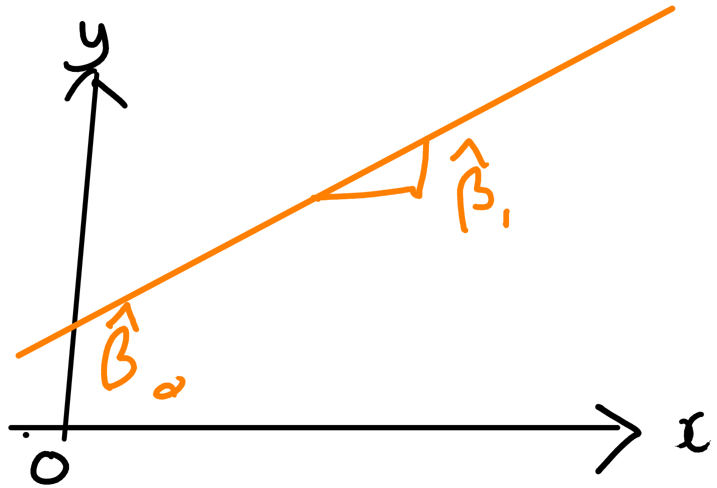
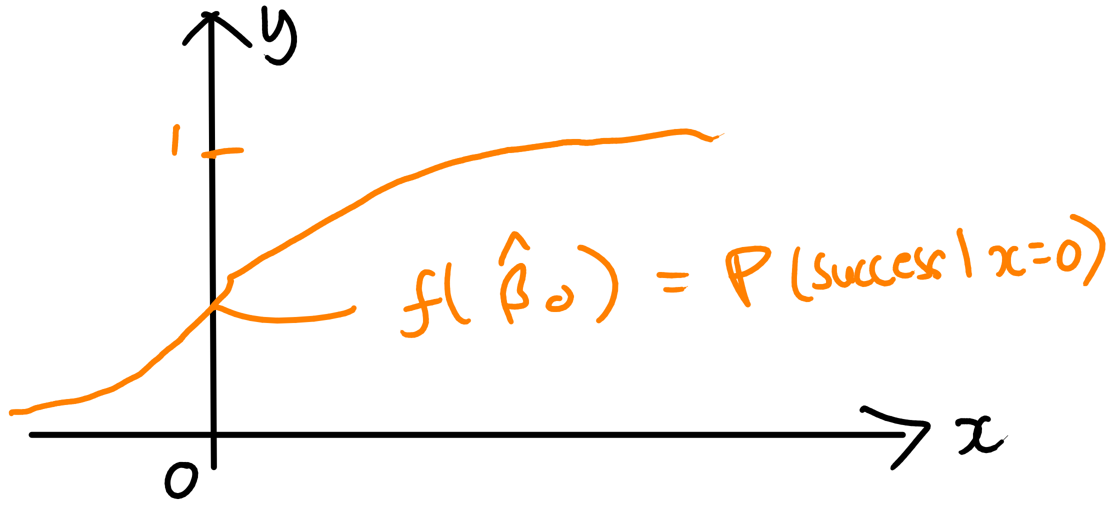
# Foundations of Data Science: Logistic regression - Interpretation of logistic regression coefficients

# Interpretation of $\hat{\beta}_0$

Lin reg



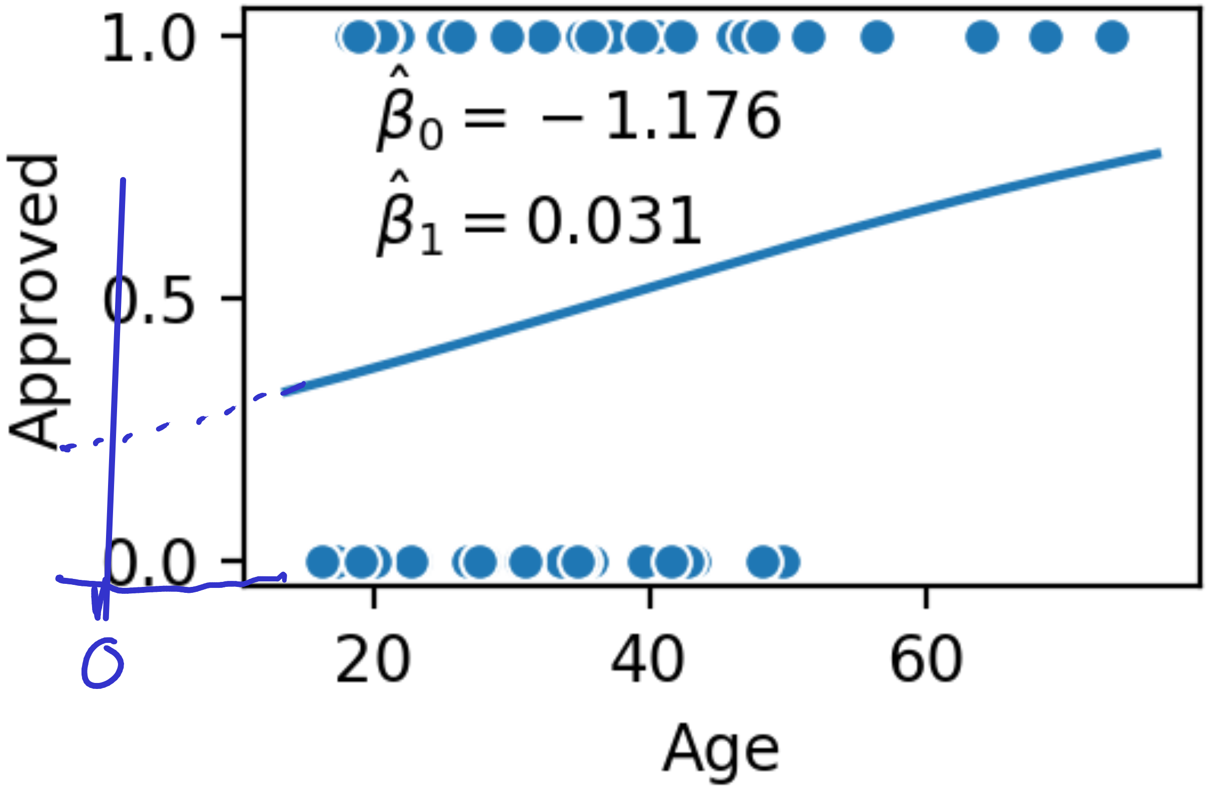

Log. reg




$x = 0$

$$\Rightarrow f(\hat{\beta}_0 + 0 \cdot \hat{\beta}_1) = f(\hat{\beta}_0)$$

$$= \frac{1}{1+e^{-\beta_0}}$$




$f(-1.176)$
$= 0.236$

# Log odds

$$\text{Log Odds(success)} = \ln \frac{P(\text{Success})}{P(\text{Failure})}$$

$$= \ln \frac{P(\text{Success})}{1 - P(\text{success})}$$

$$\boxed{\begin{array}{ll} \text{Log odds} & +1 \\ \Rightarrow \text{Odds} & \times e \end{array}}$$

| $P$ | Odds | Log odds | |
|---|---|---|---|
| 0.5 | 1 | 0 | Equally likely |
| > 0.5 | > 1 | > 0 | Success more likely than failure |
| < 0.5 | < 1 | < 0 | " less " " " |
| 1 | $\infty$ | $\infty$ | |
| 0 | 0 | $-\infty$ | |

# Logit scale

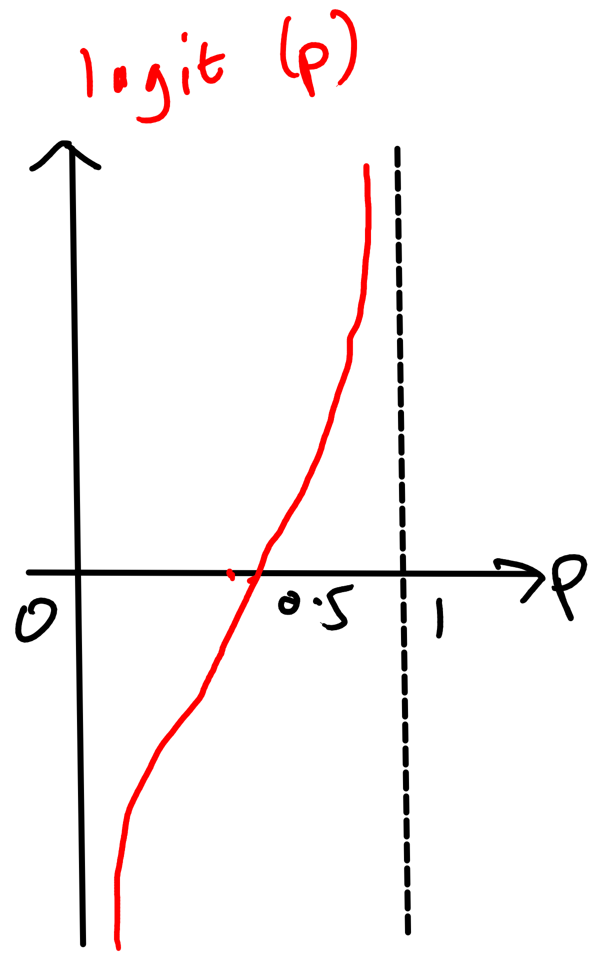
Log odds $+1$ $\Rightarrow$ odds increase by factor $e$

logistic unit $=$ logit

$f(\hat{\beta}_0) = 0.236$

$\hat{\beta}_0 = -1.176$

$$\text{logit}(p) = \ln \frac{p}{1-p}$$



# Logistic Regression in terms of log odds

Success $P(Y=1|x) = f(\beta_0+\beta_1 x) = \dfrac{1}{1+e^{-\beta_0-\beta_1 x}}$ ①

Failure $P(Y=0|x) = 1- f(\beta_0+\beta_1 x) = 1 - \dfrac{1}{1+e^{-\beta_0-\beta_1 x}}$

$$= \frac{e^{-\beta_0-\beta_1 x}}{1+e^{-\beta_0-\beta_1 x}}$$ ②

Odds $\dfrac{P(Y=1|x)}{P(Y=0|x)} = \dfrac{1}{e^{-\beta_0-\beta_1 x}} = e^{\beta_0+\beta_1 x}$

Log odds $\ln \dfrac{P(Y=1|x)}{P(Y=0|x)} = \beta_0+\beta_1 x = \text{logit}(P(Y=1|x))$

# Interpretation of $\hat{\beta}_1$

$$\text{Odds} = e^{\hat{\beta}_0 + \hat{\beta}_1 x}$$

$$= e^{\hat{\beta}_0} e^{\hat{\beta}_1 x}$$

$$OR(x) = e^{\hat{\beta}_1}$$

Credit e.g. $OR(Age) = e^{0.03} \simeq 1.03$

$\Rightarrow$ Every year of age 1.03 times more likely to have loan approved.