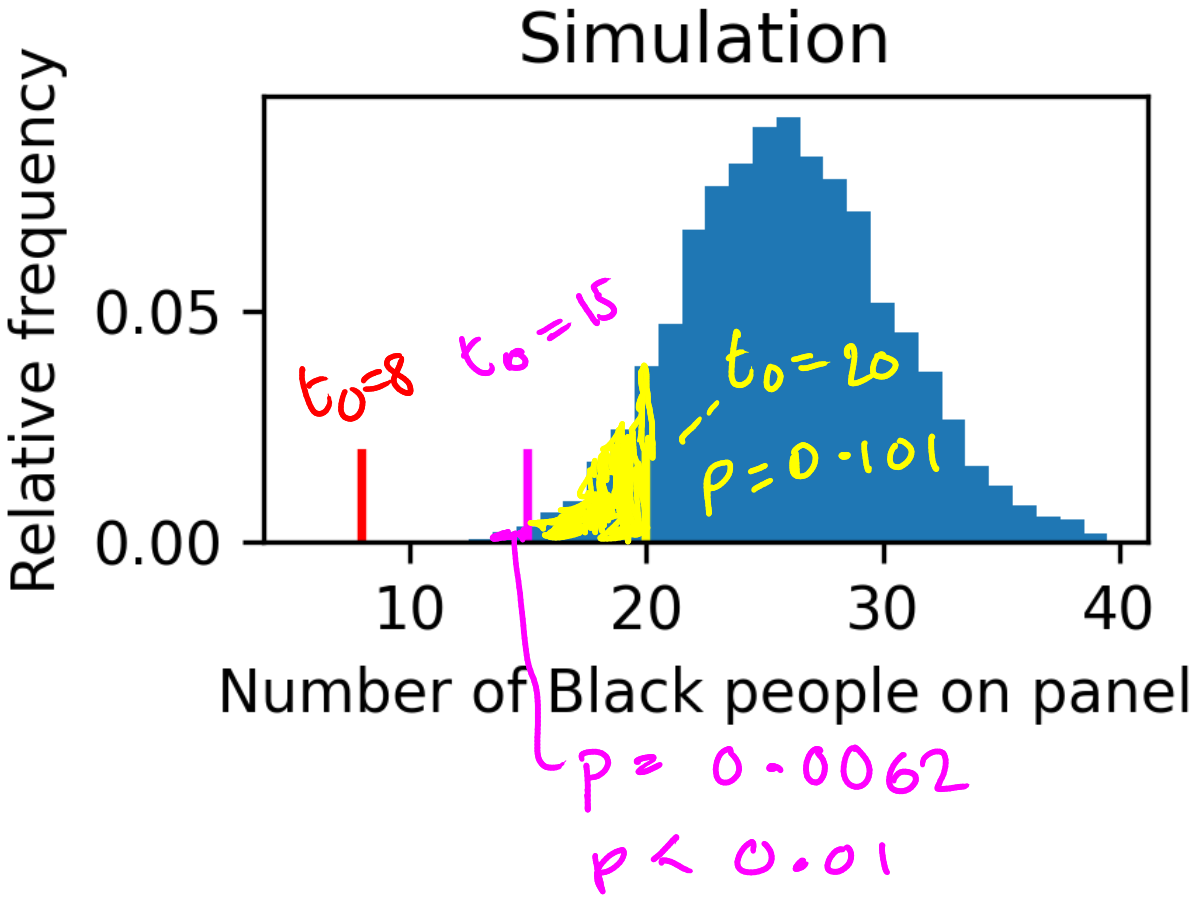
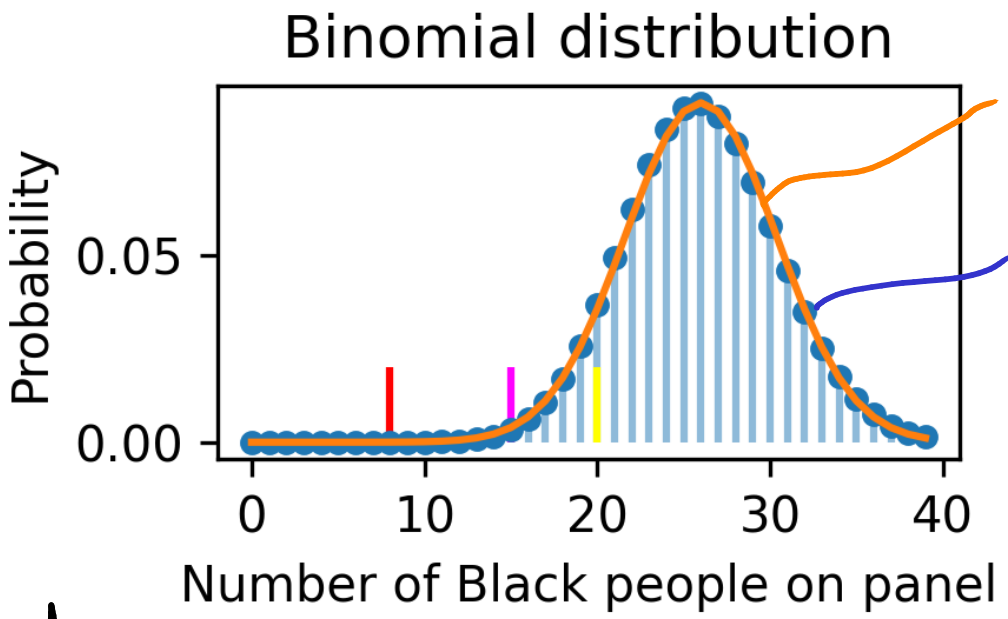
Foundations of Data Science: Hypothesis testing - p-values

Principle of p-values

Observed data ⇒ boundary of rejection region



Determining p-values from probability dists.



Normal dist
$\mu = np$
$\sigma = \sqrt{np(1-p)}$

$b(t_0; n, p)$

$n = 100$

$p = 0.26$

p-val =

Binomial: $P(T_0 \leq t_0) = B(t_0; n, p) = \sum_{t=0}^{t_0} b(t; n, p)$

Normal approx: $z = \frac{t_0 - \mu}{\sigma}$ p-val $= \Phi(z)$ - cumulative dist func. of normal

P-values computed by various methods

	t_0	Simulation	Binomial	Normal
0	8	0	4.73e-06	2.03e-05
1	15	0.0067	0.0061	0.0061
2	20	0.1020	0.1030	0.0857

Definition of p-value

The *p*-value is the probability, calculated assuming the null hypothesis is true, of obtaining a value of the test statistic at least as contradictory to H_0 as the value calculated from the available sample. (*Modern Mathematical Statistics with Applications*, p. 456)

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EDITORIAL

The ASA's Statement on *p*-Values: Context, Process, and Purpose

What p-values are

***P*-values can indicate how incompatible the data are with a specified statsistical model**. . .

The smaller the *p*-value, the greater the statistical incompatibility of the data with the null hypothesis, if the underlying assumptions used to calculate the *p*-value hold. This incompatibility can be interpreted as casting doubt on or providing evidence against the null hypothesis or the underlying assumptions. (*ASA Statement on Statistical Significance and P-values*)

What p-values are not

***P*-values do not measure the probability that the studied hypothesis is true, or the probability that the data were produced by random chance alone.**

Researchers often wish to turn a p-value into a statement about the truth of a null hypothesis, or about the probability that random chance produced the observed data. The p-value is neither. It is a statement about data in relation to a specified hypothetical explanation, and is not a statement about the explanation itself. (*ASA Statement on Statistical Significance and P-values*)

"Statistical significance"

$p < 0.05 \Rightarrow$ "statistly significant"

* significant at the $p < 0.05$ level

** " " " $p < 0.01$ "

*** " " " $p < 0.001$ "

Statistical significance

In February 2014, George Cobb, Professor Emeritus of Mathematics and Statistics at Mount Holyoke College, posed these questions to an ASA discussion forum:

Q: Why do so many colleges and grad schools teach $p = 0.05$?
A: Because that's still what the scientific community and journal editors use.
Q: Why do so many people still use $p = 0.05$?
A: Because that's what they were taught in college or grad school.