Homework 4 - Statistical Analysis

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Part 1 a: Derivation of V(Xbar)

$$
\begin{aligned}
Var(\overline{X}) &= V(\frac{1}{n}\sum_{i=1}^{n} X_i) && (1)\\
&= (\frac{1}{n})^2 \sum_{i=1}^{n} Var(X_i) && (2)\\
&= (\frac{1}{n})^2 \sum_{i=1}^{n} E(X_i)^2 - [E(X_i)]^2 && (3)\\
&= (\frac{1}{n})^2 \sum_{i=1}^{n} p_A - p_A^2 && (4)\\
&= (\frac{1}{n})^2 n(p_A)(1-p_A) && (5)\\
&= \frac{(p_A)(1-p_A)}{n} && (6)
\end{aligned}
$$

Part 1 b: Computing values for EXbar and V(Xbar) based on pA = 0.5 and n = 50

```
p_a <- 0.5
q_a <- 1 - p_a
n = 50
ex_a = p_a
var_a = ((p_a)*(q_a))/n

ex_a #Expectation
```

```
## [1] 0.5
```

```
var_a #Variance
```

```
## [1] 0.005
```

Part 1 c: Using CLT to Approximate Xbar

```
p_bhat <- 0.6

se <- sqrt(var_a) #standard error

p_val <- (1 - pnorm(p_bhat, mean = ex_a, sd = se))

p_val #result of p(Xbar > p_bhat)
```

```
## [1] 0.0786496
```

Part 1 d: Based on our p-value calculated by testing pB <= pA = 0.5, we should not reject the null hypothesis. If we use alpha = 0.05 as the rejection level, we want to reject the null hypothesis only if p(Xbar > p__bhat)

<= alpha. If this condition was satisfied, our estimate would have been far out of our confidence interval. Since our p value was 0.078, the null hypothesis can not be rejected.

Part 1 e: If the same observation was observed with p_bhat being 0.6 but with n = 100, the null hypothesis would have been rejected. By increasing the number of participants in our sample, the standard error and p-value decreases. After plugging in n = 100 into the above formulas, the new p-value will be 0.0025 (which is below our rejection threshold).

Part 1 f: The smallest value p_bhat to reject the null hypothesis with n = 100 (using one tailed test)

```
se = sqrt(0.5*(1-0.5)) / sqrt(100)

qb <- 0.5 + -qnorm(0.05, sd = se)

qb #Smallest value to reject the null hypothesis with
```

```
## [1] 0.5822427
```

Part 1 g: Small detectable improvement with n = 50

```
se = sqrt(0.5*(1-0.5)) / sqrt(50)

qb <- 0.5 + -qnorm(0.05, sd = se)

qb #Smallest value to reject the null hypothesis with n = 50
```

```
## [1] 0.6163087
```

```
sde <- qb - 0.5

sde #Smallest detectable movement
```

```
## [1] 0.1163087
```

Part 2 - Comparing to known click rate (pA = 0.75)

Part 2 (a + b): E(Xbar) and V(Xbar) and the computation of p(Xbar > p_bhat)

```
p_a <- 0.75
q_a <- 1 - 0.75
ex_a <- p_a
n = 50
var_a = ((p_a)*(q_a))/n
ex_a #Expectation
```

```
## [1] 0.75
```

```
var_a #Variance
```

```
## [1] 0.00375
```

```
p_bhat <- 0.6
se <- sqrt(var_a) #standard error
p_val <- (1 - pnorm(p_bhat, mean = ex_a, sd = se))
p_val #result of p(Xbar > p_bhat)
```

```
## [1] 0.9928471
```

Part 2 c: The null hypothesis can not be rejected. The value of p(Xbar > p_bhat) is well above our alpha threshold of 0.05 in this case

Part 2 d: The null hypothesis can still not be rejected. Instead of decreasing, the p-value ended up increasing towards a probability of 1. This means that our p hat estimation is still within our confidence interval to reject.

Part 2 e: Smallest value p_bhat that can reject the null hypothesis with n = 100

```
se = sqrt(0.75*(1-0.75)) / sqrt(100)

qb <- 0.75 + -qnorm(0.05, sd = se)

qb #Smallest value to reject the null hypothesis with
```

```
## [1] 0.8212243
```

Part 2 f: Smallest detectable improvement for pA = 0.75 with n = 50

```
se = sqrt(0.75*(1-0.75)) / sqrt(50)

qb <- 0.75 + -qnorm(0.05, sd = se)

qb #Smallest value to reject the null hypothesis with n = 50
```

```
## [1] 0.8507263
```

```
sde <- qb - 0.75

sde #Smallest detectable movement
```

```
## [1] 0.1007263
```

Part 3: The smallest detectable improvement in Part 1 g was larger than the SDE in Part 2f. This makes sense mathematically because our hypothesized probability in Part 1 was smaller than the one in Part 2. Therefore, the standard error is going to be higher in Part 1 than Part 2. When the standard error is higher, there is going to be more spread which explains why you have a wider spread of values that would disallow the rejection based on the threshold. When there's a higher variance there's going to be a larger spread from the expected value.

Part 4 a: Derivation of Var(Y)

$$
\begin{aligned}
Var(Y) &= Var(\overline{X}_B - \overline{X}_A) && (7)\\
&= Var(\overline{X}_B) + Var(\overline{X}_A) && (8)\\
&= V(\frac{1}{n}\sum_{i=1}^{n} X_iB) + V(\frac{1}{n}\sum_{i=1}^{n} X_iA) && (9)\\
&= (\frac{1}{n})^2\sum_{i=1}^{n} Var(X_iB) + (\frac{1}{n})^2\sum_{i=1}^{n} Var(X_iA) && (10)\\
&= (\frac{1}{n})^2\sum_{i=1}^{n} E(X_iB)^2 - [E(X_iB)]^2 + (\frac{1}{n})^2\sum_{i=1}^{n} E(X_iA)^2 - [E(X_iA)]^2 && (11)\\
&= (\frac{1}{n})^2\sum_{i=1}^{n} p_B - p_B^2 + (\frac{1}{n})^2\sum_{i=1}^{n} p_A - p_A^2 && (12)\\
&= (\frac{1}{n})^2 n(p_B)(1-p_B) + (\frac{1}{n})^2 n(p_A)(1-p_A) && (13)\\
&= \frac{(p_B)(1-p_B)}{n_B} + \frac{(p_A)(1-p_A)}{n_A} && (14)\\
&= \frac{(p_B)(1-p_B) + (p_A)(1-p_A)}{n} && (15)
\end{aligned}
$$

Part 4 b: Due to p_a and p_b being equal in our null hypothesis, we can add their results together. This gives n = 100 and s = 70, therefore giving a p value of .7

Part 4 c + d: Computed Variance for Y and find y hat estimation

```
p <- 0.7

var <- ((p)*(1-p)/45) + (p*(1-p)/55)

se = sqrt(var)

y_hat = (35/45)-(35/55)

y_hat #Estimate of y hat based on the data
```

```
## [1] 0.1414141
```

Part 4 e + f: After finding p(Y > y_hat), the the p value is above the threshold. Therefore, the null hypothesis can not rejected because our new p_value is still within the range.

```
p_val <- (1 - pnorm(y_hat, mean = 0, sd = se))

p_val #p(Y > y_hat)
```

```
## [1] 0.06236482
```