PHY 415 Fall 2023

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PHY 415, called, "Mathematical Methods for Physicists" is a course the brings together many of the mathematical approaches that we commonly use in physics and apply them to variety of problems. In this course, we will take a modeling-based approach where we focus on the mathematical descriptions of physical phenomenon and determine what mathematical and analytical approaches are useful in exploring those models.

To get a sense of the course, please read all the pages associated with our syllabus.

Learning Objectives

In this course, you will learn to:

- investigate physical systems using a variety of tools and approaches,
- construct and document a reproducible process for those investigations,
- use analytical, computational, and graphical approaches to answer specific questions in those investigations,
- provide evidence of the quality of work using a variety of sources, and
- collaborate effectively and contribute to a inclusive learning environment

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Part I

0 - Intro and Syllabus

CHAPTER

ONE

SYLLABUS AND OVERVIEW OF PHY 415

In designing this course, we plan to emphasize more independent learning on your part and greater agency for you in determining what you learn and how you demonstrate you have learned. So you should expect:

- to read a variety of pieces of information to coordinate information
- to present your ideas publicly and to discuss them
- to learn new approaches and novel techniques on your own
- to become more expert than me in the areas of your interest
- to learn more about scientists that you have not learned about

This is not to say that you are on your own. Here's what you can expect from us:

- resources, information, and tools to help you learn
- support and scaffolding to move you towards more independence in your learning
- timely and detailed feedback to help you along
- a commitment to an inclusive classroom

Statement on the use of Generative AI tools

1.1 Contact Information

1.1.1 Web page

- Web page for this class: https://dannycab.github.io/phy415fall23/content/intro

1.1.2 Instructor

- Prof. Danny Caballero (he/him/his)
- Class Meetings: Tuesdays and Thursdays 10:20am-12:10pm (Location: 1300 BPS)
- Email: caball14@msu.edu, office: 1310-A BPS
- Office hrs: To be scheduled, but I also have an open door policy. I enjoy visiting and talking with you about physics.

1.2 Teaching Assistant/Grader

- Ian Neuhart (he/him/his)
- Email: neuharti@msu.edu
- Office hrs: TBD

1.2.1 Learning Assistant

- Alia Valentine (she/her/hers)
- Email: valen176@msu.edu
- Office hrs: TBD, feel free to email me if you want to set up a time. I too enjoy visiting and talking with you about math and physics.

1.3 Grading

Details about course activities are here and information regarding assessment is here. Your grade will be comprised of completing weekly discussion questions, seven of eight worked problems, and three of four projects that you will complete in the form of a Jupyter notebook (a "computational essay", which we will discuss later).

Your grade is comprised of the following:

Activity	Percent of Grade
Weekly Reading Questions (completion)	5%
Seven of Eight Worked Problems (5% each)	35%
Three of Four Projects (see below)	60%
Total	**100%**

Your grade on each project is split between completion (50%) and quality (50%). We will collectively define "quality" in class, but we have provided a preliminary rubric for us to work from for the first project. Your final grade will be scaled based on your best performances; there will be slightly more projects than what comprises your grade. *The intent here is to to allow you space to explore a model or project that you really enjoy, and to reward you for doing that.* How your project grade is calculated appears below.

Activity	Percent of Grade
Best Project Grade	25%
2nd Best Project Grade	20%
3rd Best Project Grade	15%

While attendance is not required, you are unlikely to succeed with your projects without regular attendance and engagement.

1.4 "Extra" Credit

We get that you might want to do more of the projects or maybe you fell a little behind during the semester. If you complete 8 of the 8 worked problems, you can earn up to another 5%. If you complete four projects, then your lowest scoring project will be included up to an additional 10%. These have to be completed within the usual due dates for the worked problems and projects; not at the end of the semester.

1.5 Course Objectives

This course emphasizes making models of physical phenomenon and how we use various tools at our disposal to investigate those models. Hence, we have learning objectives for making models of these systems and for learning specific tools.

Principle Learning Objectives

Students will demonstrate they can:

- investigate physical systems of their choosing using a variety of tools and approaches
- construct and document a reproducible process for those investigations
- use analytical, computational, and graphical approaches to answer specific questions in those investigations
- provide evidence of the quality of their work using a variety of sources
- collaborate effectively and contribute to a inclusive learning environment

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Each of these learning objectives contributes to your development as a physicist. I recognize that these are **big** ideas to think about. What I mean is that the objectives above are quite broad and you might be able to see a little about what or why they are included. But, below, I added more detail about each one along with a smaller scale list of objectives that you will engage with. Throughout our course, you will have opportunities to demonstrate these objectives in your work. *My aim is to make what you are assessed on in this course something you are interested in, so these objectives reflect that.*

1.5.1 Investigate physical systems

Clearly, one of our central goals is learning how to make models of physical systems. This means learning about and developing fluency with a wide variety of mathematical and computational tools. In this courses, we will make extensive use of Jupyter notebooks for homework and projects. In fact, what you are reading is a set of Jupyter notebooks! Below, you will see the list of objectives for this principal objective.

Investigating Physical Systems Learning Objectives

Students will demonstrate they can:

- use mathematical techniques to predict or explain some physical phenomenon
- employ computational models and algorithms to investigate physical systems
- compare analytical and computational approaches to these investigations
- provide coherent explanations for their investigations buttressed by physical, mathematical, and/or computational knowledge and principles

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1.5.2 Construct and document a reproducible process

A critical element of physics work is making sure that with the same setup and approach, others can reproduce the work you have done. This provides validity to your work and evidences how we develop collective understanding of physics. Physics is a social enterprise and the ensuring the reproducibility of work supports that enterprise. Below are the learning objectives for this principal objective.

Reproducibility Learning Objectives

Students will demonstrate they can:

- document their work and analysis such that others can reproduce their work
- consistently reproduce their work and results in a variety of contexts
- provide an explanation for why certain work or results are not (or should not be) reproducible

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1.5.3 Use analytical, computational, and graphical approaches

The main approaches that we use to make models are mathematical, computational, and graphical. In this class, we will aim to leverage the benefits of each to learn more about the physical systems that we are investigating. Indeed, much of the "knowledge" that you are going to develop will be about specific analytical, computational, or graphical approaches to investigate physical systems. Below are the learning objectives for this principal objective.

Modeling Approaches Learning Objectives

Students will demonstrate they can:

- Use a wide variety of modeling techniques to investigate different physical systems
- Choose and employ appropriate approaches to modeling physical systems of their choosing
- Explain how those approaches lead to different results or conclusions

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1.5.4 Provide evidence of the quality of their work

The definition of the quality of a piece of science is a collective decision by the scientific community. In established communities, like physics, there are commonly-accepted ways of defining the quality of work (norms, customs, and rules all play a role). But that is not to mean those ways can't change; papers describing quantum physics and relativity brushed up hard against this issue of quality and were both dismissed and celebrated. Newer disciplines are still establishing those norms and rules. And in some cases, disciplines are pushing back against Western norms of quality. In our class, we will collectively decide what we mean by "high quality" work. Below are the learning objectives for this principal objective.

Quality Control Learning Objectives

Students will demonstrate they can:

- describe what it means to have high quality work in our class
- look for and evaluate when work meets those standards
- provide suggestions (or act on suggestions) to improve the quality of their work

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1.5.5 Collaborate effectively

Physics is a social enterprise that relies on effective and productive collaborations. Very little (if any) science is done alone; the scale of science is too grand for individuals to effectively work – everyone needs a team. In this spirit, in this classroom, we deeply encourage collaboration. We will try to develop effective collaboration through your work on projects and our in-class activities. Below are the learning objectives for this principal objective.

Collaboration Learning Objectives

Students will demonstrate they can:

- Collaborate on a variety of activities in and out of class
- Document the contributions in these collaborations and make changes if contributions are unbalanced
- Develop personally effective strategies for collaboration

1.6 Course Design

For most of you, 415 is an elective course that you are taking to learn more about how we use mathematical techniques in physics. As such, this course is designed under several different principles than a standard course. Below, I provide those principles and their rationale.

- 415 should help you learn the central tenets of modeling physical systems
  - The sheer volume of mathematical and computational physics knowledge out there is immense and impossible for any one person to learn. However, the central elements of making models, how to learn about specific techniques, and how to debug your approaches are things we can learn and employ broadly as well as to specific problems.
- 415 should be a celebration of your knowledge
  - For most of you, this course is part of your senior level coursework. What you have achieved in the last three to four years should be celebrated and enjoyed. This course will provide ample opportunities for you to share what things you know and what things you are learning with me and with each other.
- 415 should give you opportunities to engage in professional practice
  - As you start towards your professional career, it's important to learn what professional scientists do. You have probably already begun this work in advanced lab and research projects that you have worked on. We will continue developing your professional skills in this course through the use of course projects.
- 415 will illustrate that we can learn from each other
  - Even though I've been learning physics for almost 20 years, I don't know everything. I am excited to learn from you and I hope that you are excited to learn from me and each other.

1.6.1 Optional purchases:

The core readings and work for this course will be this jupyterbook. I will find resources online, make my own, and provide as much organized free material as possible. If you want to have a textbook that helps you organize your readings, please obtain copies of:

1. Mary Boas, *Mathematical Methods in the Physical Sciences* (Wiley; 2005). This book is the definitive text on mathematical approaches, written by Dr. Boas originally in 1966! Any 3rd edition will be useful and I will put the section numbers from Boas in the online readings.
2. Mark Newman, *Computational Physics* (CreateSpace Independent Publishing Platform; 2012). This book is a great introduction to a variety of computational physics techniques, written by UMich professor Mark Newman for a computational physics course. I will put section numbers from Newman in the online readings.

Additional sources

In addition, I will draw from the following books. I have copies of them if you want or need scans of sections. But they can found online in Google Books and other places as well. No need to purchase unless you want a copy for your personal library.

Mechanics

- Edwin Taylor, Mechanics
- Jerry Marion and Stephen Thornton, Classical Dynamics of Particles and Systems
- Charles Kittel, Walter D. Knight, Malvin A. Ruderman, A. Carl Helholtz, and Burton J. Moyer, Mechanics

Electromagnetism

- Edward Purcell, Electricity and Magnetism
- David J. Grriffths, Introduction to Electromagnetism

Quantum Mechanics

- David McIntyre, Quantum Mechanics
- David J. Griffiths, Introduction to Quantum Mechanics

Waves and Thermal Physics

- Frank S. Crawford, Waves
- Charles Kittel, Thermal Physics
- Ashley Carter, Classical and Statistical Thermodynamics
- Daniel Schroeder, Thermal Physics

Additional Physics Topics

- Steven H. Strogatz, Nonlinear Dynamics and Chaos
- B Lautrup, Physics of Continuous Matter
- Frank L. Pedrotti and Leno S. Pedrotti, Introduction to Optics

Mathematics

- Susan M. Lea, Mathematics for Physicists
- William E. Boyce and Richard C. DiPrima, Elementary Differential Equations
- James Brown and Ruel Churchill, Complex Variables and Applications
- Jerrold Marsden and Anthony Tromba, Vector Calculus
- Sheldon Ross, A First Course in Probability

Presenting (Visual) Information

- Edward Tufte, The Visual Display of Quantitative information
- Albert Cairo, The Truthful Art
- Stephen E. Toulmin, The Uses of Argument

1.7 Course Activities

1.7.1 “Readings”

“Reading” is an essential part of 415! Reading the notes before class is very important. I use “reading” in quotes, because in our class this idea goes beyond just reading text and includes understanding figures and watching videos. These should help inform the basis of your understating that we will draw on in class to clarify your understanding and to help you make sense of the material. I will assume you have done the required readings in advance! It will make a huge difference if you spend the time and effort to carefully read and follow the resources posted. The calendar has the details on videos and readings that will be updated.

Weekly Questions: To encourage and reward you for keeping up with the “readings”, there will be weekly questions about the readings posted for you to respond to. These are not meant to test your knowledge, but rather to focus your “reading” towards what you understand, and what you don’t yet understand. I will ask you about those things weekly and use that information to tailor in-class activities based on what I am hearing is confusing, unclear, or challenging. These questions are only graded for completion, but I do want your honest attempt.

1.7.2 Class Meetings

Classroom Etiquette: Please silence your electronic devices when entering the classroom. I don't mind you using them (in fact, see below, we will use them). But, sometimes, they can very distracting to your neighbors, so use your judgement. I appreciate that you might have questions or comments about things in class. We are going to be having short lectures combined with longer project work in class. So you will have plenty of time to catch up with social media and the news.

If you and/or your group mates are confused, just raise your hand and ask questions. If you are confused, you are likely not the only one and it's better to chat about it, then move on. Questions are always good, and are strongly encouraged! *The only way we learn is to question what we know and how we know it.*

Computing Devices: Please bring some sort of computing device to class everyday. You might be researching information online, reviewing work you have done, or actively building models of systems together. This device can be a computer, a tablet, or a phone. You can also partner up with folks because we will use them in groups. *If you need a computing device brought to class for you or your group mates to use, let me know. I will organize for some small collection of laptops if we need it.*

In-Class: We will have some short lectures about topics or concepts; some of those will be in-the-moment as needed. The idea is that you are developing a basic understanding through readings and videos, practicing using those new ideas with me and with your classmates in class, and then applying what you are learning to new ideas. So, we will also use a variety of in-class activities that help you construct an understanding of a particular topic or concept. These will not be collected or graded, but we will discuss the solutions in class. *I will not post solutions for these activities as we have no exams or quizzes.*

1.7.3 "Homework"

Worked Problems: We will spend much of our time learning specific techniques and approaches to use withh many different kinds of problems and models. Your "weekly" homework will be to select a particular example where the method or model applies, and work that problem yourself. You will need to explain your approach and findings on paper or in a notebook. You will turn in seven of eight of these worked problems. *I will not post solutions for these problems as we have no exams or quizzes.*

1.7.4 Projects

In-class Projects: The class is designed to support your independent research into ideas that you are excited about. So in-class projects are meant to equip you with the knowledge and practice to learn new things for your projects. These in-class projects will be short demonstrations of models that you complete in groups. We will circulate around the room and check on you and your group's progress and understanding. At the end of the class period, we will share the results of the in-class project and discuss any sticking points. These in-class activities will not be graded, but they will be essential for your out-of-class projects.

Out-of-class Projects: For this class, we anticipate 6 projects to be turned in roughly every 2-3 weeks, with a weeklong turn-in window (see calendar). Except for the first project, up to 3 of these projects can be completed as partner projects. Partner projects are subject to a different grading rubric that evaluates collaborative efforts and increases the expectation for other areas compared to an individual project. A preliminary rubric appears here, but we will define these collectively after the first project.

These projects will take the form a computational essay, which provides documentation and rationale for the exploration that you are completing. We will model a computational essay project in our first project and we will reflect on the rubric after it, and make changes collectively as a class to it.

I strongly encourage collaboration, an essential skill in science and engineering (and highly valued by employers!) Social interactions are critical to scientists' success – most good ideas grow out of discussions with colleagues, and essentially all physicists work as part of a group. Find partners and work together. However, it is also important that you OWN the material. I strongly suggest you start working by yourself (and that means really making an extended effort on every activity). Then work with a group, and finally, finish up on your own – write up your own work, in your own way. There will also be time for peer discussion during classes – as you work together, try to help your partners get over confusions, listen to them, ask each other questions, critique, teach each other. You will learn a lot this way! For all assignments, the work you turn in must in the end be your own: in your own words, reflecting your own understanding. (If, at any time, for any reason, you feel disadvantaged or isolated, contact me and I can discretely try to help arrange study groups.)

Help Session

Help sessions/office hours are to facilitate your learning. We encourage attendance - plan on working in small groups, our role will be as learning coaches. The sessions are concept and project-centric, but we will not be explicitly telling anyone how to do your project (how would that help you learn?) I strongly encourage you to start all projects on your own. If you come to help sessions "cold", the value of the project to you will be greatly reduced.

1.8 Assessments

1.8.1 Formative Assessment

Formative assessment is often ungraded and reflective assessment. It is meant to help you make changes to your thinking, approaches, or practice. It is not evaluative, it's corrective; to help you make changes. We will make heavy use of ungraded formative feedback throughout the course. Our weekly readings and your worked problems are meant to be formative; meaning most folks will receive full credit, but the important part to review the feedback and reflect on it.

1.8.2 Summative Assessment

Summative assessment is typically evaluative and will take the form of course projects completed out of class. These projects will take the form of a computational essay in which you write mathematics and code to investigate and explain a given phenomenon of interest. We will explore those essays in class and talk about what makes a useful one as we define a rubric for evaluation.

Preliminary Rubric

A preliminary rubric has been posted. We will use this rubric for the first out-of-class project evaluation. We will then reflect on it and make changes to collectively as a class.

Resources for Computational Essays

If you want to read more about computational essays, here's a few links in the order utility/readability:

- Steven Wolfram - What is a Computational Essay?
- University of Oslo Physics - Examples of Computational Essays
- Odden and Burk, The Physics Teacher - Computational Essays in the Physics Classroom
- Odden, Lockwood, and Caballero, Physical Review PER - Physics computational literacy: An exploratory case study using computational essays

1.9 Project Rubrics

1.9.1 Preliminary (For first out of class project)

We have worked together to define elements of a rubric that matter for making physical models. These elements appear as part of major learning goals below.

Goal	Fractional Importance
Investigate physical systems	0.30
Construct and document a reproducible process	0.10
Use analytical, computational, and graphical approaches	0.30
Provide evidence of the quality of their work	0.10
Collaborate effectively	0.20

Goal: Investigate physical systems (0.30)

- How well does your computational essay predict or explain the system of interest?
- How well does your computational essay allow the user to explore and investigate the system?

Goal: Construct and document a reproducible process (0.10)

- How well does your computational essay reproduce your results and claims?
- How well documented is your computational essay?

Goal: Use analytical, computational, and graphical approaches (0.30)

- How well does your computational essay document your assumptions?
- How well does your computational essay produce an understandable and parsimonious model?
- How well does your computational essay explain the limitations of your analysis?

Goal: Provide evidence of the quality of their work

- How well does your computational essay present the case for its claims?
- How well validated is your model?

Goal: Collaborate effectively

- How well did you share in the class's knowledge?
  - How well is that documented in your computational essay?
- How well did you work with your partner ? *For those choosing to do so*

1.10 Calendar

In this course, we will cover four principal topics in physics (in this order):

1. Classical Mechanics and Ordinary Differential Equations
2. Electromagnetism and Partial Differential Equations
3. Waves and Fourier Analysis
4. Statistical Mechanics and Monte Carlo models

The class is roughly broken into equal parts for each section. Below we indicate how each week will be spent. This is subject to change based on your input and feedback. But roughly you can expect the following:

1.10.1 Classical Mechanics and Ordinary Differential Equations

- Week 1 - Introduction to the Course, Python, and Jupyter notebooks; Modeling; Coordinate Systems and Frames
- Week 2 - Calculus of Variations; Lagrangian Mechanics; Equations of Motion; Numerical Integration and Trajectories
  - **Worked Problem 1 Assigned; due end of Week 3; 9/15**
- Week 3 - Dynamical Systems; Phase Space; Stability
  - **Worked Problem 2 Assigned; due end of Week 4; 9/22**
- Week 4 - Chaos; Dynamical Analysis of ODEs
- **Project 1 (CM and ODEs) Due 9/29**

1.10.2 Electromagnetism and Partial Differential Equations

- Week 5 - Introduction to Electrostatics; Vector fields; Numerical superposition; Laplace's Equation; Separation of Variables
  - **Worked Problem 3 Assigned; due end of Week 6; 10/6**
- Week 6 - 2D Partial Differential Equations; Boundary Conditions; Numerical Solutions; Method of Relaxation
  - **Worked Problem 4 Assigned; due end of Week 7; 10/13**
- Week 7 - Magnetic Fields; Maxwell's Equations; Vector Calculus; EM Wave Equation
- **Project 2 (EM and PDEs) Due 10/20**

1.10.3 Waves and Fourier Analysis

- Week 8 - Introduction to Waves; Normal Modes; Matrix Methods; Beats and Superposition
  - **Worked Problem 5 Assigned; due end of Week 9; 10/27**
- Week 9 - Wave formulations; Complex analysis; the Fourier Transform
  - **Worked Problem 6 Assigned; due end of Week 10; 11/3**
- Week 10 - Discrete Fourier Transform; Fast Fourier Transform; Applications of Fourier Analysis
- **Project 3 (Waves and Fourier) Due 11/10**

1.10.4 Statistical Mechanics and Monte Carlo models

- Week 11 - Distributions; Counting and Probability; Boltzmann Distribution
  - **Worked Problem 7 Assigned; due end of Week 12; 11/17**
- Week 12 - Statistical Mechanics; The Ideal Gas; Monte Carlo methods
  - **Worked Problem 8 Assigned; due end of Week 14; 12/1**
- Week 13 - Thanksgiving break
- Week 14 - Markov Chains; Metropolis Algorithm; Applications
- **Project 4 (Distributions and Monte Carlo) Due 12/8**

1.11 Classroom Environment

1.11.1 Commitment to an Inclusive Classroom

I am deeply committed to creating an inclusive classroom - one where you and your classmates feel comfortable, intellectually challenged, and able to speak up about your ideas and experiences. This means that our classroom, our virtual environments, and our interactions need to be as inclusive as possible. Mutual respect, civility, and the ability to listen and observe others are central to creating a classroom that is inclusive. I will strive to do this and I ask that you do the same. If I can do anything to make the classroom a better learning environment for you, please let me know.

If you observe or experience behaviors that violate our commitment to inclusivity, please let me know as soon as possible.

If I violate this principle, please let me know or please tell the undergraduate department chair, Stuart Tessmer (tessmer@pa.msu.edu), who I have informed to tell me about any such incidents without conveying student information to me.

1.11.2 Comments on preparation:

Physics 415 covers material you might have seen before. Many of the topics stem from a wide variety of physics courses you might have already taken. But, we might be applying them at a higher level of conceptual and mathematical sophistication.

Therefore you should expect:

- a large amount of material to review and digest.
- no recitations, and few examples covered in lecture. Most of the learning will be done through projects and questions you and your group mates raise.
- long, hard problems that usually cannot be completed by one individual alone.
- challenging projects.
- to learn more about being a physicist that you have in another class (I hope!).

Physics 415 is a challenging, upper-division physics course. Unlike more introductory courses, you are fully responsible for your own learning. In particular, you control the pace of the course by asking questions in class. I tend to speak quickly, and questions are important to slow down. This means that if you don't understand something, it is your responsibility to ask questions. Attending class and the help sessions gives you an opportunity to ask questions. I am here to help you as much as possible, but I need your questions to know what you don't understand.

Physics 415 covers some of the most important physics and mathematical methods in the field. Your reward for the hard work and effort will be learning important and elegant material that you will use over and over as a physics major. Here is what I have experienced, and heard from other faculty teaching upper division physics in the past:

- most students reported spending a minimum of 10 hours per week on the homework (!!)
- students who didn't attend the help sessions often did poorly in the class.
- students reported learning a tremendous amount in this class.

The course topics that we will cover in Physics 415 are among the greatest intellectual achievements of humans. Don't be surprised if you have to think hard and work hard to master the material.

1.11.3 Use of Generative AI Tools

You are welcome to use generative AI tools (e.g. ChatGPT, Dall-e, etc.) in this class as doing so aligns with the course learning goals. These tools can be useful in gathering information, troubleshooting code, and developing potential directions. However, you are responsible for the information you submit based on an AI query (for instance, that it does not violate intellectual property laws, or contain misinformation or unethical content). Your use of AI tools must be properly documented and cited in order to stay within university policies on academic integrity and the Spartan Code of Honor Academic Pledge.

For example, if generative AI is used to develop code or make sense of results, the original query, the resulting text, and your discussion of how that information was synthesized and used is **required** to be submitted with your work. This can be in an appendix if it distracts from the presentation of your work. Remember, AI is not likely to generate a response that would be seen as quality work and should be modified and improved. AI cannot think critically, so you must do that work. More details or the rationale for this policy can be found here: https://msu-cmse-courses.github.io/cmse202-F23-jb/course_materials/CMSE202_GenerativeAI_Policy.html

1.12 Resources

1.12.1 Confidentiality and Mandatory Reporting

College students often experience issues that may interfere with academic success such as academic stress, sleep problems, juggling responsibilities, life events, relationship concerns, or feelings of anxiety, hopelessness, or depression. As your instructor, one of my responsibilities is to help create a safe learning environment and to support you through these situations and experiences. I also have a mandatory reporting responsibility related to my role as a University employee. It is my goal that you feel able to share information related to your life experiences in classroom discussions, in written work, and in one-on-one meetings. I will seek to keep information you share private to the greatest extent possible. However, under Title IX, I am required to share information regarding sexual misconduct, relationship violence, or information about criminal activity on MSU's campus with the University including the Office of Institutional Equity (OIE).

Students may speak to someone confidentially by contacting MSU Counseling and Psychiatric Service (CAPS) (caps.msu.edu, 517-355-8270), MSU's 24-hour Sexual Assault Crisis Line (endrape.msu.edu, 517-372-6666), or Olin Health Center (olin.msu.edu, 517-884-6546).

1.12.2 Spartan Code of Honor Academic Pledge

As a Spartan, I will strive to uphold values of the highest ethical standard. I will practice honesty in my work, foster honesty in my peers, and take pride in knowing that honor is worth more than grades. I will carry these values beyond my time as a student at Michigan State University, continuing the endeavor to build personal integrity in all that I do.

1.12.3 Handling Emergency Situations

In the event of an emergency arising within the classroom, Prof. Caballero will notify you of what actions that may be required to ensure your safety. It is the responsibility of each student to understand the evacuation, "shelter-in-place," and "secure-in-place" guidelines posted in each facility and to act in a safe manner. You are allowed to maintain cellular devices in a silent mode during this course, in order to receive emergency SMS text, phone or email messages distributed by the university. When anyone receives such a notification or observes an emergency situation, they should immediately bring it to the attention of Prof. Caballero in a way that causes the least disruption. If an evacuation is ordered, please ensure that you do it in a safe manner and facilitate those around you that may not otherwise be able to safely leave. When these orders are given, you do have the right as a member of this community to follow that order. Also, if a shelter-in-place or secure-in-place is ordered, please seek areas of refuge that are safe depending on the emergency encountered and provide assistance if it is advisable to do so.

Part II

1 - Mechanics and ODEs

CHAPTER

TWO

WHAT IS MATHEMATICAL MODELING?

Nature reveals itself to us through interactions. We can tell from observations that it is nature's interactions that lead to its evolution. How nature is changing and predicting how it will change in the future is the work of science. In this work, we observe nature and its interactions to make models of those observations. We aim to predict and explain our observations of nature through this building of models.

In physics, our goals are typically to explain and predict observations of physical phenomenon. Here, we focus ourselves to those canonical things that physicists concern themselves with: motion, fields, waves, atoms, nuclei, and so on.

CHAPTER

THREE

29 SEPT 23 - ACTIVITY: WHAT IS A MODEL?

3.1 Notes

- *Class Meeting:* 29 Sept 23
- *Prior Reading:* None necessary

3.2 Models and modeling

This course focuses on common physical models and how we use mathematics and computing tools to investigate these models. But, we need to agree on an mutual understanding of a model. In this activity, you will be working in a group to define aspects of a good model for the physical sciences.

There are plenty of ideas on the internet, I can even provide you a link with plenty of information. But, the point of this activity is for us to define a good model now (given all your prior experiences with models and modeling), and to reflect on that definition as you develop expertise with models and modeling (through your learning in this class).

3.3 What is a model?

We are going to start with this short video made by Geoscientist John Aiken when he was a graduate student at Georgia Tech.

```
from IPython.display import YouTubeVideo
YouTubeVideo("dkTncoPqo5Y", width = 800, height = 600)
```

3.3.1 History and Philosophy of Science

If you would like to dive deeper into models and modeling, there's excellent work in history and philosophy of science. The field studies how science develops knowledge, practice, culture, and so on. It studies important events and provides critical information on important and, often, overlooked folks who do science. For example, historian and gender studies professor Sharon Traweek studies the high energy physics field. Her book, Beamtimes and Lifetimes: The World of High Energy Physicists [Tra09] is excellent.

Dame Nancy Cartwright (philosopher of science)

One of the more interesting scholars in Dame Nancy Cartwright who wrote a lot about the 'practice of science.' Her philosophical work informed many of the innovations in physics and broader science education – including many science courses at MSU.

Her writing is very interesting, but the style of writing can be a challenge to read. This is the nature of academic writing in different disciplines. Her book called "How The Laws of Physics Lie" [Car83] is worth a read. Here's a link to the first chapter.

3.4 What has been your experience with models?

Take 2-3 minutes to think about your prior physics class.

- What models have you used? What makes that a model?
- What modeling have you done? What makes that modeling?

```
## Take notes for yourself here

# Idea 1
# Idea 2
```

After we have discussed the above questions and generated a long list of experiences, you can move onto the next set of questions below.

Take 2-3 minutes to think about your prior work with models and modeling?

- What made a model good or not so good?
- What kinds of things could you do to make a better model?

```
## Take notes for yourself here

# Idea 1
# Idea 2
```

After we have discussed the above questions and generated a consensus list of good model features, you can move onto the next set of questions below.

Take 2-3 minutes and write down all the mathematical and/or computing models you have experienced, heard about, seen, or are otherwise familiar.

```
## Take notes for yourself here (use comments)

# Model 1
# Model 2
```

CHAPTER

FOUR

29 SEPT 23 - NOTES: SIMPLE HARMONIC OSCILLATOR

4.1 A Linear Restoring Force

The SHO model, as we will see, is the root of many more sophisticated models in physics – across many subdisciplines. There's a reason for this. The SHO is a simple model that captures the essence of many physical systems. It is a mathematical model for a "linear restoring force". Such a force gives rise to local stability, that is, a tendency for the system to return to a stable equilibrium position. The richness of this statement is not entirely clear, but we will see that it is a very powerful idea, that is related to to energy minimization, and the idea of a potential energy function.

A linear restoring force is simply one in which the size of the force is in direct proportion to some displacement and is directed opposite to the displacement. For example, a coiled spring exerts (rooughly) a linear restoring force. If you pull on a spring, the spring exerts a force on your hand that is proportional to the distance you pull the spring. It is directed opposite to the direction you pull.

In 1D, we can write this as:

$$F_{\text{spring}} = -kx$$

where k is the spring constant. The negative sign indicates that the force is directed opposite to the displacement, and x is the displacement from the equilibrium position. As we are going to consider more general motion, we can abstract this concept of a linear restoring force to three dimensions, and write:

$$\vec{F}_{\text{spring}} = -k\Delta\vec{x}$$

where $\Delta\vec{x}$ is the displacement vector from the equilibrium position.

4.2 The SHO mathematical model

We have seen that the canonical one-dimensional harmonic oscillator is described with the following **2nd order linear** differential equation:

$$m\ddot{s} = -ks$$

where s represents the distance from the oscillator's equilibrium position. It's important to note that this model for a linear restoring force gives rise to at **second order** differential equation. As you will see, there would be no ability for cycles or recurrent behavior without two dimensions (the two dimensions here are the position, s, and velocity of the oscillator, $\dot{s}$). It is also important that the differential equation is **linear**.

4.2.1 Linearity of differential equations

This is something that is easy to confuse because we use the term **linear** in different ways at different times. In many cases this prompts us to think of a line, so that we might think the only linear differential equations are ones where the variable of interest (in this case the position of the oscillator) appears only *linearly* in the equation.

Warning: So you might think that Nth order differential equations of the form:

$$\frac{d^n x}{dt^n} = c\,x$$

are the only ones that are linear. *Note for the harmonic oscillator* $n = 2$*;* $c < 0$.

That is not the case and this is key.

The linearity refers to the fact that we can write the differential equation as a **linear combination of the derivatives**.

Linear Differential Equations

So an Nth linear differential equation (e.g., for position and time) can have the general form:

$$\sum_{i=0}^{N} a_i(t)\frac{d^i x}{dt} = f(t)$$

where $\frac{d^i x}{dt}$ is the ith derivative of x (e.g., $i = 0$, $\frac{d^0 x}{dt} = x$; $i = 1$, $\frac{d^1 x}{dt} = \frac{dx}{dt}$) The coefficients are allowed to depend explicitly on time (i.e., $a_i(t)$; you have probably seen *constant coefficients* in the past (where $\frac{da_i(t)}{dt} = 0$, i.e., $a_i(t) =$ constant). In addition, as you can explore later, the system can be driven by a function that depends only on time (e.g., some forcing function, $f(t)$).

4.2.2 Why does linearity matter?

Because much of nature can be modeled with linear differential equations (or approximately so) and linear differential equations have really nice solution properties. *The solutions to linear differential equations are **holonomic functions**. This is just a fancy word for functions you know like polynomials, sine, cosine, exponentiale, and some special functions (e.g., Airy, Bessel, hypergeometric). But these solutions are really important because they are smooth as are their derivatives, so they have really nice properties.

In many cases, we can lean on strong properties derived from linearity:

- Uniqueness: If we find a solution to the differential equation and it satisfies the boundary/initial conditions, we are guaranteed that this is the only solution.
- Linear combinations: The linear nature of the differential equation means linear combinations of general solutions are unchanged. We can build solutions from known general solutions.

The Cauchy-Kowaleski theorem

This theorem is typically called the Cauchy theorem (or the uniqueness theorem) for Augustin Louis-Cachy, a French mathematician who contributed much to the field of complex analysis. However, Cauchy only proved this theorem in a special case. A Russian Mathematician, Sofya Kovalevskaya, who was the first woman to earn a PhD in Mathematics (earned 1874, University of Göttingen), proved the theorem, in general, in her dissertation.

So, it is really the "Kovalevskaya-Cauchy" theorem.

4.2.3 General Solution to the SHO

We can start with the differential equation:

$$\ddot{x} = -\omega^2 x$$

where x is the position relative to equilibrium and $\omega^2 = k/m$. We know this is a linear, second-order ODE, so we expect the solutions to be holonomic functions - that is, many of the functions we have seen before. Moreover, we need something that when we take two derivatives, we get the same functions back.

This is a key to suggest sin *and* cos *as our proposed linear combination.*

Now, we have to decide how to do that. Because **ANY** linear combination is a good starting guess, but not all of them a reasonable guesses. How do we decide?

Discussion Question

Below are several choices of potential general solution guesses, called "ansatz" in many books and classes.

Which of them will work for us? How do you know? Demonstrate that one solution you chose works.

$$x(t) = A\cos(\omega t)$$

$$x(t) = B\sin(\omega t)$$

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$x(t) = B\sin(\omega t + \phi_B)$$

$$x(t) = \sum_i^N A_i \cos(\omega_i t) + B_i \sin(\omega_i t)$$

What other general solution forms will work?

Hint

Can you think of other functions that when you take two derivatives, you get the same function back with a sign change?

What if the function were complex (recall: $i^2 = -1$)?

4.2.4 Particular Solutions

Let's pick two general solutions. Notice that both have two unknown constants. We can use the initial conditions to determine the values of these constants, and thus our particular solutions.

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$x(t) = C\sin(\omega t + \phi_C)$$

To determine a particular solution is to solve for the **trajectory** of the system given a set of initial conditions. You can think about a falling object, but any physical system that is evolving in time will have a trajectory in the general sense of various changing observable quantities. We will revisit this idea later once we make the connection to Dynamical Systems.

Find a particular solution

For an SHO that is pulled to a point x_0 and let go at $t = 0$, let's find the particular solutions using both general forms.

Compare and contrast these solutions with your neighbor. What do you notice about them? How can they be made compatible with each other?

- Make a plot (in this notebook) of both solutions for an SHO with frequency of 100Hz that starts at $x_0 = 0.1$m
- Adjust the starting location and frequency, what do you notice?
- **If you have time**, create another solution for an SHO that is put into motion with a velocity of +0.1m/s at $x_0 = 0$ at $t = 0$.

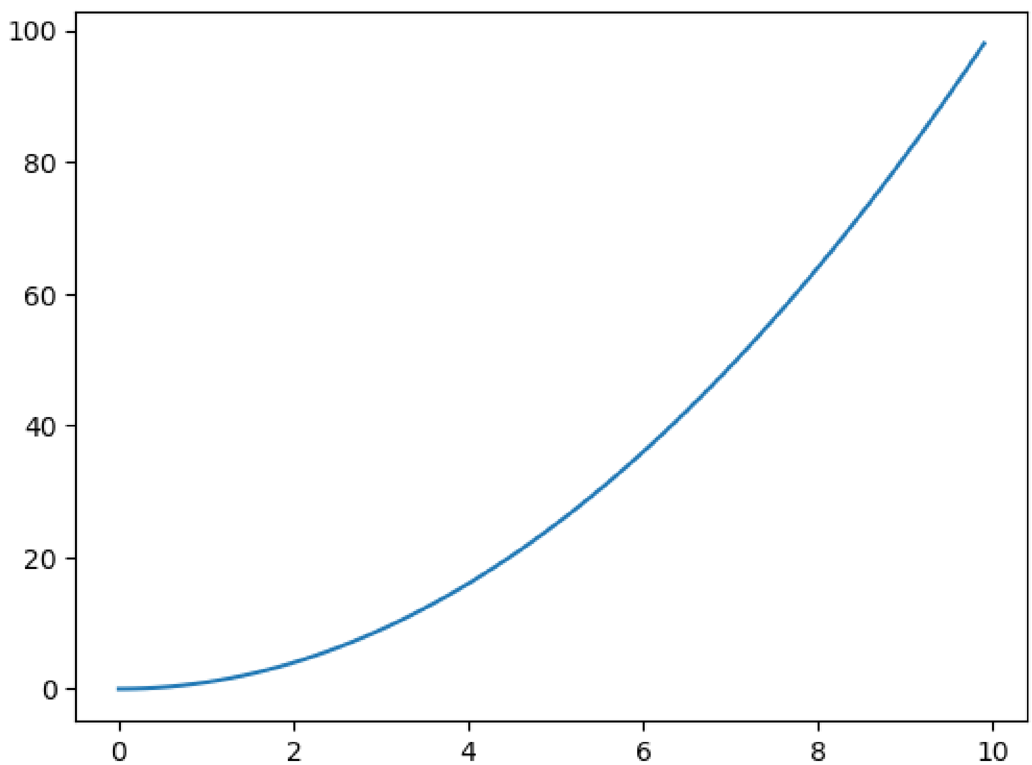
```
import numpy as np
import matplotlib.pyplot as plt

##################################
# Adjust code to fit your needs #
##################################

t = np.arange(0,10,0.1)
x = t**2

plt.plot(t,x)
```

```
[<matplotlib.lines.Line2D at 0x1161ded00>]
```



4.2.5 SHO properties

You have likely work with the SHO in other contexts. You have learned about a variety of aspects of the SHO. Talk with your neighbor about different things you all know about the properties of the SHO (it's motion, it's energy, etc.).

✎ Do this

Decide on one or two things to investigate and plot with respect to the SHO. **Be ready to explain your figure, to discuss how to improve it, and what we can claim from it.**

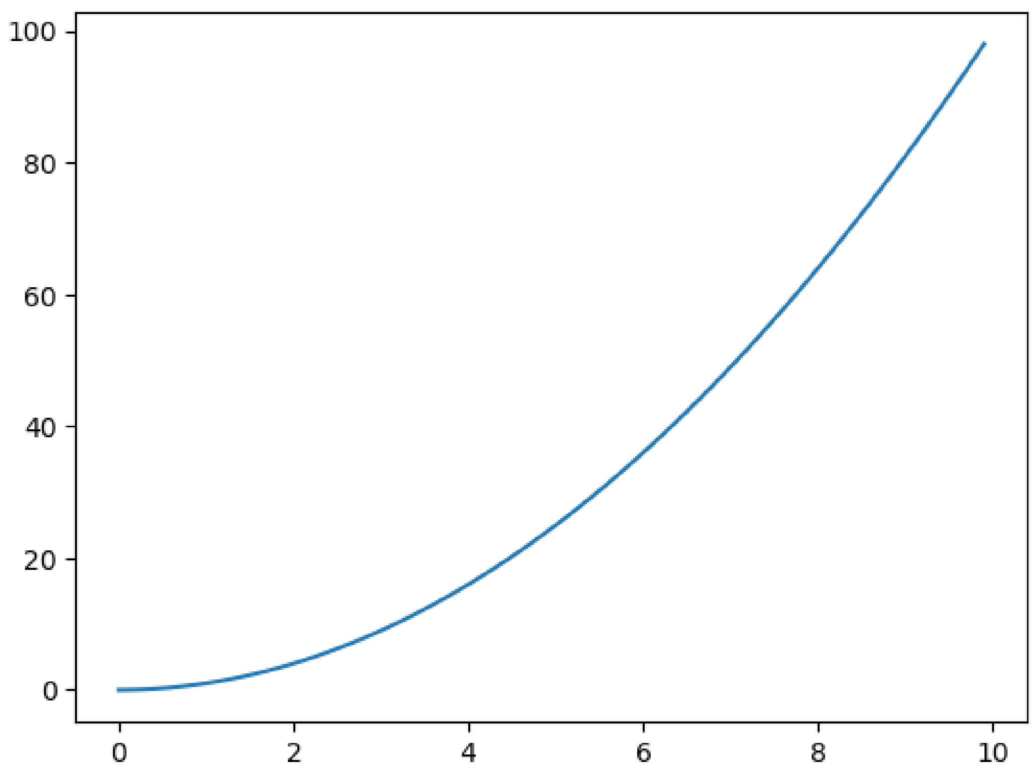
```
import numpy as np
import matplotlib.pyplot as plt

##################################
# Adjust code to fit your needs #
##################################

t = np.arange(0,10,0.1)
x = t**2

plt.plot(t,x)
```

```
[<matplotlib.lines.Line2D at 0x1162b9c10>]
```



4.3 Resources

Here are scans of four sections of four books (that you can look over at my office, or find elsewhere) that are useful for our study of the SHO and will help you with your project ideas and plans.

Sorry for the potato quality, and apparently the orientations, which I corrected but github refuses to keep the same orientation. I will do better next class and try to fix these later.

- Boas, Mathematical Methods, Sec 8.5
  - This section goes into all the math (in general) for ODEs like the SHO, which are second order ($\ddot{x}$), have constant coefficients (e.g., no explicit time dependence of the coefficients), and that are "homogenous" (i.e., have no constant terms or explicit functions of time).
  - If you want to be reminded about all the ODE things, it's a good read.
- Crawford, Waves, Secs. 1.1-1.2
  - This is an excellent book in general, but this a good reminder of all the things we did today in book form. His analysis and presenttation are formal, but really jammed packed with information. Crawford doesn't waste words. This book is out of print, but really worth reading if you are interested in waves. I tried to reproduce some of this in my own notes below.
- Marion and Thorton, Classical Mechanics, Secs. 3.1-3.2
  - This is a canonical text, that is boring. But it was all the things we did today and some stuff about the energetics.
- Strogatz, Nonlinear Dynamics, Sec. 5.1
  - This book is great. We are jumping into the middle of it, but the description of phase space analysis is so important and useful. I have tried to reproduce this in my own notes below.

4.4 Handwritten Notes

Below, I have worked up some additional notes that describe the conceptual and procedural aspects of what we did today. Think: derivations and examples for learning from for your project. They are linked below:

- The SHO

Part III

Appendices

CHAPTER

FIVE

WORKS CITED

BIBLIOGRAPHY

[Tra09] Sharon Traweek. *Beamtimes and Lifetimes*. Harvard University Press, 2009.

[Car83] Nancy Cartwright. *How the Laws of Physics Lie*. OUP Oxford, 1983.