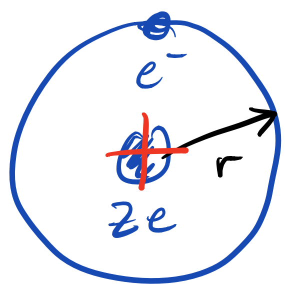
From our 3D Analysis,

$$H|nlm\rangle = E_n|nlm\rangle$$

$$\boxed{E_n = -\frac{1}{2n^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\frac{\mu}{\hbar^2} \qquad a = \frac{4\pi\varepsilon_0\hbar^2}{\mu Z e^2}}$$

energy eigenstates

modified Bohr radius



① $\frac{mv^2}{r} = F_{elec}$

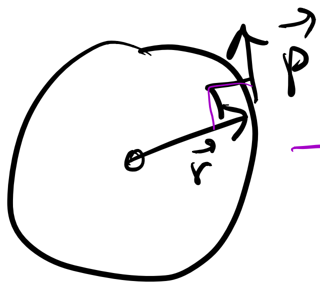
$$F_{elec} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

V = constant speed
r = constant radius } Uniform circular motion

$$\frac{m_e v^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} \qquad v^2 = \frac{Ze^2}{4\pi\varepsilon_0 r m_e}$$

$$\boxed{v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 r m_e}}}$$

② $|\vec{L}| = m_e v r$ $\qquad \vec{L} = \vec{r} \times \vec{p}$



$L = m_e v r \sin\theta = m_e v r$ ✓

③ $\underline{L = n\hbar}$ ← Bohr

$$L = m_e v r = n\hbar$$

$$r = \frac{n\hbar}{m_e v} = \frac{n\hbar}{m_e}\sqrt{\frac{4\pi\varepsilon_0 r m_e}{Ze^2}}$$

$$r^2 = \frac{n^2\hbar^2}{m_e^2}\,\frac{4\pi\varepsilon_0 m_e}{Ze^2}\,r$$

$$r = \frac{n^2\hbar^2\, 4\pi\varepsilon_0}{m_e Z e^2} = \frac{n^2(4\pi\varepsilon_0\hbar^2)}{m_e Z e^2}$$

$$\boxed{r = n^2 \frac{(4\pi\varepsilon_0\hbar^2)}{m_e Z e^2}} \qquad a = \frac{4\pi\varepsilon_0\hbar^2}{\mu Z e^2}$$

Hydrogen like $\quad m_{nuc} \gg m_e$

$$\boxed{\mu \simeq m_e}$$

(4) $KE = \frac{1}{2} m_e v^2 \qquad v^2 = \frac{Ze^2}{4\pi\varepsilon_0 r m_e}$

$$KE = \frac{1}{2} m_e \left(\frac{Ze^2}{4\pi\varepsilon_0 r m_e}\right)$$

$$KE = \frac{1}{2}\left(\frac{Ze^2}{4\pi\varepsilon_0 r}\right) \qquad r = \frac{n^2(4\pi\varepsilon_0\hbar^2)}{m Z e^2}$$

$$\boxed{KE = \frac{1}{2n^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{m_e}{\hbar^2}}$$

1D Bohr

$KE > 0$

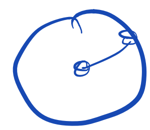
$$E_n = -\frac{1}{2n^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)\frac{\mu}{\hbar^2}$$

3D QM

Bound Energies

3D $\mu \Leftarrow$ $\rightarrow$ **Bohr** m_e

$H|nlm\rangle$



$L = n\hbar$

$$a = \frac{4\pi\varepsilon_0\hbar^2}{\mu Z e^2} = \frac{a_0}{Z} \longleftrightarrow r = \frac{4\pi\varepsilon_0\hbar^2}{m_e Z e^2} = \frac{a_0}{Z}$$

$\mu \simeq m_e$

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2}$$

$$E_n = -\frac{1}{2n^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\frac{\mu}{\hbar^2} \qquad KE = \frac{1}{2n^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\frac{m_e}{\hbar^2}$$

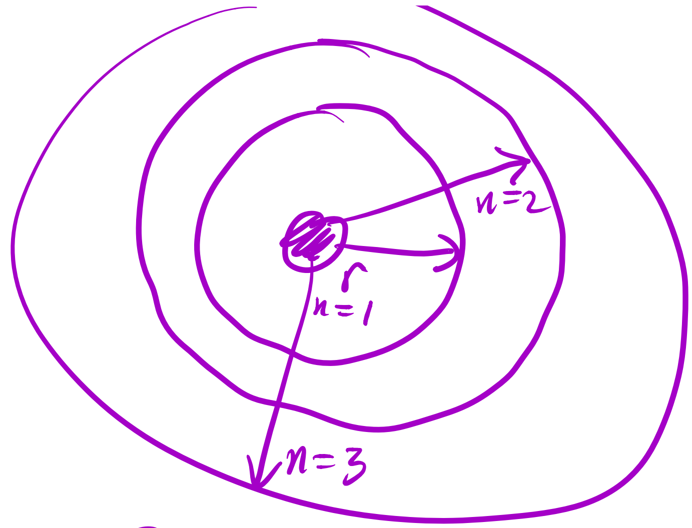
(−) Bound States

QM done before

(+) KE

Meaning of a (3D QM) & r (Bohr)

Bohr



3D QM   a?   length scale

$$\Delta x \, \Delta p_x \geq \hbar/2$$

$n=1 \quad m=0 \quad l=0$