LECTURE 2: MATHEMATICAL PRELIMINARIES

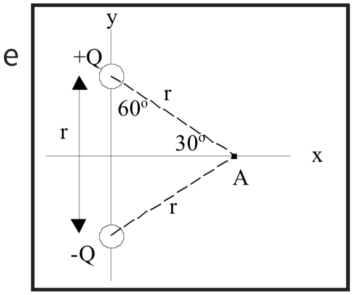
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \int \mathbf{E} \cdot d\mathbf{A} = \int \frac{\rho}{\epsilon_0} d\tau$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \int \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \int \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{A}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \qquad \int \mathbf{B} \cdot d\mathbf{A} = \mu_0 \int \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$$

Two charges +Q and -Q are fixed a distance r apart. The direction of the force on a test charge -q at A is...



A. Up
B. Down
C. Left
D. Right
E. Some other direction, or $F = 0$

In a typical Cartesian coordinate system, vector $\mathbf{A}$ lies along the $+\hat{x}$ direction and vector $\mathbf{B}$ lies along the $-\hat{y}$ direction. What is the direction of $\mathbf{A} \times \mathbf{B}$?

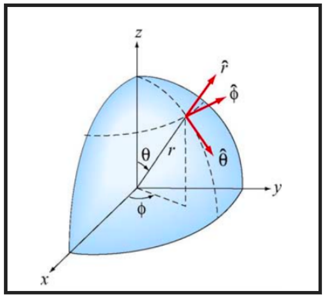
A. $-\hat{x}$
B. $+\hat{y}$
C. $+\hat{z}$
D. $-\hat{z}$
E. Can't tell

In a typical Cartesian coordinate system, vector $\mathbf{A}$ lies along the $+\hat{x}$ direction and vector $\mathbf{B}$ lies along the $-\hat{y}$ direction. What is the direction of $\mathbf{B} \times \mathbf{A}$?

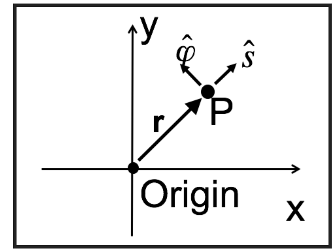
A. $-\hat{x}$
B. $+\hat{y}$
C. $+\hat{z}$
D. $-\hat{z}$
E. Can't tell

YOU DERIVE IT

Consider the radial unit vector ($\hat{r}$) in the spherical coordinate system as shown in the figure to the right. Determine the components of this unit vector in the Cartesian (x, y, z) system.



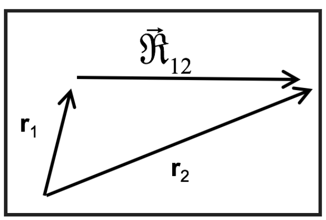
In cylindrical (2D) coordinates, what would be the correct description of the position vector $\mathbf{r}$ of the point P shown at $(x, y) = (1, 1)$?



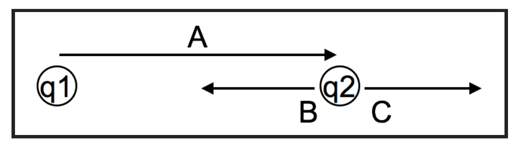
A. $\mathbf{r} = \sqrt{2}\hat{s}$
B. $\mathbf{r} = \sqrt{2}\hat{s} + \pi/4\hat{\phi}$
C. $\mathbf{r} = \sqrt{2}\hat{s} - \pi/4\hat{\phi}$
D. $\mathbf{r} = \pi/4\hat{\phi}$
E. Something else entirely

How is the vector $\mathfrak{R}_{12}$ related to $\mathbf{r}_1$ and $\mathbf{r}_2$?

A. $\mathfrak{R}_{12} = \mathbf{r}_1 + \mathbf{r}_2$
B. $\mathfrak{R}_{12} = \mathbf{r}_1 - \mathbf{r}_2$
C. $\mathfrak{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1$
D. None of these



Coulomb's Law: $\mathbf{F} = \frac{kq_1q_2}{|\mathfrak{R}|^2}\hat{\mathfrak{R}}$ where $\mathfrak{R}$ is the relative position vector. In the figure, q_1 and q_2 are 2 m apart. Which arrow **can** represent $\hat{\mathfrak{R}}$?



A. A
B. B
C. C
D. More than one (or NONE) of the above
E. You can't decide until you know if q_1 and q_2 are the same or opposite charges

You are trying to compute the work done by a force, $\mathbf{F} = a\hat{x} + x\hat{y}$, along the line $y = 2x$ from $\langle 0, 0\rangle$ to $\langle 1, 2\rangle$. What is $d\mathbf{l}$?

A. dl
B. $dx\ \hat{x}$
C. $dy\ \hat{y}$
D. $2dx\ \hat{x}$
E. Something else

You are trying to compute the work done by a force, $\mathbf{F} = a\hat{x} + x\hat{y}$, along the line $y = 2x$ from $\langle 0, 0\rangle$ to $\langle 1, 2\rangle$. Given that $d\mathbf{l} = dx\ \hat{x} + dy\ \hat{y}$, which of the following forms of the integral is correct?

A. $\int_0^1 a\ dx + \int_0^2 x\ dy$
B. $\int_0^1 (a\ dx + 2x\ dx)$
C. $\frac{1}{2} \int_0^2 (a\ dy + y\ dy)$
D. More than one is correct

A certain fluid has a velocity field given by $\mathbf{v} = x\hat{x} + z\hat{y}$. Which component(s) of the field contributed to "fluid flux" integral ($\int_S \mathbf{v} \cdot d\mathbf{A}$) through the x-z plane?

A. v_x
B. v_y
C. both
D. neither

For the same fluid with velocity field given by $\mathbf{v} = x\hat{x} + z\hat{y}$. What is the value of the "fluid flux" integral ($\int_S \mathbf{v} \cdot d\mathbf{A}$) through the entire x-y plane?

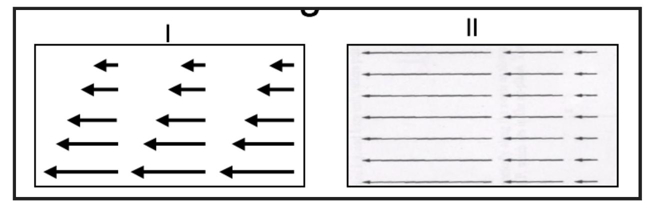
A. It is zero
B. It is something finite
C. It is infinite
D. I can't tell without doing the integral

A rod (radius R) with a hole (radius r) drilled down its entire length L has a mass density of $\frac{\rho_0 \phi}{\phi_0}$ (where ϕ is the normal polar coordinate).

To find the total mass of this rod, which coordinate system should be used (take note that the mass density varies as a function of angle):

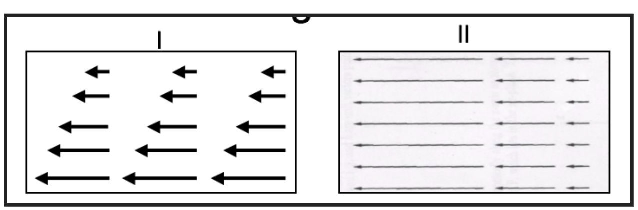
A. Cartesian (x, y, z)
B. Spherical (r, ϕ, θ)
C. Cylindrical (s, ϕ, z)
D. It doesn't matter, just pick one.

Which of the following two fields has zero divergence?



A. Both do.
B. Only I is zero
C. Only II is zero
D. Neither is zero
E. ???

Which of the following two fields has zero curl?



A. Both do.
B. Only I is zero
C. Only II is zero
D. Neither is zero
E. ???

Consider a vector field defined as the gradient of some well-behaved scalar function:

$$\mathbf{v}(x, y, z) = \nabla T(x, y, z).$$

What is the value of $\oint_C \mathbf{v} \cdot d\mathbf{l}$?

A. Zero
B. Non-zero, but finite
C. Can't tell without a function for T