True or False The following mathematical operation makes sense and is technically valid.

$$\nabla \cdot \nabla T(x, y, z)$$

A. Yes, it will produce a vector field.
B. Yes, it will produce a scalar field.
C. No, you can not take the divergence of a scalar field.
D. I don't remember what this means.

ANNOUNCEMENTS

- Homework 1 is due Friday in class
- Homework 2 is posted and covers through section 2.1
  - It is due next Wednesday
  - We will come back to section 1.5 later

You are trying to compute the work done by a force, $\mathbf{F} = a\hat{x} + x\hat{y}$, along the line $y = 2x$ from $\langle 0, 0 \rangle$ to $\langle 1, 2 \rangle$. What is $d\mathbf{l}$?

A. dl
B. $dx\ \hat{x}$
C. $dy\ \hat{y}$
D. $2dx\ \hat{x}$
E. Something else

You are trying to compute the work done by a force, $\mathbf{F} = a\hat{x} + x\hat{y}$, along the line $y = 2x$ from $\langle 0, 0\rangle$ to $\langle 1, 2\rangle$. Given that $d\mathbf{l} = dx\ \hat{x} + dy\ \hat{y}$, which of the following forms of the integral is correct?

A. $\int_0^1 a\ dx + \int_0^2 x\ dy$
B. $\int_0^1 (a\ dx + 2x\ dx)$
C. $\frac{1}{2}\int_0^2 (a\ dy + y\ dy)$
D. More than one is correct

A certain fluid has a velocity field given by $\mathbf{v} = x\hat{x} + z\hat{y}$. Which component(s) of the field contributed to "fluid flux" integral ($\int_S \mathbf{v} \cdot d\mathbf{A}$) through the x-z plane?

A. v_x

B. v_y

C. both

D. neither

For the same fluid with velocity field given by $\mathbf{v} = x\hat{x} + z\hat{y}$.
What is the value of the "fluid flux" integral ($\int_S \mathbf{v} \cdot d\mathbf{A}$)
through the entire x-y plane?

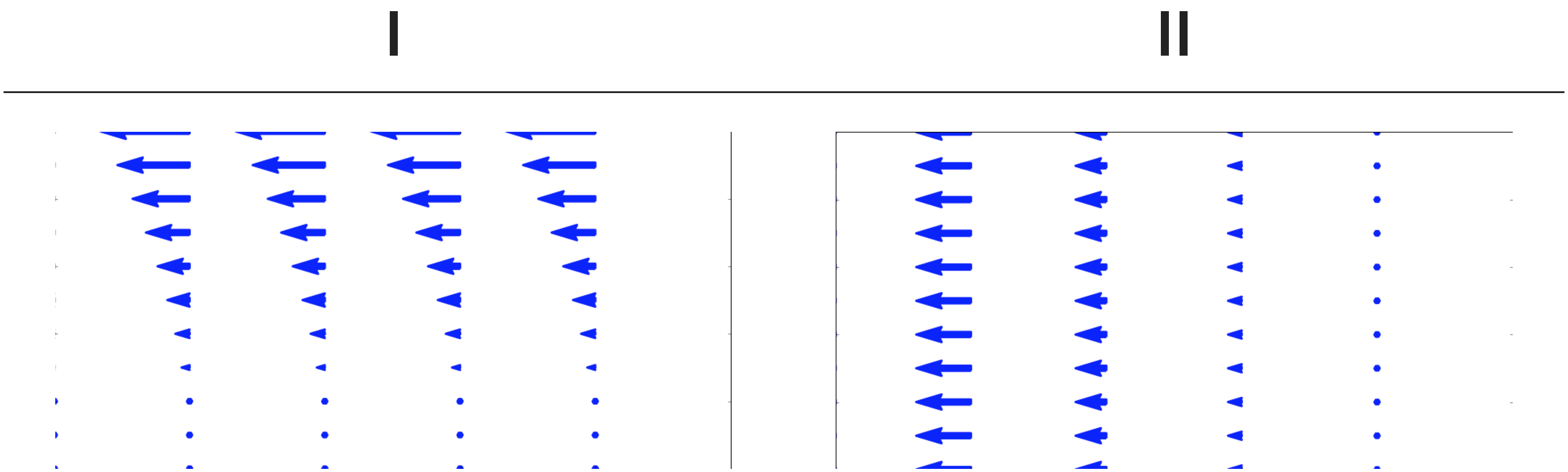
A. It is zero
B. It is something finite
C. It is infinite
D. I can't tell without doing the integral

A rod (radius R) with a hole (radius r) drilled down its entire length L has a mass density of $\frac{\rho_0 \phi}{\phi_0}$ (where ϕ is the normal polar coordinate).

To find the total mass of this rod, which coordinate system should be used (take note that the mass density varies as a function of angle):

A. Cartesian (x, y, z)
B. Spherical (r, ϕ, θ)
C. Cylindrical (s, ϕ, z)
D. It doesn't matter, just pick one.

Which of the following two fields has zero divergence?



A. Both do.

B. Only I is zero

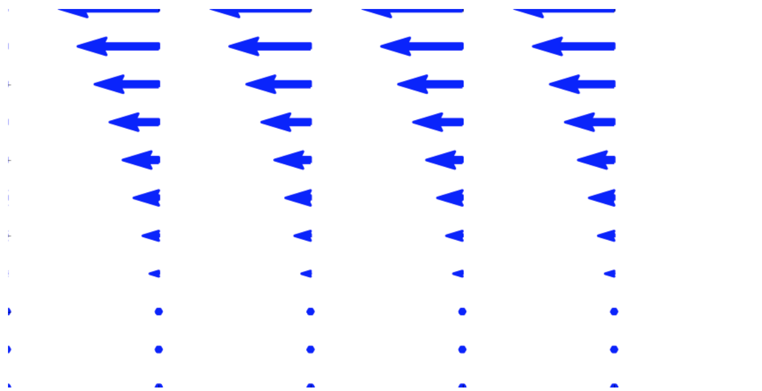
C. Only II is zero

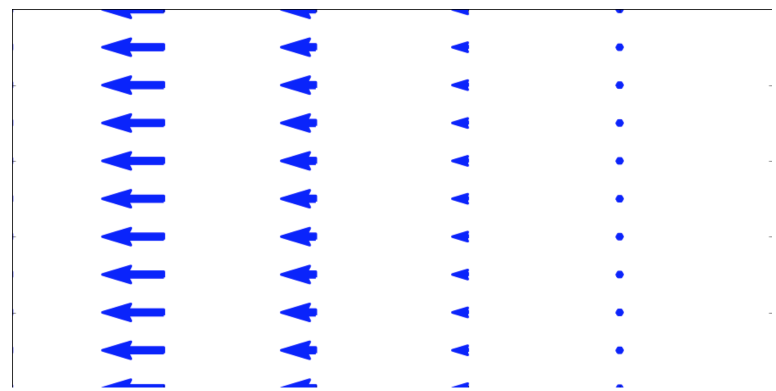
D. Neither is zero

E. ???

Which of the following two fields has zero curl?

I	II





A. Both do.

B. Only I is zero

C. Only II is zero

D. Neither is zero

E. ???

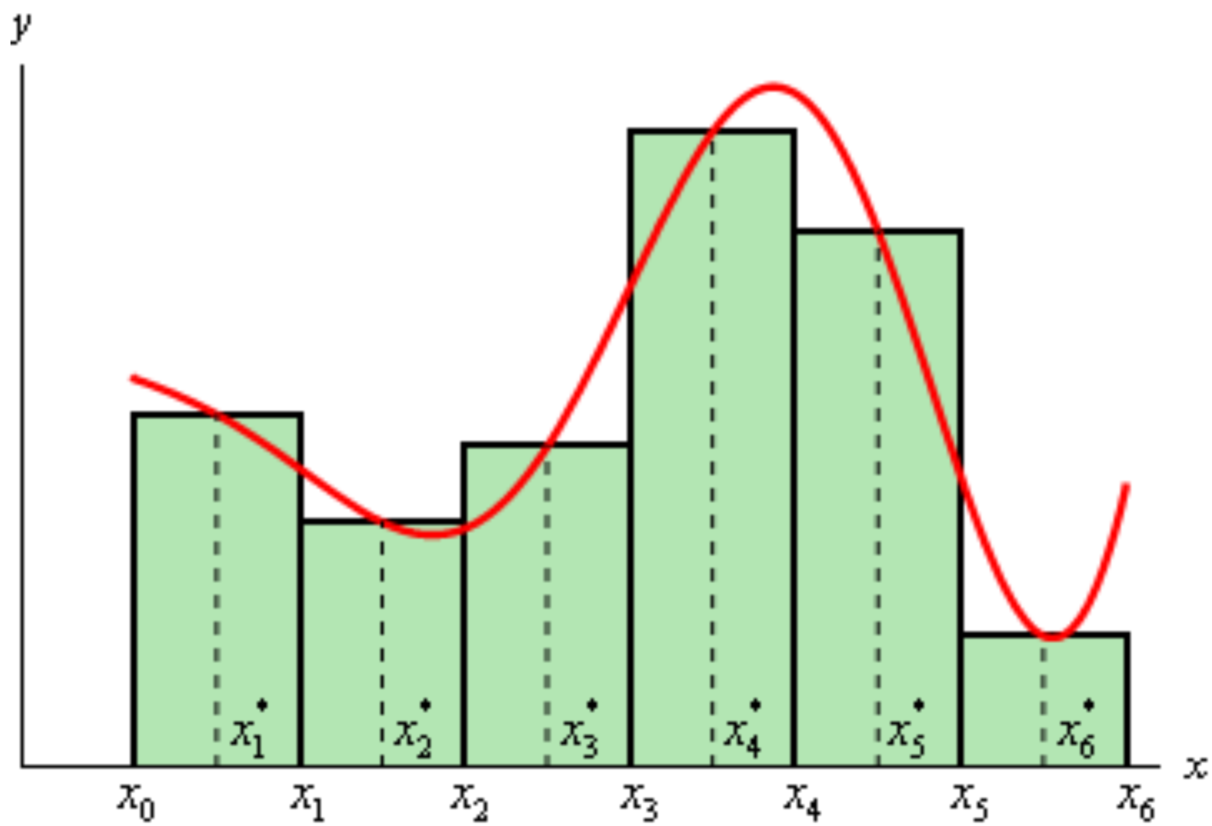
Consider a vector field defined as the gradient of some well-behaved scalar function:

$$\mathbf{v}(x, y, z) = \nabla T(x, y, z).$$

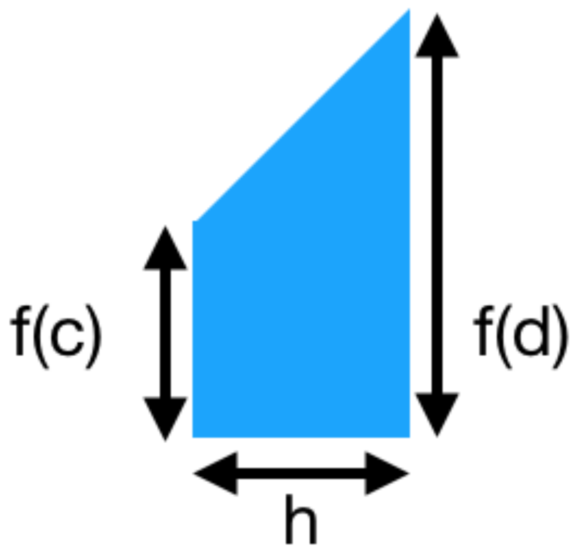
What is the value of $\oint_C \mathbf{v} \cdot d\mathbf{l}$?

A. Zero
B. Non-zero, but finite
C. Can't tell without a function for T

NUMERICAL INTEGRATION



Consider this trapezoid



What is the area of this trapezoid?

A. $f(c)h$

B. $f(d)h$

C. $f(c)h + \frac{1}{2}f(d)h$

D. $\frac{1}{2}f(c)h + \frac{1}{2}f(d)h$

E. Something else

The trapezoidal rule for a function $f(x)$ gives the area of the k th slice of width h to be,

$$A_k = \frac{1}{2} h \left(f(a + (k-1)h) + f(a + kh) \right)$$

What is the approximate integral, $I(a,b) = \int_a^b f(x) dx$,

$I(a,b) \approx$

A. $\sum_{k=1}^{N} \frac{1}{2} h \left(f(a + (k-1)h) + f(a + kh) \right)$
B. $h \left(\frac{1}{2} f(a) + \frac{1}{2} f(b) + \frac{1}{2} \sum_{k=1}^{N-1} f(a + kh) \right)$
C. $h \left(\frac{1}{2} f(a) + \frac{1}{2} f(b) + \sum_{k=1}^{N-1} f(a + kh) \right)$
D. None of these is correct.
E. More than one is correct.

The trapezoidal rule takes into account the value and slope of the function. The next "best" approximation will also take into account:

A. Concavity of the function
B. Curvature of the function
C. Unequally spaced intervals
D. More than one of these
E. Something else entirely