Given the two diff. eq's :

$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

where $C_1 + C_2 = 0$. Given the boundary conditions in the figure, which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

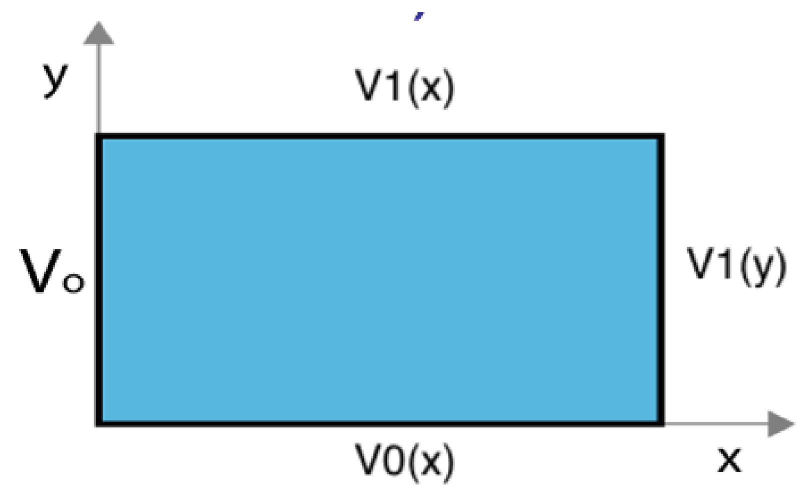
A. x

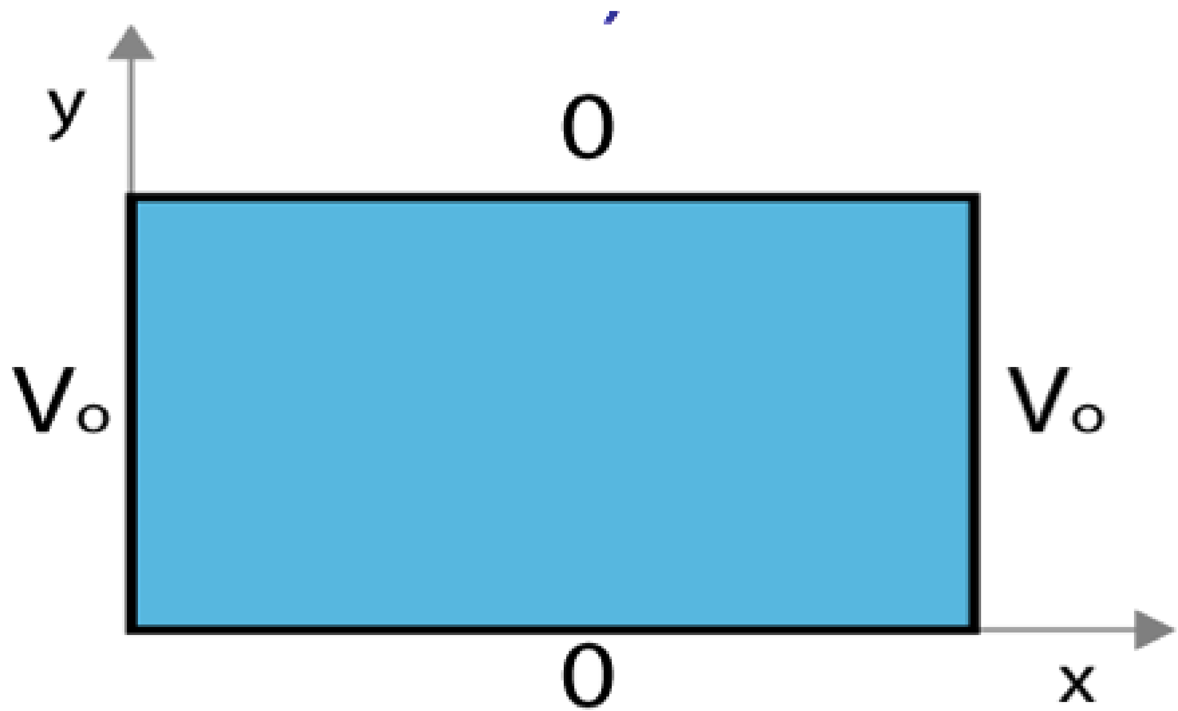
B. y

C. $C_1 = C_2 = 0$ here

D. It doesn't matter.

E. I don't know.





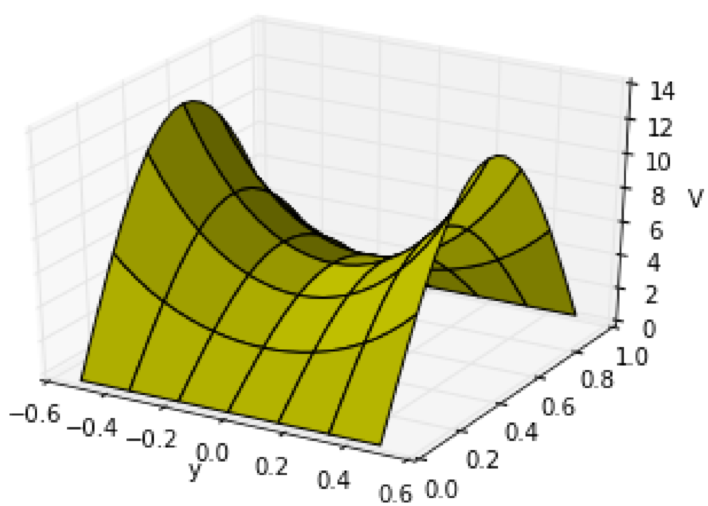
y
0
V o
V o
0
x

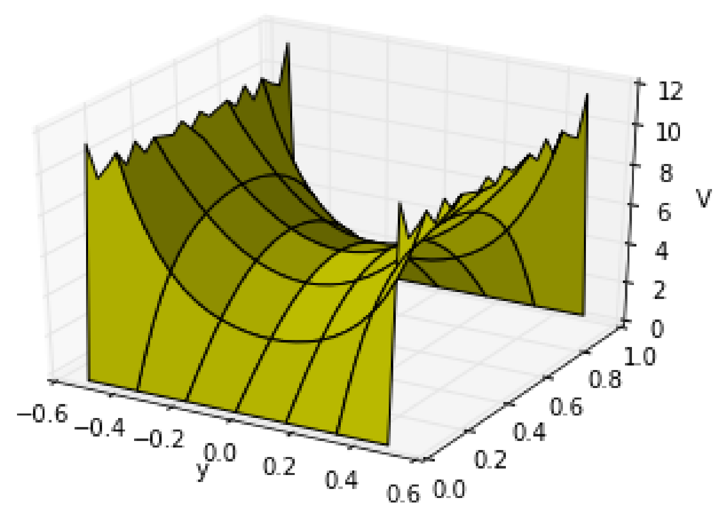
EXACT SOLUTIONS:

$$V(x,y) = \sum_{n=1}^{\infty} \frac{4V_0}{n\pi} \frac{1}{\cosh\left(\frac{n\pi}{2}\right)} \cosh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

APPROXIMATE SOLUTIONS:

(1 TERM; 20 TERMS)





Given that we want to solve Laplace's equation in 2D and that we have a description for the numerical second derivative of one variable,

$$f''(x) \approx \frac{f(x+a) - 2f(x) + f(x-a)}{a^2}$$

what is the appropriate numerical partial derivative for $V(x, y), \partial V / \partial x \approx$,

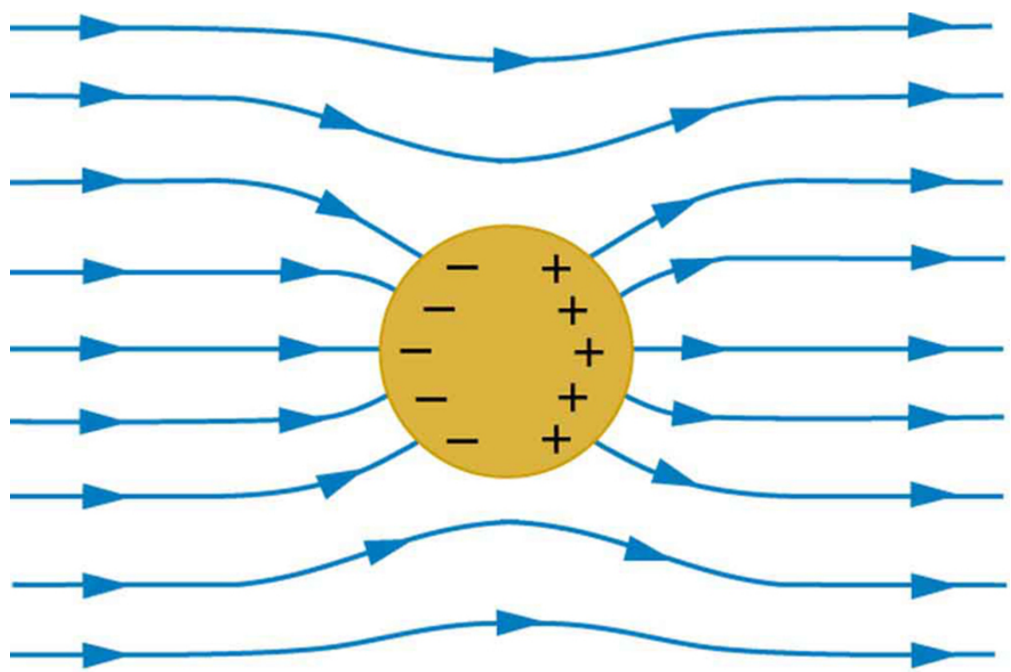
A. $[V(x+a) - 2V(x) + V(x-a)]\,/a^2$
B. $[V(x+a, y) - 2V(x, y) + V(x-a, y)]\,/a^2$
C. $[V(y+a) - 2V(y) + V(y-a)]\,/a^2$
D. $[V(x, y+a) - 2V(x, y) + V(x, y-a)]\,/a^2$
E. More than one is correct

Given that the potential at any point is given by the average of the surrounding points,

$$V(x, y) \approx \frac{1}{4}[V(x+a, y) + V(x, y+a) \\ +V(x-a, y) + V(x, y-a)]$$

Draft the psuedocode for finding the approximate potential.

SEPARATION OF VARIABLES (SPHERICAL)



Given $\nabla^2 V = 0$ in Cartesian coords, we separated $V(x, y, z) = X(x)Y(y)Z(z)$. Will this approach work in spherical coordinates, i.e. can we separate $V(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$?

A. Sure.
B. Not quite - the angular components cannot be isolated, e.g.,$f(r, \theta, \phi) = R(r)Y(\theta, \phi)$
C. It won't work at all because the spherical form of Laplace's Equation has cross terms in it (see the front cover of Griffiths)

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

V everywhere on a spherical shell is a given constant, i.e. $V(R,\theta) = V_0$. There are no charges inside the sphere. Which terms do you expect to appear when finding V(inside)?

A. Many A_l terms (but no B_l's)
B. Many B_l terms (but no A_l's)
C. Just A_0
D. Just B_0
E. Something else!

The general solution for the electric potential in spherical coordinates with azimuthal symmetry (no ϕ dependence) is:

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Consider a metal sphere (constant potential in and on the sphere, remember). Which terms in the sum vanish outside the sphere? (Recall: $V \to 0$ as $r \to \infty$)

A. All the A_l's
B. All the A_l's except A_0
C. All the B_l's
D. All the B_l's except B_0
E. Something else

Given $V_0(\theta) = \sum_l C_l P_l(\cos\theta)$, we want to get to the integral:

$$\int_{-1}^{+1} P_l(u)\, P_m(u)\, du = \frac{2}{2+1} \ (\text{for } l = m)$$

we can do this by multiplying both sides by:

A. $P_m(\cos\theta)$
B. $P_m(\sin\theta)$
C. $P_m(\cos\theta)\sin\theta$
D. $P_m(\sin\theta)\cos\theta$
E. $P_m(\sin\theta)\sin\theta$

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Suppose V on a spherical shell is:

$$V(R,\theta) = V_0 \left(1 + \cos^2\theta\right)$$

Which terms do you expect to appear when finding **V(inside)**?

A. Many A_l terms (but no B_l's)
B. Many B_l terms (but no A_l's)
C. Just A_0 and A_2
D. Just B_0 and B_2
E. Something else!

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Suppose V on a spherical shell is:

$$V(R,\theta) = V_0 \left(1 + \cos^2\theta\right)$$

Which terms do you expect to appear when finding **V(outside)**?

A. Many A_l terms (but no B_l's)
B. Many B_l terms (but no A_l's)
C. Just A_0 and A_2
D. Just B_0 and B_2
E. Something else!

How many boundary conditions (on the potential V) do you use to find V inside the spherical plastic shell?

A. 1
B. 2
C. 3
D. 4
E. It depends on $V_0(\theta)$