

The Method of Relaxation also works for Poisson's equation (i.e., when there is charge!).

$$\text{Given, } \nabla^2 V \approx \frac{V(x+a) - 2V(x) + V(x-a)}{a^2}$$

Which equations describes the appropriate "averaging" that we must do:

$$\text{A. } V(x) = \frac{1}{2}(V(x+a) - V(x-a))$$

$$\text{B. } V(x) = \frac{\rho}{\epsilon_0} + \frac{1}{2}(V(x+a) - V(x-a))$$

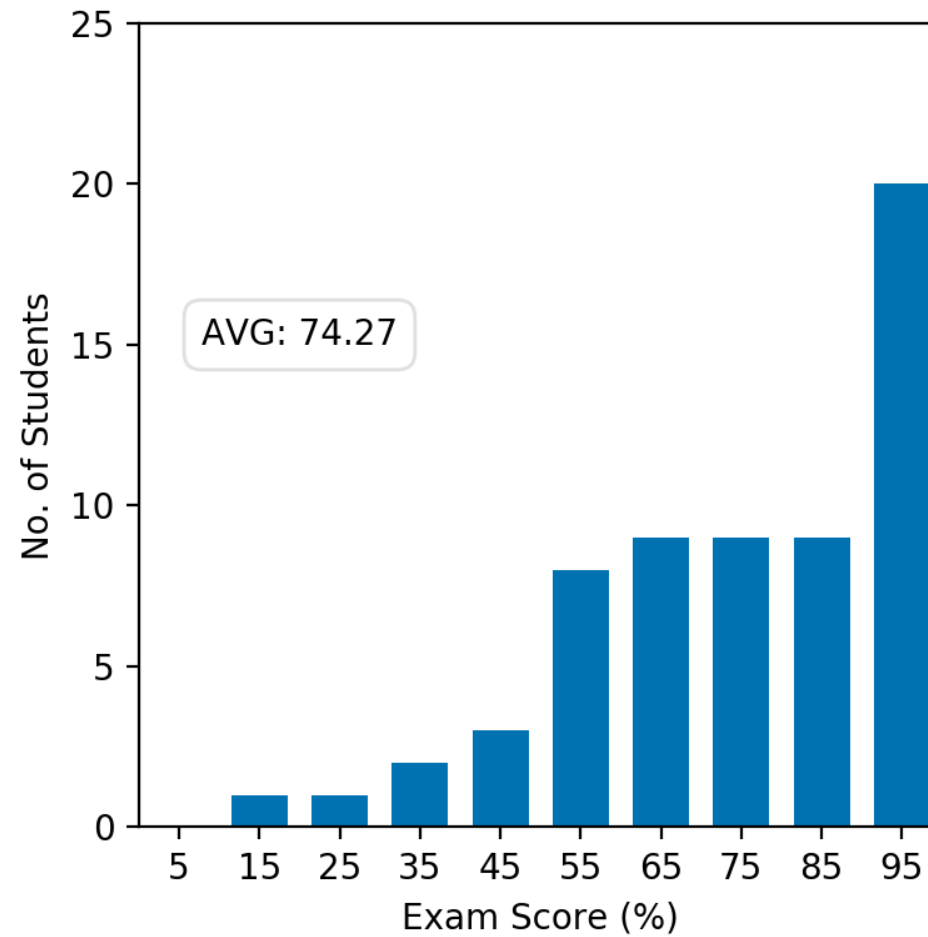
$$\text{C. } V(x) = \frac{a^2 \rho}{2\epsilon_0} + \frac{1}{2}(V(x+a) - V(x-a))$$

D. Something else

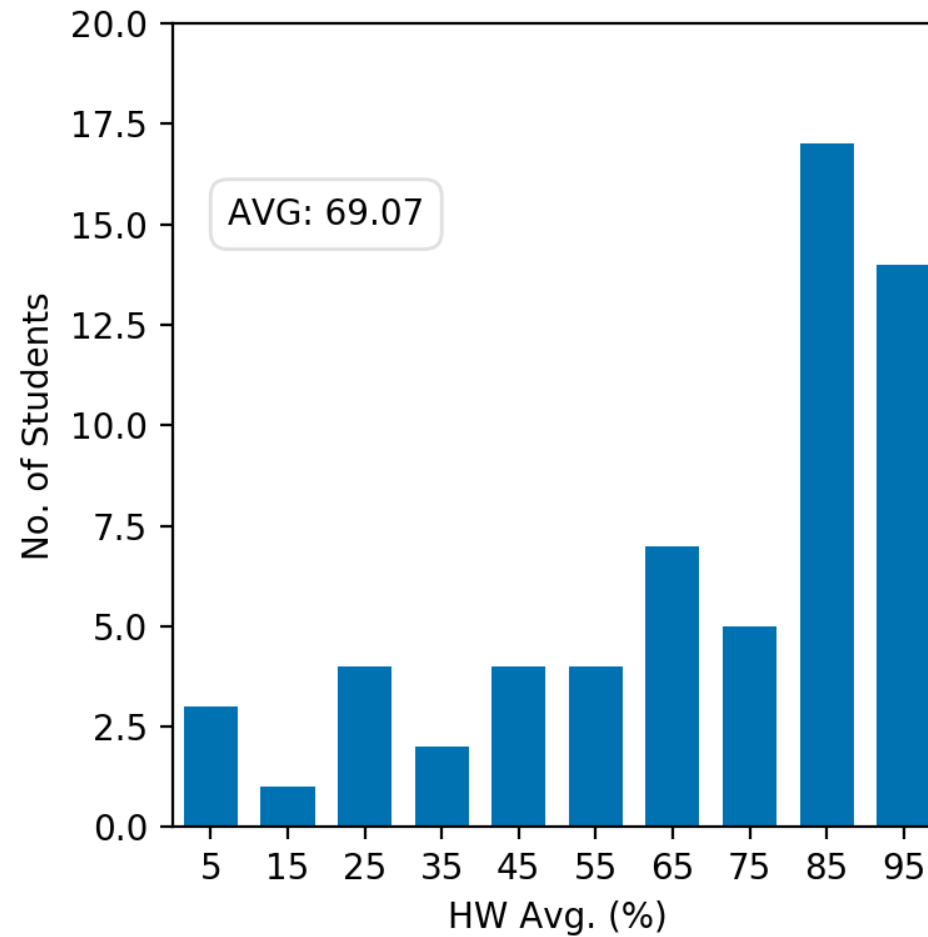
ANNOUNCEMENTS

- Exam 1 is graded
 - Should have received email this morning with updated grades
- Danny out of town Friday
 - Norman Birge will substitute

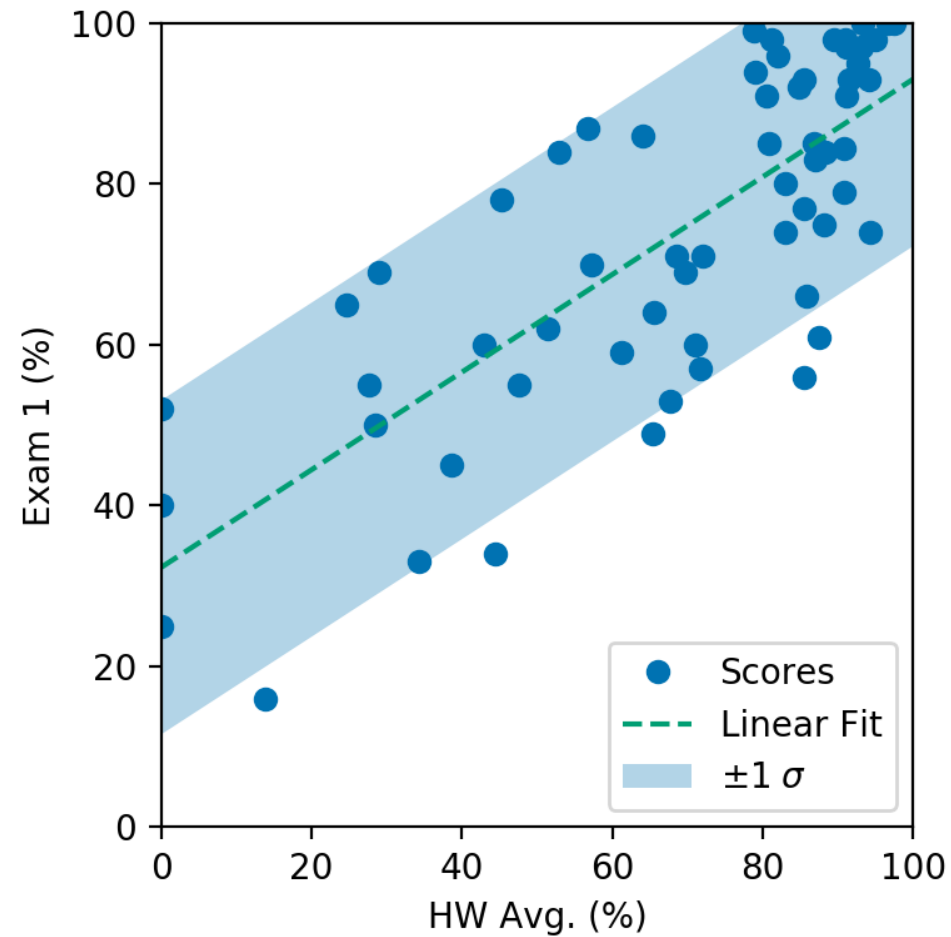
EXAM 1 GRADES



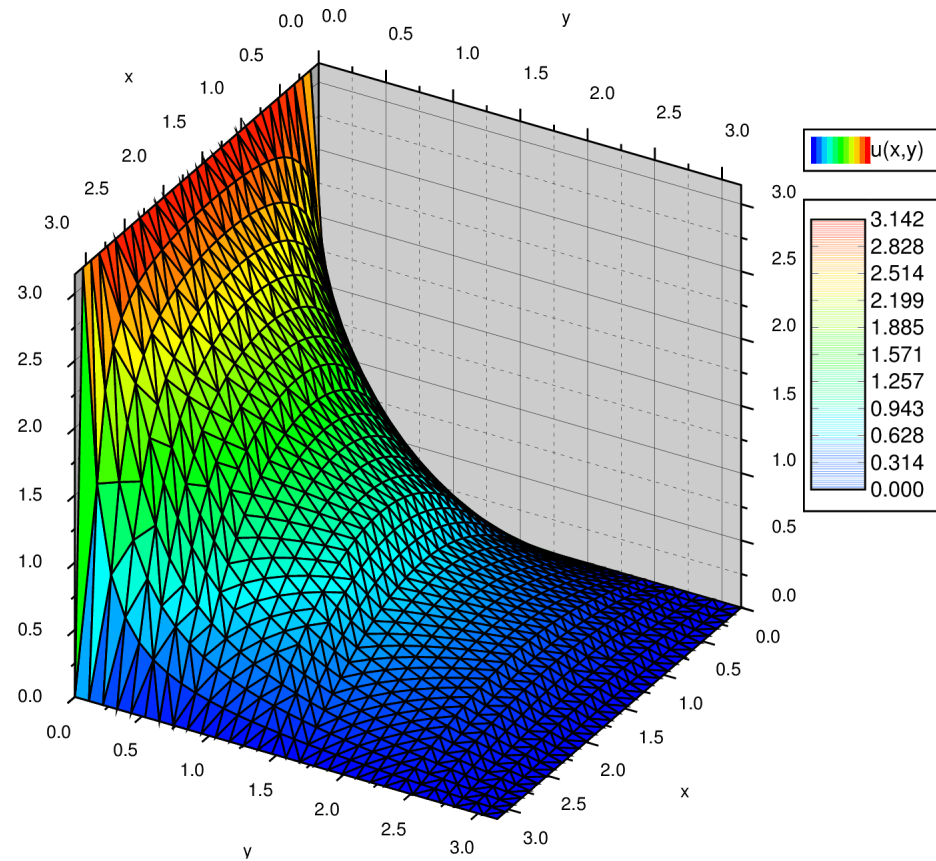
HOMEWORK AVERAGES



PLEASE DO YOUR HOMEWORK



SEPARATION OF VARIABLES (CARTESIAN)



Say you have three functions $f(x)$, $g(y)$, and $h(z)$. $f(x)$ depends on x but not on y or z . $g(y)$ depends on y but not on x or z . $h(z)$ depends on z but not on x or y .

If $f(x) + g(y) + h(z) = 0$ for all x, y, z , then:

- A. All three functions are constants (i.e. they do not depend on x, y, z at all.)
- B. At least one of these functions has to be zero everywhere.
- C. All of these functions have to be zero everywhere.
- D. All three functions have to be linear functions in x, y , or z respectively (such as $f(x) = ax + b$)

If our general solution contains the function,

$$X(x) = Ae^{\sqrt{c}x} + Be^{-\sqrt{c}x}$$

What does our solution look like if $c < 0$; what about if $c > 0$?

- A. Exponential; Sinusoidal
- B. Sinusoidal; Exponential
- C. Both Exponential
- D. Both Sinusoidal
- E. ???

Our example problem has the following boundary conditions:

- $V(0, y > 0) = 0; V(a, y > 0) = 0$
- $V(x_{0 \rightarrow a}, y = 0) = V_0; V(x, y \rightarrow \infty) = 0$

If $X'' = c_1 X$ and $Y'' = c_2 Y$ with $c_1 + c_2 = 0$, which is constant is positive?

A. c_1

B. c_2

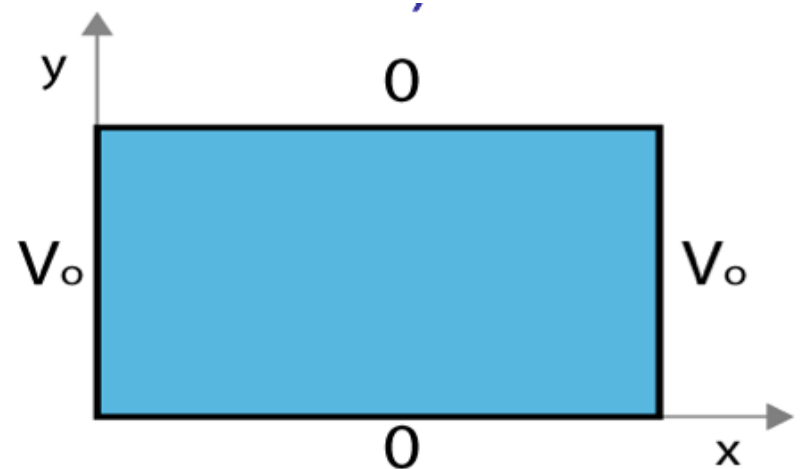
C. It doesn't matter either can be

Given the two diff. eq's :

$$\frac{1}{X} \frac{d^2 X}{dx^2} = C_1 \quad \frac{1}{Y} \frac{d^2 Y}{dy^2} = C_2$$

where $C_1 + C_2 = 0$. Given the boundary conditions in the figure, which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

- A. x
- B. y
- C. $C_1 = C_2 = 0$ here
- D. It doesn't matter.

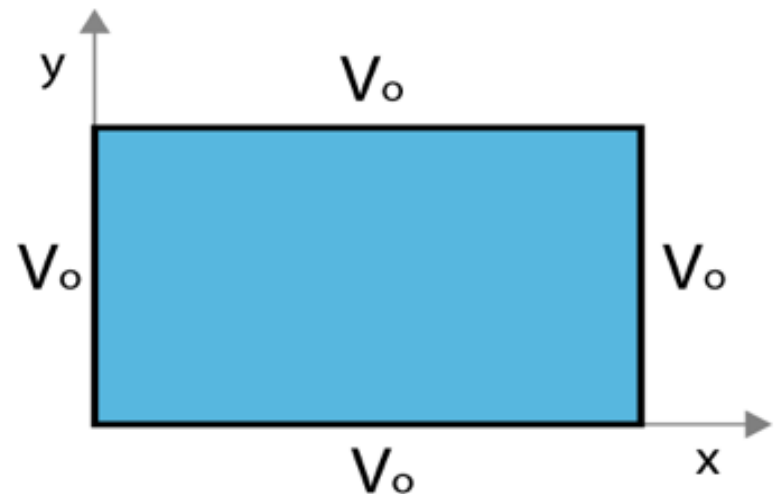


Given the two diff. eq's :

$$\frac{1}{X} \frac{d^2 X}{dx^2} = C_1 \qquad \frac{1}{Y} \frac{d^2 Y}{dy^2} = C_2$$

where $C_1 + C_2 = 0$. Given the boundary conditions in the figure, which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

- A. x
- B. y
- C. $C_1 = C_2 = 0$ here
- D. It doesn't matter.



When does $\sin(ka)e^{-ky}$ vanish?

A. $k = 0$

B. $k = \pi/(2a)$

C. $k = \pi/a$

D. A and C

E. A, B, C

Suppose $V_1(r)$ and $V_2(r)$ are linearly independent functions which both solve Laplace's equation, $\nabla^2 V = 0$.

Does $aV_1(r) + bV_2(r)$ also solve it (with a and b constants)?

- A. Yes. The Laplacian is a linear operator
- B. No. The uniqueness theorem says this scenario is impossible, there are never two independent solutions!
- C. It is a definite yes or no, but the reasons given above just aren't right!
- D. It depends...

What is the value of $\int_0^{2\pi} \sin(2x) \sin(3x) \, dx$?

A. Zero

B. π

C. 2π

D. other

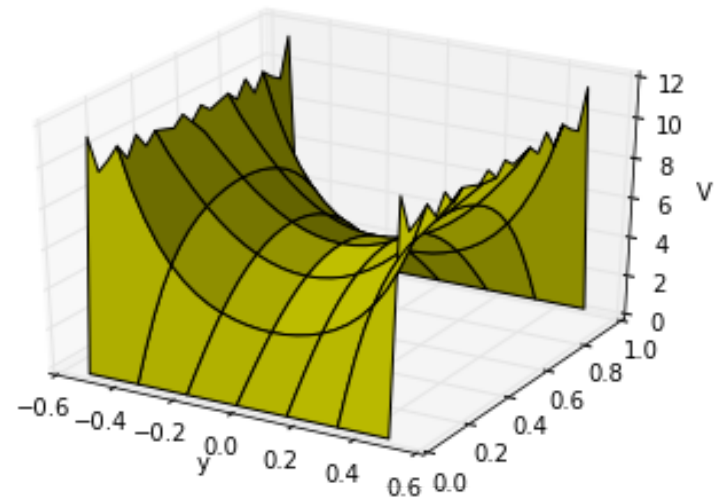
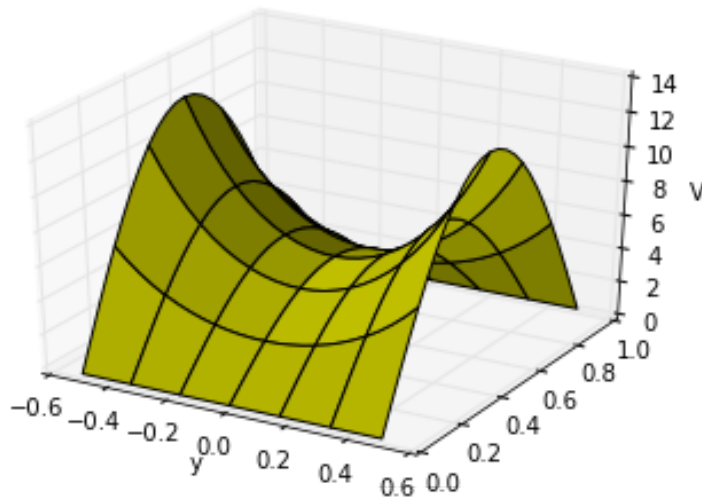
E. I need resources to do an integral like this!

EXACT SOLUTIONS:

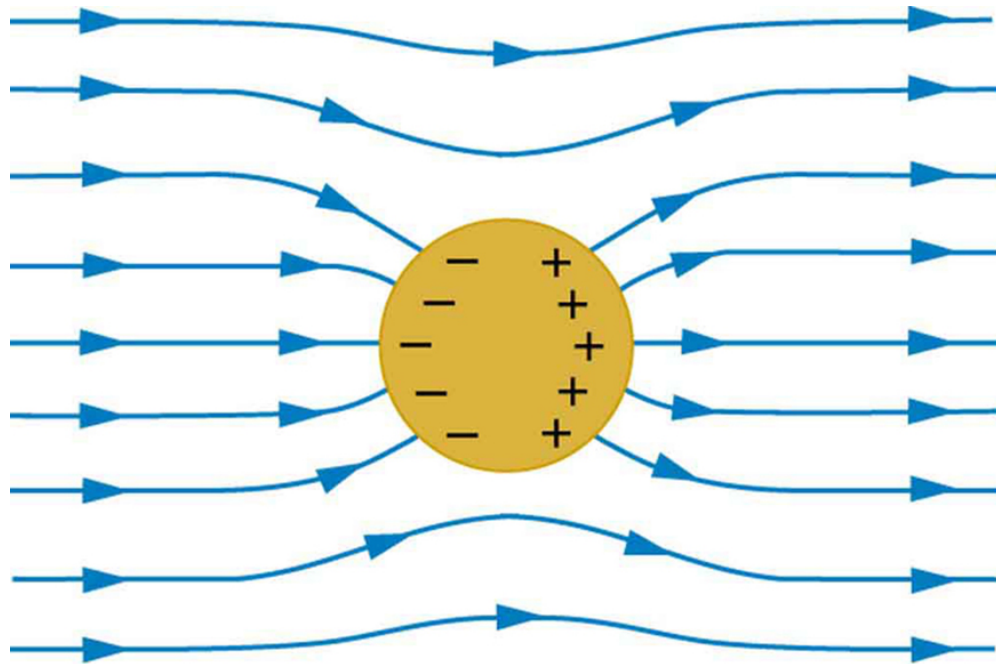
$$V(x, y) = \sum_{n=1}^{\infty} \frac{4V_0}{n\pi} \frac{1}{\cosh\left(\frac{n\pi}{2}\right)} \cosh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

APPROXIMATE SOLUTIONS:

(1 TERM; 20 TERMS)



SEPARATION OF VARIABLES (SPHERICAL)



Given $\nabla^2 V = 0$ in Cartesian coords, we separated $V(x, y, z) = X(x)Y(y)Z(z)$. Will this approach work in spherical coordinates, i.e. can we separate $V(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$?

A. Sure.

B. Not quite - the angular components cannot be isolated, e.g., $f(r, \theta, \phi) = R(r)Y(\theta, \phi)$

C. It won't work at all because the spherical form of Laplace's Equation has cross terms in it (see the front cover of Griffiths)