Given the two diff. eq's :

$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

where $C_1 + C_2 = 0$. Given the boundary conditions in the figure, which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

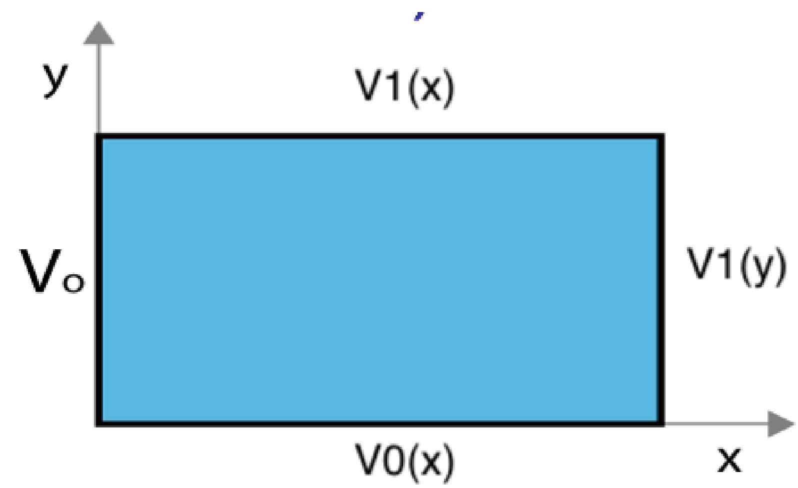
A. x

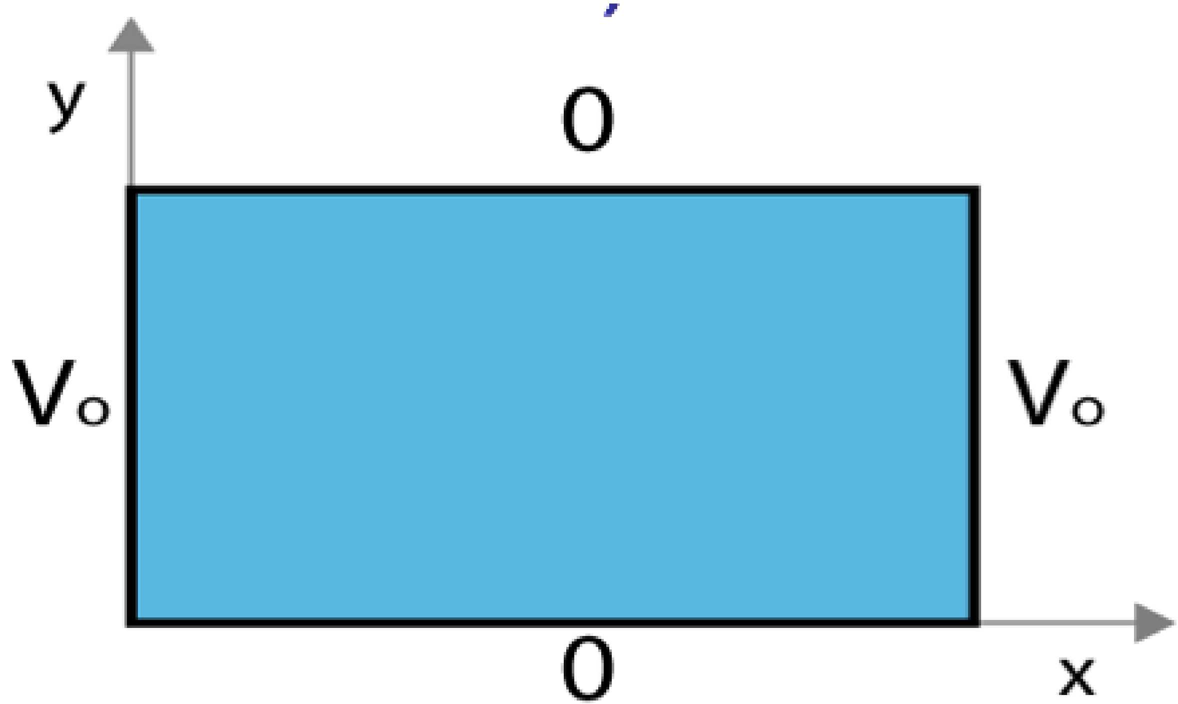
B. y

C. $C_1 = C_2 = 0$ here

D. It doesn't matter.

E. I don't know.



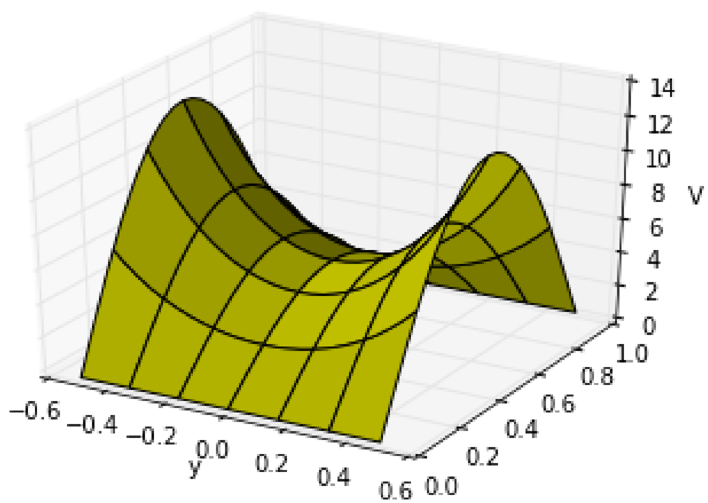

y
0
Vo
Vo
0
x

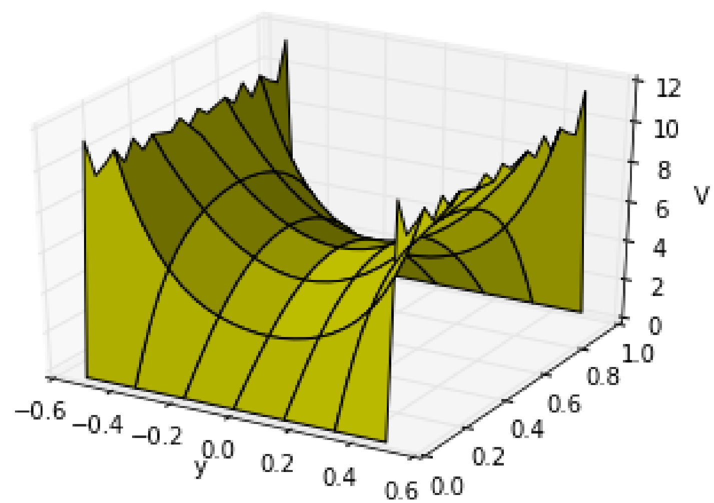
EXACT SOLUTIONS:

$$V(x,y) = \sum_{n=1}^{\infty} \frac{4V_0}{n\pi} \frac{1}{\cosh\left(\frac{n\pi}{2}\right)} \cosh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

APPROXIMATE SOLUTIONS:

(1 TERM; 20 TERMS)





Given that we want to solve Laplace's equation in 2D and that we have a description for the numerical second derivative of one variable,

$$f''(x) \approx \frac{f(x+a) - 2f(x) + f(x-a)}{a^2}$$

what is the appropriate numerical partial derivative for $V(x, y)$, $\partial V/\partial x \approx$,

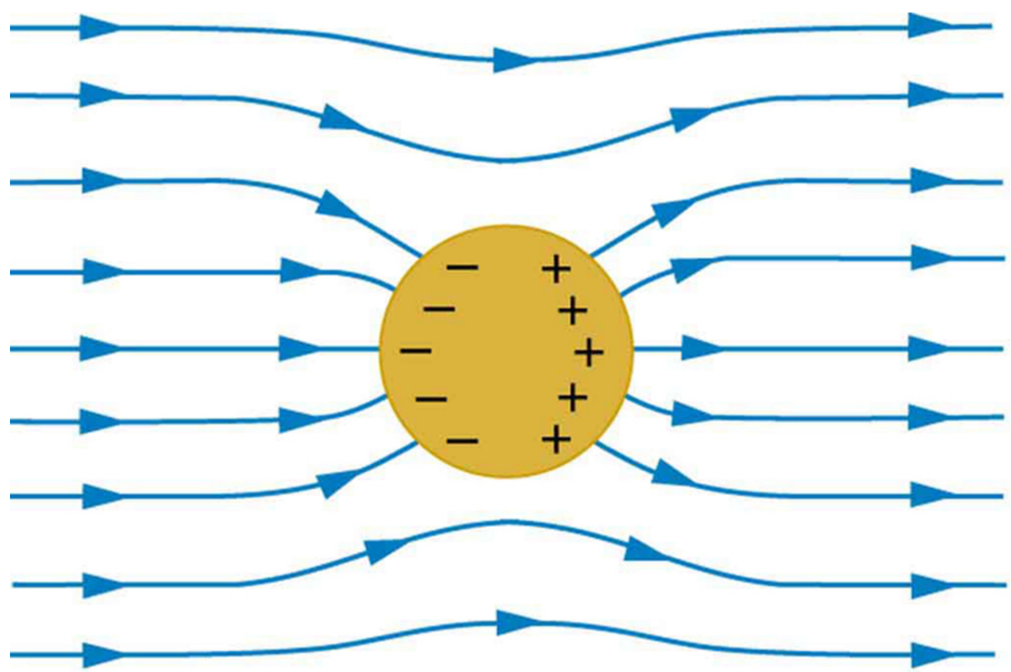
A. $[V(x+a) - 2V(x) + V(x-a)]\,/a^2$
B. $[V(x+a, y) - 2V(x, y) + V(x-a, y)]\,/a^2$
C. $[V(y+a) - 2V(y) + V(y-a)]\,/a^2$
D. $[V(x, y+a) - 2V(x, y) + V(x, y-a)]\,/a^2$
E. More than one is correct

Given that the potential at any point is given by the average of the surrounding points,

$$V(x, y) \approx \frac{1}{4}[V(x + a, y) + V(x, y + a) \\ + V(x - a, y) + V(x, y - a)]$$

Draft the psuedocode for finding the approximate potential.

SEPARATION OF VARIABLES (SPHERICAL)



$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

V everywhere on a spherical shell is a given constant, i.e. $V(R,\theta) = V_0$. There are no charges inside the sphere. Which terms do you expect to appear when finding V(inside)?

A. Many A_l terms (but no B_l's)
B. Many B_l terms (but no A_l's)
C. Just A_0
D. Just B_0
E. Something else!

The general solution for the electric potential in spherical coordinates with azimuthal symmetry (no ϕ dependence) is:

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$

Consider a metal sphere (constant potential in and on the sphere, remember). Which terms in the sum vanish outside the sphere? (Recall: $V \to 0$ as $r \to \infty$)

A. All the A_l's
B. All the A_l's except A_0
C. All the B_l's
D. All the B_l's except B_0
E. Something else

Given $V_0(\theta) = \sum_l C_l P_l(\cos\theta)$, we want to get to the integral:

$$\int_{-1}^{+1} P_l(u)\, P_m(u)\, du = \frac{2}{2+1} \text{ (for } l = m\text{)}$$

we can do this by multiplying both sides by:

A. $P_m(\cos\theta)$
B. $P_m(\sin\theta)$
C. $P_m(\cos\theta)\sin\theta$
D. $P_m(\sin\theta)\cos\theta$
E. $P_m(\sin\theta)\sin\theta$

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Suppose V on a spherical shell is:

$$V(R,\theta) = V_0 \left(1 + \cos^2\theta\right)$$

Which terms do you expect to appear when finding **V(inside)**?

A. Many A_l terms (but no B_l's)
B. Many B_l terms (but no A_l's)
C. Just A_0 and A_2
D. Just B_0 and B_2
E. Something else!

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Suppose V on a spherical shell is:

$$V(R,\theta) = V_0 \left(1 + \cos^2\theta\right)$$

Which terms do you expect to appear when finding **V(outside)**?

A. Many A_l terms (but no B_l's)
B. Many B_l terms (but no A_l's)
C. Just A_0 and A_2
D. Just B_0 and B_2
E. Something else!

How many boundary conditions (on the potential V) do you use to find V inside the spherical plastic shell?

A. 1

B. 2

C. 3

D. 4

E. It depends on $V_0(\theta)$