#### I have seen Separation of Variables before.

- A. Yes, and I'm comfortable with it.
- B. Yes, but I don't quite remember.
- C. Nope
- D. I'm triggered.

PS. Hi from San Antonio -DC

# Our example problem has the following boundary conditions:

• 
$$V(0, y > 0) = 0$$
;  $V(a, y > 0) = 0$ 

• 
$$V(x_{0\to a}, y = 0) = V_0; V(x, y \to \infty) = 0$$

If 
$$X'' = c_1 X$$
 and  $Y'' = c_2 Y$  with  $c_1 + c_2 = 0$ , which is constant is positive?

A. *c*<sub>1</sub>

B. *c*<sub>2</sub>

C. It doesn't matter either can be

Given the two diff. eq's:

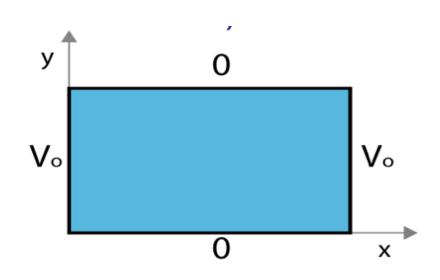
$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

where  $C_1 + C_2 = 0$ . Given the boundary conditions in the figure, which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

B. y

C.  $C_1 = C_2 = 0$  here

D. It doesn't matter.



Given the two diff. eq's:

$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

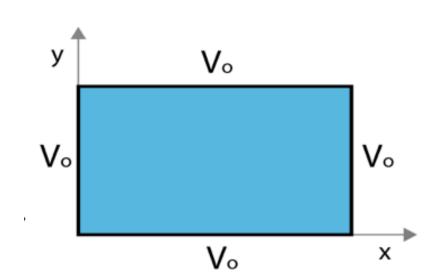
where  $C_1 + C_2 = 0$ . Given the boundary conditions in the figure, which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

A. x

B. y

C.  $C_1 = C_2 = 0$  here

D. It doesn't matter.



### When does $\sin(ka)e^{-ky}$ vanish?

A. 
$$k = 0$$

B. 
$$k = \pi/(2a)$$

$$C. k = \pi/a$$

D. A and C

E. A, B, C

Suppose  $V_1(r)$  and  $V_2(r)$  are linearly independent functions which both solve Laplace's equation,  $\nabla^2 V = 0$ .

Does  $aV_1(r) + bV_2(r)$  also solve it (with a and b constants)?

- A. Yes. The Laplacian is a linear operator
- B. No. The uniqueness theorem says this scenario is impossible, there are never two independent solutions!
- C. It is a definite yes or no, but the reasons given above just aren't right!
- D. It depends...

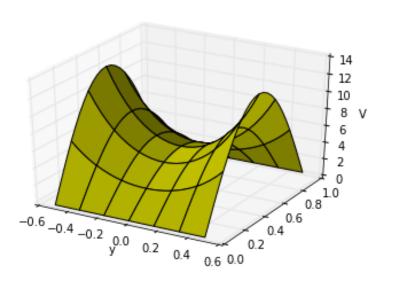
What is the value of  $\int_0^{2\pi} \sin(2x) \sin(3x) dx$ ?

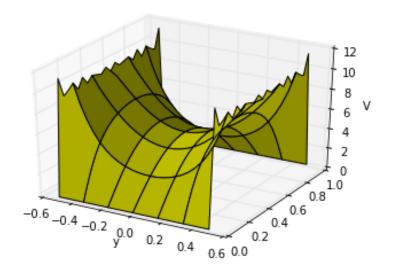
- A. Zero
- $B. \pi$
- $C. 2\pi$
- D. other
- E. I need resources to do an integral like this!

#### **EXACT SOLUTIONS:**

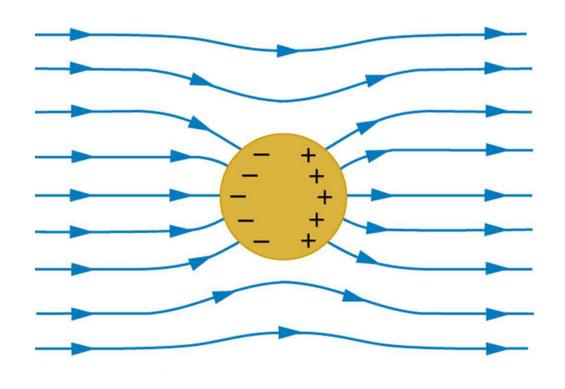
$$V(x,y) = \sum_{n=1}^{\infty} \frac{4V_0}{n\pi} \frac{1}{\cosh(\frac{n\pi}{2})} \cosh(\frac{n\pi x}{a}) \sin(\frac{n\pi y}{a})$$

## APPROXIMATE SOLUTIONS: (1 TERM; 20 TERMS)





## SEPARATION OF VARIABLES (SPHERICAL)



Given  $\nabla^2 V = 0$  in Cartesian coords, we separated V(x,y,z) = X(x)Y(y)Z(z). Will this approach work in spherical coordinates, i.e. can we separate  $V(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$ ?

- A. Sure.
- B. Not quite the angular components cannot be isolated, e.g.,  $f(r, \theta, \phi) = R(r)Y(\theta, \phi)$
- C. It won't work at all because the spherical form of Laplace's Equation has cross terms in it (see the front cover of Griffiths)