

The ODE that describes the $R(r)$ part of our solution is:

$$\frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) = l(l+1)R$$

I claim this ODE gives rise to polynomial solutions.

Find a general solution for $R(r)$ in terms of l .

I still have questions about what we are trying to do with separation of variables in spherical coordinates.

- A. Yes, definitely, let's talk about what we are trying to do (briefly).
- B. I have some questions, but I think I got the gist of it. We can move on.
- C. I got it, let's move on.

ANNOUNCEMENTS

- Homework 8 has 2D relaxation problem
 - It is OK to post code on Slack and get help
 - Solution to HW7 (1D relaxation) is linked (you may work from it)
- DC out of town Friday; Rachel will cover

Let's take the Θ ODE term by term starting with $l = 0$

$$\frac{d}{d\theta} \left(\sin \theta \frac{d\Theta}{d\theta} \right) = 0$$

What are some possible solutions?

Hint: This is not as tricky as it might seem.

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

V everywhere on a spherical shell is a given constant, i.e.

$V(R, \theta) = V_0$. There are no charges inside the sphere.

Which terms do you expect to appear when finding
V(inside)?

- A. Many A_l terms (but no B_l 's)
- B. Many B_l terms (but no A_l 's)
- C. Just A_0
- D. Just B_0
- E. Something else!

The general solution for the electric potential in spherical coordinates with azimuthal symmetry (no ϕ dependence) is:

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

Consider a metal sphere (constant potential in and on the sphere, remember). Which terms in the sum vanish outside the sphere? (Recall: $V \rightarrow 0$ as $r \rightarrow \infty$)

- A. All the A_l 's
- B. All the A_l 's except A_0
- C. All the B_l 's
- D. All the B_l 's except B_0
- E. Something else

Given $V_0(\theta) = \sum_l C_l P_l(\cos \theta)$, we want to get to the integral:

$$\int_{-1}^{+1} P_l(u) P_m(u) du = \frac{2}{2l+1} \quad (\text{for } l = m)$$

we can do this by multiplying both sides by:

- A. $P_m(\cos \theta)$
- B. $P_m(\sin \theta)$
- C. $P_m(\cos \theta) \sin \theta$
- D. $P_m(\sin \theta) \cos \theta$
- E. $P_m(\sin \theta) \sin \theta$

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

Suppose V on a spherical shell is:

$$V(R, \theta) = V_0 (1 + \cos^2 \theta)$$

Which terms do you expect to appear when finding **$V(\text{inside})$** ?

- A. Many A_l terms (but no B_l 's)
- B. Many B_l terms (but no A_l 's)
- C. Just A_0 and A_2
- D. Just B_0 and B_2
- E. Something else!

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

Suppose V on a spherical shell is:

$$V(R, \theta) = V_0 (1 + \cos^2 \theta)$$

Which terms do you expect to appear when finding **$V(\text{outside})$** ?

- A. Many A_l terms (but no B_l 's)
- B. Many B_l terms (but no A_l 's)
- C. Just A_0 and A_2
- D. Just B_0 and B_2
- E. Something else!