

What is the value of  $\int_0^a \sin(n\pi x/a) \sin(m\pi x/a) dx$  ?

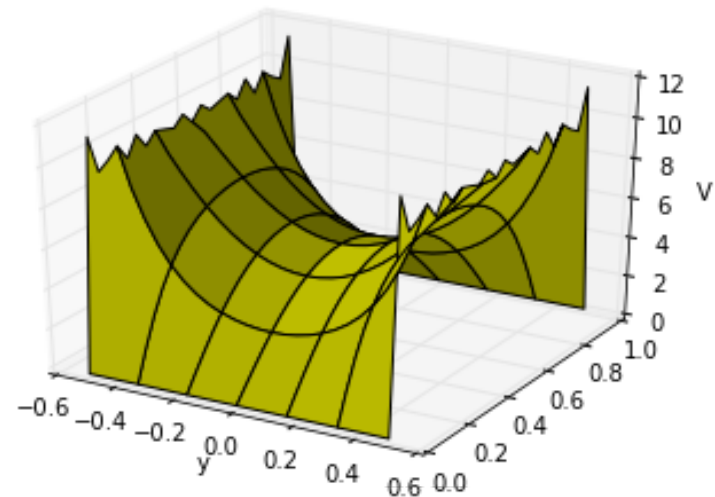
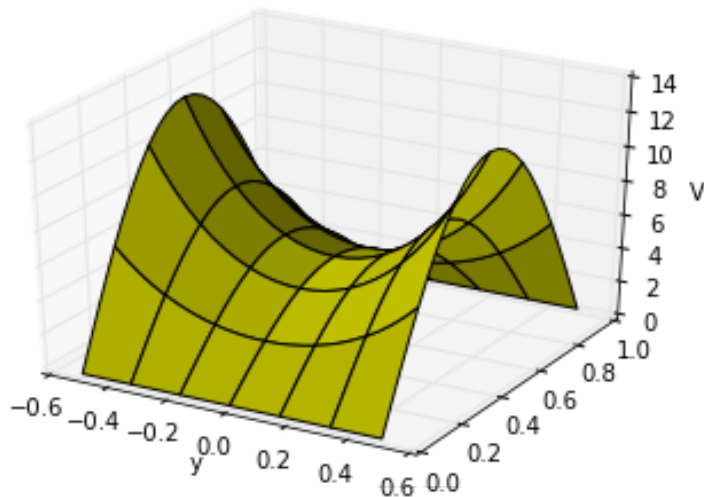
- A. Zero
- B. Non-zero
- C. Depends on  $n$  and  $m$

# EXACT SOLUTIONS:

$$V(x, y) = \sum_{n=1}^{\infty} \frac{4V_0}{n\pi} \frac{1}{\cosh\left(\frac{n\pi}{2}\right)} \cosh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

# APPROXIMATE SOLUTIONS:

(1 TERM; 20 TERMS)



Given that we want to solve Laplace's equation in 2D and that we have a description for the numerical second derivative of one variable,

$$f''(x) \approx \frac{f(x+a) - 2f(x) + f(x-a)}{a^2}$$

what is the appropriate numerical partial derivative for

$$V(x, y), \partial^2 V / \partial x^2 \approx,$$

- A.  $[V(x+a) - 2V(x) + V(x-a)] / a^2$
- B.  $[V(x+a, y) - 2V(x, y) + V(x-a, y)] / a^2$
- C.  $[V(y+a) - 2V(y) + V(y-a)] / a^2$
- D.  $[V(x, y+a) - 2V(x, y) + V(x, y-a)] / a^2$
- E. More than one is correct

Given that the potential at any point is given by the average of the surrounding points,

$$V(x, y) \approx \frac{1}{4} [V(x + a, y) + V(x, y + a) + V(x - a, y) + V(x, y - a)]$$

Draft the psuedocode for finding the approximate potential.

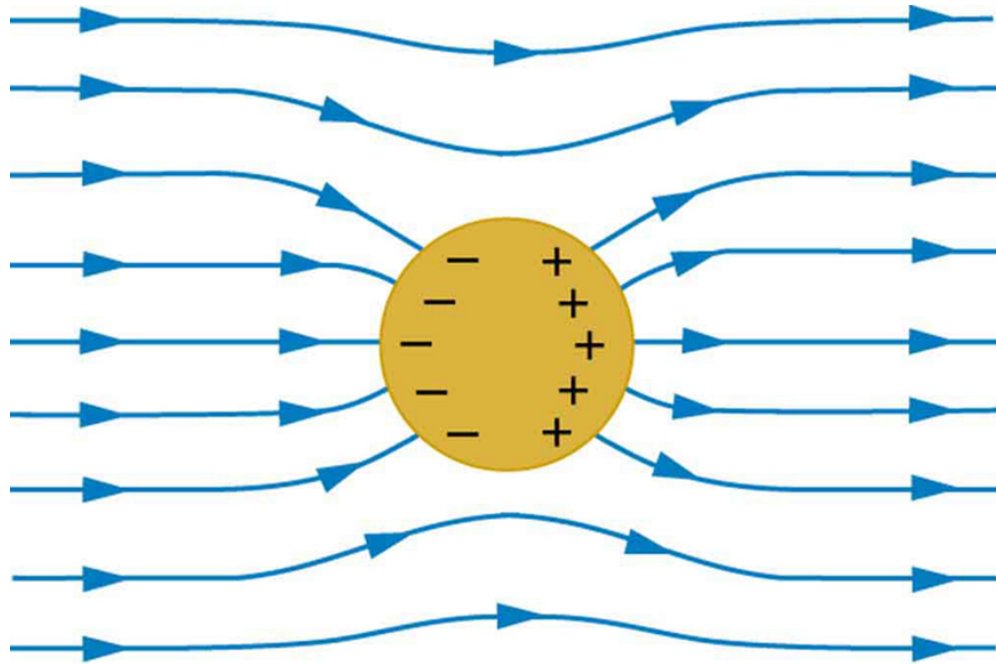
Given  $\nabla^2 V = 0$  in Cartesian coords, we separated  $V(x, y, z) = X(x)Y(y)Z(z)$ . Will this approach work in spherical coordinates, i.e. can we separate  $V(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$ ?

A. Sure.

B. Not quite - the angular components cannot be isolated, e.g.,  $f(r, \theta, \phi) = R(r)Y(\theta, \phi)$

C. It won't work at all because the spherical form of Laplace's Equation has cross terms in it (see the front cover of Griffiths)

# SEPARATION OF VARIABLES (SPHERICAL)



The ODE that describes the  $R(r)$  part of our solution is:

$$\frac{d}{dr} \left( r^2 \frac{dR}{dr} \right) = l(l + 1)R$$

I claim this ODE gives rise to polynomial solutions.

Find a general solution for  $R(r)$  in terms of  $l$ .