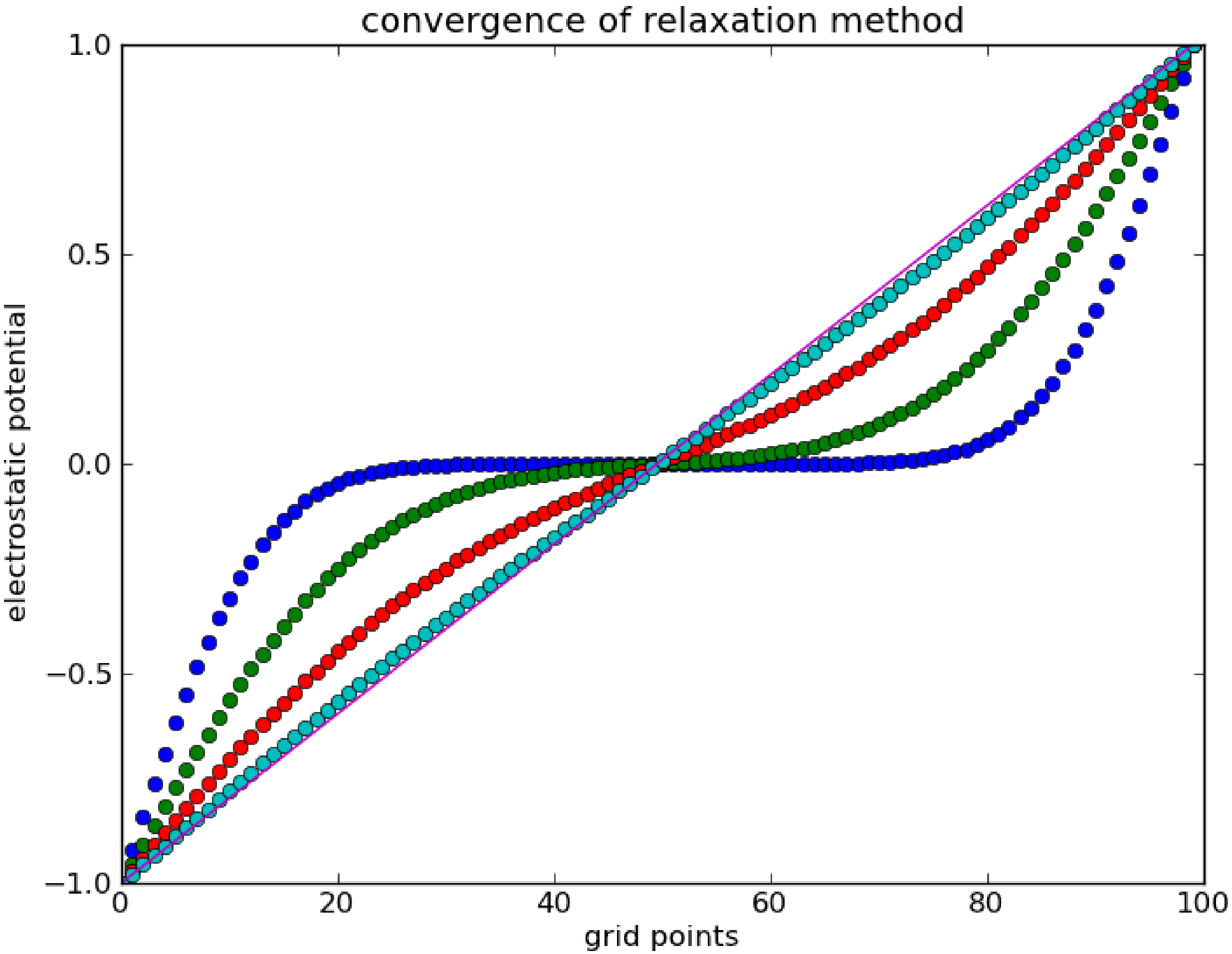
The solution to Laplace's equation in one Cartesian dimension is always linear. Why?

A. The general solution for $\frac{d^2V(x)}{dx^2} = 0$ is always a line.
B. The only way for a given point $V(x_i)$ (in 1d) to be the arithmetic average of its neighbors is for all point to be on a line.
C. Given boundary conditions, it's a unique solution.
D. None of these.
E. More than one of these.

METHOD OF RELAXATION



Consider a function $f(x)$ that is both continuous and continuously differentiable over some domain. Given a step size of a, which could be an approximate derivative of this function somewhere in that domain? $df/dx \approx$

A. $f(x_i + a) - f(x_i)$
B. $f(x_i) - f(x_i - a)$
C. $\frac{f(x_i+a)-f(x_i)}{a}$
D. $\frac{f(x_i)-f(x_i-a)}{a}$
E. More than one of these

If we choose to use:

$$\frac{df}{dx} \approx \frac{f(x_i + a) - f(x_i)}{a}$$

Where are we computing the approximate derivative?

A. a

B. x_i

C. $x_i + a$

D. Somewhere else

Taking the second derivative of $f(x)$ discretely is as simple as applying the discrete definition of the derivative,

$$f''(x_i) \approx \frac{f'(x_i + a/2) - f'(x_i - a/2)}{a}$$

Derive the second derivative in terms of f.

With the approximate form of Laplace's equation:

$$\frac{V(x_i + a) - 2V(x_i) + V(x_i - a)}{a} \approx 0$$

What is a the appropriate estimate of $V(x_i)$?

A. $1/2(V(x_i + a) - V(x_i - a))$
B. $1/2(V(x_i + a) + V(x_i - a))$
C. $a/2(V(x_i + a) - V(x_i - a))$
D. $a/2(V(x_i + a) + V(x_i - a))$
E. Something else

To investigate the convergence, we must compare the estimate of V before and after each calculation. For our `1D` relaxation code, V will be a `1D` array. For the kth estimate, we can compare V_k against its previous value by simply taking the difference.

Store this in a variable called `err`. What is the type for `err`?

A. A single number
B. A 1D array
C. A 2D array
D. ???

The Method of Relaxation also works for Poisson's equation (i.e., when there is charge!).

Given, $\nabla^2 V \approx \dfrac{V(x+a) - 2V(x) + V(x-a)}{a^2}$

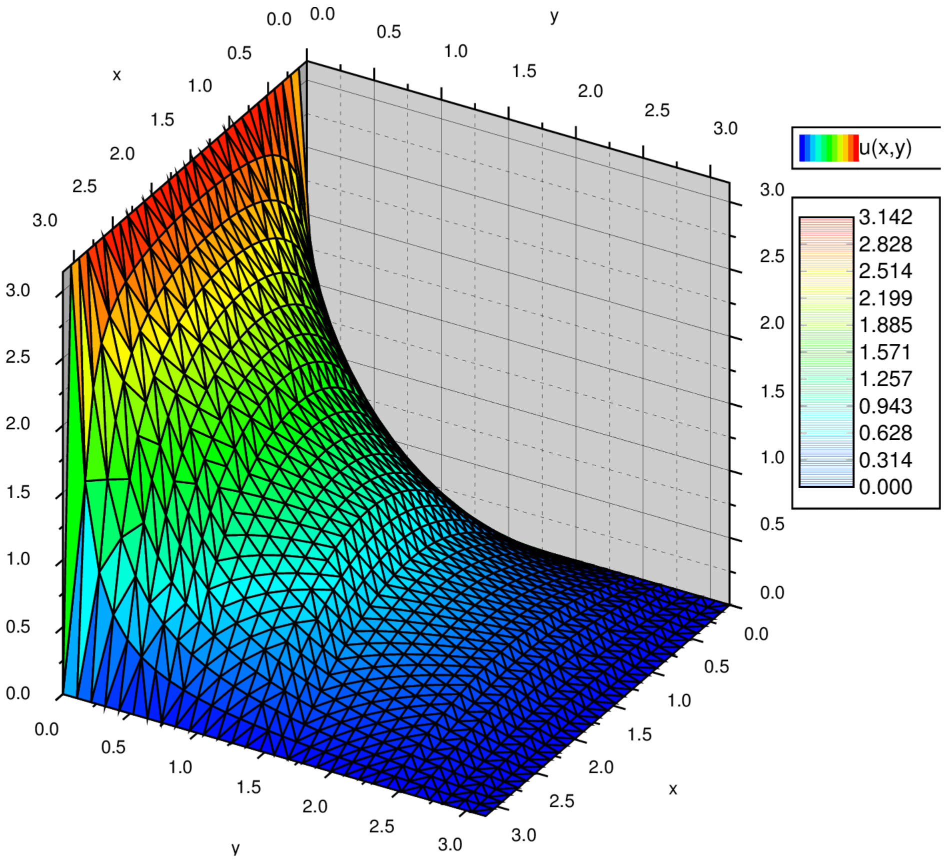
Which equations describes the appropriate "averaging" that we must do:

A. $V(x) = \dfrac{1}{2}(V(x+a) - V(x-a))$

B. $V(x) = \dfrac{\rho(x)}{\varepsilon_0} + \dfrac{1}{2}(V(x+a) + V(x-a))$

C. $V(x) = \dfrac{a^2 \rho(x)}{2\varepsilon_0} + \dfrac{1}{2}(V(x+a) + V(x-a))$

SEPARATION OF VARIABLES (CARTESIAN)



Say you have three functions $f(x)$, $g(y)$, and $h(z)$. $f(x)$ depends on x but not on y or z. $g(y)$ depends on y but not on x or z. $h(z)$ depends on z but not on x or y.

If $f(x) + g(y) + h(z) = 0$ for all x, y, z, then:

A. All three functions are constants (i.e. they do not depend on x, y, z at all.)
B. At least one of these functions has to be zero everywhere.
C. All of these functions have to be zero everywhere.
D. All three functions have to be linear functions in x, y, or z respectively (such as $f(x) = ax + b$)

If our general solution contains the function,

$$X(x) = Ae^{\sqrt{c}x} + Be^{-\sqrt{c}x}$$

What does our solution look like if $c < 0$; what about if $c > 0$?

A. Exponential; Sinusoidal
B. Sinusoidal; Exponential
C. Both Exponential
D. Both Sinusoidal
E. ???