

# Homework 11 (Due. Apr. 3)

## 1. Star Wars with relativity

Space Probe #1 passes very close to earth at a time that both we (on earth) and the onboard computer on Probe 1 decide to call  $t = 0$  in our respective frames. The probe moves at a constant speed of  $0.5c$  away from earth. When the clock aboard Probe 1 reads  $t = 60\text{sec}$ , it sends a light signal straight back to earth.

1. At what time was the signal sent, according to the earth's rest frame?
2. At what time in the earth's rest frame do we receive the signal?
3. At what time in Probe 1's rest frame does the signal reach earth?
4. Space Probe #2 passes very close to earth at  $t = 1\text{sec}$  (earth time), chasing Probe 1. Probe 2 is only moving at  $0.3c$  (as viewed by us). Probe 2 launches a proton beam (which moves at  $v = 0.21c$  relative to Probe 2) directed at Probe 1. Does this proton beam strike Probe 1? Please answer twice, once ignoring relativity theory, and then again using Einstein!

## 2. Muon calculations

The mean lifetime of muons is  $2\ \mu\text{s}$  in their rest frame. Muons are produced in the upper atmosphere, as cosmic-ray secondaries.

1. Calculate the mean distance traveled by muons with speed  $v = 0.99c$ , assuming classical physics (i.e. without special relativity).
2. Under this assumption, what percentage of muons produced at an altitude of 10 km reach the ground, assuming they travel downward at  $v = 0.99c$ ? Careful here, you will have to think about the distribution of lifetimes given that particle decay is a Poisson process.
3. Calculate the mean distance traveled by muons with speed  $v = 0.99c$ , taking into account special relativity.
4. Under this assumption, what percentage of muons produced at an altitude of 10 km reach the ground, assuming they travel downward at  $v = 0.99c$ ?

## 3. $\pi$ Mesons

A group of  $\pi$  mesons (pions) are observed to be traveling at  $u' = 0.8c$  in a physics laboratory. The mean lifetime (let's call it  $\lambda$ ) for unstable particles undergoing exponential decay is the average time for a group of particles to be reduced to  $1/e$  of their original number. We can express this mathematically as:  $N_t = N_0 \exp(-\Delta t/\lambda)$ . We see then that for  $\Delta t = \lambda$ ,  $N_t = N_0 \exp(-1) = (1/e) \cdot N_0$

1. What is the  $\gamma$ -factor for these pions?
2. If the pions' proper mean lifetime is  $\lambda = 2.6 \times 10^{-8}$  s, what is the lifetime ( $\lambda'$ ) as observed in the laboratory frame?
3. If there were initially 32,000 pions, how many will be left after they have traveled 36 meters, from the source to a detector (as measured in the laboratory frame)?
4. Show that this number is the same, as calculated in the rest frame of the pions. Explain this calculation.
5. What would the answer to part 4 be if there were no time dilation (i.e.,  $\Delta\tau = \Delta t'$ )?
6. How long does the distance of 36 meters in the laboratory frame appear to the pions in their rest frame?

7. Use the result from part 6 and the proper time elapsed (as calculated in the rest frame of the pions) to find the velocity of the laboratory frame, as calculated in the rest frame of the pions.

#### 4. Paired Project Problem

1. Read the feedback that you received on your plan and think about how you and your partner are going to work through your new plan.
2. Project work - Provide a detailed explanation of the models and theoretical calculations needed to set up your work. The produce that you produce for this should be presented as a “graphic” that would appear in a poster under “background or model.”
3. Self-reflection - Think about how the project is going and how you are both contributing. Write out a document for the last couple of weeks worth of work including this one that describes: Who did what? Hoes does it feel like the contributions for the members of your pair are equal? Regarding the project specifically, what questions do you need to answer to continue to move forward and what help do you need from me or others?

You will turn in both your “graphic” and your self-reflection using the same GitHub repository you started for Project 2. **Make sure that you sync your repository first to get the new feedback!**