Homework 10 (Due. Mar. 20)

1. Different kinds of conductors

In Griffiths (and in class), we derived equations for EM waves in conductors, assuming a "good conductor." Interestingly, it turns out that the formulas we got can be pushed a good deal further than you might naively expect, into regimes where σ is not so large ("poor conductors.") In this case, you will need to use Griffiths more careful results for the real and imaginary parts of the k vector and work with those.

1. Based on the above comments, show that the skin depth for a "poor conductor" (i.e., $\sigma \ll \varepsilon\omega$) is $d \approx ??\sqrt{\varepsilon/\mu}$, independent of frequency or wavelength. Work out what the "??" is in this equation. (Also, check the units of your answer explicitly, please!)
2. Show that the skin depth for a "good conductor" (i.e., $\sigma \gg \varepsilon\omega$) is $d = \lambda/??$, where λ is the wavelength in the conductor. (Work out the ?? in that formula.) Find the skin depth for microwaves in Cu ($f = 2.5$ GHz). Briefly, how do you interpret that answer physically?
3. About how thick does aluminum foil have to be, to be optically opaque? Comment briefly.
4. For biological tissues (like skin), ε and σ depend on frequency, you can't use their free space values. But, μ on the other hand is close to its free space value. At microwave frequencies (say, about 2.5 GHz), their values are $\sim \varepsilon = 47\varepsilon_0$, and $\sigma = 2.2\Omega^{-1}m^{-1}$. Is skin (at this frequency) the "good conductor" or "poor conductor" case, or neither? Evaluate the skin depth for microwaves hitting your body.
5. If the EM wave from part 4 (e.g. from a radar station) hits your body, what fraction of the incident power do you absorb? (Hint: think about "R" first, then you can get "T" easily!)
6. Let's think about contacting submarines by radio. For low frequency radio waves (say, f=3 kHz) estimate the skin depth in the sea, and comment on the feasibility/issues of such radio communication. What is the wavelength of this same radio wave in free space, by the way? (Hint: Treating seawater as like a human body is ok for this one- just use the given values from part 4 as needed. But, as mentioned there, in reality they'll be different at this very different frequency and slightly different material. Small bonus credit if you can find more appropriate values, and give the reference!)

2. Wave Packets

We keep saying that you can always sum up plane waves to get real wave packets. Let's try it! Consider a localized wavepacket that satisfies the one-dimensional wave equation from a sum of sinusoidal waves using Fourier's integral method:

$$f(x,t) = \int_{-\infty}^{\infty} A(k)e^{ik(x-ct)}dk$$

1. Show that the $f(x,t)$ satfies the one-dimensional wave equation with wave speed, c.
2. Assume that $A(k)$ is given by the a Gaussian distribution centered at some positive wavevector k_0:

$$A(k) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(k-k_0)^2}{2\sigma^2}\right)$$

Sketch this function, Roughly what range of wavevectors Δk contribute signficantly to the wave packet?

3. Calculate $f(x,t)$ from the above $A(k)$. There is a famous and handy Gaussian integral that can be helpful here that works for any z even complex ones!

$$\int_{-\infty}^{\infty} \exp\left(-\frac{q^2 y^2}{2} + zy\right) dy = \sqrt{\frac{2\pi}{q^2}} \exp\left(\frac{z^2}{2q^2}\right)$$

4. Describe $f(x,t)$ physically as best you can. How is the x-width, Δx of the "wavepacket" related to the k-width Δk? Does this relationship between Δx and Δk remind you of anything from quantum mechanics (PHY 215 or 471)?
5. Pick a visible wavelength, and plot, using Jupyter, a Gaussian pulse that lasts 1 femtosecond. (When I say plot, I'm thinking of $Re[f(x, t = 0)]$, and then think about what happens as time goes by) Try to get as many details reasonably correct as you can.

As usual, you will turn part 5 in using your GitHub repo

3. Spring constant of an atom

In class we have modeled an atom as an electron with mass m attached to the heavy nucleus with a spring. This atomic spring has zero equilibrium length and spring constant $k = m\omega_0$. Here we show that this model, though crude, is not crazy. Quantum mechanics tells us that in the ground state of hydrogen, the electron wave function is a spherically-symmetric cloud of charge centered on the atom. So, without too much violence to reality, we can model the electron as a uniform sphere of charge of radius R = Bohr radius = 0.53 angstroms). When an external E-field is applied, the nucleus is pushed one way and the electron cloud is pushed the other way, so there is an induced dipole moment. We assume that the electron's sphere of negative charge is not distorted in shape when it is displaced. We also assume that the nucleus is so heavy that it remains stationary, while the electron cloud can oscillate about the stationary nucleus.
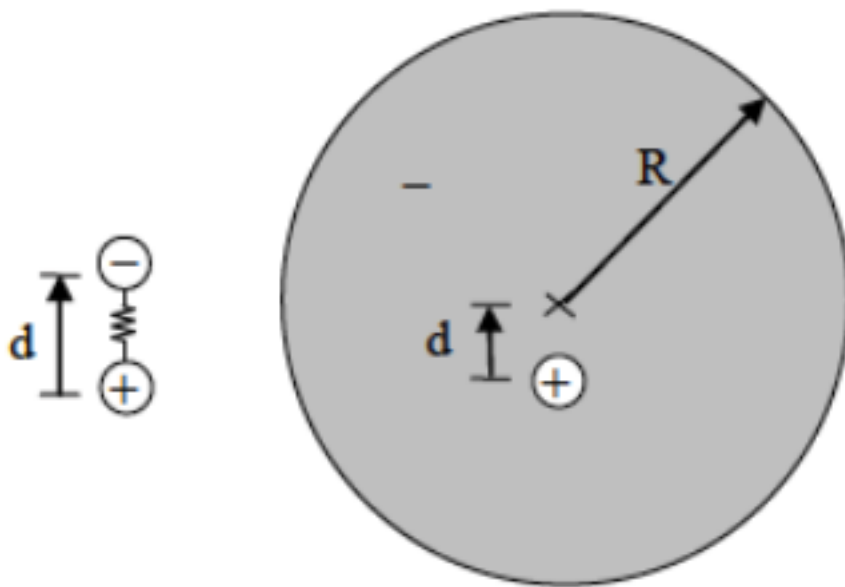

Figure 1: ecloud

1. Show that when the nucleus is displaced from the center of the electron cloud by a distance $d < R$), there is a restoring force that is proportional to d. Derive an expression the spring constant k in this situation.
2. Compute the value of the spring constant k, for the case of a hydrogen atom. Give the answer in both SI units (N/m = J/m^2) and in slightly more natural units of electron-volts per angstrom^2 (eV/Å^2). What is the natural frequency $f_0 = \omega_0/2\pi$ (in Hz) for this mass-spring system?

3. What is the wavelength of light that has this frequency? (Give your answer in nanometers.) What part of the spectrum does this correspond to? (Visible?, IR?, UV?, radio?, etc…)

4. Paired Project Problem - Planning your project

After reviewing the feedback you received on your project idea, work with your partner to consider the plan for the next several weeks. In doing this answer the question:

- How do you intend to structure the work? Explain the details of what will be done and who will be doing what.

The expectation is that you have written 2-3 paragraphs describing the work and a detailed timeline. Where applicable you should also describe the roles and responsibilities of each member of the group at different points in the timeline.

Each team will turn in a single repository, so put your work there.