four quarks, scalar things, and more.

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Abstract

"Young man, in mathematics you don't understand things. You just get used to them." — John von Neumann

Dedication

For my dog.

Declaration

I declare bankruptcy!

Acknowledgements

I want to thank my dog.

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introduction

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QCD = hard

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ϕ/ρ

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Investigating the tetraquark content of the light scalar mesons κ and $a_0(980)$

In this chapter, we examine the effect of including tetraquark operators on the spectrum in the scalar meson sectors containing the $K_0^*(700)$ (here and often elsewhere referred to as the κ) and the $a_0(980)$ in $N_f = 2+1$ QCD, using an anisotropic lattice with gauge field configurations generated by the Hadron Spectrum Collaboration []. Preliminary results of additional states found using tetraquark operators are shown, and possible implications of these states are discussed. This is the first work to study the tetraquarks in the κ and $a_0(980)$ sectors of $N_f = 2+1$, $m_\pi \approx 230$ MeV QCD with proper evaluation of all diagrams in the correlator Wick contractions. Previous studies of the κ have invalidly neglected the evaluation of disconnected contributions, and only one $N_f = 2+1$ study of the $a_0(980)$ to date, by Alexandrou et al. [], has included disconnected diagrams. That study, done at $m_\pi \approx 300$ MeV, identified an additional tetraquark-associated level in the range of 1100 to 1200 MeV, which they claim to be a candidate for the $a_0(980)$ meson. We find an additional state in the range of ... to ...

4.1 Operator construction

We include single- and two- meson operators, as well as tetraquark operators, in the basis of interpolating operators. We construct our elemental operators using building blocks of smeared, gauge-covariantly displaced quark fields, and stout-smeared link variables. To form the final operators out of our elemental operators, we project the elemental operators onto various symmetry channels according to isospin, parity, G-parity, octahedral little group, etc. For example, to form a meson operator $M_l(t)$ that transforms irreducibly under all symmetries of interest (labeled by the compound index l) at time t, we must take a linear combination of our elemental meson operators, $M_l(t) = c^{(l)}_{\alpha\beta}\Phi^{AB}_{\alpha\beta}(\boldsymbol{p}, t)$. To form a two-meson operator $\mathcal{O}_l(t)$, we would follow a similar procedure and project the product of two final meson operators $M^a_{l_a}(t)M^b_{l_a}(t)$ onto a final symmetry channel l: $\mathcal{O}_l(t) = c^{(l)}_{l_a l_b} M^a_{l_a}(t)M^b_{l_a}(t)$.

In order to construct a tetraquark operator, we must consider the various ways to construct a color-singlet four-quark object out of four quark fields. We can see from the Clebsch-Gordon decompositions that the only way to construct a color-singlet is with two quarks and two antiquarks, and that doing so yields two linearly independent color singlet objects:

$$\begin{aligned}
3 \otimes 3 \otimes 3 \otimes 3 &= 3 \oplus 3 \oplus 3 \oplus \overline{6} \oplus \overline{6} \oplus 15 \oplus 15 \oplus 15 \oplus 15, \\
3 \otimes 3 \otimes 3 \otimes \overline{3} &= \overline{3} \oplus \overline{3} \oplus \overline{3} \oplus 6 \oplus 6 \oplus 6 \oplus \overline{15} \oplus \overline{15} \oplus 24, \\
3 \otimes 3 \otimes \overline{3} \otimes \overline{3} &= 1 \oplus 1 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27.
\end{aligned} \tag{4.1}$$

There are 81 basis vectors formed by the quark fields, $p^*_\alpha(x)q^*_\beta(x)r_\gamma(x)s_\mu(x)$, where each r, s transforms as a color vector in the fundamental 3 irrep, and so, p^*, q^* transform in the $\overline{3}$ irrep. We need two linearly independent and gauge-invariant combinations of these to exhaust all possible tetraquark operators. It is easy to see that the following combinations are both linearly independent and gauge-invariant (and thus form our elemental tetraquark operators):

$$\begin{aligned}
T_S &= (\delta_{\alpha\gamma}\delta_{\beta\mu} + \delta_{\alpha\mu}\delta_{\beta\gamma})\, p^*_\alpha(x)q^*_\beta(x)r_\gamma(x)s_\mu(x) \\
T_A &= (\delta_{\alpha\gamma}\delta_{\beta\mu} - \delta_{\alpha\mu}\delta_{\beta\gamma})\, p^*_\alpha(x)q^*_\beta(x)r_\gamma(x)s_\mu(x).
\end{aligned} \tag{4.2}$$

While these elemental tetraquark operators each ostensibly appear to be a combination of two individual mesons, they differ in that the gauge-invariant pieces do need to transform irreducibly under symmetry operations; only the combination of the two quark-antiquark pairs must transform irreducibly. In other words, we project the entire elemental tetraquark operator onto relevant symmetry channels, rather than each individual "meson" operator.

4.2 Lattice Spectra Results (Preliminary)

4.2.1 κ Channel

We summarize results obtained by fitting a spectrum in the κ at-rest symmetry channel for two operator bases: one including only single-meson and two-meson operators, and one including single-meson, two-meson, and tetraquark operators. Figure 4.1 shows the spectrum with and without the inclusion of a tetraquark operator in the basis. The tetraquark operator is of the flavor structure *antistrange-light-antistrange-strange*, and was of the antisymmetric form in (4.2). We see that including a tetraquark operator yields an additional state not

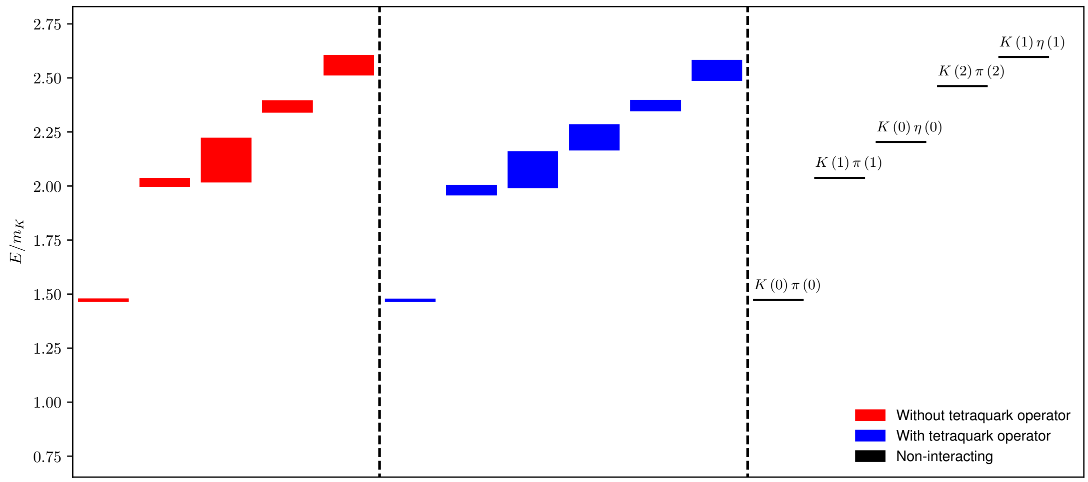

Figure 4.1: The first five and six levels of the spectrum in the κ at-rest symmetry channel. On the left: the spectrum obtained using a basis with no tetraquark operators. In the middle: the spectrum obtained using one tetraquark operator. On the right: non-interacting levels shown for reference, where (0) denotes to particles at rest, and (1) denotes particles with unit momentum, and (2) denotes particles with two units of squared momentum.

present when only single- and two-meson operators are used. Additionally, a plot of the overlap factors for the tetraquark operator (Figure 4.2) shows significant overlap onto this extra state. This suggests that there is a state in our lattice spectrum that shares quantum numbers with the κ resonance, and that has tetraquark content. Whether or not this is evidence of the κ resonance having tetraquark content, however, will have to wait for future scattering studies using this data.

4.2.2 $a_0(980)$ Channel

We summarize results obtained by fitting a spectrum in the $a_0(980)$ at-rest symmetry channel for again for two operator bases as in the κ channel. Figure 4.3 shows the spectrum with and without the inclusion of a tetraquark operator in the basis. The tetraquark operator is of the flavor structure *antilight-light-antilight-light*, and was also of the antisymmetric form in (4.2). We again see an extra level appear when we include a tetraquark operator. Again, overlap factors are shown for the tetraquark operator, and significant overlaps with the additional level can be seen in Figure 4.4. This again suggests there is a state in our lattice spectrum that shares quantum numbers with the $a_0(980)$ resonance, and that has tetraquark content. As in the κ-channel case, evidence for or against the $a_0(980)$ having tetraquark content will have to wait for future scattering studies done using this data.

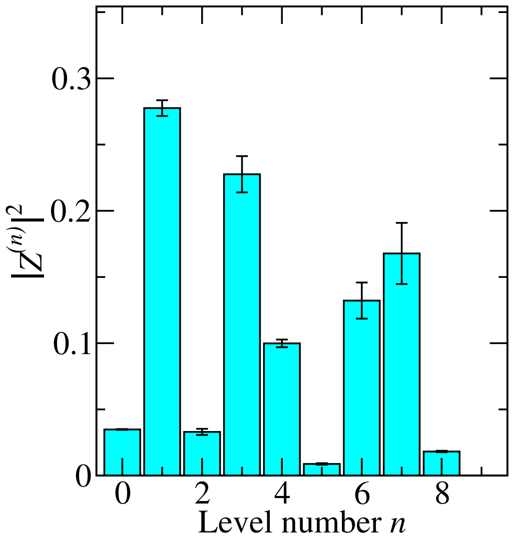

Figure 4.2: The overlap factors for the tetraquark operator used to produce the extra level in the κ symmetry channel.

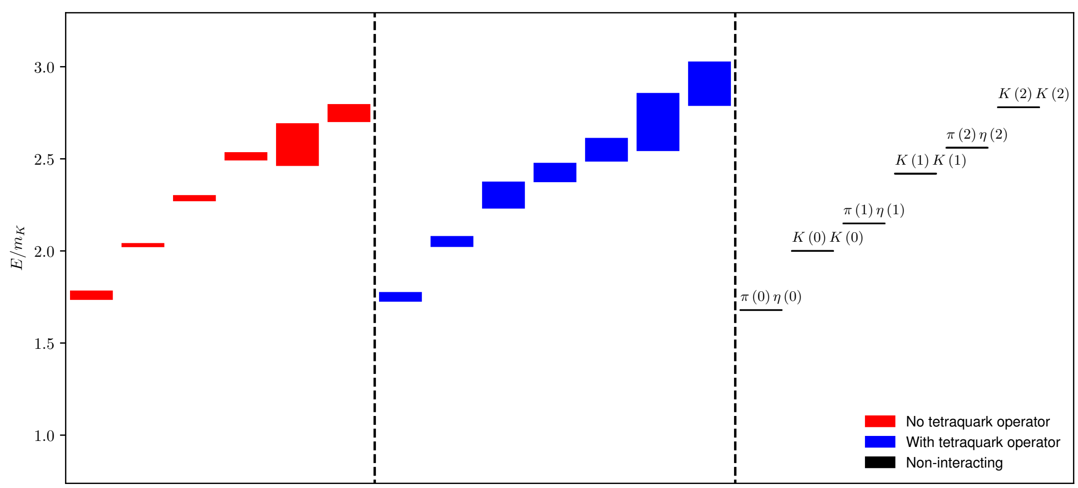

Figure 4.3: The first six and seven levels of the spectrum in the $a_0(980)$ at-rest symmetry channel. On the left: the spectrum obtained using a basis with no tetraquark operators. In the middle: the spectrum obtained using one tetraquark operator. On the right: non-interacting levels shown for reference, where (0) denotes to particles at rest, and (1) denotes particles with unit momentum, and (2) denotes particles with two units of squared momentum.

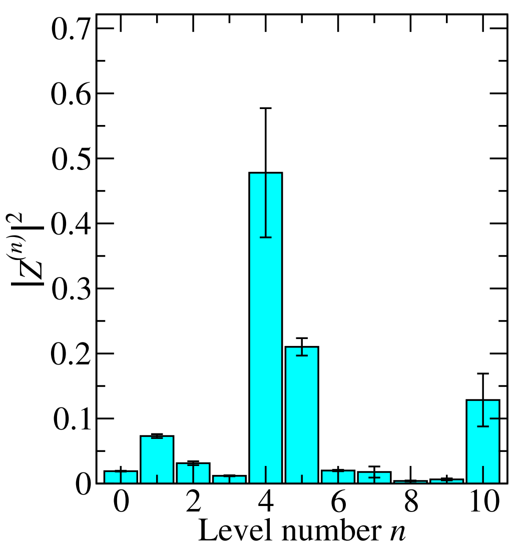

Figure 4.4: The overlap factors for the tetraquark operator used to produce the extra level in the $a_0(980)$ symmetry channel.