1 Boolean Logic

Such simple things, And we make of them something so complex it defeats us, Almost.
—John Ashbery (b. 1927), American poet

Every digital device—be it a personal computer, a cellular telephone, or a network router—is based on a set of chips designed to store and process information. Although these chips come in different shapes and forms, they are all made from the same building blocks: Elementary *logic gates*. The gates can be physically implemented in many different materials and fabrication technologies, but their logical behavior is consistent across all computers. In this chapter we start out with one primitive logic gate—Nand—and build all the other logic gates from it. The result is a rather standard set of gates, which will be later used to construct our computer's processing and storage chips. This will be done in chapters 2 and 3, respectively.

All the hardware chapters in the book, beginning with this one, have the same structure. Each chapter focuses on a well-defined task, designed to construct or integrate a certain family of chips. The prerequisite knowledge needed to approach this task is provided in a brief Background section. The next section provides a complete Specification of the chips' abstractions, namely, the various services that they should deliver, one way or another. Having presented the *what*, a subsequent Implementation section proposes guidelines and hints about *how* the chips can be actually implemented. A Perspective section rounds up the chapter with concluding comments about important topics that were left out from the discussion. Each chapter ends with a technical Project section. This section gives step-by-step instructions for actually building the chips on a personal computer, using the hardware simulator supplied with the book.

This being the first hardware chapter in the book, the Background section is somewhat lengthy, featuring a special section on *hardware description and simulation tools*.

1.1   Background

This chapter focuses on the construction of a family of simple chips called *Boolean gates*. Since Boolean gates are physical implementations of *Boolean functions*, we start with a brief treatment of Boolean algebra. We then show how Boolean gates implementing simple Boolean functions can be interconnected to deliver the functionality of more complex chips. We conclude the background section with a description of how hardware design is actually done in practice, using software simulation tools.

1.1.1   Boolean Algebra

Boolean algebra deals with Boolean (also called binary) values that are typically labeled true/false, 1/0, yes/no, on/off, and so forth. We will use 1 and 0. A Boolean function is a function that operates on binary inputs and returns binary outputs. Since computer hardware is based on the representation and manipulation of binary values, Boolean functions play a central role in the specification, construction, and optimization of hardware architectures. Hence, the ability to formulate and analyze Boolean functions is the first step toward constructing computer architectures.

Truth Table Representation   The simplest way to specify a Boolean function is to enumerate all the possible values of the function's input variables, along with the function's output for each set of inputs. This is called the *truth table* representation of the function, illustrated in figure 1.1.

The first three columns of figure 1.1 enumerate all the possible binary values of the function's variables. For each one of the 2^n possible tuples $v_1 \ldots v_n$ (here $n = 3$), the last column gives the value of $f(v_1 \ldots v_n)$.

Boolean Expressions   In addition to the truth table specification, a Boolean function can also be specified using Boolean operations over its input variables. The basic Boolean operators that are typically used are "And" (x And y is 1 exactly when both x and y are 1) "Or" (x Or y is 1 exactly when either x or y or both are 1), and "Not" (Not x is 1 exactly when x is 0). We will use a common arithmetic-like notation for these operations: $x \cdot y$ (or xy) means x And y, $x + y$ means x Or y, and $\bar{x}$ means Not x.

To illustrate, the function defined in figure 1.1 is equivalently given by the Boolean expression $f(x, y, z) = (x + y) \cdot \bar{z}$. For example, let us evaluate this expression on the

x	y	z	$f(x,y,z)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Figure 1.1   Truth table representation of a Boolean function (example).

inputs $x = 0$, $y = 1$, $z = 0$ (third row in the table). Since y is 1, it follows that $x + y = 1$ and thus $1 \cdot \bar{0} = 1 \cdot 1 = 1$. The complete verification of the equivalence between the expression and the truth table is achieved by evaluating the expression on each of the eight possible input combinations, verifying that it yields the same value listed in the table's right column.

Canonical Representation   As it turns out, every Boolean function can be expressed using at least one Boolean expression called the *canonical representation*. Starting with the function's truth table, we focus on all the rows in which the function has value 1. For each such row, we construct a term created by And-ing together *literals* (variables or their negations) that fix the values of all the row's inputs. For example, let us focus on the third row in figure 1.1, where the function's value is 1. Since the variable values in this row are $x = 0$, $y = 1$, $z = 0$, we construct the term $\bar{x}y\bar{z}$. Following the same procedure, we construct the terms $x\bar{y}\bar{z}$ and $xy\bar{z}$ for rows 5 and 7. Now, if we Or-together all these terms (for all the rows where the function has value 1), we get a Boolean expression that is equivalent to the given truth table. Thus the canonical representation of the Boolean function shown in figure 1.1 is $f(x,y,z) = \bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z}$. This construction leads to an important conclusion: Every Boolean function, no matter how complex, can be expressed using three Boolean operators only: And, Or, and Not.

Two-Input Boolean Functions   An inspection of figure 1.1 reveals that the number of Boolean functions that can be defined over n binary variables is 2^{2^n}. For example, the sixteen Boolean functions spanned by two variables are listed in figure 1.2. These functions were constructed systematically, by enumerating all the possible 4-wise combinations of binary values in the four right columns. Each function has a conventional

Function	x	0	0	1	1
	y	0	1	0	1
Constant 0	0	0	0	0	0
And	$x \cdot y$	0	0	0	1
x And Not y	$x \cdot \bar{y}$	0	0	1	0
x	x	0	0	1	1
Not x And y	$\bar{x} \cdot y$	0	1	0	0
y	y	0	1	0	1
Xor	$x \cdot \bar{y} + \bar{x} \cdot y$	0	1	1	0
Or	$x + y$	0	1	1	1
Nor	$\overline{x + y}$	1	0	0	0
Equivalence	$x \cdot y + \bar{x} \cdot \bar{y}$	1	0	0	1
Not y	$\bar{y}$	1	0	1	0
If y then x	$x + \bar{y}$	1	0	1	1
Not x	$\bar{x}$	1	1	0	0
If x then y	$\bar{x} + y$	1	1	0	1
Nand	$\overline{x \cdot y}$	1	1	1	0
Constant 1	1	1	1	1	1

Figure 1.2   All the Boolean functions of two variables.

name that seeks to describe its underlying operation. Here are some examples: The name of the Nor function is shorthand for Not-Or: Take the Or of x and y, then negate the result. The Xor function—shorthand for "exclusive or"—returns 1 when its two variables have opposing truth-values and 0 otherwise. Conversely, the Equivalence function returns 1 when the two variables have identical truth-values. The If-x-then-y function (also known as $x \rightarrow y$, or "x Implies y") returns 1 when x is 0 or when both x and y are 1. The other functions are self-explanatory.

The Nand function (as well as the Nor function) has an interesting theoretical property: Each one of the operations And, Or, and Not can be constructed from it, and it alone (e.g., x Or $y = (x$ Nand $x)$ Nand $(y$ Nand $y))$. And since every Boolean function can be constructed from And, Or, and Not operations using the canonical representation method, it follows that every Boolean function can be constructed from Nand operations alone. This result has far-reaching practical implications: Once we have in our disposal a physical device that implements Nand, we can use many copies of this device (wired in a certain way) to implement in hardware any Boolean function.

1.1.2   Gate Logic

A *gate* is a physical device that implements a Boolean function. If a Boolean function *f* operates on *n* variables and returns *m* binary results (in all our examples so far, *m* was 1), the gate that implements *f* will have *n input pins* and *m output pins*. When we put some values $v_1 \ldots v_n$ in the gate's input pins, the gate's "logic"—its internal structure—should compute and output $f(v_1 \ldots v_n)$. And just like complex Boolean functions can be expressed in terms of simpler functions, complex gates are composed from more elementary gates. The simplest gates of all are made from tiny switching devices, called *transistors*, wired in a certain topology designed to effect the overall gate functionality.

Although most digital computers today use electricity to represent and transmit binary data from one gate to another, any alternative technology permitting switching and conducting capabilities can be employed. Indeed, during the last fifty years, researchers have built many hardware implementations of Boolean functions, including magnetic, optical, biological, hydraulic, and pneumatic mechanisms. Today, most gates are implemented as transistors etched in silicon, packaged as *chips*. In this book we use the words *chip* and *gate* interchangeably, tending to use the term *gates* for simple chips.

The availability of alternative switching technology options, on the one hand, and the observation that Boolean algebra can be used to abstract the behavior of *any* such technology, on the other, is extremely important. Basically, it implies that computer scientists don't have to worry about physical things like electricity, circuits, switches, relays, and power supply. Instead, computer scientists can be content with the abstract notions of Boolean algebra and gate logic, trusting that someone else (the physicists and electrical engineers—bless their souls) will figure out how to actually realize them in hardware. Hence, a *primitive gate* (see figure 1.3) can be viewed as a black box device that implements an elementary logical operation in one way or another—we don't care how. A hardware designer starts from such primitive gates and designs more complicated functionality by interconnecting them, leading to the construction of *composite* gates.
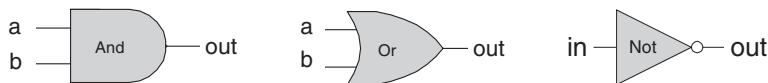
Figure 1.3   Standard symbolic notation of some elementary logic gates.

Gate interface   **Gate implementation**

If a=b=c=1 then out=1
else out=0
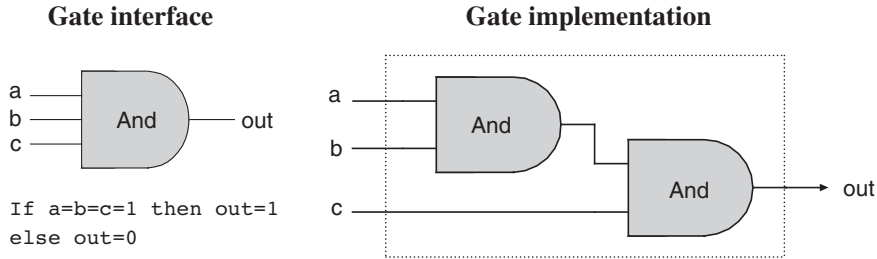
Figure 1.4 Composite implementation of a three-way And gate. The rectangle on the right defines the conceptual boundaries of the gate interface.

Primitive and Composite Gates   Since all logic gates have the same input and output semantics (0's and 1's), they can be chained together, creating *composite gates* of arbitrary complexity. For example, suppose we are asked to implement the 3-way Boolean function $And(a, b, c)$. Using Boolean algebra, we can begin by observing that $a \cdot b \cdot c = (a \cdot b) \cdot c$, or, using prefix notation, $And(a, b, c) = And(And(a, b), c)$. Next, we can use this result to construct the composite gate depicted in figure 1.4.

The construction described in figure 1.4 is a simple example of *gate logic*, also called *logic design*. Simply put, logic design is the art of interconnecting gates in order to implement more complex functionality, leading to the notion of *composite gates*. Since composite gates are themselves realizations of (possibly complex) Boolean functions, their "outside appearance" (e.g., left side of figure 1.4) looks just like that of primitive gates. At the same time, their internal structure can be rather complex.

We see that any given logic gate can be viewed from two different perspectives: external and internal. The right-hand side of figure 1.4 gives the gate's internal architecture, or *implementation*, whereas the left side shows only the gate *interface*, namely, the input and output pins that it exposes to the outside world. The former is relevant only to the gate designer, whereas the latter is the right level of detail for other designers who wish to use the gate as an abstract off-the-shelf component, without paying attention to its internal structure.

Let us consider another logic design example—that of a Xor gate. As discussed before, $Xor(a, b)$ is 1 exactly when either a is 1 and b is 0, or when a is 0 and b is 1. Said otherwise, $Xor(a, b) = Or(And(a, Not(b)), And(Not(a), b))$. This definition leads to the logic design shown in figure 1.5.

Note that the gate *interface* is unique: There is only one way to describe it, and this is normally done using a truth table, a Boolean expression, or some verbal specifica-
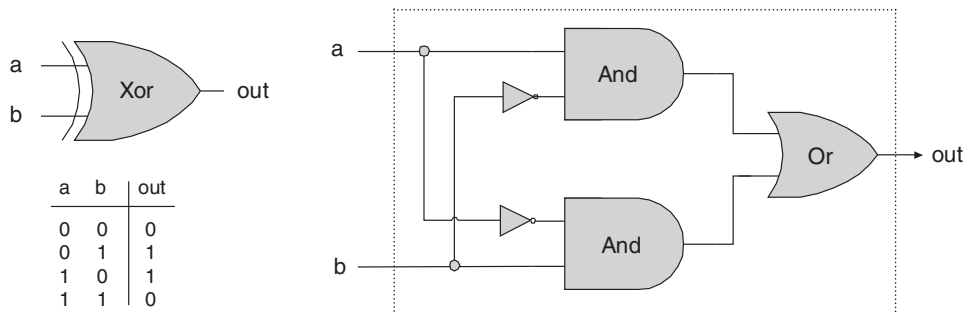
Figure 1.5 Xor gate, along with a possible implementation.

tion. This interface, however, can be realized using many different *implementations*, some of which will be better than others in terms of cost, speed, and simplicity. For example, the Xor function can be implemented using four, rather than five, And, Or, and Not gates. Thus, from a functional standpoint, the fundamental requirement of logic design is that *the gate implementation will realize its stated interface, in one way or another*. From an efficiency standpoint, the general rule is to try to *do more with less*, that is, use as few gates as possible.

To sum up, the art of logic design can be described as follows: Given a gate specification (interface), find an efficient way to implement it using other gates that were already implemented. This, in a nutshell, is what we will do in the rest of this chapter.

1.1.3   Actual Hardware Construction

Having described the logic of composing complex gates from simpler ones, we are now in a position to discuss how gates are actually built. Let us start with an intentionally naïve example.

Suppose we open a chip fabrication shop in our home garage. Our first contract is to build a hundred Xor gates. Using the order's downpayment, we purchase a soldering gun, a roll of copper wire, and three bins labeled "And gates," "Or gates," and "Not gates," each containing many identical copies of these elementary logic gates. Each of these gates is sealed in a plastic casing that exposes some input and output pins, as well as a power supply plug. To get started, we pin figure 1.5 to our garage wall and proceed to realize it using our hardware. First, we take two And gates, two Not gates, and one Or gate, and mount them on a board according to the

figure's layout. Next, we connect the chips to one another by running copper wires among them and by soldering the wire ends to the respective input/output pins. Now, if we follow the gate diagram carefully, we will end up having three exposed wire ends. We then solder a pin to each one of these wire ends, seal the entire device (except for the three pins) in a plastic casing, and label it "Xor." We can repeat this assembly process many times over. At the end of the day, we can store all the chips that we've built in a new bin and label it "Xor gates." If we (or other people) are asked to construct some other chips in the future, we'll be able to use these Xor gates as elementary building blocks, just as we used the And, Or, and Not gates before.

As the reader has probably sensed, the garage approach to chip production leaves much to be desired. For starters, there is no guarantee that the given chip diagram is correct. Although we can prove correctness in simple cases like Xor, we cannot do so in many realistically complex chips. Thus, we must settle for empirical testing: Build the chip, connect it to a power supply, activate and deactivate the input pins in various configurations, and hope that the chip outputs will agree with its specifications. If the chip fails to deliver the desired outputs, we will have to tinker with its physical structure—a rather messy affair. Further, even if we will come up with the right design, replicating the chip assembly process many times over will be a time-consuming and error-prone affair. There must be a better way!

1.1.4   Hardware Description Language (HDL)

Today, hardware designers no longer build anything with their bare hands. Instead, they plan and optimize the chip architecture on a computer workstation, using structured modeling formalisms like *Hardware Description Language*, or HDL (also known as VHDL, where V stands for *Virtual*). The designer specifies the chip structure by writing an *HDL program*, which is then subjected to a rigorous battery of tests. These tests are carried out virtually, using computer simulation: A special software tool, called a *hardware simulator*, takes the HDL program as input and builds an image of the modeled chip in memory. Next, the designer can instruct the simulator to test the virtual chip on various sets of inputs, generating simulated chip outputs. The outputs can then be compared to the desired results, as mandated by the client who ordered the chip built.

In addition to testing the chip's correctness, the hardware designer will typically be interested in a variety of parameters such as speed of computation, energy consumption, and the overall cost implied by the chip design. All these param-
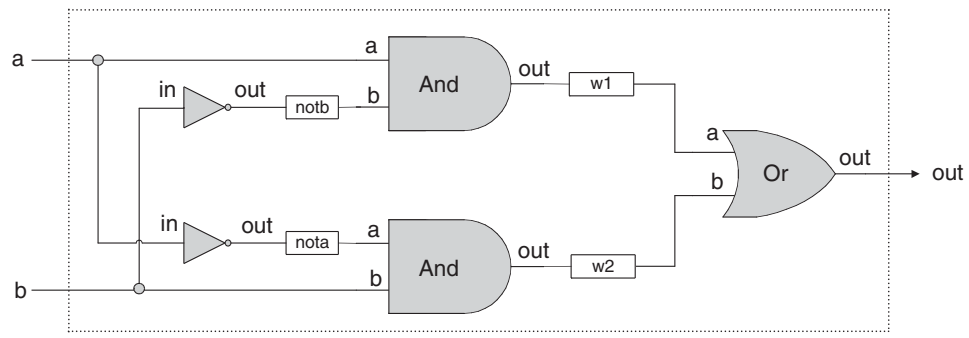
eters can be simulated and quantified by the hardware simulator, helping the designer optimize the design until the simulated chip delivers desired cost/performance levels.

Thus, using HDL, one can completely plan, debug, and optimize the entire chip before a single penny is spent on actual production. When the HDL program is deemed complete, that is, when the performance of the simulated chip satisfies the client who ordered it, the HDL program can become the blueprint from which many copies of the physical chip can be stamped in silicon. This final step in the chip life cycle—from an optimized HDL program to mass production—is typically outsourced to companies that specialize in chip fabrication, using one switching technology or another.

Example: Building a Xor Gate   As we have seen in figures 1.2 and 1.5, one way to define *exclusive or* is $\mathrm{Xor}(a, b) = \mathrm{Or}(\mathrm{And}(a, \mathrm{Not}(b)), \mathrm{And}(\mathrm{Not}(a), b))$. This logic can be expressed either graphically, as a gate diagram, or textually, as an HDL program. The latter program is written in the HDL variant used throughout this book, defined in appendix A. See figure 1.6 for the details.

Explanation   An HDL definition of a chip consists of a *header* section and a *parts* section. The header section specifies the chip *interface*, namely the chip name and the names of its input and output pins. The parts section describes the names and topology of all the lower-level parts (other chips) from which this chip is constructed. Each part is represented by a *statement* that specifies the part name and the way it is connected to other parts in the design. Note that in order to write such statements legibly, the HDL programmer must have a complete documentation of the underlying parts' *interfaces*. For example, figure 1.6 assumes that the input and output pins of the Not gate are labeled `in` and `out`, and those of And and Or are labeled `a`, `b` and `out`. This API-type information is not obvious, and one must have access to it before one can plug the chip parts into the present code.

Inter-part connections are described by creating and connecting *internal pins*, as needed. For example, consider the bottom of the gate diagram, where the output of a Not gate is piped into the input of a subsequent And gate. The HDL code describes this connection by the pair of statements `Not(...,out=nota)` and `And(a=nota,...)`. The first statement creates an internal pin (outbound wire) named `nota`, feeding `out` into it. The second statement feeds the value of `nota` into the `a` input of an And gate. Note that pins may have an unlimited fan out. For example, in figure 1.6, each input is simultaneously fed into two gates. In gate

HDL program (`Xor.hdl`)	*Test script* (`Xor.tst`)	*Output file* (`Xor.out`)

```
/* Xor (exclusive or) gate:
   If a<>b out=1 else out=0. */
CHIP Xor {
   IN a, b;
   OUT out;
   PARTS:
   Not(in=a, out=nota);
   Not(in=b, out=notb);
   And(a=a, b=notb, out=w1);
   And(a=nota, b=b, out=w2);
   Or(a=w1, b=w2, out=out);
}
```

```
load Xor.hdl,
output-list a, b, out;
set a 0, set b 0,
eval, output;
set a 0, set b 1,
eval, output;
set a 1, set b 0,
eval, output;
set a 1, set b 1,
eval, output;
```

```
a | b | out
- - - - - - -
0 | 0 | 0
0 | 1 | 1
1 | 0 | 1
1 | 1 | 0
```

Figure 1.6   HDL implementation of a Xor gate.

diagrams, multiple connections are described using forks. In HDL, the existence of forks is implied by the code.

Testing   Rigorous quality assurance mandates that chips be tested in a specific, replicable, and well-documented fashion. With that in mind, hardware simulators are usually designed to run *test scripts*, written in some scripting language. For example, the test script in figure 1.6 is written in the scripting language understood by the hardware simulator supplied with the book. This scripting language is described fully in appendix B.

Let us give a brief description of the test script from figure 1.6. The first two lines of the test script instruct the simulator to load the `Xor.hdl` program and get ready to

print the values of selected variables. Next, the script lists a series of testing scenarios, designed to simulate the various contingencies under which the Xor chip will have to operate in "real-life" situations. In each scenario, the script instructs the simulator to bind the chip inputs to certain data values, compute the resulting output, and record the test results in a designated output file. In the case of simple gates like Xor, one can write an exhaustive test script that enumerates all the possible input values of the gate. The resulting output file (right side of figure 1.6) can then be viewed as a complete empirical proof that the chip is well designed. The luxury of such certitude is not feasible in more complex chips, as we will see later.
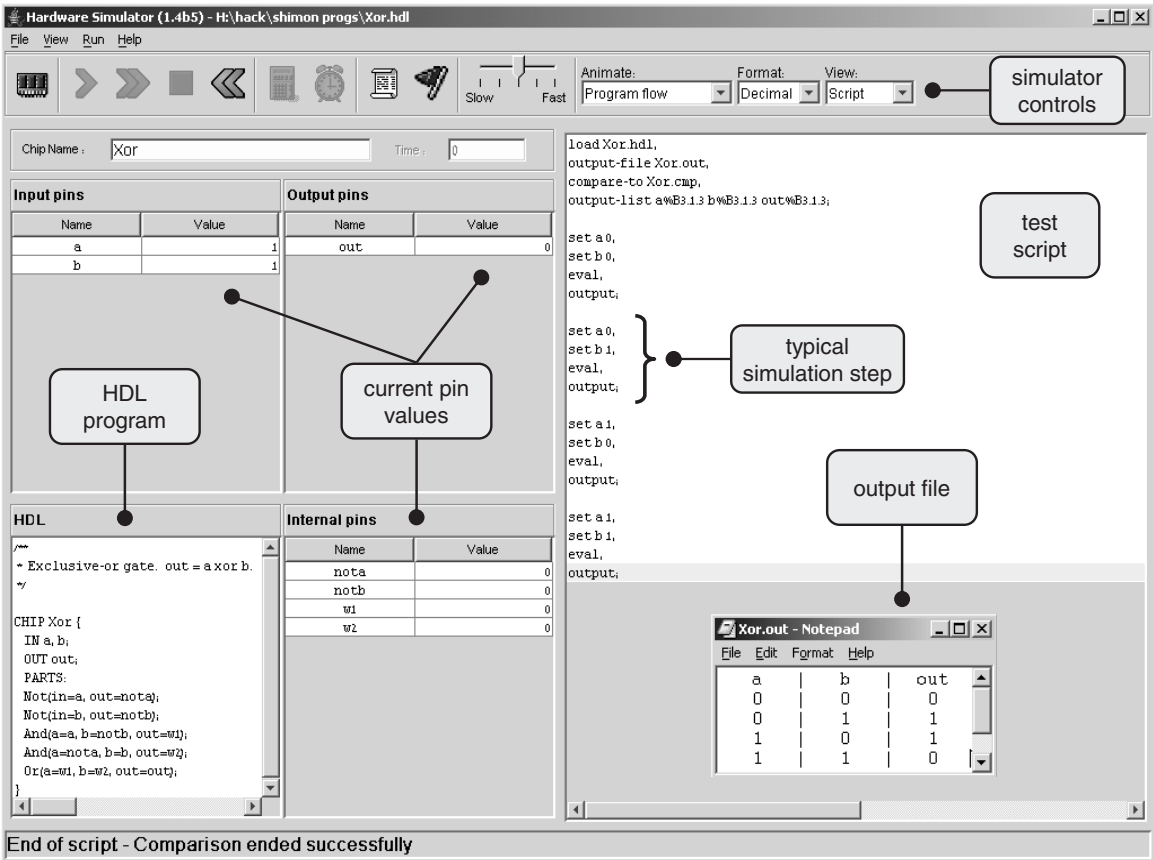
1.1.5   Hardware Simulation

Since HDL is a hardware construction *language*, the process of writing and debugging HDL programs is quite similar to software development. The main difference is that instead of writing code in a language like Java, we write it in HDL, and instead of using a compiler to translate and test the code, we use a *hardware simulator*. The hardware simulator is a computer program that knows how to parse and interpret HDL code, turn it into an executable representation, and test it according to the specifications of a given test script. There exist many commercial hardware simulators on the market, and these vary greatly in terms of cost, complexity, and ease of use. Together with this book we provide a simple (and free!) hardware simulator that is sufficiently powerful to support sophisticated hardware design projects. In particular, the simulator provides all the necessary tools for building, testing, and integrating all the chips presented in the book, leading to the construction of a general-purpose computer. Figure 1.7 illustrates a typical chip simulation session.

1.2   Specification

This section specifies a typical set of gates, each designed to carry out a common Boolean operation. These gates will be used in the chapters that follow to construct the full architecture of a typical modern computer. Our starting point is a single primitive Nand gate, from which all other gates will be derived recursively. Note that we provide only the gates' specifications, or interfaces, delaying implementation details until a subsequent section. Readers who wish to construct the specified gates in HDL are encouraged to do so, referring to appendix A as needed. All the gates can be built and simulated on a personal computer, using the hardware simulator supplied with the book.

Figure 1.7   A screen shot of simulating an Xor chip on the hardware simulator. The simulator state is shown just after the test script has completed running. The pin values correspond to the last simulation step ($a = b = 1$). Note that the *output file* generated by the simulation is consistent with the Xor truth table, indicating that the loaded HDL program delivers a correct Xor functionality. The *compare file*, not shown in the figure and typically specified by the chip's client, has exactly the same structure and contents as that of the output file. The fact that the two files agree with each other is evident from the status message displayed at the bottom of the screen.

1.2.1   The Nand Gate

The starting point of our computer architecture is the Nand gate, from which all other gates and chips are built. The Nand gate is designed to compute the following Boolean function:

a	*b*	Nand(a, b)
0	0	1
0	1	1
1	0	1
1	1	0

Throughout the book, we use "chip API boxes" to specify chips. For each chip, the API specifies the chip name, the names of its input and output pins, the function or operation that the chip effects, and an optional comment.

```
Chip name: Nand
Inputs:    a, b
Outputs:   out
Function:  If a=b=1 then out=0 else out=1
Comment:   This gate is considered primitive and thus there is
           no need to implement it.
```

1.2.2   Basic Logic Gates

Some of the logic gates presented here are typically referred to as "elementary" or "basic." At the same time, every one of them can be composed from Nand gates alone. Therefore, they need not be viewed as primitive.

Not   The single-input Not gate, also known as "converter," converts its input from 0 to 1 and vice versa. The gate API is as follows:

```
Chip name: Not
Inputs:    in
Outputs:   out
Function:  If in=0 then out=1 else out=0.
```

And   The And function returns 1 when both its inputs are 1, and 0 otherwise.

```
Chip name: And
Inputs:    a, b
Outputs:   out
Function:  If a=b=1 then out=1 else out=0.
```

Or   The Or function returns 1 when at least one of its inputs is 1, and 0 otherwise.

```
Chip name: Or
Inputs:    a, b
Outputs:   out
Function:  If a=b=0 then out=0 else out=1.
```

Xor   The Xor function, also known as "exclusive or," returns 1 when its two inputs have opposing values, and 0 otherwise.

```
Chip name: Xor
Inputs:    a, b
Outputs:   out
Function:  If a≠b then out=1 else out=0.
```

Multiplexor   A multiplexor (figure 1.8) is a three-input gate that uses one of the inputs, called "selection bit," to select and output one of the other two inputs, called "data bits." Thus, a better name for this device might have been *selector*. The name *multiplexor* was adopted from communications systems, where similar devices are used to serialize (multiplex) several input signals over a single output wire.

```
Chip name: Mux
Inputs:    a, b, sel
Outputs:   out
Function:  If sel=0 then out=a else out=b.
```

a	b	sel	out
0	0	0	0
0	1	0	0
1	0	0	1
1	1	0	1
0	0	1	0
0	1	1	1
1	0	1	0
1	1	1	1

sel	out
0	a
1	b
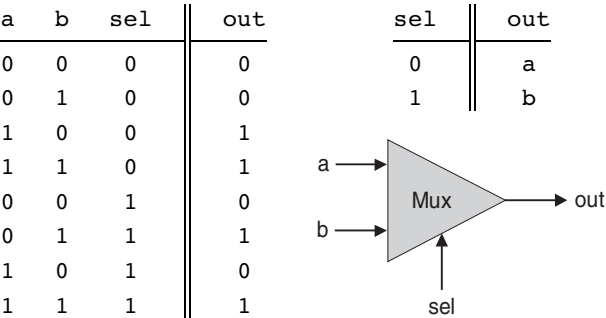
Figure 1.8 Multiplexor. The table at the top right is an abbreviated version of the truth table on the left.

sel	a	b
0	in	0
1	0	in
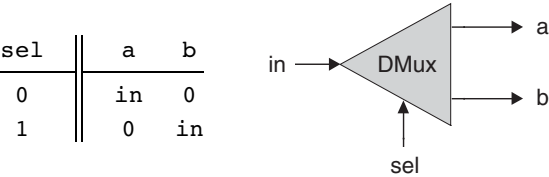
Figure 1.9 Demultiplexor.

Demultiplexor A demultiplexor (figure 1.9) performs the opposite function of a multiplexor: It takes a single input and channels it to one of two possible outputs according to a selector bit that specifies which output to chose.

```
Chip name: DMux
Inputs:    in, sel
Outputs:   a, b
Function:  If sel=0 then {a=in, b=0} else {a=0, b=in}.
```

1.2.3   Multi-Bit Versions of Basic Gates

Computer hardware is typically designed to operate on multi-bit arrays called "buses." For example, a basic requirement of a 32-bit computer is to be able to compute (bit-wise) an And function on two given 32-bit buses. To implement this operation, we can build an array of 32 binary And gates, each operating separately

on a pair of bits. In order to enclose all this logic in one package, we can encapsulate the gates array in a single chip interface consisting of two 32-bit input buses and one 32-bit output bus.

This section describes a typical set of such multi-bit logic gates, as needed for the construction of a typical 16-bit computer. We note in passing that the architecture of *n*-bit logic gates is basically the same irrespective of *n*'s value.

When referring to individual bits in a bus, it is common to use an array syntax. For example, to refer to individual bits in a 16-bit bus named `data`, we use the notation `data[0], data[1],..., data[15]`.

Multi-Bit Not   An *n*-bit Not gate applies the Boolean operation Not to every one of the bits in its *n*-bit input bus:

```
Chip name: Not16
Inputs:    in[16] // a 16-bit pin
Outputs:   out[16]
Function:  For i=0..15 out[i]=Not(in[i]).
```

Multi-Bit And   An *n*-bit And gate applies the Boolean operation And to every one of the *n* bit-pairs arrayed in its two *n*-bit input buses:

```
Chip name: And16
Inputs:    a[16], b[16]
Outputs:   out[16]
Function:  For i=0..15 out[i]=And(a[i],b[i]).
```

Multi-Bit Or   An *n*-bit Or gate applies the Boolean operation Or to every one of the *n* bit-pairs arrayed in its two *n*-bit input buses:

```
Chip name: Or16
Inputs:    a[16], b[16]
Outputs:   out[16]
Function:  For i=0..15 out[i]=Or(a[i],b[i]).
```

Multi-Bit Multiplexor   An *n*-bit multiplexor is exactly the same as the binary multiplexor described in figure 1.8, except that the two inputs are each *n*-bit wide; the selector is a single bit.

```
Chip name: Mux16
Inputs:    a[16], b[16], sel
Outputs:   out[16]
Function:  If sel=0 then for i=0..15 out[i]=a[i]
           else for i=0..15 out[i]=b[i].
```

1.2.4   Multi-Way Versions of Basic Gates

Many 2-way logic gates that accept two inputs have natural generalization to multi-way variants that accept an arbitrary number of inputs. This section describes a set of multi-way gates that will be used subsequently in various chips in our computer architecture. Similar generalizations can be developed for other architectures, as needed.

Multi-Way Or   An *n*-way Or gate outputs 1 when at least one of its *n* bit inputs is 1, and 0 otherwise. Here is the 8-way variant of this gate:

```
Chip name: Or8Way
Inputs:    in[8]
Outputs:   out
Function:  out=Or(in[0],in[1],...,in[7]).
```

Multi-Way/Multi-Bit Multiplexor   An *m*-way *n*-bit multiplexor selects one of *m* *n*-bit input buses and outputs it to a single *n*-bit output bus. The selection is specified by a set of *k* control bits, where $k = \log_2 m$. Figure 1.10 depicts a typical example.

The computer platform that we develop in this book requires two variations of this chip: A 4-way 16-bit multiplexor and an 8-way 16-bit multiplexor:

sel[1]	sel[0]	out
0	0	a
0	1	b
1	0	c
1	1	d
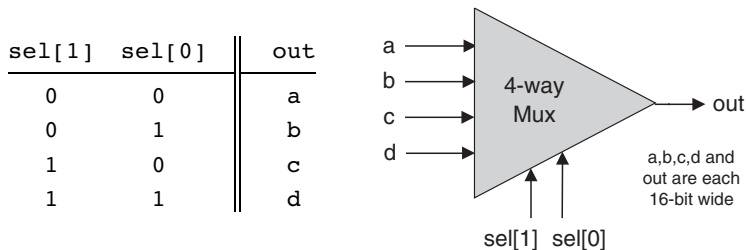
Figure 1.10   4-way multiplexor. The width of the input and output buses may vary.

```
Chip name: Mux4Way16
Inputs:    a[16], b[16], c[16], d[16], sel[2]
Outputs:   out[16]
Function:  If sel=00 then out=a else if sel=01 then out=b
           else if sel=10 then out=c else if sel=11 then out=d
Comment:   The assignment operations mentioned above are all
           16-bit. For example, "out=a" means "for i=0..15
           out[i]=a[i]".
```

```
Chip name: Mux8Way16
Inputs:    a[16],b[16],c[16],d[16],e[16],f[16],g[16],h[16],
           sel[3]
Outputs:   out[16]
Function:  If sel=000 then out=a else if sel=001 then out=b
           else if sel=010 out=c ... else if sel=111 then out=h
Comment:   The assignment operations mentioned above are all
           16-bit. For example, "out=a" means "for i=0..15
           out[i]=a[i]".
```

Multi-Way/Multi-Bit Demultiplexor   An m-way n-bit demultiplexor (figure 1.11) channels a single n-bit input into one of m possible n-bit outputs. The selection is specified by a set of k control bits, where $k = \log_2 m$.

The specific computer platform that we will build requires two variations of this chip: A 4-way 1-bit demultiplexor and an 8-way 1-bit multiplexor, as follows.
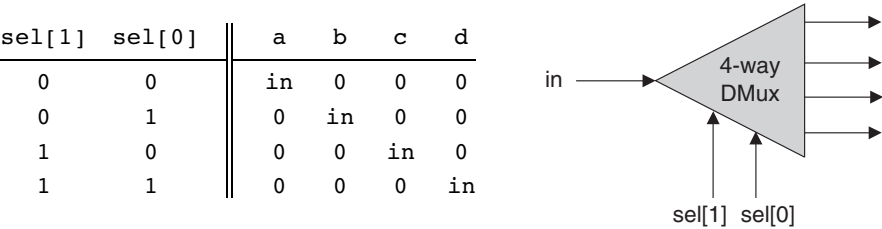
sel[1]	sel[0]	a	b	c	d
0	0	in	0	0	0
0	1	0	in	0	0
1	0	0	0	in	0
1	1	0	0	0	in

Figure 1.11   4-way demultiplexor.

```
Chip name: DMux4Way
Inputs:    in, sel[2]
Outputs:   a, b, c, d
Function:  If sel=00 then      {a=in, b=c=d=0}
           else if sel=01 then {b=in, a=c=d=0}
           else if sel=10 then {c=in, a=b=d=0}
           else if sel=11 then {d=in, a=b=c=0}.
```

```
Chip name: DMux8Way
Inputs:    in, sel[3]
Outputs:   a, b, c, d, e, f, g, h
Function:  If sel=000 then      {a=in, b=c=d=e=f=g=h=0}
           else if sel=001 then {b=in, a=c=d=e=f=g=h=0}
           else if sel=010 ...
           ...
           else if sel=111 then {h=in, a=b=c=d=e=f=g=0}.
```

1.3   Implementation

Similar to the role of *axioms* in mathematics, *primitive gates* provide a set of elementary building blocks from which everything else can be built. Operationally, primitive gates have an "off-the-shelf" implementation that is supplied externally. Thus, they can be used in the construction of other gates and chips without worrying about their internal design. In the computer architecture that we are now beginning

to build, we have chosen to base all the hardware on one primitive gate only: Nand. We now turn to outlining the first stage of this bottom-up hardware construction project, one gate at a time.

Our implementation guidelines are intentionally partial, since we want you to discover the actual gate architectures yourself. We reiterate that each gate can be implemented in more than one way; the simpler the implementation, the better.

Not: The implementation of a unary Not gate from a binary Nand gate is simple. Tip: Think positive.

And: Once again, the gate implementation is simple. Tip: Think negative.

Or/Xor: These functions can be defined in terms of some of the Boolean functions implemented previously, using some simple Boolean manipulations. Thus, the respective gates can be built using previously built gates.

Multiplexor/Demultiplexor: Likewise, these gates can be built using previously built gates.

Multi-Bit Not/And/Or Gates: Since we already know how to implement the elementary versions of these gates, the implementation of their n-ary versions is simply a matter of constructing arrays of n elementary gates, having each gate operate separately on its bit inputs. This implementation task is rather boring, but it will carry its weight when these multi-bit gates are used in more complex chips, as described in subsequent chapters.

Multi-Bit Multiplexor: The implementation of an n-ary multiplexor is simply a matter of feeding the same selection bit to every one of n binary multiplexors. Again, a boring task resulting in a very useful chip.

Multi-Way Gates: Implementation tip: Think forks.

1.4   Perspective

This chapter described the first steps taken in an applied digital design project. In the next chapter we will build more complicated functionality using the gates built here.

Although we have chosen to use Nand as our basic building block, other approaches are possible. For example, one can build a complete computer platform using Nor gates alone, or, alternatively, a combination of And, Or, and Not gates. These constructive approaches to logic design are theoretically equivalent, just as all theorems in geometry can be founded on different sets of axioms as alternative points of departure. The theory and practice of such constructions are covered in standard textbooks about *digital design* or *logic design*.

Throughout the chapter, we paid no attention to efficiency considerations such as the number of elementary gates used in constructing a composite gate or the number of wire crossovers implied by the design. Such considerations are critically important in practice, and a great deal of computer science and electrical engineering expertise focuses on optimizing them. Another issue we did not address at all is the physical implementation of gates and chips using the laws of physics, for example, the role of transistors embedded in silicon. There are of course several such implementation options, each having its own characteristics (speed, power requirements, production cost, etc.). Any nontrivial coverage of these issues requires some background in electronics and physics.

1.5   Project

Objective   Implement all the logic gates presented in the chapter. The only building blocks that you can use are primitive Nand gates and the composite gates that you will gradually build on top of them.

Resources   The only tool that you need for this project is the hardware simulator supplied with the book. All the chips should be implemented in the HDL language specified in appendix A. For each one of the chips mentioned in the chapter, we provide a skeletal `.hdl` program (text file) with a missing implementation part. In addition, for each chip we provide a `.tst` script file that tells the hardware simulator how to test it, along with the correct output file that this script should generate, called `.cmp` or "compare file." Your job is to complete the missing implementation parts of the supplied `.hdl` programs.

Contract   When loaded into the hardware simulator, your chip design (modified `.hdl` program), tested on the supplied `.tst` file, should produce the outputs listed in the supplied `.cmp` file. If that is not the case, the simulator will let you know.

Tips   The Nand gate is considered primitive and thus there is no need to build it: Whenever you use Nand in one of your HDL programs, the simulator will automatically invoke its built-in `tools/builtIn/Nand.hdl` implementation. We recommend implementing the other gates in this project in the order in which they appear in the chapter. However, since the `builtIn` directory features working versions of all the chips described in the book, you can always use these chips without defining them first: The simulator will automatically use their built-in versions.

For example, consider the skeletal `Mux.hdl` program supplied in this project. Suppose that for one reason or another you did not complete this program's implementation, but you still want to use Mux gates as internal parts in other chip designs. This is not a problem, thanks to the following convention. If our simulator fails to find a `Mux.hdl` file in the current directory, it automatically invokes a built-in Mux implementation, pre-supplied with the simulator's software. This built-in implementation—a Java class stored in the `builtIn` directory—has the same interface and functionality as those of the Mux gate described in the book. Thus, if you want the simulator to ignore one or more of your chip implementations, simply move the corresponding `.hdl` files out of the current directory.

Steps   We recommend proceeding in the following order:

0.   The *hardware simulator* needed for this project is available in the `tools` directory of the book's software suite.

1.   Read appendix A, sections A1–A6 only.

2.   Go through the *hardware simulator tutorial*, parts I, II, and III only.

3.   Build and simulate all the chips specified in the `projects/01` directory.