Problem Set #1

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1. Consider a market in which the goods are homogenous.

   (a) The elasticity of demand, $\varepsilon < 0$, can be written as:

   $$\varepsilon = \left(P'(Q)\right)^{-1} \frac{P(Q)}{Q}$$

   Thus, letting ε remain constant, we can derive:

   $$\begin{aligned} P(Q) &= QP'(Q)\varepsilon \\ P'(Q) &= \left(QP''(Q) + P'(Q)\right)\varepsilon \end{aligned}$$

   Thus, with $P'(Q) < 0$ and $\varepsilon < 0$, $QP''(Q) + P'(Q) > 0$ for all Q.

   (b) Under Cournot competition, each firm, i, solves the following problem:

   $$\max_{q_i} \Pi_i = P(Q)q_i - c(q_i), \qquad Q = \sum_{j=1}^{N} q_j$$

   Which yields the following FOC, which is identical for all firms:

   $$P(Q) + P'(Q)q_i = c'(q_i) \Rightarrow q_i = \left(P'(Q)\right)^{-1}\left(c'(q_i) - P(Q)\right)$$

   Since cost functions are identical by assumption, $q_i = q_j = q \ \forall i, j$ in equilibrium, so we use the implicit function theorem to solve:[1]

   $$\begin{aligned} qP'(Nq) + c'(q) &= P(Nq) \\ \frac{\partial q}{\partial N}\left[P'(Nq) + c''(q)\right] &= \left(q + N\frac{\partial q}{\partial N}\right)\left[P'(Nq) - qP''(Nq)\right] \\ \frac{\partial q}{\partial N} &= \frac{q\left[P'(Nq) - qP''(Nq)\right]}{P'(Nq) + c''(q) + N\left[qP''(Nq) - P'(Nq)\right]} \end{aligned}$$

[1] Due to algebraic errors, I had to redo this several times, spending a long time on it. As a result, many intermediate steps are omitted below.

By assumption (A1), we know $c''(q) \geq P'(Nq)$, and by assumption (A2), we know $-qP''(Nq) \geq P'(Nq)$. Thus, we can reduce the equation as follows:

$$\frac{\partial q}{\partial N} \leq \frac{q^2 P''(Nq)}{c''(q) - NP'(Nq)}$$
$$\frac{\partial q}{\partial N} \leq \frac{q^2 c''(q)}{c''(q) - Nc''(q)}$$
$$\frac{\partial q}{\partial N} \leq \frac{q^2}{1 - N} < 0 \quad \forall N > 1$$

Since demand slopes downward and $Q = Nq$, an increase in q necessarily increases Q, decreasing price. Thus, equilibrium price and price per firm quantity are decreasing in N.

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