Problem Set #2

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Question 1

(a) Using the demand function, (1), we can solve:

$$\frac{dQ}{dP}\frac{P}{Q} = -\frac{1}{a_1}\frac{a_0 - a_1 Q + \nu}{Q} = 1 - \frac{a_0 + \nu}{a_1 Q}$$

Thus, elasticity is increasing in Q and decreasing in ν.

(b) A Cournot equilibrium with homogenous firms is characterized by:

$$\frac{dC}{dq} - \frac{Q}{N}\frac{dP}{dQ} = P$$

Solving for Q^* and P^* with a fixed N and F yields:

$$Q^* = \left(\frac{a_0 - b_0 + \nu - \eta}{2(a_1 - b_1)}\right)N, \qquad P^* = a_0 - \left(\frac{a_0 - b_0 + \nu - \eta}{2(a_1 - b_1)}\right)a_1 N + \nu$$

(c) If firms enter until it is no longer profitable, then we can determine the equilibrium number of firms, N^*, by setting profit, given P^* and Q^*, equal to zero:

$$\begin{aligned} P^*(Q^*/N) &= F + (b_0 + b_1 Q^* + \eta)(Q^*/N) \\ N^* &= \frac{2(a_1 - b_1)}{a_1 + b_1} - \frac{4F(a_1 - b_1)^2}{(a_0 - b_0 + \nu - \eta)^2(a_1 - b_1)} \end{aligned}$$

Letting $b_1 = 0$, the equilibrium value for N reduces to:

$$N^* = 2 - \frac{4Fa_1}{a_0 - b_0 + \nu - \eta}$$

(d) Using the values calculated above, we can calculate the Lerner index, L_I, and Herfindahl index, H, as follows (letting $b_1 = 0$ in the final step):

$$
\begin{aligned}
L_I = -1/\varepsilon = -\left(1 - \frac{a_0 + \nu}{a_1 Q^*}\right)^{-1} &= \frac{a_1 Q^*}{a_0 + \nu - a_1 Q^*} \\
&= \frac{\left(\frac{a_0 - b_0 + \nu - \eta}{2(a_1 - b_1)}\right) a_1 N}{a_0 + \nu - \left(\frac{a_0 - b_0 + \nu - \eta}{2(a_1 - b_1)}\right) a_1 N} \\
&= \frac{(a_0 - b_0 + \nu - \eta) N}{2(a_0 + \nu) - (a_0 - b_0 + \nu - \eta) N} \\
H = \sum_{i=1}^{N} \left(\frac{q^*}{Q^*}\right)^2 &= \sum_{i=1}^{N} \frac{1}{N^2} = \frac{1}{N}
\end{aligned}
$$

(e) Equilibrium elasticity (letting $b_1 = 0$ in the final step) is:

$$
\varepsilon^* = 1 - \frac{2(a_0 + \nu)(a_1 - b_1)}{(a_0 - b_0 + \nu - \eta) a_1 N} = 1 - \frac{2(a_0 + \nu)}{(a_0 - b_0 + \nu - \eta) N}
$$

Thus, we can calculate:

$$
\begin{aligned}
\frac{\partial \varepsilon^*}{\partial F} &= 0 \\
\frac{\partial \varepsilon^*}{\partial \nu} &= \frac{2(b_0 + \eta)}{(a_0 - b_0 + \nu - \eta)^2 N} \\
\frac{\partial \varepsilon^*}{\partial \eta} &= \frac{2(a_0 + \nu)}{(a_0 - b_0 + \nu - \eta)^2 N}
\end{aligned}
$$

Using the equations from (d), we can calculate $\log(L_1)$ and $\log(H)$:

$$
\begin{aligned}
\log(L_1) &= \log(a_0 - b_0 + \nu - \eta) + \log(N) - \log(2(a_0 + \nu) - (a_0 - b_0 + \nu - \eta) N) \\
\log(H) &= -\log(N)
\end{aligned}
$$

Thus, neither index changes with F, and $\log(H)$ does not change with any variable other than N.

(f) If firms collude and split the profits, the new equilibrium will be determined by:

$$
\max_Q \ (a_0 - a_1 Q + \nu) Q - F - (b_0 - b_1 Q + \nu) Q
$$

Which results in the following equilibrium price and quantity:

$$
Q^* = \frac{b_0 - a_0}{2(b_1 - a_1)}, \quad P^* = a_0 - \left(\frac{b_0 - a_0}{b_1 - a_1}\right) \frac{a_1}{2} + \nu
$$

Assuming that the colluding firms split profit equally, we can determine the endogenous number of firms in equilibrium as follows:

$$\begin{aligned}
0 &= \pi(Q^*/N, P^*) \\
FN^2 &= (a_0 - b_0 + \nu - \eta)QN - a_1 Q^2 N + b_1 Q^2 \\
N^* &= \frac{a_1 Q^2 - (a_0 - b_0 + \nu - \eta)Q \pm \sqrt{[-a_1 Q^2 + (a_0 - b_0 + \nu - \eta)Q]^2 + 4Fb_1 Q^2}}{-2F}
\end{aligned}$$

Letting $b_1 = 0$, this problem simplifies nicely, with $N^* = 0$ as one solution and, for the other:

$$N^* = \frac{a_0 - b_0 + \nu - \eta - a_1 Q^2}{F} = \frac{a_0 - b_0 + \nu - \eta}{F} - \frac{(b_0 - a_0)^2}{4Fa_1}$$

The new Lerner index, L_I, under collusion is (letting $b1 = 0$ in the final step):

$$L_I = \frac{a_1 Q^*}{a_0 + \nu - a_1 Q^*} = \frac{a_1 \frac{b_0 - a_0}{2(b_1 - a_1)}}{a_0 + \nu - a_1 \frac{b_0 - a_0}{2(b_1 - a_1)}} = \frac{a_0 - b_0}{a_0 + b_0 + 2\nu}$$

While the Herfindahl index, H, is simply the reciprocal of N^*:

$$H = \frac{F}{a_0 - b_0 + \nu - \eta - \frac{(b_0 - a_0)^2}{4a_1}}$$

(g) The elasticity of (3) is solved as follows:

$$\begin{aligned}
P &= e^{c_0 + \xi} Q^{-c_1} \\
\frac{dQ}{dP} &= -\frac{1}{c_1} \left[e^{c_0 + \xi} Q^{-c_1} \right]^{\frac{-1}{c_1} - 1} e^{\frac{c_0 + \xi}{c_1}} = -\frac{1}{c_1} Q^{1 + c_1} \\
\frac{P}{Q} &= e^{c_0 + \xi} Q^{c_1 - 1} \\
\varepsilon &= \frac{dQ}{dP} \frac{P}{Q} = -\frac{1}{c_1} e^{c_0 + \xi}
\end{aligned}$$

This does not change with Q or ξ. Using the same Cournot equilibrium formula from (a), we can solve for the equilbrium under (3):

$$\begin{aligned}
\frac{dc}{dq} - \frac{Q}{N} \frac{dQ}{dP} &= P \\
\frac{c_1}{N} + \frac{Q}{e^{c_0 + \xi}} \left(b_0 + \eta - 2b_1 \frac{Q}{N} \right) &= 1
\end{aligned}$$

Again letting $b_1 = 0$, we can solve:

$$Q^* = e^{\frac{c_0 - \xi}{c_1}} \left(\frac{N - c_1}{N(b_0 + \eta)} \right)^{\frac{1}{c_1}}, \qquad P^* = \frac{N(b_0 + \eta)}{N - c_1}$$

The Lerner index, L_I, and Herfindahl index, H, for this system are:

$$L_I = -1/\varepsilon = c_1 e^{-c_0-\xi}, \qquad\qquad H = 1/N$$

Equilibrium elasticity does not depend on F or η, but is is decreasing in ξ:

$$\frac{\partial \varepsilon^*}{\partial \xi} = -\frac{1}{c_1} e^{c_0+\xi}$$

The Herfindahl index (and its log) are not changing in F, η, or ξ, as it is only changing in N. However, the log of the Lerner index is decreasing in ξ:

$$\log(L_I) = c_1 - c_0 - \xi, \quad \frac{\partial \log(L_I)}{\partial \xi} = -1$$

Question 2

The table below displays the results from the requested analyses.[1]

	$\beta_{\log(H)}$	$\text{se}\big(\beta_{\log(H)}\big)$	F-score, $\beta_{\log(H)} = 1$	$\Pr\big(\beta_{\log(H)} = 1\big)$	N
Equation (3)					
Cournot	0.002	0.002	334,362	0.000	583
Collusion	-0.005	0.002	210,439	0.000	417
Pooled	-0.001	0.001	566,764	0.000	1000
Equation (1)					
Cournot	-1.985	0.008	128,342	0.000	105
Collusion	-0.006	0.002	204,808	0.000	355
Pooled	0.136	0.028	946	0.000	460

(a)

(b)

(c)

(d)

(e)

(f)

[1] Note that there are fewer than 1000 observations for equation 1's results. This is due to domain restrictions for the observed Lerner index and the equation 1 equilibrium Lerner index, which is undefined if $N = 3$ and negative if $N > 3$.

Question 3

(a)

(b)

(c)