Problem Set #6

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Question 1

The time Bob takes to walk to Happy Cow Farm[1] is given by:

$$T = \frac{1}{5}D_R + \frac{1}{3}D_F$$

Where D_R is the distance Bob travels on the road, and D_F is the distance he travels through the forest. Letting x represent the difference between D_R and the maximim distance Bob would travel on the road, we can solve the problem as:

$$\min_x \frac{1}{5}(12 - x) + \frac{1}{6}\sqrt{25 + x^2}$$

$$\begin{aligned}\frac{dT}{dx} &= 0 \\ \frac{x}{3\sqrt{25+x^2}} &= \frac{1}{5} \\ 25x^2 &= 9(25 + x^2) \\ x^2 &= \frac{225}{16}\end{aligned}$$

Since x cannot be negative, and $\frac{x}{3\sqrt{25+x^2}} - \frac{1}{5}$ is non-decreasing for $x \geq 0$, we know that only $x = \frac{15}{4}$ minimizes T. Thus,

$$T = \frac{1}{5}(12 - \frac{15}{4}) + \frac{1}{6}\sqrt{25 + \left(\frac{15}{4}\right)^2} = \frac{56}{15}$$

Thus, the shortest amount of time it will take Bob to walk to Happy Cow Farm is 224 minutes, or 3 hours and 11 minutes.

[1] Whether "Happy Cow" is an appropriate name for a place that exists primarily to harvest cows is an ethical one and thus beyond the scope of this question.

Question 2

It is not possible for x_0 to be a local optimum of f. Suppose $f'(x_0) = 0$. Then, x_0 is an inflection point of f but not a local optimum.

Proof.

1. Let $x_1 < x_0 < x_2 \in B_\varepsilon(x_0)$. By assumption, $f'(x_1) < 0$ and $f'(x_2) < 0$.
2. Since f is differentiable for all $x \in B_\varepsilon(x_0)$, f is also continuous $\forall x \in B_\varepsilon(x_0)$. Thus, $f(x_1) > f(x_0) > f(x_2)$

$\therefore x_0$ is not a local optimum of f $\blacksquare$

Question 3

By the chain rule, for $\beta \in \{r, s, t\}$, $\frac{\partial w}{\partial \beta} = \sum_{\alpha \in \{x,y,z\}} \frac{\partial w}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial \beta}$. Thus,

$$\frac{\partial w}{\partial r} = (1)y^2z + (2)(2)xyz + (3)xy^2 = y^2z + 4xyz + 3xy^2$$
$$\frac{\partial w}{\partial s} = (2)y^2z + (3)(2)xyz + (1)xy^2 = 2y^2z + 6xyz + xy^2$$
$$\frac{\partial w}{\partial t} = (1)y^2z + (1)(2)xyz + (1)xy^2 = y^2z + 2xyz + xy^2$$

Question 4

Question 5

Question 6