

Problem Set #3

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1 Nevo's Code

Multinomial Logit

The table below displays the estimates for the coefficient on price, α , for each specification. It is generated by the attached code, `edgel_ps3.tex`. Note that there are fewer observations than are in the data provided; this is due to the specification requiring an “outside option”, for which I chose the first brand.

	(1)	(2)	(3)	(4)
	OLS	OLS	IV	IV
α	-30.036 (0.216)	-27.988 (0.918)	-30.071 (0.216)	-39.603 (0.743)
FE?		X		X
R^2	-0.32	0.46	-0.32	0.39
N	2256	2256	2256	2256

I calculate the multi-product Bertrand-Nash markups, $\mu \in \mathbb{R}^{j \times t}$, as

$$\mu = \Omega^{-1} s \mu_j = \frac{s_j}{\partial s_j / \partial \delta_j} = \frac{1}{(1 - s_j) \alpha}$$

Where s is the vector of market shares for each brand in each quarter and city, and

$$\Omega = \Omega^* \odot H, \quad H_{ij} = \frac{\partial s_j}{p_j}$$
$$\Omega_{ij}^* = \begin{cases} 1, & i \text{ and } j \text{ are owned by the same firm} \\ 0, & \text{otherwise} \end{cases}$$

Thus, by decomposing price into marginal cost and markup, we can also back out firm j 's marginal cost, c_j , and calculate its margin, m_j :

$$c_j = p_j - \mu_j, \quad m_j = \frac{p_j}{c_j} - 1$$

Using each $\hat{\alpha}$ from the table above, the mean, median, and standard deviation of markups, margins, and implied marginal costs under each specification are given in the table below.

	(1)	(2)	(3)	(4)
$\mathbb{E}[\mu_{jt}]$	0.004	0.005	0.004	0.003
$\text{Med}(\mu_{jt})$	0.002	0.003	0.002	0.002
$\text{Var}(\mu_{jt})$	0.000	0.000	0.000	0.000
$\mathbb{E}[c_{jt}]$	0.121	0.121	0.121	0.122
$\text{Med}(c_{jt})$	0.119	0.119	0.119	0.120
$\text{Var}(c_{jt})$	0.001	0.001	0.001	0.001
$\mathbb{E}[m_{jt}]$	0.041	0.044	0.041	0.029
$\text{Med}(m_{jt})$	0.018	0.019	0.018	0.014
$\text{Var}(m_{jt})$	0.007	0.009	0.007	0.003

With the pre-merger marginal cost and α estimates, we have $J \times C \times T$ equations and as many unknowns for the post-merger equilibrium using the Bertrand-Nash equilibrium conditions. Following Nevo (2000), the new price vector, p , solves

$$p = \hat{m}c + \Omega_m^{-1}s(p)$$

Where Ω_m is the post-merger Ω matrix. However, the solution does not have a closed form. Instead, we can simulate any merger by:

1. Changing Ω^* according to the new ownership structure
2. Using the pre-merger price vector, p_0 , and its associated shares, s_0 , as an initial guess for the equilibrium
3. Updating prices using the equilibrium condition and estimated values $\hat{m}c$ and $\hat{\alpha}$ with the post-merger ownership matrix, Ω_{post}^* to calculate $p_1 = \hat{m}c + \Omega^{-1}s_0$
4. If $\|p_1 - p_0\|$ is greater than some negligible tolerance value, updating shares and Ω according to p_1 and repeating until convergence to p 's fixed point

The results for post-merger prices and shares are displayed in the table below.

	Post-Nabisco				GM-Quaker			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$\mathbb{E}[p]$	0.124	0.124	0.124	0.124	0.125	0.125	0.125	0.125
$\text{Med}(p)$	0.122	0.122	0.122	0.122	0.123	0.123	0.123	0.123
$\text{Var}(p)$	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
$\mathbb{E}[s]$	0.029	0.020	0.019	0.017	0.030	0.021	0.019	0.016
$\text{Med}(s)$	0.013	0.011	0.011	0.010	0.016	0.012	0.011	0.009
$\text{Var}(s)$	0.002	0.001	0.001	0.001	0.002	0.001	0.001	0.001

The problem with this analysis is that we assume that any heterogeneity in consumer preferences and price sensitivity is independent, identically distributed, and orthogonal to utility. These assumptions are dubious and likely result in unrealistic estimations of counter-factuals. We can remedy these issues with a random coefficients model that interacts consumer demographics with prices and product characteristics, which will capture the observable heterogeneity in consumer preferences and price sensitivity.

Random Coefficients

The results of the random coefficients estimation are provided below and are consistent with those in results.txt.

	Mean	Σ	Income	Income ²	Age	$\mathbb{1}\{\text{Child}\}$
Constant	-2.314 (0.246)	0.377 (0.122)	3.089 (1.007)	0.000 (0.000)	1.186 (1.010)	0.000 (0.000)
Price	-32.430 (4.826)	1.848 (0.978)	16.598 (104.545)	-0.659 (5.457)	0.000 (0.000)	11.624 (5.137)
Sugar	0.117 (0.246)	-0.003 (0.012)	-0.193 (0.040)	0.000 (0.000)	0.030 (0.034)	0.000 (0.000)
Mushy	0.271 (0.012)	0.081 (0.198)	1.468 (0.676)	0.000 (0.000)	-1.514 (1.095)	0.000 (0.000)

GMM objective: 14.998

MD R^2 : 0.395

MD weighted R^2 : 0.077

Using this demand specification, the new estimates for markups and marginal costs are summarized below.

	Mean	Median	Variance
Markups	-0.043	-0.042	0.000
MC	0.168	0.166	0.001

I do not have the time to repeat the merger simulations with these results.

2 pyBLP

The screenshot below displays the random coefficients estimates using the py-BLP package.

```

Problem Results Summary:
=====
GW Step   Objective      Gradient      Hessian      Hessian      Clipped      Weighting Matrix  Covariance Matrix
      Value      Norm      Min Eigenvalue  Max Eigenvalue  Shares      Condition Number  Condition Number
-----
1      +3.791626E+00  +7.734942E-06  +1.376600E-05  +1.484249E+04    0      +5.455147E+07      +1.606032E+09
=====

Cumulative Statistics:
=====
Computation Time  Optimizer  Optimization  Objective  Fixed Point  Contraction
                  Converged  Iterations  Evaluations  Iterations  Evaluations
-----
00:00:39          Yes          55          61          50760        157432
=====

Nonlinear Coefficient Estimates (Robust SEs in Parentheses):
=====
Sigma:      1      prices      sugar      mushy      Pi:      income      income_squared      age      child
-----
1      +6.993214E-01      +4.390604E+00      -1.744827E-02      +1.517420E-01      1      +2.065387E+00      +0.000000E+00      +1.422622E+00      +0.000000E+00
      (+2.378130E-01)      (+2.127527E+00)      (+1.642874E-02)      (+2.113733E-01)      (+1.705928E+00)      (+2.306151E+01)      (+6.910632E-01)      (+4.543802E+00)
prices      +0.000000E+00      +0.000000E+00      +0.000000E+00      +0.000000E+00      prices      +8.542775E+02      -4.389096E+01      +0.000000E+00      +1.146863E+01
      (+0.000000E+00)      (+0.000000E+00)      (+0.000000E+00)      (+0.000000E+00)      (+4.402449E+02)      (+2.306151E+01)      (+3.951282E-02)      (+4.543802E+00)
sugar      +0.000000E+00      +0.000000E+00      +0.000000E+00      +0.000000E+00      sugar      -4.627917E-01      +0.000000E+00      +5.620248E-02      +0.000000E+00
      (+0.000000E+00)      (+0.000000E+00)      (+0.000000E+00)      (+0.000000E+00)      (+1.889797E-01)      (+0.000000E+00)      (+3.951282E-02)      (+0.000000E+00)
mushy      +0.000000E+00      +0.000000E+00      +0.000000E+00      +0.000000E+00      mushy      +2.513253E-01      +0.000000E+00      -1.319953E+00      +0.000000E+00
      (+0.000000E+00)      (+0.000000E+00)      (+0.000000E+00)      (+0.000000E+00)      (+1.206436E+00)      (+0.000000E+00)      (+7.404005E-01)      (+0.000000E+00)
=====

Beta Estimates (Robust SEs in Parentheses):
=====
prices
-----
-7.862774E+01
(+2.379252E+01)
=====

```

The markup and marginal cost estimates are displayed below, confirming that my estimates from the Matlab code are far off.

	Mean	Median	Variance
Markups	0.372	0.345	0.018
MC	0.081	0.080	0.001

Notably, these markup estimates are far higher than those derived from the logit regression. Unfortunately, I do not have the time to repeat the merger simulations with these results.