Problem Set #1

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Question 1

The social planner's problem is:

$$\max_{\{I_t, C_t, K_{t+1}\}_{t=0}^{\infty}} \beta^t U(C_t) \text{ s.t. } K_{t+1} = (1-\delta)K_t + I_t - D_t,\ C_t + I_t \leq F(K_t)$$

At any optimum, the resource constraint will hold with equality, so we can combine the law of motion and the resource constraint to obtain a single constraint:

$$F(K_t) = K_{t+1} - (1-\delta)K_t + C_t + D_t$$

We can derive the Euler equation by taking the first-order conditions of the Lagrangian function.:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t U(C_t) - \lambda_t \left(F(K_t) - K_{t+1} + (1-\delta)K_t - C_t - D_t \right)$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t U'(C_t) + \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \lambda_t - \lambda_{t+1} \left(F'(K_{t+1}) + 1 - \delta \right) = 0$$

Taken together, these first-order conditions give us the Euler equation for the SPP:

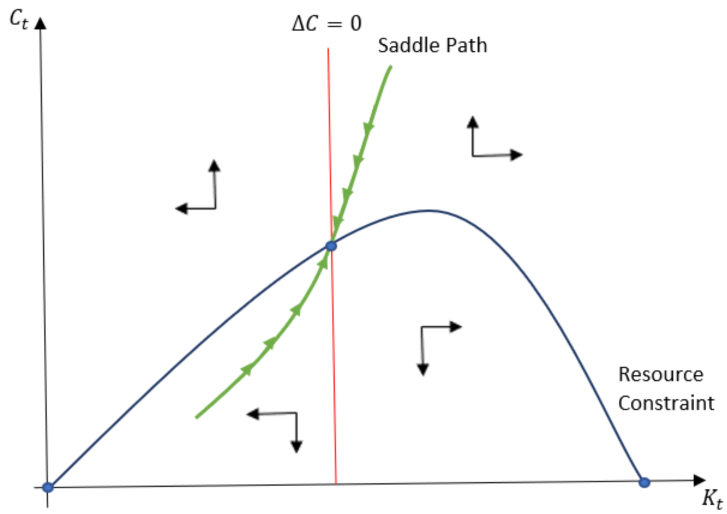
$$U'(C_t) = \beta U'(C_{t+1}) \left[F(K_{t+1}) + 1 - \delta \right]$$

Question 2

Given the steady-state value D, the steady-state values of capital, $K_t = K_{t+1} = \overline{K}(D)$, and consumption, $C_t = C_{t+1} = \overline{C}(D)$ are detemined by the intersection of the resource constraint and Euler equation when capital and consumption are held constant:

$$
\begin{aligned}
U'(\overline{C}) = \beta U'(\overline{C}) \left[F(\overline{K}) + 1 - \delta\right] &\Rightarrow \beta \left[F(\overline{K}) + 1 - \delta\right] = 1 \\
F(\overline{K}) = \overline{K} - (1-\delta)\overline{K} + \overline{C} + D &\Rightarrow F(\overline{K}) = \delta\overline{K} + \overline{C} + D
\end{aligned}
$$

The phase diagram for this steady-state is displayed below, with the three steady states marked by blue dots.



Question 3

Question 4