Econ 712 Macroeconomics I

Fall 2020, University of Wisconsin-Madison

Instructor: Dean Corbae, TAs: Duong Dang and Katya Kazakova

Problem Set #7 - Due 10/22/20

1 Overlapping generations with housing

Consider the following 2 period OG model. Agents earn y when young and 0 when old. There is a fixed supply of housing $H^s = 1$. Agents utility function is given by

$$U\left(c_t^t, h_t, c_{t+1}^t\right) = \ln\left(c_t^t\right) + \alpha h_t + \beta c_{t+1}^t$$

where c_t^t is the period t consumption, h_t is the period t housing choice, and c_{t+1}^t is the period $t+1$ consumption of a person born in period t. The initial old hold the stock of housing. Assume that $1 + \alpha > \beta y$.

1. Write down and solve the planner's problem.
2. If p_t is the period t price of a house, solve for a competitive equilibrium with housing in the following parts:
   (a) What is the optimization problem facing a young agent?
   (b) What are the market clearing conditions?
   (c) Define a competitive general equilibrium
   (d) Solve for an agent's optimal housing and consumption decision rules. How does housing depend on the current and future price of houses? Note: if you choose to solve the problem without imposing non-negativity constraints, you should verify the conditions under which consumption and housing choices are non-negative.
   (e) Solve for the law of motion for house prices in equilibrium and graph it in (p_t, p_{t+1}) space. Assume that $p_t < y$.
   (f) Solve for a steady state house price level.
   (g) Does the competitive equilibrium implement the planner's allocation in a steady state?