Problem Set #7

Danny Edgel
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Collaborated with Sarah Bass, Emily Case, Michael Nattinger, and Alex Von Hafften

Question 1

Let $X \subset \mathbb{R}^n$ be convex. We can prove that, for any $k \in \mathbb{N}$, $\lambda_1, ..., \lambda_k \geq 0$, $\sum_{i=1}^k \lambda_i = 1$, if $x_1, ..., x_k \in X$, then $\sum_{i=1}^k \lambda_i x_1 \in X$.

Proof.

1. *Base step.* Suppose $x_1, x_2 \in X$. Since X is convex, $(1-\lambda)x_1 + \lambda x_2$ is also in X for all $\lambda \in [0, 1]$

2. *Induction Step.* Assume that, for some $k \in \mathbb{N}$, $\sum_{i=1}^k \lambda_i x_i \in X$, where $\sum_{i=1}^k = 1$. Let $x_{k+1} \in X$ and $\lambda' \in [0, 1]$. Then, since X is convex,

$$(1-\lambda')x_{k+1} + \lambda' \sum_{i=1} \lambda_i x_i$$

is also in X. Now, define

$$\lambda'_i = \begin{cases} \lambda' \lambda_i, & i \in \{1, ..., k\} \\ 1-\lambda', & i = k+1 \end{cases}$$

Then, $\sum_{i=1}^{k+1} \lambda'_i x_i \in X$ and $\sum_{i=1}^{k+1} \lambda'_i = 1$

$\therefore \sum_{i=1}^k \lambda_i x_i \in X$ for any $k \in \mathbb{N}$ ■