Problem Set #4

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Discussed and/or compared answers with Sarah Bass, Emily Case, Katherine Kwok, Michael Nattinger, and Alex Von Hafften

Question 1

To set up the Ramsey problem, we must first solve for the resource constraint and implementability contraint of this economy. The resource constraint is simply:

$$c_t + k_{t+1} = F(k_t, 1 - l_t) + (1 - \delta)k_t$$

We can derive the implementability constraint by solving the household problem:

$$\max_{c_t, l_t, \tau_t, k_t} \sum_{t=1}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \nu(l_t) \right] \text{ s.t. } (1-\tau)c_t + k_{t+1} = w_t(1-l_t) + (1-\delta+r_t)k_t$$

Question 2

1. A competitive equilibrium is a policy, (M_t, B_t); allocation, (c_{1t}, c_{2t}, n_t); and price system, (p_t, w_t, R_t), such that:
   (a) Given the policy and price system, the allocation solves the household problem
   (b) The allocation satisfies the government budget constraint

2. The first order conditions of the household problem are:

$$\frac{\beta^t}{c_{1t}} - \lambda_{t+1}p_t - \mu_t p_t = 0 \tag{c_{1t}}$$
$$\frac{\alpha\beta^t}{c_{2t}} - \lambda_{t+1}p_t = 0 \tag{c_{2t}}$$
$$-\frac{\gamma\beta^t}{1-n_t} + \lambda_{t+1}w_t = 0 \tag{n_t}$$
$$-\lambda_t + \lambda_{t+1}R = 0 \tag{B_t}$$
$$-\lambda_t + \lambda_{t+1} + \mu_t = 0 \tag{M_t}$$

From the FOCs for M_t, B_t, and c_{1t}, we can solve:

$$\begin{aligned} &\frac{\lambda_t}{\lambda_{t+1}} = 1 + \frac{\mu_t}{\lambda_{t+1}} \\ \Rightarrow &R_t = \frac{\lambda_t}{\lambda_{t+1}} = 1 + \frac{\mu_t}{\lambda_{t+1}} \\ \Rightarrow &\frac{\beta^t}{\lambda_{t+1}c_{1t}} = p_t\left(1 + \frac{\mu_t}{\lambda_{t+1}}\right) \\ \Rightarrow &\frac{\beta^t}{c_{1t}} = \lambda_{t+1}p_t R \end{aligned}$$

Combining this with the FOC for c_{2t} gives us:

$$\frac{c_{2t}}{\alpha c_{1t}} = R$$

Combining the FOCs for c_{2t} and n_t yields:

$$\frac{\gamma}{\alpha}\left(\frac{c_{2t}}{1-n_t}\right) = \frac{w_t}{p_t}$$

Since production is linear in labor, $w_t = p_t$, so the righthand side of the equation becomes 1. To observe the relationship between n_t and R, combine our two optimization conditions with the resource constraint to solve for n_t as a function of R:

$$\begin{aligned}
c_{1t} &= \frac{c_{2t}}{\alpha R} \\
c_{1t} + c_{2t} &= n_t \\
c_{2t} &= \frac{\alpha R n_t}{1+\alpha R} \\
\frac{\gamma}{\alpha}\left[\frac{\frac{n_t}{1+\frac{1}{\alpha R}}}{1-n_t}\right] &= 1 \\
\gamma n_t &= \left(\alpha + \frac{1}{R}\right)(1-n_t) \\
n_t &= \frac{1+\alpha R}{1+(\gamma+\alpha)R}
\end{aligned}$$

Taking the derivative yields:

$$\frac{dn_t}{dR} = -\frac{\gamma}{\left[1+(\gamma+\alpha)R\right]^2} < 0$$

Thus, labor decreases when R increases.