Problem Set #10

Danny Edgel
Econ 710: Economic Statistics and Econometrics II
Spring 2021

April 7, 2021

Discussed and/or compared answers with Sarah Bass, Emily Case, Katherine Kwok, Michael Nattinger, and Alex Von Hafften

Exercise 22.1

(a) The conditional CDF of Y is $Pr(Y \leq y | X = x)$. Given our model, we can solve:

$$Pr(Y \leq y|X = x) = Pr(x'\theta + e \leq y|X = x) = Pr(e \leq y - x'\theta | X = x) = F(y - x'\theta)$$

(b) Since the distribution of Y is known, we can solve for θ with MLE. Thus,

$$\rho(Y, X, \theta) = -\log\left(f(Y|X, \theta)\right) = -\log\left(f(y - x'\theta)\right)$$

$$\phi(Y, X\theta) = \frac{\partial}{\partial\theta}\rho(Y, X, \theta) = -\left(\frac{f'(y - x'\theta)}{f(y - x'\theta)}\right)x$$

(c) The asymptotic distribution of $\hat{\theta}$ is given by:

$$\sqrt{n}(\hat{\theta} - \theta_0) \to_d \mathcal{N}\left(0, Q^{-1}\Omega Q^{-1}\right)$$

Where:

$$Q = \mathbb{E}\left[\frac{\partial^2}{\partial\theta\partial\theta'}\rho_i(\theta)\right] = \mathbb{E}\left[\left(\frac{f'(e_i)^2}{f(e_i)^2}\right)x_i x_i'\right]$$

$$\Omega = \mathbb{E}\left[\phi_i\phi_i'\right] = \mathbb{E}\left[\left(\frac{\partial}{\partial\theta}\rho_i\right)\left(\frac{\partial}{\partial\theta'}\rho_i'\right)\right] = \mathbb{E}\left[\frac{\partial^2}{\partial\theta\partial\theta'}\rho_i\right] = Q$$

$$\Rightarrow V = Q^{-1}\Omega Q^{-1} = Q^{-1} = \mathbb{E}\left[\left(\frac{f'(e_i)^2}{f(e_i)^2}\right)^{-1}(x_i x_i')^{-1}\right]$$

Exercise 23.1

(a) The conditional mean is not linear in θ, since $\mathbb{E}[Y] = \exp(\theta)$. Thus, this is a nonlinear regression model.

(b) Yes. We can run OLS on the model as-is, treating $\exp(\theta)$ as our parameter of interest, then transform it post-estimation by taking logs.

(c) My answer to part (b) is exactly non-linear least squares, with $m(\theta) = \exp(\theta)$.

Exercise 23.2

Exercise 23.7

Exercise 23.8

Exercise 24.3

Exercise 24.4

Exercise 24.5

Exercise 24.14