Problem Set #2

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Question 1

The social planner in this problem seeks to maximize utility subject to the production function, resource constraint, and law of motion:

$$\max_{\{C_t,K_t,I_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(C_t), \text{ s.t. } Y_t = AK_t^{\alpha},\ K_{t+1} = K_t^{1-\delta} I_t^{\delta},\ Y_t = C_t + I_t$$

Combining the production function, resource constraint, and law of motion gives the following Lagrangian function:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \log(C_t) - \lambda_t \left(K_{t+1} - K_t^{1-\delta}(AK_t^{\alpha} - C_t)^{\delta}\right)$$

Which has the following first-order conditions:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= \frac{\beta^t}{C_t} - \lambda_t \delta \left(AK_t^{\alpha} - C_t\right)^{\delta-1} = 0 \\
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= -\lambda_t + \lambda_{t+1} K_t^{-\delta} (AK_t^{\alpha} - C_t)^{\delta} \left[1 - \delta + \alpha\delta AK_t^{\alpha}(AK_t^{\alpha} - C_t)^{-1}\right] = 0 \\
\Rightarrow \frac{C_{t+1}}{C_t} &= \beta K_t^{\delta} \left[\frac{(AK_{t+1}^{\alpha} - C_{t+1})^{1-\delta}}{AK_t^{\alpha} - C_t}\right] \left(1 - \delta + \frac{\alpha\delta AK_t^{\alpha}}{AK_t^{\alpha} - C_t}\right)^{-1}
\end{aligned}$$

Which simplifies to the following Euler equation:

$$\frac{C_{t+1}}{C_t} = \beta K_t^{\delta} \left[\frac{(AK_{t+1}^{\alpha} - C_{t+1})^{1-\delta}}{(1-\delta)(AK_t^{\alpha} - C_t) + \alpha\delta AK_t^{\alpha}}\right] = \beta K_t^{\delta} \left[\frac{(Y_{t+1} - C_{t+1})^{1-\delta}}{(1-\delta)(Y_t - C_t) + \alpha\delta Y_t}\right]$$

Question 2

Question 3

Question 4

Question 5

Question 6

Question 7

Question 8