Problem Set #6

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Discussed and/or compared answers with Sarah Bass, Emily Case, Katherine Kwok, Michael Nattinger, and Alex Von Hafften

Question 1

(i) The direct representation of the sample average is:

$$\mu_0 = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{T_i}\sum_{t=1}^{T_i} Y_{it}$$

Since 1_i contains T_i elements, $1_i'1_i = T_i$, and $1_i'Y_i = \sum_{t=1}^{T_i} Y_i$. It is clear, then, that

$$\mathbb{E}\left[\hat{\mu}_{OLS}\right] = \frac{\sum_{i=1}^{n} 1_i'Y_i}{\sum_{i=1}^{n} 1_i'1_i}$$

(ii) We can solve for the variance of $\hat{\mu}_{IV}$ as follows:

$$Var(\hat{\mu}_{IV}) = Var\left(\frac{\sum_{i=1}^{n} Z_i'Y_i}{\sum_{i=1}^{n} Z_i'1_i}\right) = \frac{\sum_{i=1}^{n} Z_i'Var(Y_i)Z_i}{\left(\sum_{i=1}^{n} Z_i'1_i\right)^2}$$

Where:

$$Var(Y_i) = Var(\mu_0 + \alpha_i + \varepsilon_{it}) = Var(\alpha_i) + Var(\varepsilon_{it}) + 2Cov(\alpha_i, \varepsilon_{it})$$
$$\Rightarrow \Omega_i = \sigma_\alpha^2 1_i 1_i' + \sigma^2 I_{T_i}$$

(iii) To determine how we can show that $Var(\hat{\mu}_{IV} \geq \left(\sum_{i=1}^{n} 1_i'\Omega_i^{-1}1_i\right)^{-1}$, we simply need to find that the following inequality holds:

$$\left(\sum_{i=1}^{n} Z_i'1_i\right)^2 \leq \left(\sum_{i=1}^{n} Z_i'\Omega_i Z_i\right)\left(\sum_{i=1}^{n} 1_i'\Omega_i^{-1}1_i\right)$$

Once this inequality is established, it follows that:

$$Var(\hat{\mu}_{IV}) = \frac{\sum_{i=1}^{n} Z_i' Var(Y_i) Z_i}{\left(\sum_{i=1}^{n} Z_i' 1_i\right)^2} \geq \frac{\sum_{i=1}^{n} Z_i' Var(Y_i) Z_i}{\left(\sum_{i=1}^{n} Z_i' \Omega_i Z_i\right)\left(\sum_{i=1}^{n} 1_i' \Omega_i^{-1} 1_i\right)} = \left(\sum_{i=1}^{n} 1_i' \Omega_i^{-1} 1_i\right)^{-1}$$

We can establish the inquality using the Cauchy-Schwarz inequality:

$$\left(\sum_{i=1}^{n} Z_i' 1_i\right)^2 = \left(\sum_{i=1}^{n} Z_i' \Omega^{1/2} \Omega^{-1/2} 1_i\right)^2 \leq \left(\sum_{i=1}^{n} Z_i' \Omega_i Z_i\right)\left(\sum_{i=1}^{n} 1_i' \Omega_i^{-1} 1_i\right)$$

This variance is achieved by $\overline{Z}_i = \Omega_i^{-1} 1_i$

(iv)

(v)

(vi)

(vii)

Question 2

(i)

(ii)

Question 3

(i)

(ii)

(iii)