Problem Set #2

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Question 1

(a) Using the demand function, (1), we can solve:

$$\frac{dQ}{dP}\frac{P}{Q} = -\frac{1}{a_1}\frac{a_0 - a_1 Q + \nu}{Q} = 1 - \frac{a_0 + \nu}{a_1 Q}$$

Thus, elasticity is increasing in Q and decreasing in ν.

(b) A Cournot equilibrium with homogenous firms is characterized by:

$$\frac{dC}{dq} - \frac{Q}{N}\frac{dP}{dQ} = P$$

Solving for Q^* and P^* with a fixed N and F yields:

$$Q^* = \left(\frac{a_0 - b_0 + \nu - \eta}{2(a_1 - b_1)}\right) N, \qquad P^* = a_0 - \left(\frac{a_0 - b_0 + \nu - \eta}{2(a_1 - b_1)}\right) a_1 N + \nu$$

(c) If firms enter until it is no longer profitable, then we can determine the equilibrium number of firms, N^*, by setting profit, given P^* and Q^*, equal to zero:

$$P^*(Q^*/N) = F + (b_0 + b_1 Q^* + \eta)(Q^*/N)$$
$$N^* = \frac{2(a_1 - b_1)}{a_1 + b_1} - \frac{4F(a_1 - b_1)^2}{(a_0 - b_0 + \nu - \eta)^2 (a_1 - b_1)}$$

Letting $b_1 = 0$, the equilibrium value for N reduces to:

$$N^* = 2 - \frac{4Fa_1}{a_0 - b_0 + \nu - \eta}$$

(d) Using the values calculated above, we can calculate the Lerner index, L_I, and Herfindahl index, H, as follows (letting $b_1 = 0$ in the final step):

$$
\begin{aligned}
L_I &= -1/\varepsilon = -\left(1 - \frac{a_0 + \nu}{a_1 Q^*}\right)^{-1} = \frac{a_1 Q^*}{a_0 + \nu - a_1 Q^*} \\
&= \frac{\left(\frac{a_0 - b_0 + \nu - \eta}{2(a_1 - b_1)}\right) a_1 N}{a_0 + \nu - \left(\frac{a_0 - b_0 + \nu - \eta}{2(a_1 - b_1)}\right) a_1 N} \\
&= \frac{(a_0 - b_0 + \nu - \eta)N}{2(a_0 + \nu) - (a_0 - b_0 + \nu - \eta)N} \\
H &= \sum_{i=1}^{N} \left(\frac{q^*}{Q^*}\right)^2 = \sum_{i=1}^{N} \frac{1}{N^2} = \frac{1}{N}
\end{aligned}
$$

(e) Equilibrium elasticity (letting $b_1 = 0$ in the final step) is:

$$
\varepsilon^* = 1 - \frac{2(a_0 + \nu)(a_1 - b_1)}{(a_0 - b_0 + \nu - \eta)a_1 N} = 1 - \frac{2(a_0 + \nu)}{(a_0 - b_0 + \nu - \eta)N}
$$

Thus, we can calculate:

$$
\begin{aligned}
\frac{\partial \varepsilon^*}{\partial F} &= 0 \\
\frac{\partial \varepsilon^*}{\partial \nu} &= \frac{2(b_0 + \eta)}{(a_0 - b_0 + \nu - \eta)^2 N} \\
\frac{\partial \varepsilon^*}{\partial \eta} &= \frac{2(a_0 + \nu)}{(a_0 - b_0 + \nu - \eta)^2 N}
\end{aligned}
$$

Using the equations from (d), we can calculate $\log(L_1)$ and $\log(H)$:

$$
\begin{aligned}
\log(L_1) &= \log(a_0 - b_0 + \nu - \eta) + \log(N) - \log(2(a_0 + \nu) - (a_0 - b_0 + \nu - \eta)N) \\
\log(H) &= -\log(N)
\end{aligned}
$$

Thus, neither index changes with F, and $\log(H)$ does not change with any variable other than N.

(f) If firms collude and split the profits, the new equilibrium will be determined by:

$$
\max_{Q}\ (a_0 - a_1 Q + \nu)Q - F - (b_0 - b_1 Q + \nu)Q
$$

Which results in the following equilibrium price and quantity:

$$
Q^* = \frac{b_0 - a_0}{2(b_1 - a_1)}, \quad P^* = a_0 - \left(\frac{b_0 - a_0}{b_1 - a_1}\right) \frac{a_1}{2} + \nu
$$

Assuming that the colluding firms split profit equally, we can determine the endogenous number of firms in equilibrium as follows:

$$\begin{aligned}
0 &= \pi(Q^*/N, P^*) \\
FN^2 &= (a_0 - b_0 + \nu - \eta)QN - a_1 Q^2 N + b_1 Q^2 \\
N^* &= \frac{a_1 Q^2 - (a_0 - b_0 + \nu - \eta)Q \pm \sqrt{[-a_1 Q^2 + (a_0 - b_0 + \nu - \eta)Q]^2 + 4F b_1 Q^2}}{-2F}
\end{aligned}$$

Letting $b_1 = 0$, this problem simplifies nicely, with $N^* = 0$ as one solution and, for the other:

$$N^* = \frac{a_0 - b_0 + \nu - \eta - a_1 Q^2}{F} = \frac{a_0 - b_0 + \nu - \eta}{F} - \frac{(b_0 - a_0)^2}{4Fa_1}$$

(g) The elasticity of (3) is solved as follows:

$$\begin{aligned}
P &= e^{c_0 + \xi} Q^{-c_1} \\
\frac{dQ}{dP} &= -\frac{1}{c_1}\left[e^{c_0+\xi}Q^{-c_1}\right]^{\frac{-1}{c_1}-1} e^{\frac{c_0+\xi}{c_1}} = -\frac{1}{c_1}Q^{1+c_1} \\
\frac{P}{Q} &= e^{c_0+\xi}Q^{c_1 - 1} \\
\varepsilon &= \frac{dQ}{dP}\frac{P}{Q} = -\frac{1}{c_1}e^{c_0+\xi}
\end{aligned}$$

This does not change with Q or ξ. Using the same Cournot equilibrium formula from (a), we can solve for the equilbrium under (3):

$$\begin{aligned}
\frac{dc}{dq} - \frac{Q}{N}\frac{dQ}{dP} &= P \\
\frac{c_1}{N} + \frac{Q}{e^{c_0+\xi}}\left(b_0 + \eta - 2b_1\frac{Q}{N}\right) &= 1
\end{aligned}$$

Again letting $b_1 = 0$, we can solve:

$$Q^* = e^{\frac{c_0 - \xi}{c_1}}\left(\frac{N - c_1}{N(b_0+\eta)}\right)^{\frac{1}{c_1}}, \qquad P^* = \frac{N(b_0+\eta)}{N - c_1}$$

The Lerner index, L_I, and Herfindahl index, H, for this system are:

$$L_I = -1/\varepsilon = c_1 e^{-c_0 - \xi}, \qquad H = 1/N$$

Equilibrium elasticity does not depend on F or η, but is is decreasing in ξ:

$$\frac{\partial \varepsilon^*}{\partial \xi} = -\frac{1}{c_1}e^{c_0+\xi}$$

The Herfindahl index (and its log) are not changing in F, η, or ξ, as it is only changing in N. However, the log of the Lerner index is decreasing in ξ:

$$\log(L_I) = c_1 - c_0 - \xi, \quad \frac{\partial \log(L_I)}{\partial \xi} = -1$$

Question 2

(a)

(b)

(c)

(d)

(e)

(f)

Question 3

(a)

(b)

(c)