Homework #1

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Question 1

Suppose $t_i \in S_i$ is strictly dominated by $s_i \in S_i$, but that $\sigma_i \in \Delta S_i$, which is supported by t_i is not strictly dominated. Let $\sigma_i' \in \Delta S_i$ be a mixed strategy that has the same support as σ_i, but with s_i played played with the same frequency as t_i instead of t_i. Since s_i strictly dominates t_i, this strategy results in a strictly higher payoff than σ_i. Therefore, σ_i' strictly dominates σ_i.

$\therefore$ by contradiction, any mixed strategy that contains a strictly-dominated pure strategy in its support is strictly dominated $\blacksquare$

Question 2

(a) This scenario is a game with two players ($N = \{1, 2\}$) with identical strategy sets $S_i = \{2, 3, ..., 499, 500\}$, $i = 1, 2$, and payoff functions:

$$u_i(s_i, s_j) = \begin{cases} s_i + 2, & s_i < s_j \\ s_i, & s_i = s_j \\ s_j - 2, & s_i > s_j \end{cases}, \; i \in \{1, 2\}, \; j \neq i$$

(b) Player 1's payoff maximization problem is

$$\max_{s_1} u_1(s_1, s_2)$$

Where player 1's payoff matrix is:

	$s_2 < \overline{s}_2$	$s_2 = \overline{s}_2$
$s_1 < \overline{s}_2$	$[s_2 - 2, s_1 + 2]$	$s_1 + 2$
$s_1 = \overline{s}_2$	$s_2 - 2$	s_1
$s_1 > \overline{s}_2$	$s_2 - 2$	$s_2 - 2$

Thus, if $s_2 = \overline{s}_2$, then player 1's best response is clearly less than $\overline{s}$, and if $s_2 < \overline{s}_2$, then choosing $s_1 < \overline{s}_2$ will, at worst, make player 1 as poor-off as if they chose $s_1 \geq \overline{s}_2$ but will possibly make them better-off. Thus, player 1's best response is $s_1 < \overline{s}_2$.

(c) Player 2 faces the same best response function that player 1 does. If you begin with the presumption that player 1 believes $\overline{s}_2 \in [2, 500]$ and iteratively remove strictly dominated strategies for each player, then you arrive at $s_1 = s_2 = 2$ regardless of your initial choice of $\overline{s}_2$.

Question 3

(a) No pure strategies in this game dominate any other pure strategies. However, player 2's pure strategy of C is dominated by a mixed strategy of L and R. Specifically, any strategy $\sigma_2 = \pi L + (1-\pi)R$, where $\pi \in (\frac{1}{3}, \frac{1}{2})$ dominates C.

(b) If there is common knowledge of rationality between the players, then no player will play a strictly dominated player, and each player will choose a strategy as if the other player will not play a strictly dominated strategy. Thus, player 2 will choose π such that player 1 is indifferent between T and B. Player 1's payoff, based on pi is:

$$u_1(T) = 8(1-\pi),\ u_1(B) = 2\pi + (1-\pi)5$$

Player 1 is indifferent between T and B when $\pi = \frac{3}{5}$. Meanwhile, player 2 is indifferent between L and R when player 1 plays T with probability $\frac{8}{11}$. Thus, our prediction of play in this game is

$$\left(\frac{8}{11}T + \frac{3}{11}B, \frac{3}{5}L + \frac{2}{5}R\right)$$

Question 4

All pure strategies in this game are rationalizable because, for each player, each pure strategy is a best response to one of the other player's actions. Thus, neither player can eliminate a pure strategy by playing a mixed strategy that doesn't include it. Under complete knowledge of rationality, then, this game will result in a mixed strategy Nash equilibrium that includes all possible moves for both players.

Question 5

(a) Each player, $i \in \{1,2\}$, in this game has strategy set $S_i = \{\text{seek}, \text{don't seek}\}$, each with the payoff function (where $j \neq i$,

$$u_i(s_i, s_j) = \begin{cases} 0, & s_i = \text{ don't seek} \\ r - c, & s_i = s_j = \text{ seek} \\ R - c, & s_i = \text{, seek, } s_j = \text{ don't seek} \end{cases}$$

Which results in the following payoff matrix:

		2: Seek	2: Don't Seek
1	Seek	$(r-c, r-c)$	$(R-c, 0)$
1	Don't Seek	$(0, R-c)$	$(0,0)$

(b) If $r > c$, then seeking approval will be a strictly dominant strategy for each firm. If $r < c < R$, then there will be a mixed-strategy Nash equilibrium, in which each firm seeks approval some of the time. If $R < c$, then not seeking approval is the clearly dominant strategy for both players, resulting in a pure strategy Nash equilibrium at $(\text{don't seek}, \text{don't seek})$. However, if $c < r < R$, then both players are always strictly better off when they seek approval, regardless of the other players decision, resulting in a pure strategy Nash equilibrium of $(\text{seek}, \text{seek})$.