Homework #1

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Question 1

Suppose $t_i \in S_i$ is strictly dominated by $s_i \in S_i$, but that $\sigma_i \in \Delta S_i$, which is supported by t_i is not strictly dominated. Let $\sigma_i' \in \Delta S_i$ be a mixed strategy that has the same support as σ_i, but with s_i played played with the same frequency as t_i instead of t_i. Since s_i strictly dominates t_i, this strategy results in a strictly higher payoff than σ_i. Therefore, σ_i' strictly dominates σ_i.

$\therefore$ by contradiction, any mixed strategy that contains a strictly-dominated pure strategy in its support is strictly dominated ■

Question 2

(a) This scenario is a game with two players ($N = \{1, 2\}$) with identical strategy sets $S_i = \{2, 3, ..., 499, 500\}$, $i = 1, 2$, and payoff functions:

$$u_i(s_i, s_j) = \begin{cases} s_i + 2, & s_i < s_j \\ s_i, & s_i = s_j \\ s_j - 2, & s_i > s_j \end{cases}, \; i \in \{1, 2\}, \; j \neq i$$

(b) Player 1's payoff maximization problem is

$$\max_{s_1} u_1(s_1, s_2)$$

Where player 1's payoff matrix is:

	$s_2 < \overline{s}_2$	$s_2 = \overline{s}_2$
$s_1 < \overline{s}_2$	$[s_2 - 2, s_1 + 2]$	$s_1 + 2$
$s_1 = \overline{s}_2$	$s_2 - 2$	s_1
$s_1 > \overline{s}_2$	$s_2 - 2$	$s_2 - 2$

Thus, if $s_2 = \overline{s}_2$, then player 1's best response is clearly less than $\overline{s}$, and if $s_2 < \overline{s}_2$, then choosing $s_1 < \overline{s}_2$ will, at worst, make player 1 as poor-off as if they chose $s_1 \geq \overline{s}_2$ but will possibly make them better-off. Thus, player 1's best response is $s_1 < \overline{s}_2$.

(c) Player 2 faces the same best response function that player 1 does. If you begin with the presumption that player 1 believes $\overline{s}_2 \in [2, 500]$ and iteratively remove strictly dominated strategies for each player, then you arrive at $s_1 = s_2 = 2$ regardless of your initial choice of $\overline{s}_2$.