Problem Set #4

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Question 1

(a) The table below displays the coefficient estimates, alongside robust standard errors.

VARIABLES	(1) log(wage)
Education	0.144***
	(0.0118)
Experience	0.0426***
	(0.0125)
Experience^2	-0.0951***
	(0.0341)
Constant	0.531***
	(0.202)
Observations	267
R-squared	0.389
Sum-of-squared Errors	82.50

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

(b) In terms of the model parameters, with $experience = 10$,

$$\theta = \frac{\beta_1}{\beta_2 + \frac{1}{5}\beta_3}$$

Using the parameter estimates from the model,

$$\hat{\theta} \approx 6.109$$

(c) The asymptotic standard error of $\hat{\theta}$ is the square root of its asymptotic variance. Since θ is a function of β, we can use the delta method to solve for the variance of $\hat{\theta}$ as a function of the variance-covariance matrix of $\hat{\beta}$:

$$\sqrt{n}\left(\hat{\theta}-\theta\right) \rightarrow_d f'(\hat{\beta})\mathcal{N}\left(0,V\right) \equiv \mathcal{N}\left(0, f'(\hat{\beta})'Vf'(\hat{\beta})\right)$$

Where V is the variance-covariance matric of $\hat{\beta}$ and

$$f'(\hat{\beta}) = \begin{pmatrix} \frac{\partial f(\hat{\beta})}{\partial \hat{\beta}_1} \\ \frac{\partial f(\hat{\beta})}{\partial \hat{\beta}_2} \\ \frac{\partial f(\hat{\beta})}{\partial \hat{\beta}_3} \\ \frac{\partial f(\hat{\beta})}{\partial \hat{\beta}_4} \end{pmatrix} = \begin{pmatrix} \frac{1}{\hat{\beta}_2+\frac{1}{5}\hat{\beta}_3} \\ -\frac{\hat{\beta}_1}{\left(\hat{\beta}_2+\frac{1}{5}\hat{\beta}_3\right)^2} \\ -\frac{\hat{\beta}_1}{5\left(\hat{\beta}_2+\frac{1}{5}\hat{\beta}_3\right)^2} \\ 0 \end{pmatrix}$$

(d) Using the results from the regression summarized in part (a),

$$\begin{aligned} s(\hat{\theta}) &\approx 1.63 \\ 90\% \text{ c.i.} &= [\hat{\theta}-1.645s(\hat{\theta}), \hat{\theta}+1.645s(\hat{\theta})] \approx [3.428, 8.790] \end{aligned}$$

Question 2

According to equation (8.3),

$$\widetilde{\beta}_{CLS} = \underset{R'\beta=c}{\arg\min}\ \text{SSE}(\beta)$$

Where $R = \begin{pmatrix} 0 \\ I_{k_2} \end{pmatrix}$ and $c = 0$. Then, (8.3) can be simplified as the following unconstrained optimization problem:

$$\widetilde{\beta}_{CLS} = \text{argmin SSE}\begin{pmatrix} \beta_1 \\ 0 \end{pmatrix}$$

WAnd where $\hat{\beta}_{OLS}$ from the regression of Y on X_1 is defined as:

$$\hat{\beta}_{OLS} = \text{argmin SSE}(\beta_1)$$

Question 3

By equation (8.3),

$$\widetilde{\beta}_{CLS} = \underset{R'\beta=c}{\arg\min}\ \text{SSE}(\beta)$$

Where, $\text{SSE}(\beta) = (Y - X\beta)'(Y - X\beta) = (Y - X_1\beta_1 - X_2\beta_2)'(Y - X_1\beta_1 - X_2\beta_2)$ and, in this case, $R = \begin{pmatrix} I_k \\ I_k \end{pmatrix}$ and $c = 0$. Then,

$$\begin{aligned}
\widetilde{\beta}_{CLS} &= \arg\min_{R'\beta=c} \text{SSE}(\beta)(Y - X\beta)'(Y - X\beta) \\
\mathscr{L} &= (Y - X_1\beta_1 - X_2\beta_2)'(Y - X_1\beta_1 - X_2\beta_2) - \lambda(\beta_1 + \beta_2) \\
\frac{\partial \mathscr{L}}{\partial \beta_1} &= -2X_1'(Y - X_1\beta_1 - X_2\beta_2) - \lambda = 0 \\
\frac{\partial \mathscr{L}}{\partial \beta_2} &= -2X_2'(Y - X_1\beta_1 - X_2\beta_2) - \lambda = 0 \\
\frac{\partial \mathscr{L}}{\partial \lambda} &= \beta_1 + \beta_2 = 0 \\
\beta_1 &= -\beta_2 \\
-2X_1'(Y - X_1\beta_1 - X_2\beta_2) &= -2X_2'(Y - X_1\beta_1 - X_2\beta_2) \\
-2X_1'(Y + X_1\beta_2 - X_2\beta_2) &= -2X_2'(Y + X_1\beta_2 - X_2\beta_2) \\
-2X_1'Y - 2X_1'(X_1 - X_2)\beta_2 &= -2X_2'Y - 2X_2'(X_1 - X_2)\beta_2 \\
2(X_2 - X_1)'(X_1 - X_2)\beta_2 &= 2(X_1 - X_2)'Y \\
\beta_2 &= [(X_2 - X_1)'(X_1 - X_2)]\,(X_1 - X_2)'Y \\
\beta_1 &= [(X_2 - X_1)'(X_2 - X_1)]\,(X_2 - X_1)'Y
\end{aligned}$$

Thus,

$$\widetilde{\beta}_{CLS} = \begin{pmatrix} [(X_2 - X_1)'(X_2 - X_1)]\,(X_2 - X_1)'Y \\ [(X_2 - X_1)'(X_1 - X_2)]\,(X_1 - X_2)'Y \end{pmatrix}$$

Question 4

8.4(a)

Question 5

8.22

Question 6

9.1

9.2

Question 7

9.4

Question 8

9.7