Micro Quarter 3 Review

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1 Matching Foundations of Markets

1.1 Non-Transferable Utility (NTU) Matching

- Positive-Assortative Matching (PAM): The derivative of each side's payoff function has the same sign
- Negative-Assortative Matching (NAM): The derivative of each side's payoff function has the opposite sign
- Gale-Shapley Theorem:
  - Male-pessimal outcome is the female-optimal outcome, and vice versa
  - If male-pessimal and male-optimal outcomes are the same, then the stable matching is unique
- The DAA can take no more than $n^2 - 2n + 2$ rounds, where there are n men and n women
- Solving DAA algorithm in discrete case: Example from first question of 2020 midterm below, showing the DAA matching from women proposing.

	Round									
	1	2	3	4	5	6	7	8	9	10
A	S	R*				P*				M*
B	R*		S*						P*	
C	P*				R*			S*		
D	S*			P*			R*			

1.2 Welfare Theorems of Matching

1. A competitive equilibrium yields an efficient matching
2. An efficient matching is a competitive equilibrium for a suitable set of wages

1.3 Transferable Utility (TU) Matching

- PAM: Supermodular
  - If differentiable, cross-derivative is positive
  - If not differentiable, increasing differences
- NAM: Submodular
  - If differentiable, cross-derivative is negative
  - If not differentiable, decreasing differences
- Competitive mechanics of a TU matching market: there exists an industry of "match-makers" who get the output from the matches they make and pay wages to each side.
  - If matchmakers are making a profit, then free entry of new match-makers hold that new match-makers will enter and out-compete incumbent match-makers by either making better matches or paying higher wages. Thus, the profit of match-makers has a maximum of 0
  - If matchmakers make inefficient matches, then they will be unable to pay high enough wages to at least one side of the match to stave off more efficient matchmakers from offering higher wages for a more efficient match
- Finding wages (differentiable case)
  1. Let $\pi = h(x, y) - v(x) - w(y)$ be the profit function for matchmakers in this market, where $h(x, y)$ is the output of a match. Find FOC for one side of the market[1]
  2. If PAM, solve FOC for first derivative of wage function using $y = x$. If NAM, solve using $y = 1 - x$.
  3. Take antiderivative to determine wage function, including some constant, c. Let k be the constant for the other side's wage function
  4. Impose free entry/exit condition to let $\pi = 0$ at its maximum; solve $\pi(x, y) = 0$ for the relationship between c and k.
  5. Suppose $c + k = S$. Then, $k = S - c$ and the range of market-decentralizing wages is given by the range of c such that the wage of each side is weakly greater than the side's outside option
     - In the typical case where the value of not matching for each side is zero, $c \in [0, S]$
     - Suppose D is the cost of matching for the x side. Then $c \in [-D, S]$

[1] if FOCs aren't symmetric, repeat steps 1-3 for other side of market

  - If there is a short side of the market, pin wages down uniquely by setting c and k such that the short side captures all surplus. For example, if the mass of type y exceeds that of type x, then $c = 0$, $k = S$.

- Finding wages (discrete case)

  1. Determine the efficient match (i.e. the one that maximimizes total output)
  2. Set up the inequalities for the wages in this match:
     - The sum of the two sides in each efficient match can be no greater than the output of that match
     - The sum of each individual's wage and the wage of their next-best match must be weakly greater than their match output (in order to dissuade that match)
  3. Due to competitive mechanics of TU matching, match condition (1) will hold with equality
  4. Use these two sets of inequalities to derive the intervals for each individual's wage

1.4 Double Auctions

- In a double auction, there are n buyers with valuations v_i for a homogenous good and m sellers of the good with costs c_j

- NAM theorem: NAM of buyers and sellers in a double auction results in a total surplus that is weakly greater than any other matching method

- Solving for equilibrium in a double auction:

  1. Determine market demand by sorting valuations in descending order and summing demand at each price
  2. Determine market supply by sorting costs in ascending order and summing supply at each price
  3. Tip for steps 1 and 2: Draw up a table of each side of the market, with columns representing the open-closed interval of demand/supply at each price/cost/valuation—e.g., for $v_i = \$10$, if adding i's demand at a price of \$10 takes demand from 20 units to 25, then have one column that represents pre-\$10 demand and another with demand at \$10. This makes graphing the staircase-looking function much easier.
  4. Either visually identfy on the tables where the two curves intersect, or begin testing plausbile prices, raising or lowering price "guesses" according to supply and demand at each guessed price
  5. After determining an equilibrium price/quantity, determine whether it is unique by graphing supply and demand around that point

- Collusion outcomes:
  - Sellers collude: Total seller profit is maximized. To solve:
    1. Begin with the highest price that won't decrease demand
    2. Calculate the change in total seller profit if you raise to the next buyer's valuation
    3. If (2) is negative, then the price from (1) is optimal. If (2) is positive, repeat step 2 until you find the price that results in negative marginal profit
  - Buyers collude: Total consumer surplus is maximized. To solve, use same method as solving for the seller collusion outcome, but with progressively lowering the price

2 Partial Equilibrium

2.1 Supply and Demand

2.1.1 Intensive vs. Extensive Margins

- **Extensive Margins** reflect who is in the market
  - e.g. fixed costs decrease $\Rightarrow$ new firms enter the market $\Rightarrow$ supply increases on the extensive margin
- **Intensive Margins** reflect how much each actor in the market buys/sells
  - e.g. incomes increase $\Rightarrow$ consumers have more money $\Rightarrow$ demand increases on the intensive margin
- Supply:
  - Marginal costs determine intensive margin ($MR = MC$ sets optimal quantity)
  - Average costs determine extensive margin ($P = AVC$ determines shutdown decision in short run, $P = ATC$ determines entry/exit in long run)
- Supply and demand curves reflect both intensive and extensive margins

2.1.2 Competitive Equilibrium

- **Walrasian Price Stability**: Use net demand to determine whether to raise/lower price
  - During price adjustment, the short side of the market determines the quantity
  - $Q^D > Q^S \Rightarrow$ market quantity is Q^S, price increases
  - $Q^S > Q^D \Rightarrow$ market quantity is Q^D, price decreases
  - $Q^S = Q^D = Q^* \Rightarrow$ market is Walrasian stable
- **Marshallian Quantity Stability**: Use gap between demand price and supply price to raise/lower supply prices
  - During price adjustment, the short side of the market determines the quantity
  - $P^D > P^S \Rightarrow$ Suppliers raise prices
  - $P^S > P^D \Rightarrow$ Suppliers lower prices
  - $P^S = P^D = P^* \Rightarrow$ market is Marshallian stable

2.1.3 Elasticity

- Demand: $\varepsilon = \frac{dQ^D}{dP}\frac{P}{Q} = \frac{\mathrm{dlog}(Q^S)}{\mathrm{dlog}(P)} \approx \frac{\%\Delta Q^D}{\%\Delta P} < 0$ (usually)
- Supply: $\eta = \frac{dQ^S}{dP}\frac{P}{Q} = \frac{\mathrm{dlog}(Q^S)}{\mathrm{dlog}(P)} \approx \frac{\%\Delta Q^S}{\%\Delta P} > 0$ (usually)
- Le Chatellier's Principle: demand/supply is more elastic (i.e. the absolute value is greater) in the long run than in the short run
- Elasticity and price volatility:
  - More elastic $\Rightarrow$ greater quantity volatility, lower price volatility
  - Less elastic $\Rightarrow$ greater price volatility, lower quantity volatility
- Elasticity and tax incidence:
  - Incidence Theorem: The more elastic side of the market bears the lower burden, regardless of who "pays" the tax
    * Impose excise tax $\tau \equiv dP^D - dP^S$. Then:

$$dP^D \approx \left(\frac{\eta}{\eta - \varepsilon}\right)\tau > 0 \text{ and } dP^S \approx \left(\frac{\varepsilon}{\eta - \varepsilon}\right)\tau < 0$$

      Where $\frac{\eta}{\eta-\varepsilon}$ is the share of the tax paid by consumers, and so on.
  - Deadweight loss for small taxes: Using ε and tax τ,

$$DWL = \frac{1}{2}dQ(dP^D - DP^S) = \left(\frac{1}{1/\varepsilon - 1/\eta}\right)\left(\frac{Q}{2P^D}\right)\tau^2$$

  - Tax Irrelevance Theorem: Regardless of whether demand or supply "pays" the tax, the demand and supply prices, market quantity, and efficiency loss are the same.
  - Ramsey Inverse Elasticity Rule: taxes should be proportional to the sum of the reciprocals of its supply and demand elasticities
    * In other words, DWL is lower and tax revenue is higher if you tax less elastically-supplied or -demanded goods.

2.2 Market Power

2.2.1 Barriers to Entry

- Technical Barriers to Entry: Barriers that are due to the specific nature of production, distribution, etc.
  - Minimum efficient scale, i.e. fixed costs that are so high, you can only profitably operate if sales and/or prices are sufficiently high
  - Ownership of unique resources - exhaustible resources or a location (e.g. ski resorts)

- Legal Barriers to Entry:
  - Natural monopolies chartered by the government (e.g. the post office, public utilities)
  - Patents, trademarks, copyrights
- Cartels (legal and illegal)
- Noncompete Agreements
- Network Externalities: the value of a good/service increases with scale (e.g. social media)

2.2.2 Monopoly

- 101 stuff: $MC = MR$ comes from maximizing profit where the price, rather than being taken as given, is the inverse demand curve
- Inverse Elasticity Rule: Rewriting the FOC yields:

$$P(Q)\left(1 - \frac{1}{|\varepsilon|}\right) = C'(Q)$$

  From which markups are an inversely related to demand elasticity. Which gives us the Lerner index of market power, which goes from 0 (perfect competition) to 1 (captive market):

$$L = \frac{P(Q) - C'(Q)}{P(Q)} = \frac{1}{|\varepsilon|} < 1$$

2.2.3 Monopsony

- Rather than a single seller having full control over quantity supplied, a single buyer with full control over quantity demanded
- Canonical example: employers with monospony power hiring at $W < VMP$
  - Inverse elasticity rule: $VMP(L) = w(L)\left(1 + \frac{1}{\eta}\right)$

2.2.4 Oligopoly

- Cartels
  - Successful cartels operate as a multiplant firm, setting market quantity equal to the monopoly's profit-maximizing quantity
  - Problem: individual firms face $MR > MC$, tempting them to "chisel" by increasing supply

- Cournot competition: $i \in \{1, ..., n\}$ firms solve:

$$\max_{q_i} P(Q)q_i - C(q_i),\ Q = \sum_{j=1}^{n} q_j$$

  - The Cournot equilibrium converges to the competitive equilibrium as $n \to \infty$
  - The Cournot game is submodular ("strategic substitutes"), because each firm i's best reply function is monotonically decreasing in other firms' choice variables (q_j, $j \neq i$)

- Stackelberg competition: a large, dominant firm (L) moves first, followed by another firm (or set of firms), F. Solved via backward induction by the leading firm:

$$\max_{q_F} P(q_L + q_F)q_F - C(q_F)$$

- Stackelberg leads to a higher equilibrium supply than Cournot, which is higher than monopoly, and all of them are lower than perfect competition

2.2.5 Price Discrimination

- Def: Charging different prices to different consumers.
- First degree: Charging different prices at the individual level (profit-maximizing 1st degree PD: charging each consumer their willingness to pay)
- Second degree: Charging different prices based on quantity
  - Two-part tariff: paying a fixed fee for the right to trade at a linear price (e.g. Costco memberships)
  - Quantity discounts (e.g. lower per-unit price when you buy in bulk)
- Third degree: Charging different prices to different consumer groups (e.g. senior or student discounts)

2.3 Externalities

Pecuniary vs. technical externalities:

- Pecuniary externality: an increase in demand for some good increases the price, harming existing consumers
  - The price system reallocates gains from trade, maximizing welfare
- Technical externality: A transaction between two agents imposes a cost or benefit on a third party, which is not intenalized in the price of that transaction.

- An uninternalized benefit is a *positive externality* (e.g. beekeepers near a flower garden)
- An uninternalized cost is a *negative externality* (e.g. carbon pollution)

- The efficient allocation comes from social marginal cost being equal to social marginal benefit: ($SMC = SMB$)
  - Without externalities, private marginal costs/benefits are equal to social marginal costs/benefits

2.3.1 Coase Theorem

- Big idea: If property rights are well-defined and agents can costlessly bargain, then the efficient allocation will be reached even in the prescence of externalities.
- E.g.: If a cattle rancher is given cattle-grazing rights, then the farmer pays the rancher to reduce grazing; If the farmer is given the right not to have cattle graze on their farm, then the rancher pays the farmer for grazing access.

2.3.2 Pigouvian Taxation

- The efficient outcome can be attained in the presence of externalities by taxing activities with negative externalities and subsidizing those with positive externalities such that $PMC(Q) - \tau(Q) = SMC(Q)$ or $PMB(Q) + \tau(Q) = SMB(Q)$
- The efficient tax is set at the marginal damage (or marginal benefit) of the activity
- Alternatively, tradable permits for the activity/good with negative externalities can be issued; the permits should be equal to the optimal quantity and will trade at cost of marginal damage

2.4 Public Goods

	Rival	Nonrival
Excludable	Private Goods	Club Goods
Non-Excludable	Congestion Public Goods	Pure Public Goods

- Rival: One agent's consumption of a good diminishes another agent's benefit (e.g. you and I can't both drink the same 1ml of coca-cola)
- Excludable: Providers of a good/service cannot prevent those who haven't paid for it from consuming it (e.g. a fireworks display)
- For nonrival goods, the aggregate demand curve is calculated by adding inverse demand curves: $P^D_{agg} = \sum_i P^D_i$

2.4.1 Efficient Provisioning

- Tragedy of the Commons: Because a consumers return on (or cost of) a public good is equal to the average return (or cost) rather than the marginal return (or cost) $PMC \neq SMC$ and/or $PMB \neq SMB \Rightarrow$ the competitive equilibrium will not be efficient

- The efficient allocation can be reached via Pigouvian taxes

  - Example: fishing between to lakes, A and B, with returns $F(X_A)$ and $G(X_B)$, with $X_A + X_B = 1$.
  - SPP: $F'(X_A) = G'(X_B) \Rightarrow X_A^*, X_B^*$
  - Suppose $X_B > X_B^*$ in the competitive equilibrium. Determine Pigouvian tax, τ^* with:
  $$\frac{G(X_B^*)}{X_B^*} - \tau^* = \frac{F(X_A^*)}{X_A^*}$$

- Discrete nonrival goods: provision of one more unit of the good at cost c is efficient if $\exists\{t_i\}_1^n$ such that $\sum_i^n t_i = c$ and each agent i is weakly better off paying t_i and having one more unit of the good, and at least one agent is strictly better off.

- Continuous nonrival goods: maximize some social welfare function (SWF), W

  - a strictly quasi-concave one $\Rightarrow \exists!$ solution
  - Lemma: Samuelson Condition The opimal consumption of a public good, G is given by $\sum_{i=1}^n MRS_{G,w}^i = MRT_{G,w}$
    - Reduces to $\sum_i MB^i(G) = MC(G)$
  - Lindahl Equilibrium: Let G be a public good and each i of n consumers are endowed with w_i of some private good. Then, a LE is an allocation, $(G^*, x_1^*, ..., x_n^*)$, and set of prices, $\{p_1, ..., p_n\}$, paid by each consumer for the public good, such that, for each consumer:
  $$(G^*, x_i^*) = \underset{x_i, G}{\operatorname{argmax}}\, U^i(G, x_i) \text{ s.t. } x_i + p_i G = w_i$$

    The Lindahl equilibrium satisfied the Samuelson condition, because each consumer contributes their marginal benefit.
  - Example: Preak Load Pricing This is an interesting problem that I don't yet understand and want to come back to

3 General Equilibrium

3.1 GE in Exchange Economies

Notation:

- $\mathcal{E} = (\{u^i\}, \overline{x})$
- $L \geq 2$ goods, $\ell \in \{1, ..., L\}$
- $n \geq 2$ traders, $i \in \{1, ..., n\}$
- Endowments $\overline{x}^i = (\overline{x}^i_1, ..., \overline{x}^i_L)' \in \mathbb{R}^L_+$
- An allocation is a matrix $x = (x^1, ..., x^n) \in \mathbb{R}^{L \times n}_+$
- Price vector $p = (p_1, ..., p_L) \in \mathbb{R}^L$
- Trader i has utility $u^i : \mathbb{R}^L_+ \to \mathbb{R}$
- Trader i's income is the market value of their endowment: $p \cdot \overline{x}^i$
- Trader i's budget set is $\mathcal{B}^i(\overline{x}^i, p) = \left\{x^i \in \mathbb{R}^L_+ | p \cdot x^i \leq p \cdot \overline{x}^i\right\}$

Then, each consumer solves:

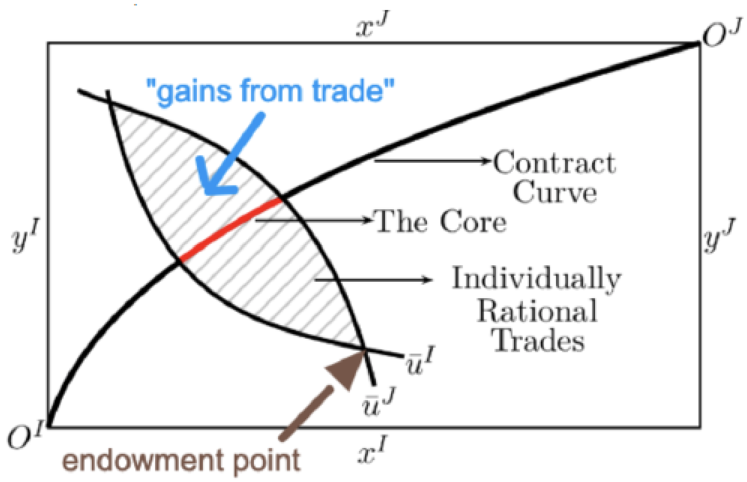
$$\max_{x^i} u^i(x^i) \text{ s.t. } x^i \in \mathcal{B}^i(\overline{x}^i, p)$$

An allocation is feasible for $\mathcal{E}$ if $\sum_{i=1}^N x^i_\ell \leq \sum_{i=1}^n \overline{x}^i_\ell \ \forall \ell$. A feasible allocation x is socially optimal if $\nexists$ a feasible allocation z such that no trader is worse off but at least one trader is strictly better off.

3.1.1 Competitive Equilibrium

Most questions on exchange economy GE appear to be given as two-person, two-good economies and involve both solving for and plotting the contract curve.

- Edgeworth box: a graph of a two-person, two-good economy:



- Contract curve: The set of all socially efficient allocations
- Any individually rational allocation is one where $u^i(x^i) \geq u^i(\overline{x}^i)\ \forall i$
- The core is the set of individually rational allocations
- A competitive equilirbium (CE) in $\mathcal{E}$ is a (x, p) s.t. x is feasible and optimal for traders, given p
  - Optimal in a 2-good, 2-person economy $\Rightarrow MRS^1_{x,y} = MRS^2_{x,y}$
- In an exchange economy, gains from trade arise from differences in preferences and/or endowments
- First Welfare Theorem: If (x, p) is a CE of $\mathcal{E}$ and preferences are locall non-satiated, then x is socially efficient
- Second Welfare Theorem: If traders have continuous, monotonic, and quasiconcave utility functions and x is a socially efficient allocation, then $\exists p$ such that (x, p) is a CE of $\mathcal{E}$
  - If at least one consumer has smooth, convex preferences, p is unique

3.1.2 Excess Demand Functions

- Since preferences are assumed to be strictly convex, each consumer has a unique demand for each good ℓ at each price p: $x^i_\ell(p)$
- Consumer i's excess demand for ℓ is their demand for ℓ, net of their endowment of ℓ: $ED^i_\ell(p) = x^i_\ell(p) - \overline{x}^i_\ell(p)$
- Market clearing implies $\sum_{i=1}^n ED^i_\ell(p) = 0 \; \forall \ell$
- Walras's Law: If traders cosume their entire income at allocation $x(p)$, then the market calue of net excess demand vanishes: $\sum_\ell^L p_\ell \sum_{i=1}^n ED^i_\ell(p) = 0$
  - Implication: If $L - 1$ markets clear, then L markets clear
  - Other implication: Can normalize all prices by a numeraire and solve for the price ratio, e.g. $\frac{p_y}{p_x}$
  - $\therefore$ CE problem is $L - 1$ equations in $L - 1$ goods
  - e.g. $L = 2$ with x and y: let $p = \frac{p_y}{p_x}$, then set $ED^1_x(p) + ED^2_x(p) = 0$. The only unknown is p.
- Theorem (existence): If every trader i has strictly monotone and convex preferences over x and y and owns a positive endowment, $(\overline{x}^i, \overline{y}^i)$, then there exists a Walrasian stable competitive equilibrium, (x, y, p)
  - Proof: monotonicity and convexity imply that $ED_x(0) < 0 < ED_x(\infty)$. By intermediate value theorem, $\exists p$ s.t. $ED_x(p) = 0$
  - Even though excess demand functions can look basically any which way, they're almost always locally unique, so you don't need to worry about cases where they're not

3.1.3 Trade Offer Curves

- Plots, for each trader i, the optimal consumption at each possible price, holding endowments constant (think of it as a best response curve)
- The TOC can be non-monotone, even with monotone preferences; with two goods, x and y:
  - If x and y are both normal, TOC strictly falls in the x-y plane
  - If x is normal and y is inferior, TOC may fall or rose
  - If x is inferior and y is normal, TOC may fall or rose
  - If x and y are both inferior, TOC may fall or rise, and may turn backward
- Solving the TOC with $L = 2$: Set the marginal rate of substitution:

$$MRS = \frac{\partial U / \partial x}{\partial U / \partial y}$$

Equal to the budget constraint in terms of endowments:

$$p = \frac{\overline{x} - x}{y - \overline{y}}$$

And solve for y as a funciton of x, with endowments fixed as TOC parameters

- The intersection of each trader's TOC yields an equilibrium, which is unique if demand has the gross substitutes property: an increase in price p_k raises the demand of every other good x_ℓ, for $\ell \neq k$
- Proposition (Uniqueness): If the aggregate excess demand function satisfies gross substitutes, the economy has at most one Walrasian equilibrium

3.1.4 Monopoly in an Exchange Economy

3.2 GE with Production

3.3 GE under Uncertainty