Homework 2

Due Monday night, November 9, at midnight.

Feel free to work together on these problems, but each student needs to write up his/her own answers at the end, rather than directly copying from one master solution.

1. Each of $n \geq 2$ players can make contributions $s_i \in [0, w]$ to the production of some public good, where $w > 0$ and s_i is the contribution of player i. Their payoff functions are given by $\pi_i(s_1, s_2, \ldots, s_n) = n \min\{s_1, s_2, \ldots, s_n\} - s_i$. Find all pure strategy Nash equilibria in the game.

2. Donald and Joe are competing for three voters. Each candidate decides how much to spend on campaigning in voter i's area. Denote Donald's expenditure in area i by a_i and Joe's expenditures by b_i. The total budget for each candidate is 1, so $\sum_{i=1}^{3} a_i = \sum_{i=1}^{3} b_i = 1$. Voter i votes for Donald if and only if $a_i > b_i$. Similarly, Voter i votes for Joe if and only if $b_i > a_i$. The candidate with more votes wins the election.

   (a) Formulate the above strategic situation as a normal form game between Donald and Joe.

   (b) Show that there are no pure strategy Nash equilibria in this game.

3. Alice and Bob are collaborating on a project and simultaneously decide on adopting one of the three available technologies T_1, T_2, T_3. For $i = \{1, 2, 3\}$, Alice and Bob get a payoff of i, when both of them use the same technology T_i. In all other cases, they receive a payoff of zero.

   (a) Formulate the above strategic situation as a normal form game between Alice and Bob.

   (b) Describe the set of rationalizable strategies and compute all Nash equilibria (pure and mixed) of the game.

4. Suppose there are two firms in a market. Each firm's cost function is the same, given by $C_i(q_i) = q_i$ for $i = 1, 2$ where q_i is the output of firm i. If the firms' total

output is Q i.e., $Q = q_1 + q_2$, then the market price is

$$P(Q) = \begin{cases} 2 - Q & \text{if } Q \leq 2, \\ 0 & \text{if } Q > 2. \end{cases}$$

For $i = 1, 2$, firm i's revenue and profit are given by $R_i(q_1, q_2) = q_i P(q_1 + q_2)$ and $\pi_i(q_1, q_2) = q_i P(q_1 + q_2) - C_i(q_i)$ respectively.

(a) Suppose that both the firms maximize their profits and this is common knowledge. Compute the equilibrium quantities and profits of both the firms.

(b) Now suppose that firm 1 maximizes $\frac{3}{4}\pi_1(q_1, q_2) + \frac{1}{4}R_1(q_1, q_2)$ (i.e., firm 1 puts a weight of $\frac{3}{4}$ on its profit and a weight of $\frac{1}{4}$ on its revenue) and firm 2 maximizes profit and this is common knowledge. Now compute the equilibrium quantities and profits of both the firms.

(c) Compare the equilibrium profits of the two firms in the above two cases.

5. Let there be a continuum of participants choosing a time $t \in [0, 1]$ to attend a seminar on Zoom which starts at $t = 1$. At $t \geq 0$, the organizer allows the participants to join the seminar. Participants do not want to join the seminar too early or too late, but they also want to arrive when there are lots of others already present. Each agent's payoff is $u(t)v(q)$, where $u(t) = t(1 - t)$ is the fundamental payoff of joining the seminar at time t, and $v(q) = 1 + q + \frac{1}{4}q^2$ is the strategic quantile payoff of being at the q quantile of those joining.

(a) Solve for the symmetric Nash equilibrium quantile function $Q(t)$ of the Zoom seminar participation game. Can there be a terminal rush?

(b) During what times do people join the seminar i.e., what is the support of Q?