Problem Set #4

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Question 1

In order for Pr(Defying) $= 0$, Z must be monotonic in X. In order for Pr(Complying) > 0, it must be the case that, in a nonzero number of cases, $X(Z=1)=1$ where $X(Z=0)=0$. Thus, $U_1 > 0$ and $\Pr(U_1 > U_0) > 0$.

Question 2

(i) The autocovariance function is defined as:

$$\gamma(k) = Cov(Y_t, Y_{t-k}) = \mathbb{E}\left[\left(Y_t - \mathbb{E}\left[Y_t\right]\right)\left(Y_{t-k} - \mathbb{E}\left[Y_{t-k}\right]\right)\right] = \mathbb{E}\left[Y_t Y_{t-k}\right] - \mathbb{E}\left[Y_t\right]\mathbb{E}\left[Y_{t-k}\right]$$

Where:

$$\begin{aligned}
\mathbb{E}\left[Y_t\right] &= \mu + \mathbb{E}\left[\varepsilon_t\right] + \theta_1 \mathbb{E}\left[\varepsilon_{t-1}\right] + ... + \theta_q \mathbb{E}\left[\varepsilon_{t-q}\right] = \mu = \mathbb{E}\left[Y_{t-k}\right], \forall k \\
Y_t Y_{t-k} &= \mu^2 + \mu\left(\varepsilon_{t-k} + \theta_1 \varepsilon_{t-k-1} + ... + \theta_q \varepsilon_{t-k-q}\right) + \varepsilon_t\left(\varepsilon_{t-k} + \theta_1 \varepsilon_{t-k-1} + ... + \theta_q \varepsilon_{t-k-q}\right) + ... \\
\mathbb{E}\left[Y_t Y_{t-k}\right] &= \mu^2 + \varepsilon_t^2 + ... + \varepsilon_{t-k}^2
\end{aligned}$$

Thus, letting $\varepsilon_t^2 = \sigma^2$ for all t and recognizing that $\theta_k = 0$ for all $k < t - q$,

$$\gamma(k) = \begin{cases} \left(\theta_k + ... + \theta_{q-k}\theta_q\right)\sigma^2, & k \leq q \\ 0, & k > q \end{cases}$$

(ii) If $q = 1$, then:

$$\gamma(k) = \begin{cases} \left(1+\theta_1^2\right)\sigma^2 & k = 0 \\ \theta_1 \sigma^2, & k = 1 \\ 0, & k > 1 \end{cases} \Rightarrow \rho(k) = \begin{cases} 1 & k = 0 \\ \frac{\theta_1}{1+\theta_1^2}, & k = 1 \\ 0, & k > 1 \end{cases}$$

(iii)

(iv)

Question 3

(i)

(ii)