Problem Set #1

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Econ 712: Macroeconomics I
Fall 2020

September 10, 2020

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The price of a share of some stock at time t with perfect forsight is given as:

$$p_t = \frac{d + p_{t+1}}{1 + r}$$

Question 1

Solve for the steady state stock price $\mathbf{p^* = p_t = p_{t+1}}$

We can easily solve for p^* by simply substituting p^* for p_t and p_{t+1} in the share price formula and solving for p^*:

$$\begin{aligned} p^* &= \frac{d + p^*}{1 + r} \\ (1 + r)p^* &= d + p^* \\ rp^* &= d \\ p^* &= \frac{d}{r} \end{aligned}$$

Question 2

Solve for the steady state stock price $\mathbf{p^* = p_t = p_{t+1}}$

Given p_t as a function of p_{t+1}, we can derive:

$$p_t = f(p_{t-1}) = (1 + r)p_{t-1} - d$$

The general, closed-form solution will take the form:

$$p_t = ca^t + M = p_t^c + p_t^p$$

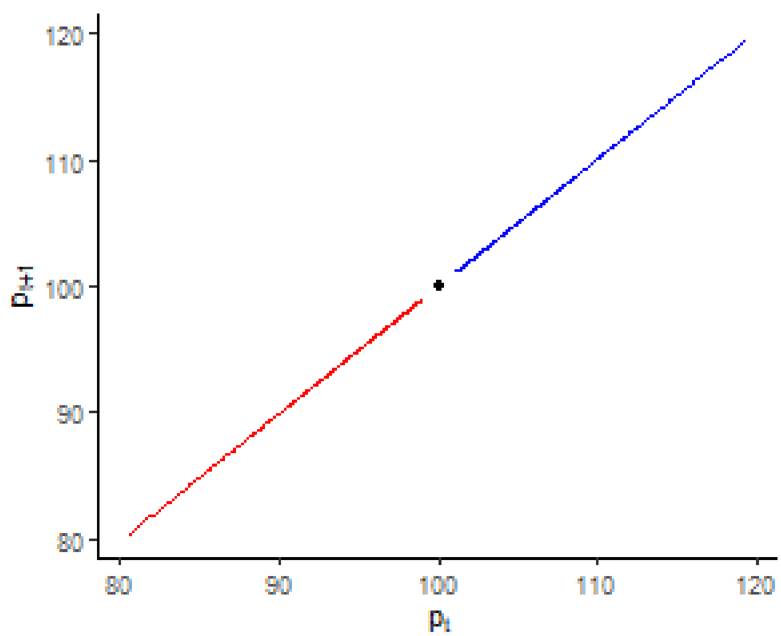
Where $p_t^c = ca^t$ is a complementary solution, p_t^p is a particular solution, and $a = 1 + r$. Since we know that $p_{t+1} = f(p^*) = p^*$, we can use p^* as a particular solution. Then, since p_0 is given, we can use $p_0^g = p_0$ as a boundary solution to solve for c:

$$p_0 = ca^0 + p^*$$
$$c = p_0 - p^*$$

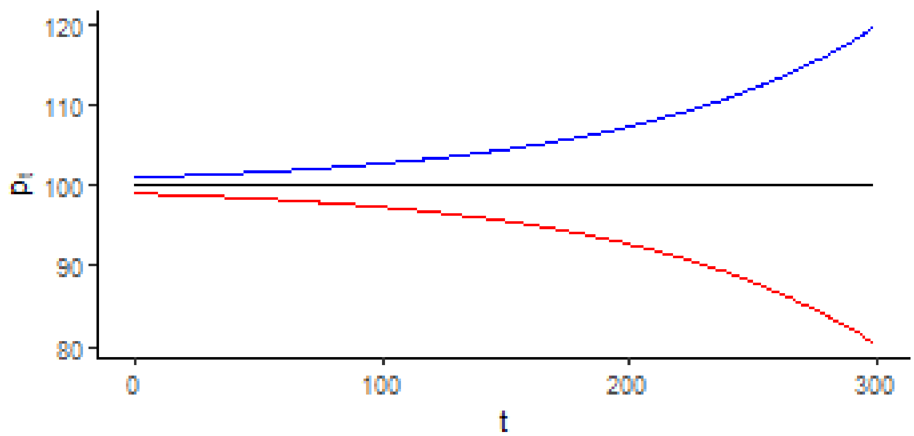
Thus, we arrive at the closed-form solution: $p_t = (p_0 - p^*)(1+r)^t + p^*$. Since $r > 0$, $1 + r > 1$. Therefore, $|p_t|$ diverges to infinity unless its initial value is equal to its steady state:

$$\lim_{t\to\infty} p_t = \begin{cases} \infty, & p_0 > p^* \\ p^*, & p_0 = p^* \\ -\infty, & p_0 < p^* \end{cases}$$

Each of these outcomes is displayed in the plot below, which plots p_t against p_{t+1} for $t = \{1, ..., 300\}$. The black line (dot) shows the relationship when $p_0 = p^*$, the blue line plots it when $p_0 = p^* + 1$, and the red line plots it when $p_0 = p^* - 1$



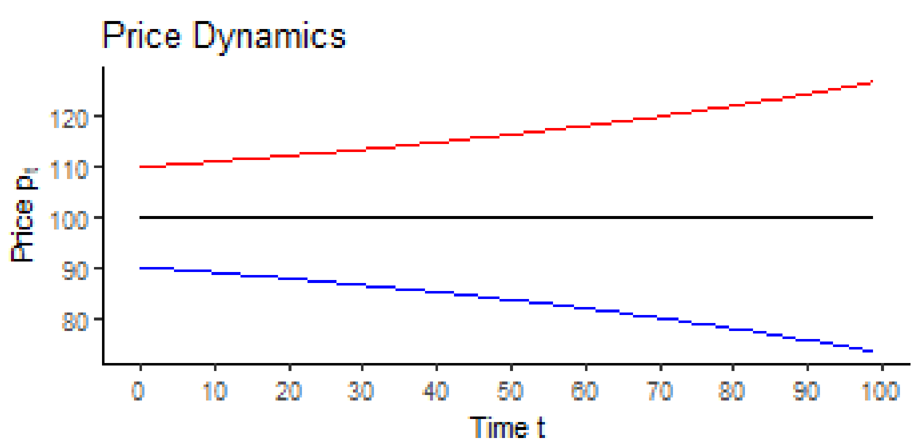
The same series are plotted below, using the same color scheme for each series, with t plotted on the x-axis and p_t plotted on the y-axis.



Question 3

Modify the code provided to plot the price dynamics over 100 periods with three different initial prices which are respectively blow, at, and above the steady state price level

The requested chart is below, which was generated using R. I know Matlab enough to translate between the two, but I prefer to use R. The red line uses $p_0 = 110$, and the blue line uses $p_0 = 90$.



Question 4

Suppose the Federal Reserve announces at t = 20 to raise the federal funds rate from 1% to **2**% at **t = 50 and remain at the new level for-**

ever. Using (1) with d = 1; how does the price respond to the policy announcement and the interest rate change over time? Plot the price dynamics from t = 0 to t = 99.

Beginning at time $t = 20$, the price at time t accounts for $p_{50} = 50$. Since $p_t = \frac{d+p_{t+1}}{1+r}$, the knowledge that $p_{50} = 50$ at $t = 20$ leads investors to price the stock lower in each period from $t = 20$ to $t = 49$ even though r has not yet changed. This takes the form of a steep, one-time drop in period $t = 20$ following the news, then a slow convergence to p_{50} over the next 30 time periods, as the plot below shows.