Problem Set #11

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Discussed and/or compared answers with Sarah Bass, Emily Case, Katherine Kwok, Michael Nattinger, and Alex Von Hafften

Exercise 25.1

The slope coefficients represent the difference in log odds between the two dependent variable outcomes generated by the independent variables. Thus, the slope coefficients will have the same absolute values in each specification, but different signs.

Exercise 25.3

Recall that $P(x) = P[Y = 1|X = x]$, where $Y \in \{0, 1\}$. Then, when we have the model:

$$Y = P(X) + e$$

It must be the case that e is bounded by 0 and 1, where:

$$e = \begin{cases} 1 - P(x), & Y = 1 \\ P(x), & Y = 0 \end{cases}$$

Where $Y = 1$ with probability $P(x)$ and $Y = 0$ with probability $1 - P(x)$. Now, we can find the conditional variance of e:

$$\begin{aligned} Var(e|X) &= \mathbb{E}\left[e^2|X\right] - \mathbb{E}\left[e|X\right]^2 = \mathbb{E}\left[e^2|X\right] \\ e^2 &= Y^2 - 2YP(x) + P(x)^2 \\ \mathbb{E}\left[e^2|X\right] &= \mathbb{E}\left[Y^2|X\right] - 2\mathbb{E}\left[Y|X\right]P(x) + P(x)^2 \\ \mathbb{E}\left[Y|X\right] &= P(x) \\ \mathbb{E}\left[Y^2|X\right] &= (1)P(x) + (0)(1 - P(x)) = P(x) \\ \Rightarrow Var(e|X) &= P(x) - P(x)^2 = P(x)\left(1 - P(x)\right) \end{aligned}$$

Exercise 25.9

The logit log-likelihood funciton is:

$$\ell_n(\beta) = \sum_{i=1}^{n} \log\left(\Lambda\left(Z_i'\beta\right)\right)$$

Exercise 25.12

Exercise 25.14

Exercise 25.15

The results of the probit estimation are displayed below, with coefficients reported in the first column and average marginal effects reported in the second column for interpretation. The AME column provides the average change in probability of union membership caused by a change in the independent variable. This suggests that each year of age adds .043% to the odds that an individual belongs to a union, while Hispanic individuals are 1.15% less likely, on average, to belong to a union than non-Black and non-Hispanic individuals (i.e. the omitted group).

	(1)	(2)
VARIABLES	Probit	AME
age	0.00841***	0.000430***
	(0.00111)	(5.74e-05)
education	0.000487	2.49e-05
	(0.00488)	(0.000249)
$\mathbb{1}\{Black\}$	-0.0606	-0.00310
	(0.0430)	(0.00220)
$\mathbb{1}\{Hispanic\}$	-0.225***	-0.0115***
	(0.0437)	(0.00225)
Constant	-2.362***	
	(0.0865)	
Observations	50,742	50,742

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Exercise 25.17

Exercise 26.1

Exercise 26.3

Exercise 26.7

Exercise 26.8