# Problem Set #1

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## Question 1

**For two events, $A, B \in S$, prove that $A \bigcup B = (A \bigcap B) \bigcup ((A \bigcap B^c) \bigcup (B \bigcap A^c))$.**

*Proof.*

1. $(a \bigcap B) \bigcup ((A \bigcap B^c) \bigcup (B \bigcap A^c)) = ((A \bigcap B) \bigcup (A \bigcap B^c)) \bigcup ((A \bigcap B) \bigcup (B \bigcap A^c))$
2.

## Question 2

**Prove that $P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B)$.**

## Question 3

**Suppose that the unconditional probability of a disease is 0.0025. A screening test for this disease has a detection rate of 0.9, and has a false positive rate of 0.01. Given that the screening test returns positive, what is the conditional probability of having the disease?**

## Question 4

**Suppose that a pair of events $A$ and $B$ are mutually exclusive, i.e., $A \bigcap B = \emptyset$, and that $P(A) > 0$ and $P(B) > 0$. Prove that** $A$ and $B$ **are not independent.**

## Question 5

**Consider the experiment of tossing two dice. Let** $A = \{\text{First die is 6}\}, B = \{\text{Second die is 6}\}$, and $C = \{\text{Both dice are the same}\}$**.**

### (a)

**Show that $A$ and $B$ are independent (unconditionally), but $A$ and $B$ are dependent given $C$.**

### (b)

**Given the urn experiment (see 5(b)), Show that $A$ and $B$ are not independent, but are conditionally independent given $C$.**

## Question 6

**Prove that if $X \sim F_X$ and $Y \sim F_Y$, then $P(X > t) \geq P(Y > t), \forall t$ and $P(X > t) > P(Y > t)$, for some $t$.**

## Question 7

**Show that the function**

$$F_X = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-x} & \text{if } x \geq 0 \end{cases}$$

**is a CDF, and find $f_X(x)$ and $F_X^{-1}(y)$.**