Problem Set #2

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Discussed and/or compared answers with Sarah Bass, Emily Case, Katherine Kwok, Michael Nattinger, and Alex Von Hafften

Question 1

1. In a decentralized environment, each agent faces the following utility maximization problem:

$$\max_{\{c_{it}, b_{i,t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_{it}) \text{ s.t. } c_{it} + b_{i,t+1} \leq y_{it} + R_t b_{it},\ b_{i,t+1} \geq \phi_t$$

Where, in our example from class, endowments alternate between a high and low endowment for each type of borrower, i. Let $i = l$ index the low-endowment borrower and $i = h$ index the high-endowment borrower, with the following primitives:

$$u(c) = \log(c) \qquad \beta = 0.5 \qquad (y_h, y_l) = (15, 4)$$

Then, the Euler equation yields:

$$\frac{1}{c_{lt}} \geq \beta R \frac{1}{c_{h,t+1}}$$
$$\frac{1}{c_{ht}} \geq \beta R \frac{1}{c_{l,t+1}}$$

Assume that the borrowing constraint binds. Since utility is monotonically increasing in consumption, the budget constraint will also bind. Then, in each period,

$$c_{ht} + \phi_{t+1} = 15 - R_t \phi_t$$
$$c_{lt} - \phi_{t+1} = 4 + R_t \phi_t$$

In equilibrium, $\phi_t = \phi t + 1$, $c_{it} = c_{i,t+1}$, and $R_t = R_{t+1}$ for all t. Then,

$$
\begin{aligned}
c_h &= 15 + \phi(1+R) \\
c_l &= 4 - \phi(1+R)
\end{aligned}
$$

The constrained efficient allocation in this problem is $(c_h, c_l) = (10, 9)$. Then, the above equations give us $\phi = -\frac{5}{1+R}$. Combining with the Euler equations from above enable us to obtain the following ranges of R that would decentralize the constrained efficient allocation:

$$R \leq \frac{1}{\beta}\frac{9}{10}$$

So each consumer's Euler equation is satisfied. Since the constrained efficient allocation by definition satisfies the voluntarity participation constraint, we need not show that it is satisfied. Now, we just need to show that markets clear. Let $R = \frac{1}{\beta}\frac{9}{10}$:

$$
\begin{aligned}
&c_h + c_l = 15 + \phi(1+R) + 4 - \phi(1+R) = y_l + y_h \\
&b_l + b_h = -\phi + \phi = 0
\end{aligned}
$$

2. The other equilibrium in this market is $\phi_t = 0$ for all t. This satisfies the voluntary participation constraint:

$$
\begin{aligned}
\frac{\log(15 - \phi(1+R))}{1-\beta^2} + \beta\frac{\log(4 + \phi(1+R))}{1-\beta^2} &= \frac{\log(15)}{1-\beta^2} + \beta\frac{\log(4)}{1-\beta^2} = V_h^d \\
\frac{\log(4 - \phi(1+R))}{1-\beta^2} + \beta\frac{\log(15 + \phi(1+R))}{1-\beta^2} &= \frac{\log(4)}{1-\beta^2} + \beta\frac{\log(5)}{1-\beta^2} = V_l^d
\end{aligned}
$$

The low endowment type cannot borrow and would not choose to save, so market clearing requires that the high endowment type doesn't save. Then each consumer's budget constraint implies that $c_{it} = y_{it}$ for all i and t. Then the goods market clears and from the Euler equation, we can obtain R:

$$
\begin{aligned}
\frac{15}{4} \geq \beta R \Rightarrow R \leq \frac{1}{\beta}\frac{15}{4} \\
\frac{4}{15} \geq \beta R \Rightarrow R \leq \frac{1}{\beta}\frac{4}{15}
\end{aligned}
$$

So the other equilibrium in this market is $\phi = 0$ and $R = \frac{1}{\beta}\frac{4}{15}$

Question 2

1. Let s be the savings of an agent in autarky. With R as the equilibrium interest rate, the new autarky utility level of an agent in the high state is given by:
$$V^d = \frac{\log(y_h - s)}{1-\beta^2} + \beta\frac{\log(y_l + Rs)}{1-\beta^2}$$

2. An equilibrium in this economy is an allocation, $\{c_{lt}, c_{ht}\}$, price system, $\{R_t\}$, and borrowing constraint, $\{\phi_t\}$ that solve the consumer's problem:
$$\max_{\{c_{it}, b_{i,t+1}\}} \sum_{t=0}^{\infty} \beta^t \log(c_{it}) \text{ s.t. } c_{it} + b_{i,t+1} \leq y_{it} + R_t b_{it},\ b_{i,t+1} \geq -\phi_t$$
In each period, markets clear:
$$c_{lt} + c_{ht} = y_{lt} + y_{ht} \qquad \text{(Goods Market)}$$
$$b_{lt} + b_{ht} = 0 \qquad \text{(Securities Market)}$$
And the not-too-tight debt constraint is satisfied:
$$\frac{\log(y_h - \phi(1+R))}{1-\beta^2} + \beta\frac{\log(y_l + \phi(1+R))}{1-\beta^2} = \frac{\log(y_h - s)}{1-\beta^2} + \beta\frac{\log(y_l + Rs)}{1-\beta^2}$$

3. The two equations that characterize the equilibrium are the not-too-tight debt constraint and high type's Euler equation (in equilibrium):
$$\frac{1}{y_h - \phi(1+R)} = \frac{\beta R}{y_l + \phi(1+R)}$$
The optimal s chosen in autarky is a function of R, determined by the agent's Euler equation:
$$\begin{aligned} \frac{1}{y_h - s} &= \frac{\beta R}{y_l + Rs} \\ \Rightarrow s &= \frac{\beta R y_h - y_l}{R(\beta+1)} \end{aligned}$$

4. From the Euler equation, we can solve for ϕ as a function of R:
$$\begin{aligned} \frac{1}{y_h - \phi(1+R)} &= \frac{\beta R}{y_l + \phi(1+R)} \\ \phi(1+R) + \beta R\phi(1+R) &= \beta R y_h - y_l \\ \Rightarrow \phi &= \frac{\beta R y_h - y_l}{(1+\beta R)(1+R)} \end{aligned}$$

Which we can plug into the not-too-tight debt constraint (with each side multiplied to remove the common denominator) to solve for R:

$$\begin{aligned}
\log\left(y_h - \frac{\beta R y_h - y_l}{(1+\beta R)(1+R)}(1+R)\right) + \beta\log\left(y_l + \frac{\beta R y_h - y_l}{(1+\beta R)(1+R)}(1+R)\right) \\
= \log\left(y_h - \frac{\beta R y_h - y_l}{R(\beta+1)}\right) + \beta\log\left(y_l + R\frac{\beta R y_h - y_l}{R(\beta+1)}\right) \\
\log\left(y_h - \frac{\beta R y_h - y_l}{1+\beta R}\right) + \beta\log\left(y_l + \frac{\beta R y_h - y_l}{1+\beta R}\right) \\
= \log\left(y_h - \frac{\beta R y_h - y_l}{R(\beta+1)}\right) + \beta\log\left(y_l + \frac{\beta R y_h - y_l}{\beta+1}\right)
\end{aligned}$$

This equation is satisfied when $R = 1$, which gives an equilibrium borrowing constraint of $\phi = \frac{\beta y_h - y_l}{2(1+\beta)}$

5. Given this result and our primitives, we can calculate $\phi = \frac{14}{3}$ and the equilibrium consumption as $(c_l, c_h) = (4 + 2\phi, 15 - 2\phi) \approx (15.3, 5.7)$. This compares to equilibrium consumption of $(10, 9)$ with the autarky punishment.

6. This result suggests an intuitive relationship between consumption smoothing and default punishment: that lower punishments result to less consumption smoothing. This comes from the fact that weaker default punishments appear in the form of tighter voluntary participation constraints, so borrowing constraints must be tighter in order to satisfy the voluntary participation constraint.