Econ 712 Macroeconomics I

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Problem Set #3

1. Consider the following overlapping generations problem. In each period $t = 1, 2, 3, ...$ a new generation of 2 period lived households are born. Each generation has a unitary mass. There is a unit measure of initial old who are endowed with $\bar{M} > 0$ units of fiat money. Each generation is endowed with w_1 in youth and w_2 in old age of non-storable consumption goods where $w_1 > w_2$. There is no commitment technology to enforce trades. The utility function of a household of generation $t \geq 1$ is

$$U\left(c_t^t, c_{t+1}^t\right) = \ln(c_t^t) + \ln(c_{t+1}^t)$$

where $\left(c_t^t, c_{t+1}^t\right)$ is consumption of a household of generation t in youth (i.e. in period t) and old age (i.e in period $t+1$). The preferences of the initial old are given by $U(c_1^0) = \ln(c_1^0)$ where c_1^0 is consumption by a household of the initial old.

   (a) State and solve the planner's problem.[1]

   (b) State the representative household's problem in period $t \geq 0$. Try to write the budget constraints in real terms.

   (c) Define and solve for an autarkic equilibrium, assuming that it exists.

   (d) Define and solve for a competitive equilibrium assuming valued money but with $w_2 = 0$.

   (e) Compare the solutions to the planners problem, the autarky equilibrium and the stationary monetary competitive equilibrium with valued money, all with $w_2 = 0$

   (f) What happens to consumption, money demand and prices in a competitive equilibrium with valued money if the initial money supply is halved, i.e. $\bar{M}' = \frac{\bar{M}}{2}$. Keep the assumption that $w_2 = 0$.

[1] Don't worry about the finiteness of the objective function. We will learn more about solutions to this problem later.

2. Plot the trade offer curves for the following utility functions where the endowment is (w_1, w_2) for goods 1 and 2, respectively.

   (a) $U = 10c_1 - 4c_1^2 + 4c_2 - c_2^2$, $(w_1, w_2) = (0, 2)$

   (b) $U = min\{2c_1 + c_2, c_1 + 2c_2\}$, $(w_1, w_2) = (1, 0)$

   (c) $U = min\{2c_1 + c_2, c_1 + 2c_2\}$, $(w_1, w_2) = (1, 10)$