# Problem Set #1

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Econ 714: Macroeconomics II
Spring 2021

February 1, 2021

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## Question 1

The social planner's problem is:

$$\max_{\{I_t, C_t, K_{t+1}\}_{t=0}^{\infty}} \beta^t U(C_t) \text{ s.t. } K_{t+1} = (1-\delta)K_t + I_t - D_t,\ C_t + I_t \leq F(K_t)$$

At any optimum, the resource constraint will hold with equality, so we can combine the law of motion and the resource constraint to obtain a single constraint:

$$F(K_t) = K_{t+1} - (1-\delta)K_t + C_t + D_t$$

We can derive the Euler equation by taking the first-order conditions of the Lagrangian function.:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t U(C_t) - \lambda_t \left(F(K_t) - K_{t+1} + (1-\delta)K_t - C_t - D_t\right)$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t U'(C_t) + \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \lambda_t - \lambda_{t+1}\left(F'(K_{t+1}) + 1 - \delta\right) = 0$$

Taken together, these first-order conditions give us the Euler equation for the SPP:

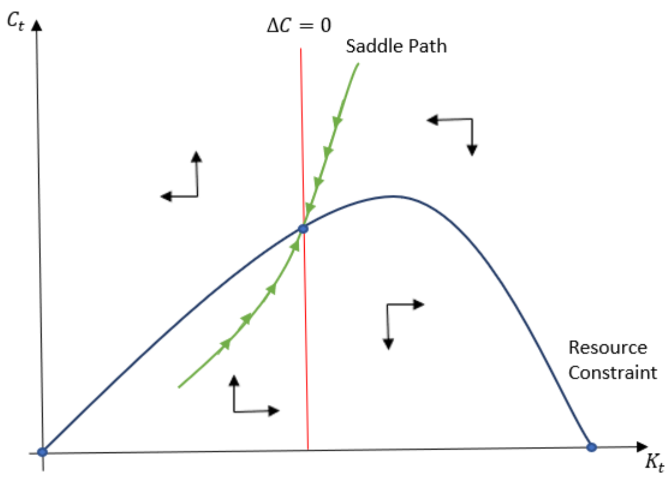
$$U'(C_t) = \beta U'(C_{t+1})\left[F'(K_{t+1}) + 1 - \delta\right]$$

## Question 2

Given the steady-state value $D$, the steady-state values of capital, $K_t = K_{t+1} = \overline{K}(D)$, and consumption, $C_t = C_{t+1} = \overline{C}(D)$ are detemined by the intersection of the resource constraint and Euler equation when capital and consumption are held constant:

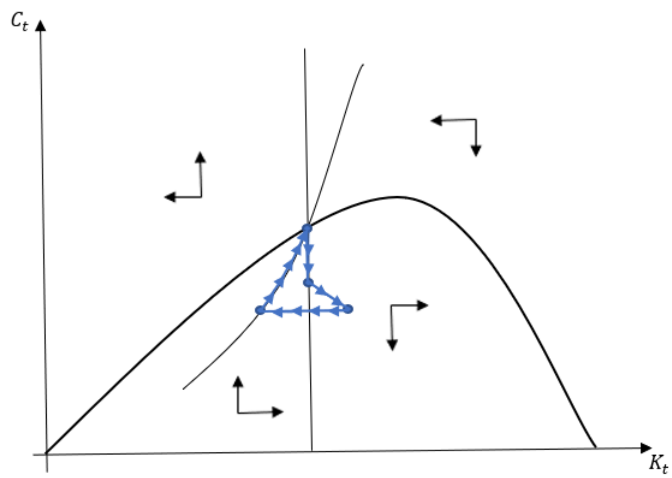
$$U'(\overline{C}) = \beta U'(\overline{C}) \left[F'(\overline{K}) + 1 - \delta\right] \Rightarrow \beta \left[F'(\overline{K}) + 1 - \delta\right] = 1$$
$$F(\overline{K}) = \overline{K} - (1-\delta)\overline{K} + \overline{C} + D \Rightarrow F(\overline{K}) = \delta\overline{K} + \overline{C} + D$$

The phase diagram for the solution is displayed below, with the three steady states marked by blue dots.



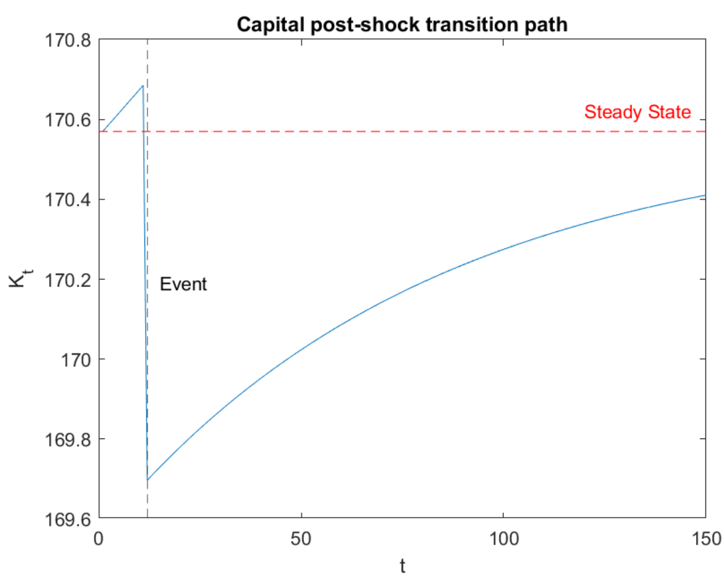

## Question 3

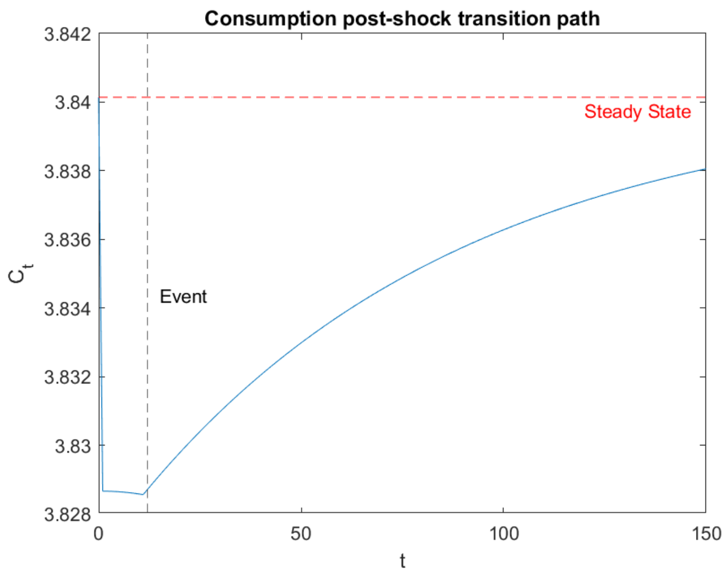
The chart below displays the predicted trajectory of consumption, beginning at the steady state, jumping downward in the period when the news is announced, then slowly trending downward (as capital increases) until the shock to capital hits, landing consumption on the saddle path, where it remains until reaching the steady-state again.




## Question 4

The charts below display capital and consumption, respectively, plotted against time, beginning with one period before the news, then plotting the next 150 periods. As projected in (3), consumption jumps down when the news hits, trends slowly downward before the shock, then begins trending upward toward the steady-state after the capital shock.





Consumption post-shock transition path
3.842
3.84
3.838
3.836
3.834
3.832
3.83
3.828
$C_t$
Steady State
Event
0
50
100
150
t