Homework #2

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Question 1

In the long run, each firm will only produce if the price is above its long run average cost. All firms have identical marginal cost curves, but variable long run average costs. The firm with the lowest long run average cost is firm $k = 1$, which will have positive output if the price is strictly greater than 1, assuming the good is divisible. If goods can only be produced in discrete quantities, then there will only be positive output if the price is weakly greater than 3.

Question 2

The total cost and total revenue of an individual firm in this market, as a function of output q, is:

$$TC(q) = 1 + \frac{q^2}{4\theta^2} \qquad\qquad TR(q) = pq = q$$

(a) Since there is a continuum of price-taking firms, this market is competitive, with each firm choosing q to maximize total profit:

$$\max_q \; (1-\tau)pq - 1 - \frac{q^2}{4\theta^2}$$

From the first-order condition of this problem, we can derive the firm supply curve:

$$\begin{aligned} (1-\tau)p - \frac{q}{2\theta^2} &= 0 \\ q &= 2\theta^2(1-\tau)p \end{aligned}$$

We can determine the mass of developers that supply this market by finding θ of the marginal developer:

$$\begin{aligned}
\frac{q}{2\theta^2} &= \frac{1}{q} + \frac{q}{4\theta^2} \\
\frac{2\theta^2(1-\tau)}{2\theta^2} &= \frac{1}{2\theta^2(1-\tau)} + \frac{2\theta^2(1-\tau)}{4\theta^2} \\
1-\tau &= \frac{1}{2\theta^2(1-\tau)} + \frac{(1-\tau)}{2} \\
\theta^* &= \frac{1}{1-\tau}
\end{aligned}$$

The aggregate supply curve is derived by summing each firm's suppy curve, where $m(\theta) = \beta\theta^{-\beta-1}$ is the probability mass function of θ:

$$\begin{aligned}
Q(p) &= \int_{\theta^*}^{\infty} m(\theta) 2\theta^2 (1-\tau) p d\theta = 2(1-\tau)\beta p \int_{\theta^*}^{\infty} \theta^{1-\beta} d\theta \\
&= 2(1-\tau)\beta p \left[\frac{1}{2-\beta}\theta^{2-\beta}\right]_{\theta^*}^{\infty} \\
Q(p) &= 2\left(\frac{\beta}{\beta-2}\right)(1-\tau)^{\beta-1} p
\end{aligned}$$

(b) When apple raises its tax rate, the θ of the marginal developer, θ^*, increases, decreasing the number of developers who supply the market. The per-firm supply also decreases, by $2\theta^2\Delta\tau$.

(c) Apple's revenue-maximizing tax is found by either maximizing its revenue or finding the value of τ for which the elasticity of supply with respect to τ is equal to negative one:

$$\begin{aligned}
\frac{dQ}{d\tau}\frac{\tau}{Q} &= -1 \\
\left(\frac{-2\left(\frac{\beta}{\beta-2}\right)(\beta-1)(1-\tau)^{\beta-2} p}{2\left(\frac{\beta}{\beta-2}\right)(1-\tau)^{\beta-1} p}\right)\tau &= -1 \\
(\beta-1)\tau &= 1-\tau \\
\tau &= \frac{1}{\beta}
\end{aligned}$$

(d) Holding τ constant, an increase in β does not change θ^*, so the same "number" of developers supply the market. However, a higher value of β is associated with a steeper pmf of θ, with lower mass for higher values of θ. This decreases the heterogeneity of developers, thus decreasing overall surplus in the market.

Question 3

Denote the parameters and price and quantity variables for each flavor with subscript $i \in \{T, F\}$, for triple-chocolate-chunk and fruit-bowl-punch, respectively. Since Ben and Jerry's is a monopoly, it chooses (Q_i, P_i) such that:

$$\max_{Q_i} \ (a_i - b_i Q_i - c) Q_i$$

Where c is the constant marginal cost for each flavor. Then, the profit-maximizing price and quantity for each flavor is:

$$Q^* = \frac{a_i - c}{2b_i} \qquad\qquad P^* = \frac{1}{2}(a_i + c)$$

Since we know that F has a higher price intercept than T, we also know that Ben and Jerry's will charge a higher price for it.

Question 4

(a)

(b)

(c)

Question 5