Midterm Review

Danny Edgel
Econ 713: Microeconomics I
Spring 2021, Quarter 1

February 16, 2021

NTU Matching

- Positive-Assortative Matching (PAM): The derivative of each side's payoff function has the same sign
- Negative-Assortative Matching (NAM): The derivative of each side's payoff function has the opposite sign
- Gale-Shapley Theorem:
  - Male-pessimal outcome is the female-optimal outcome, and vice versa
  - If male-pessimal and male-optimal outcomes are the same, then the stable matching is unique
- The DAA can take no more than $n^2 - 2n + 2$ rounds, where there are n men and n women
- Solving DAA algorithm in discrete case: Example from first question of 2020 midterm below, showing the DAA matching from women proposing.

	Round									
	1	2	3	4	5	6	7	8	9	10
A	S	R*				P*				M*
B	R*		S*						P*	
C	P*				R*			S*		
D	S*			P*			R*			

Welfare Theorems of Matching

1. A competitive equilibrium yields an efficient matching
2. An efficient matching is a competitive equilibrium for a suitable set of wages

TU Matching

- PAM: Supermodular
  - If differentiable, cross-derivative is positive
  - If not differentiable, increasing differences
- NAM: Submodular
  - If differentiable, cross-derivative is negative
  - If not differentiable, decreasing differences
- Finding wages (differentiable case)
  1. Let $\pi = h(x, y) - v(x) - w(y)$ be the profit function for matchmakers in this market, where $h(x, y)$ is the output of a match. Find FOC for one side of the market[1]
  2. If PAM, solve FOC for first derivative of wage function using $y = x$. If NAM, solve using $y = 1 - x$.
  3. Take antiderivative to determine wage function, including some constant, c. Let k be the constant for the other side's wage function
  4. Impose free entry/exit condition to let $\pi = 0$ at its maximum; solve $\pi(x, y) = 0$ for the relationship between c and k.
  5. Suppose $c + k = S$. Then, $k = S - c$ and the range of market-decentralizing wages is given by the range of c such that the wage of each side is weakly greater than the side's outside option
     - In the typical case where the value of not matching for each side is zero, $c \in [0, S]$
     - Suppose D is the cost of matching for the x side. Then $c \in [-D, S]$
     - If there is a short side of the market, pin wages down uniquely by setting c and k such that the short side captures all surplus. For example, if the mass of type y exceeds that of type x, then $c = 0$, $k = S$.

[1] if FOCs aren't symmetric, repeat steps 1-3 for other side of market