Problem Set #12

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Discussed and/or compared answers with Sarah Bass, Emily Case, Katherine Kwok, Michael Nattinger, and Alex Von Hafften

Exercise 27.1

The latent variable model is:

$$Y^* = X'\beta + e$$

We can derive the conditional means of the censored ($m(X)$) and truncated ($m^{\#}(X)$) variables by assuming that $e|X \sim \mathcal{N}(0, \sigma^2)$. Then, for the censored conditional mean,

And for the truncated conditional mean,

$$\begin{aligned} m^{\#}(X) &= \mathbb{E}\left[Y|X\right] = X'\beta\mathbb{E}\left[\mathbb{1}\left\{Y^* > 0\right\}|X\right] = X'\beta\mathbb{E}\left[\mathbb{1}\left\{e > -X'\beta\right\}|X\right] \\ &= X'\beta\left(1 - \Phi\left(-\frac{X'\beta}{\sigma^2}\right)\right) \end{aligned}$$

Exercise 27.2

No, the OLS estimate of β is biased downward in this model.

Exercise 27.4

An NLLS estimator for the conditional mean of the model in (27.2) is:

$$\min_{\beta,\sigma} \left(Y - X'\beta\Phi\left(\frac{X'\beta}{\sigma}\right) - \sigma\phi\left(\frac{X'\beta}{\sigma}\right)\right)^2$$

Exercise 27.8

The latent variable model for (27.7) is:

$$
\begin{aligned}
Y^* &= X'\beta + e \\
S^* &= Z'\gamma + uS && = \mathbb{1}\{S^* > 0\} \\
Y &= \begin{cases} Y^*, & S = 1 \\ \text{missing}, & S = 0 \end{cases}
\end{aligned}
$$

Assume:

$$
\begin{pmatrix} e \\ u \end{pmatrix} \sim \mathcal{N}\left(0, \begin{pmatrix} \sigma^2 & \sigma_{21} \\ \sigma 21 & 1 \end{pmatrix}\right)
$$

Then,

$$
\begin{aligned}
\mathbb{E}\left[Y|X, Z, S = 1\right] &= \mathbb{E}\left[Y^*|X, Z, S = 1\right] = \mathbb{E}\left[X'\beta + e|X, Z, S = 1\right] \\
&= X'\beta + \mathbb{E}\left[e|X, Z, S = 1\right] = X'\beta + \mathbb{E}\left[e|X\gamma + u > 0\right] \\
\mathbb{E}\left[e|X\gamma + u > 0\right] &= \mathbb{E}\left[e|u > -X\gamma\right] = Cov(e, u)\frac{\phi(Z'\gamma)}{\Phi(Z'\gamma)} = \sigma_{21}\lambda(Z'\gamma) \\
\mathbb{E}\left[Y|X, Z, S = 1\right] &= X'\beta + \sigma_{21}\lambda(Z'\gamma)
\end{aligned}
$$

Exercise 27.9

The results of the OLS regression (using robust standard errors) are displayed in the table at the end of this exercise. Absent of misspecification, these results would suggest that the replationship between transfers and income is linear and perfectly symmetric about \$1,000. However, the dependent variable is censored at zero, so this model is misspecified. The share of observations that are censored is 11.98% . I would expect censoring bias to be a problem in this example.

	(1)	(2)	(3)	(4)	(5)
VARIABLES	(a) OLS	(c) OLS subsample	(d) tobit	(d) tobit	(e) CLAD
Income ('000)	-42.67***	-42.82***	-42.67***		-40.84***
	(3.827)	(3.803)	(2.632)		(0.391)
(Income-1)$\mathbb{1}\{\text{Income} > 1\}$	42.67***	42.86***	42.67***		40.82***
	(3.827)	(3.804)	(2.633)		(0.391)
var(e.tinkind)				384.4***	
				(5.834)	
Constant	49.72***	50.53***	49.72***		42.45***
	(3.810)	(3.774)	(2.609)		(0.386)
Observations	8,684	6,734	8,684	8,684	8,180
R-squared	0.030	0.033			

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Exercise 28.12