Problem Set #3 (2^{nd} Half)

(Due Tuesday, December 1 before midnight)

Economics 709 **Fall 2020**

The numbered exercises listed below are from Hansen, *Econometrics*.

1. 3.24 - 3.25
2. 7.2 - 7.4
3. 7.8
4. 7.9 (a)
5. 7.10
6. 7.13 - 7.15
7. 7.17
8. 7.19
9. Suppose $y_i = 1 + x_i\gamma + \varepsilon_i$, where y_i, x_i, ε_i are scalar.
   Define $w_i = \begin{pmatrix} 1 \\ x_i \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 \\ \gamma \end{pmatrix}$.
   Assume that x_i has a discrete distribution:

$$\Pr(x_i = 1) = \Pr\left(x_i = \frac{4}{3}\right) = \Pr\left(x_i = \frac{5}{3}\right) = \Pr(x_i = 2) = \frac{1}{4}$$

We will use the following assumptions:

(A0) (y_i, x_i) i.i.d.

(A1) $E(\varepsilon_i|w_i) = 0$

(A1$'$) $E(w_i\varepsilon_i) = 0$

(A2) $Var(\varepsilon_i|w_i) = \sigma^2$

Assume that you will observe data $(y_1, x_1), \ldots, (y_n, x_n)$ (a sample of size n). Below, state any additional assumptions *needed* to obtain your answers.

Consider the following OLS estimator from regressing y_i on w_i using only observations where $x_i = 1$ or $x_i = 2$:

$$\hat{\beta} = \left[\frac{1}{n}\sum_{i=1}^{n} w_i\, w_i'\, \mathbf{1}\{x_i \in \{1,2\}\}\right]^{-1} \frac{1}{n}\sum_{i=1}^{n} w_i\, y_i\, \mathbf{1}\{x_i \in \{1,2\}\}$$

where $\mathbf{1}\{A\}$ is an indicator function for the event A.

(a) Under (A0) and (A1), does $\hat{\beta} \xrightarrow{p} \beta$?

(b) Under (A0) and (A1′), does $\hat{\beta} \xrightarrow{p} \beta$?

(c) Under (A0), (A1), and (A2), what is the asymptotic distribution of $\sqrt{n}(\hat{\beta} - \beta)$? Simplify as much as possible.

(d) Consider the following OLS estimator from regressing y_i on w_i using only observations where $x_i = \frac{4}{3}$ or $x_i = \frac{5}{3}$:

$$\hat{\hat{\beta}} = \left[\frac{1}{n} \sum_{i=1}^{n} w_i \, w_i' \, \mathbf{1}\left\{ x_i \in \left\{ \frac{4}{3}, \frac{5}{3} \right\} \right\} \right]^{-1} \frac{1}{n} \sum_{i=1}^{n} w_i \, y_i \, \mathbf{1}\left\{ x_i \in \left\{ \frac{4}{3}, \frac{5}{3} \right\} \right\}$$

Let $\hat{\beta}_2$ and $\hat{\hat{\beta}}_2$ denote the second elements of the vectors $\hat{\beta}$ and $\hat{\hat{\beta}}$. Note that $\hat{\beta}_2$ and $\hat{\hat{\beta}}_2$ are estimators for γ.

Under (A0), (A1), and (A2), which estimator for γ do you prefer $\hat{\beta}_2$ or $\hat{\hat{\beta}}_2$? Explain.

(e) Consider the OLS estimator, $\hat{\alpha}$, from regressing y_i on x_i (no constant term) using only the observations where $x_i = 1$ or $x_i = 2$.
Under (A0), (A1), and (A2), what is the probability limit of $\hat{\alpha}$?

(f) Let α denote your answer to part (e). Under (A0), (A1), and (A2), what is the asymptotic distribution of $\sqrt{n}(\hat{\alpha} - \alpha)$?

Here's the empirical problem that will be due with the next problem set ...

7.28 (a) - (d): Use the subsample of the CPS that you used for problems 3.24 and 3.25 (instead of the subsample requested in the problem)