Econ 711 – Fall 2020 – Problem Set 6

Due online Monday night October 19 at midnight.

Please feel free to work together on these problems (and all homeworks), but each student needs to write up his/her own answers at the end, rather than directly copying from one master solution.

Question 1. Rationalizing Demand

Suppose you observe the following data on prices, wealth, and chosen consumption bundles for a certain consumer at four points in time:

w	p	x
100	$(5, 5, 5)$	$(12, 4, 4)$
100	$(7, 4, 5)$	$(9, 3, 5)$
100	$(2, 4, 1)$	$(27, 9, 10)$
150	$(7, 4, 5)$	$(15, 5, 5)$

(a) Are the data consistent with Walras law?

(b) Can these data be rationalized by a continuous, monotonic and concave utility function? (*Hint: you don't need to calculate the cost of every bundle at every price; if $x^i > x^j$, then $p \cdot x^i > p \cdot x^j$ for any $p \gg 0$.*)

Question 2. Aggregating Demand

Suppose there are n consumers, and consumer $i \in \{1, 2, \ldots, n\}$ has indirect utility function

$$v^i(p, w_i) \quad = \quad a_i(p) + b(p)w_i$$

where $\{a_i\}_{i=1}^n$ and b are differentiable functions from $\mathbb{R}^k_+$ to $\mathbb{R}$.

(a) Use Roy's Identity to calculate each consumer's Marshallian demand $x^i(p, w_i)$.

(b) Calculate the Marshallian demand $X(p, W)$ of a "representative consumer" with wealth W and indirect utility function

$$V(p, W) \quad = \quad \sum_{i=1}^{n} a_i(p) + b(p)W$$

and show that $X(p, \sum_{i=1}^n w_i) \quad = \quad \sum_{i=1}^n x^i(p, w_i)$.

Question 3. Homothetic Preferences

Complete, transitive preferences $\succsim$ on $\mathbb{R}^k_+$ are called *homothetic* if for all $x, y \in \mathbb{R}^k_+$ and all $t > 0$,

$$x \succsim y \qquad \leftrightarrow \qquad tx \succsim ty$$

(a) Show that if preferences are homothetic, Marshallian demand is homogeneous of degree 1 in wealth: for any $t > 0$, $x(p, tw) = tx(p, w)$.

(b) Show that if preferences are homothetic, monotone, and continuous, they can be represented by a utility function which is homogeneous of degree 1. (Hint: try the utility function we used to prove existence of a utility function in class!)

(c) Show that given (a) and (b), the indirect utility function takes the form $v(p, w) = b(p)w$ for some function b.

Question 4. Quasilinear Utility

Let $X = \mathbb{R} \times \mathbb{R}^{k-1}_+$ (allow positive or negative consumption of the first good), suppose utility

$$u(x) \quad = \quad x_1 + U(x_2, \ldots, x_k)$$

is quasilinear, and fix the price of the first good $p_1 = 1$.

(a) Show that Marshallian demand for goods 2 through k does not depend on wealth.

(b) Show that indirect utility can be written as $v(p, w) = w + \tilde{v}(p)$ for some function $\tilde{v}$.

(c) Show the expenditure function can be written as $e(p, u) = u - f(p)$ for some function f.

(d) Show that the Hicksian demand for goods 2 through k does not depend on target utility.

(e) Show that Compensating Variation and Equivalent Variation are the same when the price of good $i \neq 1$ changes, and also equal to Consumer Surplus.