Econ 712 Macroeconomics I

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Problem Set #4

Consider the following overlapping generations problem. Each period $t = 1, 2, 3, ...$ a new generation of 2 period lived households are born. The measure of identical households born in any period grows by $1 + n$. That is, we assume population growth of rate $n \geq 0$.

There is a unit measure of initial old who are endowed with $\overline{M}_1$ units of fiat money as well as w_2 units of consumption goods. Instead of a fixed money supply, now assume that the money supply increases at the rate $z \geq 0$. The increase in money supply is handed out each period to old agents in direct proportion to the amount of money that they chose when young. In other words, if a young agent chooses $M_{t+1}^t \geq 0$, they will receive $(1 + z)M_{t+1}^t$ units of money when old.

Each generation is endowed with w_1 in youth and w_2 in old age of nonstorable consumption goods where $w_1 > w_2$. The utility function of a household of generation $t \geq 1$ is

$$U\left(c_t^t, c_{t+1}^t\right) = \ln(c_t^t) + \ln(c_{t+1}^t)$$

where $\left(c_t^t, c_{t+1}^t\right)$ is consumption of a household of generation t in youth (i.e. in period t) and old age (i.e in period $t+1$). The preferences of the initial old are given by $U(c_1^0) = \ln(c_1^0)$, where c_1^0 is consumption by a household of the initial old.

1. State and solve the planner's problem under the assumption of equal weights on each generation (i.e., on representative agents from each generation) so that the the objective of the social planner is to maximize $U(c_1^0) + \sum_{t=1}^{\infty} U\left(c_t^t, c_{t+1}^t\right)$. Since this objective may not be well defined (i.e. add up to infinity), we can apply the

"overtaking" criterion to determine optimality.[1] Hence just go ahead and maximize away as usual.

2. Let p_t be the price of consumption goods in terms of money at time t. Define a competitive equilibrium.

3. Solve for an autarkic equilibrium.

4. Solve for a steady state (non-autarkic) monetary equilibrium. As $w_1 > w_2$, we know this corresponds to the Samuelson case with $\beta = 1$. Verify the non-negativity constraint on money is not binding in the equilibrium. What is the rate of return on money in the monetary equilibrium? Give intuition why the rate of return is at the level you find. Hints:

   (a) Solve for the initial old and generation t's problems given goods prices (p_t, p_{t+1}) for $t \geq 1$. That is, solve for $(c_t^t, c_{t+1}^t, M_{t+1}^t)$ given (p_t, p_{t+1}).

   (b) Denote $q_t := \frac{p_t}{p_{t+1}}$ and use goods market clearing with the optimal consumption functions $c_t^{t-1}(p_{t-1}, p_t), c_t^t(p_t, p_{t+1})$ derived above to get a first-order difference equation in q_t.

   (c) Solve for steady states $\bar{q}$. Show that one steady state of $\bar{q}$ correspondes to a steady state monetary equilibrium with inter-generational trade.

5. Does the stationary monetary equilibrium Pareto dominate autarky? Can you use your answer in part 1 to establish that? If so, how can the government implement it?

[1]The "overtaking criterion" states that an allocation $\{c_t^{t-1}, c_t^t\}_{t=1}^T$ overtakes $\{\widehat{c}_t^{t-1}, \widehat{c}_t^t\}_{t=1}^T$ if

$$\liminf_{T\to\infty} \left[U(c_1^0) + \sum_{t=1}^{T} U(c_t^t, c_{t+1}^t) - U(\widehat{c}_1^0) + \sum_{t=1}^{T} U(\widehat{c}_t^t, \widehat{c}_{t+1}^t) \right] > 0.$$

Since the finite sum is well defined, this sequence is well defined.

6. Does money exhibit super-neutrality (i.e. the level of inflation does not change equilibrium consumption allocations)?