# Econ 711 – Fall 2020 – Problem Set 4

## Due online Monday night October 5 at midnight.

Please feel free to work together on these problems (and all homeworks), but each student needs to write up his/her own answers at the end, rather than directly copying from one master solution.

### Question 1. Choice rules from preferences

Let X be a choice set and  $\succeq$  a complete and transitive preference relation on X. Show that the choice rule induced by  $\succeq$ ,

$$C(A, \succsim) = \{x \in A : x \succsim y \ \forall y \in A\}$$

must satisfy the Weak Axiom of Revealed Preference (WARP).

#### Question 2. Preferences from choice rules

Let X be a choice set and  $C: \mathcal{P}(X) \to \mathcal{P}(X)$  a nonempty choice rule. Show that if C satisfies WARP, then the preference relation  $\succeq_C$  defined on X by

$$x \succsim_C y$$
 if and only if there exists a set  $A \subseteq X$  such that  $x, y \in A$  and  $x \in C(A)$ 

is complete and transitive, and that the choice rule it induces,  $C(\cdot, \succsim_C)$ , is equal to C.

### Question 3. Choice over finite sets

Let X be a **finite** set, and  $\succeq$  a complete and transitive preference relation on X.

- (a) Show that the induced choice rule  $C(\cdot, \succeq)$  is nonempty that  $C(A, \succeq) \neq \emptyset$  if  $A \neq \emptyset$ .
- (b) Show that a utility representation exists.

(Hint: for part (a), fix X finite, and prove that for  $A \neq \emptyset$ ,  $C(A, \succeq) \neq \emptyset$  by induction on the number of elements in the set A. For part (b), use induction on |X| to prove the stronger result that when X is finite, a utility representation exists with range  $\{1, 2, \ldots, |X|\}$ .)