# Midterm Review

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## Non-Transferable Utility (NTU) Matching

- Positive-Assortative Matching (PAM): The derivative of each side's payoff function has the same sign
- Negative-Assortative Matching (NAM): The derivative of each side's payoff function has the opposite sign
- Gale-Shapley Theorem:
  - Male-pessimal outcome is the female-optimal outcome, and vice versa
  - If male-pessimal and male-optimal outcomes are the same, then the stable matching is unique
- The DAA can take no more than $n^2 - 2n + 2$ rounds, where there are n men and n women
- Solving DAA algorithm in discrete case: Example from first question of 2020 midterm below, showing the DAA matching from women proposing.

| | Round | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| A | S | R* | | | | P* | | | | M* |
| B | R* | | S* | | | | | | P* | |
| C | P* | | | | R* | | | S* | | |
| D | S* | | | P* | | | R* | | | |

## Welfare Theorems of Matching

1. A competitive equilibrium yields an efficient matching
2. An efficient matching is a competitive equilibrium for a suitable set of wages

# Transferable Utility (TU) Matching

- PAM: Supermodular
  - If differentiable, cross-derivative is positive
  - If not differentiable, increasing differences
- NAM: Submodular
  - If differentiable, cross-derivative is negative
  - If not differentiable, decreasing differences
- Competitive mechanics of a TU matching market: there exists an industry of "match-makers" who get the output from the matches they make and pay wages to each side.
  - If matchmakers are making a profit, then free entry of new match-makers hold that new match-makers will enter and out-compete incumbent match-makers by either making better matches or paying higher wages. Thus, the profit of match-makers has a maximum of 0
  - If matchmakers make inefficient matches, then they will be unable to pay high enough wages to at least one side of the match to stave off more efficient matchmakers from offering higher wages for a more efficient match
- Finding wages (differentiable case)
  1. Let $\pi = h(x, y) - v(x) - w(y)$ be the profit function for matchmakers in this market, where $h(x, y)$ is the output of a match. Find FOC for one side of the market[1]
  2. If PAM, solve FOC for first derivative of wage function using $y = x$. If NAM, solve using $y = 1 - x$.
  3. Take antiderivative to determine wage function, including some constant, $c$. Let k be the constant for the other side's wage function
  4. Impose free entry/exit condition to let $\pi = 0$ at its maximum; solve $\pi(x, y) = 0$ for the relationship between $c$ and $k$.
  5. Suppose $c + k = S$. Then, $k = S - c$ and the range of market-decentralizing wages is given by the range of $c$ such that the wage of each side is weakly greater than the side's outside option
     - In the typical case where the value of not matching for each side is zero, $c \in [0, S]$
     - Suppose $D$ is the cost of matching for the $x$ side. Then $c \in [-D, S]$

[1] if FOCs aren't symmetric, repeat steps 1-3 for other side of market

  - If there is a short side of the market, pin wages down uniquely by setting $c$ and $k$ such that the short side captures all surplus. For example, if the mass of type $y$ exceeds that of type $x$, then $c = 0$, $k = S$.

- Finding wages (discrete case)

  1. Determine the efficient match (i.e. the one that maximimizes total output)
  2. Set up the inequalities for the wages in this match:
     - The sum of the two sides in each efficient match can be no greater than the output of that match
     - The sum of each individual's wage and the wage of their next-best match must be weakly greater than their match output (in order to dissuade that match)
  3. Due to competitive mechanics of TU matching, match condition (1) will hold with equality
  4. Use these two sets of inequalities to derive the intervals for each individual's wage

# Double Auctions

- In a double auction, there are $n$ buyers with valuations $v_i$ for a homogenous good and $m$ sellers of the good with costs $c_j$

- NAM theorem: NAM of buyers and sellers in a double auction results in a total surplus that is weakly greater than any other matching method

- Solving for equilibrium in a double auction:

  1. Determine market demand by sorting valuations in descending order and summing demand at each price
  2. Determine market supply by sorting costs in ascending order and summing supply at each price
  3. Tip for steps 1 and 2: Draw up a table of each side of the market, with columns representing the open-closed interval of demand/supply at each price/cost/valuation—e.g., for $v_i = \$10$, if adding $i$'s demand at a price of \$10 takes demand from 20 units to 25, then have one column that represents pre-\$10 demand and another with demand at \$10. This makes graphing the staircase-looking function much easier.
  4. Either visually identfy on the tables where the two curves intersect, or begin testing plausbile prices, raising or lowering price "guesses" according to supply and demand at each guessed price
  5. After determining an equilibrium price/quantity, determine whether it is unique by graphing supply and demand around that point

- Collusion outcomes:
  - Sellers collude: Total seller profit is maximized. To solve:
    1. Begin with the highest price that won't decrease demand
    2. Calculate the change in total seller profit if you raise to the next buyer's valuation
    3. If (2) is negative, then the price from (1) is optimal. If (2) is positive, repeat step 2 until you find the price that results in negative marginal profit
  - Buyers collude: Total consumer surplus is maximized. To solve, use same method as solving for the seller collusion outcome, but with progressively lowering the price