Homework #6

Danny Edgel
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Collaborated with Sarah Bass, Emily Case, Michael Nattinger, and Alex Von Hafften

Question 1

In order for $\{(C, C), (C, C), ...\}$ to be supported in a subgame perfect equilibrium, it must be the case that neither player has incentive to deviate and earn a one-time payoff of 8 despite getting a lower payoff of 1 in each period thereafter. Thus, in each period, the following condition must be satisfied:

$$\sum_{t=0}^{\infty} 2\delta^t \geq 8 + \sum_{t=1}^{\infty} \delta^t$$

Solving this for δ yields:

$$\begin{aligned} 2\left(\frac{\delta}{1-\delta}\right) &\geq 8 + \frac{\delta}{1-\delta} \\ \delta &\geq 6(1-\delta) \\ \delta &\geq \frac{6}{7} \end{aligned}$$

Thus, $\delta \in \left[\frac{6}{7}\right]$ in order for $\{(C, C), (C, C), ...\}$ to be supported in a subgame perfect equilibrium.

Question 2

(a) In order for (σ_1, σ_2) to be an SPE, it must the case that a one-time deviation to D, which earns a payoff of 3, is outweighed by a one-time penalty of 0. this is true when the following inequality is satisfied:

$$2 + 2\delta + \sum_{t=2}^{\infty} 2\delta^t \geq 3 + 0\delta + \sum_{t=2}^{\infty} 2\delta^t$$

Solving for δ,

$$2 + 2\delta \geq 3$$
$$\delta \geq \frac{1}{2}$$

(b) If (P, P) resulted in a payoff of 1/2 for both players, then the condition would become:

$$2 + 2\delta \geq 3 + \frac{1}{2}\delta$$
$$\delta \geq \frac{2}{3}$$

(c) As I explained in (a), the condition for sustaining (σ_1, σ_2) as an SPE is ensuring that the penalty of deviation to (P, P) is large enough keep it in your opponent's best interest from deviating to their higher-payoff outcome (of (C, D) or (D, C)). Increasing the payoff from the penalty outcome requires a higher weight on the difference between the equilibrium-path outcome (2) relative to the penalty outcome (1/2). Since the penalty outcome occurs in the period after the hypothetical deviation, a lower δ decreases the penalty from deviation.

(d) Presuming that the question is "for what set of (P, P) payoffs does their exist a $\delta < 1$ such that (σ_1, σ_2) can be supported as an SPE?", we must determine the symmetric payoff, x, that satisfies:

$$\lim_{\delta \to 1^-} \{2 + 2\delta\} \geq \lim_{\delta \to 1^-} \{3 + x\delta\}$$

To understand what this means for all feasible values of x given some $\delta < 1$, we can solve for x and relate that solution to $\delta = 1$:

$$x \leq 2 - \frac{1}{\delta} < 1$$

Thus, in order for (σ_1, σ_2) to be supported as an SPE, the symmetric payoff from (P, P) must be less than 1.

Question 3

(a) On this equilibrium path, the player with an incentive to deviate is the C player, who prefers a payoff of 0 to a payoff of -1. Since any player can trigger the 0 payoff, this requires no coordination and can thus be used as a punishment, since the deviating player would have otherwise received a payoff of 2 during the punishment round. The pure strategy profile for this equilibrium, then, is:

(I) Play (A, B, C) initially, or if (C, A, B) was played last. Play (B, C, A) if (A, B, C) was played last and (C, A, B) if (B, C, A) was played last

(II) If there is a deviation from (I), play (D, D, D) (or any other play that involves a non-deviating player playing D) once, then restart (I)

(III) If there is a deviation from (II), then restart (II)

An equilibrium path with this pure strategy profile can be supported if the payoff from the equilibrium path at least matches that of a deviation from it:

$$-1 + 2\delta + 0\delta^2 \geq 0 + 0\delta + 0\delta^2$$
$$\delta \geq \frac{1}{2}$$

Thus, this strategy profile is an SPE if $\delta \geq \frac{1}{2}$.

(b) In this new equilibrium, the deviating player is slated (under the equilibrium path) to play B following their deviation, rather than A. Under the old punishment scheme, each player would prefer deviating to the equilibrium path under any δ. An alternative is to adopt a new punishment scheme that deviates to (D, D, D) for some L periods where the deviating player was slated to play A. Under any of these equilibrium plays that could sustain the new path, the old path could also be sustained in the old equilibrium with an even lower δ. This is because, under any δ, it takes a lower δ to punish a deviation when the player with an incentive to deviate would get their maximum payoff (of 2, in this case) immediately following their deviation were they not to deviate.

Question 4

(a) An equivalent behavioral strategy is presented below.

$$\beta_1 = ((\beta_1(A), \beta_1(B)), (\beta_1(E), \beta_1(F), \beta_1(G))) = \left(\left(\frac{5}{6}, \frac{1}{6}\right), \left(0, \frac{1}{2}, \frac{1}{2}\right)\right)$$

(b) Each of the mixed strategies involving A must be played with frequencies that sums to $\frac{1}{3}$. In other words,

$$\sigma_1(AE) + \sigma_1(AF) + \sigma_1(AG) = \beta_1(A) = \frac{1}{3}$$

Half of the time that B is played, E is also played. The other half of the time is split between F and G. Since B is played $\frac{2}{3}$ of the time,

$$\sigma_1(BE) = \frac{1}{3},\ \sigma_1(BF) = \sigma_1(BG) = \frac{1}{6}$$

Question 5

To compute the sequential equilibria of this game, let us consider possible beliefs held by player 3, then deduce consistent and sequentially rational strategies by each player:

1. $\mu(D) = \mu(d) = 0$

   sequential rationality $\Rightarrow$ player 3's information set is never reached, so $\beta_3(R) \in [0, 1]$

   sequential rationality $\Rightarrow \beta_2(d) = 0$ if $\beta_3(R) \in \left[0, \dfrac{2}{3}\right)$

   sequential rationality $\Rightarrow \beta_1(D) = 0$ if $\beta_2(a) \geq \dfrac{1}{2}\beta_3(R)$

   consistency $\Rightarrow \mu(D) = \mu(d) = 0$

   $\therefore$ $\mu(D) = \mu(d) = 0$, $\beta_1(D) = 0$, $\beta_2(d) = 0$, and $\beta_3(R) \in [0, \frac{2}{3})$ is a sequential equilibrium

2. $\mu(D) = \mu(d) = 1$

   $$\begin{aligned}\text{sequential rationality} &\Rightarrow \beta_3(L) = 1\\ \text{sequential rationality} &\Rightarrow \beta_2(d) = 0\\ \text{sequential rationality} &\Rightarrow \beta_1(D) = 0\end{aligned}$$

   μ is not consistent, so (β, μ) is not a sequential equilibrium

3. $\mu(D) \in (0, 1) \wedge \mu(d) = 1$

   $$\begin{aligned}\text{sequential rationality} &\Rightarrow \beta_3(L) = 1\\ \text{sequential rationality} &\Rightarrow \beta_2(d) = 0\\ \text{sequential rationality} &\Rightarrow \beta_1(D) = 0\end{aligned}$$

   μ is not consistent, so (β, μ) is not a sequential equilibrium

4. $\mu(D) = 1 \wedge \mu(d) \in (0, 1)$

   $$\begin{aligned}\text{sequential rationality} &\Rightarrow \beta_3(L) = 1\\ \text{sequential rationality} &\Rightarrow \beta_2(d) = 0\\ \text{sequential rationality} &\Rightarrow \beta_1(D) = 0\end{aligned}$$

   μ is not consistent, so (β, μ) is not a sequential equilibrium

5. $\mu(D) \in (0, 1) \wedge \mu(d) \in (0, 1)$

   sequential rationality $\Rightarrow \beta_3(R) \in (0, 1)$ if $\dfrac{\beta_1(D)}{\beta_1(D) + (1 - \beta_1(D))\beta_2(d)} = \dfrac{2}{3}$

   sequential rationality $\Rightarrow \beta_2(d) \in (0, 1)$ if $\beta_3(R) = \dfrac{2}{3}$

   sequential rationality $\Rightarrow \beta_1(D) \in (0, 1)$ if $\dfrac{\beta_2(a)}{\beta_3(R)} = \dfrac{1}{2}$

   $\therefore$ there is a mixed-strategy equilibrium at:

   $$\{(\mu(D), \mu(d)), (\beta_1(D), \beta_2(d), \beta_3(R))\} = \left\{\left(\frac{1}{3}, \frac{1}{4}\right), \left(\frac{4}{7}, \frac{1}{4}, \frac{2}{3}\right)\right\}$$

Question 6

Question 7

Let x be the top node for player 2 and $\mu(x)$ be player 2's belief that she is at the top node. Regardless of which node player 2 is on, B is dominated by one of the other two moves, so $\beta_2(T) + \beta_2(M) = 1$.

After removing B from play, I is dominated by O for player one if player 1 is of type t_b. Thus, in order for beliefs to be consistent, $\mu(x) = 1$. Sequential rationality then dictates that $\beta_2(T) = 1$. Given β_2, player 1's sequentially rational move when type t_a is I. Thus, there is a separating sequential equilibrium at

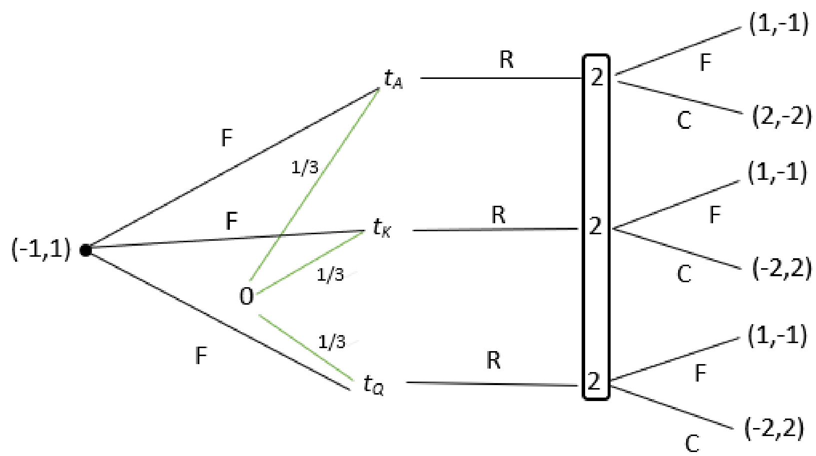
$$\{\{(\beta_{t_a}(I), \beta_{t_b}(I), \beta_2(T)), \mu(x)\} = \{\{(1, 0, 1), 1\}$$

In order for the intuitive criterion to be violated, one of player 1's types would have have a strictly dominant strategy that would enable them to make a higher payoff than the equilibrium payoff were they to deviate to it. The only possible payoff improvement for player 1 is if t_b could play I and have player 2 play B. However, following ISD, I is strictly dominated for t_b. Thus, the intuitive criterion is not violated.

Suppose that $\mu(x) \leq 0.5$. Then, player 2's sequentially rational strategy is to play $\beta_2(M) = 0$ if $\mu(x) < \frac{1}{2}$ and to mix with $\beta_2(T) \leq \frac{1}{3}$ if $\mu(x) \leq 0.5$. This is a pooling equilibrium where both types of player 1 choose to play O and player 2 is never driven to update her beliefs about the behavior of player 1. However, were player 1 to deviate, player 2 would recognize that such a deviation is dominated by O for type t_b. Thus, a best response for player 2 would be to play $\beta_2(T) = 1$. Therefore, this equilibrium does not satisfy the intuitive criterion.

Question 8

(a) An extensive form game, Γ, of this interaction is drawn below.



(b) Player 1 could be of three possible types, under each of which she can have a different strategy. Thus,

$$S_1 = \{RRR, RRF, RFF, FFF, FFR, FRR, FRF, RFR\}\,,\ S_2 = \{C, F\}$$

(c) As far as either player is concerned, player 1 drawing a queen is identical to player 1 drawing a king. F is strictly dominated for t_A, so there is no equilibrium in which $\beta_{t_A}(F) > 0$. If player 2 plays a pure strategy of always folding (say, because $\mu(t_A) = 1$), then player 1 will have an incentive to raise regardless of what card she draws.

This cannot be an equilibrium, because any beliefs with which this pure strategy from player 2 are rational would then not be consistent. On the other hand, the pure strategy $\beta_2(C) = 1$ is sequentially rational if $\mu(t_A) \leq \frac{3}{4}$. Player 1's only sequentially rational response to this strategy is to only raise if she draws an ace, rendering μ inconsistent. Thus, there are no equilibria in which player 2 plays a pure strategy.

Player 2 is willing to mix if $\mu(t_A) = \frac{3}{4}$, and these beliefs are only consistent in a pooling equilibrium. According to Bayes rule,

$$\mu(t_A) = \frac{\frac{1}{3}\beta_{t_A}(R)}{\frac{1}{3}\left(\beta_{t_A}(R) + \beta_{t_K}(R) + \beta_{t_Q}(R)\right)} = \frac{1}{1 + \beta_{t_K}(R) + \beta_{t_Q}(R)}$$

Then, in order for beliefs to be consistent, $\beta_{t_K}(R) + \beta_{t_Q}(R) = \frac{1}{3}$. In order for player 1 to mix with these weights when she doesn't draw an ace, she must be indifferent between raising and folding, based on player 2's strategy:

$$-2\beta_2(C) + 1 - \beta_2(c) = -1 \iff \beta_2(C) = \frac{2}{3}$$

Therefore, there appears to be one sequential equilbrium at:

$$\left\{\left(\left(\beta_{t_A}(R), \beta_{t_K}(R), \beta_{t_Q}(R)\right), \beta_2(C)\right), \mu(t_A)\right\} = \left\{\left(\left(1, \left[0, \frac{1}{3}\right], \frac{1}{3} - \beta_{t_K}(R)\right), \frac{2}{3}\right), \frac{3}{4}\right\}$$