Problem Set #1

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Econ 709: Economic Statistics and Econometrics I
Fall 2020

September 17, 2020

Question 1

Suppose that $Y = X^3$ and $f_X(x) = 42x^5(1-x)$, $x \in (0,1)$. Find the PDF of Y, and show that the PDF integrates to 1.

Question 2

Consider the CDF $F_X(x) = \begin{cases} 1.2x & \textbf{if } x \in [0, 0.5) \\ 0.2 + 0.8x & \textbf{if } x \in [0.5, 1] \end{cases}$, and the function

$$f_X(x) = \begin{cases} 1.2 & \text{if } x \in [0, 0.5) \\ a & \text{if } x = 0.5 \\ 0.8 & \text{if } x \in [0.5, 1] \end{cases}$$

Show that f_X is the density function of F_X as long as $a \geq 0$. That is, show that for all $x \in [0,1]$, $F_X(x) = \int_0^x f_X(t)dt$.

Question 3

Let X have the PDF $F_X(x) = \frac{2}{9}(x+1)$, $x \in [-1, 2]$. Find the PDF of $Y = X^2$. Note that this is a bit different from the exercise in the lecture note.

Question 4

A median of a distribution is a value m such that $P(X \leq m) \geq \frac{1}{2}$ and $P(X \geq m) \geq \frac{1}{2}$. Find the median of the distribution $f(x) = \frac{1}{\pi(1+x^2)}$, $x \in \mathbb{R}$.

Question 5

Show that if X is a continuous random variable, then $\min\limits_a E|X - a| = E|X - m|$, where m is the median of X.

(hint: work out the integral expression of $E|X - a|$ and notice that it is differentiable)

Question 6

Let μ_n denote the nth central moment of a random variable X. Two quantities of interest, in addition to the mean and variance are

$$\alpha_3 = \frac{\mu_3}{\mu_2^{3/2}} \text{ and } \alpha_4 = \frac{\mu_4}{\mu_2^2}$$

The value α_3 is called the skewness and α_4 is called the kurtosis. The skewness measures the lack of symmetry in the density function. The kurtosis measures the peakedness or flatness of the density function.

1. **Show that if a density function is symmetric about a point a, then $\alpha_3 = 0$**

2. **Calculate α_3 for $f(x) = e^{-x}$, $x \geq 0$, a density function that is skewed to the right.**

3. **Calculate α_4 for the following density functions and commend on the peakedness of each:**

   (a) $\mathbf{f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, x \in \mathbb{R}}$

   (b) $\mathbf{f(x) = 1/2, x \in (-1, 1)}$

(c) $\mathbf{f(x) = \frac{1}{2}e^{-|x|}}$, $\mathbf{x} \in \mathbb{R}$