Problem Set #6

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Question 1

Find $\mathbb{E}[\mathbb{E}[\mathbb{E}[\mathbf{Y}|\mathbf{X_1},\mathbf{X_2},\mathbf{X_3}]|\mathbf{X_1},\mathbf{X_2}]|\mathbf{X_1}]$

By the Law of Iterated Expectation,

$$\begin{aligned}\mathbb{E}[\mathbb{E}[\mathbb{E}[Y|X_1,X_2,X_3]|X_1,X_2]|X_1] &= \mathbb{E}[\mathbb{E}[Y|X_1,X_2]|X_1] \\ \mathbb{E}[\mathbb{E}[Y|X_1,X_2]|X_1] &= \mathbb{E}[Y|X_1]\end{aligned}$$

Thus, $\mathbb{E}[\mathbb{E}[\mathbb{E}[Y|X_1,X_2,X_3]|X_1,X_2]|X_1] = \mathbb{E}[Y|X_1]$

Question 2

Prove that for any function $h(x)$ such that $\mathbb{E}\,|h(X)e| < \infty$ then $\mathbb{E}[h(X)e] = 0$, where $e = Y - m(X)$ and $m(X) = \mathbb{E}[Y|X]$

According to the conditioning theorem, if $\mathbb{E}\,|Y| < \infty$, then

$$\mathbb{E}[g(X)Y|X] = g(X)\,\mathbb{E}[Y|X]$$

Thus, Since $\mathbb{E}\,|h(X)e| < \infty$ trivially implies $\mathbb{E}\,|Y| < \infty$, we can use the Law of Iterated Expectation to solve:

$$\begin{aligned}\mathbb{E}[h(X)e] &= \mathbb{E}[h(X)Y - h(X)m(X)] = \mathbb{E}[h(X)Y] - \mathbb{E}[h(X)m(X)] \\ &= \mathbb{E}[\mathbb{E}[h(X)Y|X]] - \mathbb{E}[h(X)m(X)] = \mathbb{E}[h(X)\,\mathbb{E}[Y|X]] - \mathbb{E}[h(X)m(X)] \\ &= \mathbb{E}[h(X)m(X)] - \mathbb{E}[h(X)m(X)] = 0\end{aligned}$$

$\therefore$ for any function $h(x)$ such that $\mathbb{E}\,|h(X)e| < \infty$ then $\mathbb{E}[h(X)e] = 0$ ■

Question 3

$$\mathbb{E}[Y|X] = \begin{cases} .4, & X = 0 \\ .3, & X = 1 \end{cases}$$

$$\mathbb{E}[Y^2|X] = \begin{cases} .4, & X = 0 \\ .3, & X = 1 \end{cases}$$

$$Var(Y|X) = \mathbb{E}[Y^2|X] - (\mathbb{E}[Y|X])^2 = \begin{cases} .24, & X = 0 \\ .21, & X = 1 \end{cases}$$

Question 4

Show that $\sigma^2(X)$ minimizes the mean-squared error and is thus the best predictor.

Question 5

2.8

Question 6

2.10 - 2.14 Explain your answers.

Question 7

2.16

Question 8

4.1 - 4.6