Problem Set #2 (2^{nd} Half)

(Due Sunday, November 22 before midnight)

Economics 709 **Fall 2020**

You may work in groups. However, please note that you must write up your own solutions in your own words.

The exercises listed below are from Hansen, *Econometrics*.

1. 3.2
2. 3.5 - 3.7
3. 3.11 - 3.13
4. 3.16
5. 3.21 - 3.23
6. 3.24 - 3.25 (due with the next problem set)
7. Given the $n \times 1$ vector y and the $n \times k$ matrix X. Assume: $\text{rank}(X) = k$; $E(y|X) = X\beta$; and $\text{Var}(y|X) = \sigma^2 I$.

   Partition X: $X = [X_1 \;\; X_2]$ where X_1 is $n \times k_1$, X_2 is $n \times k_2$, and $k_1 + k_2 = k$. And similarly partition β: $\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$, where β_1 is $k_1 \times 1$ and β_2 is $k_2 \times 1$.

   (a) Consider the OLS regression of y on X that yields the OLS estimator $\hat{\beta}$. What is $E(\hat{\beta}_1|X)$? Simplify your answer.

   (b) Let $\hat{y} = X\hat{\beta}$. Now, consider the OLS regression of $\hat{y}$ on X_1 that yields the OLS estimator $\hat{\tilde{\beta}}_1$. What is $E(\hat{\tilde{\beta}}_1|X)$? (Simplify your answer.) Is $\hat{\tilde{\beta}}_1$ an unbiased estimator of β_1?

   (c) Consider the OLS regression of y on X_1 that yields the OLS estimator $\tilde{\beta}_1$. Let $\tilde{y} = X_1\tilde{\beta}_1$.

   Now consider the OLS regression of $\tilde{y}$ on X that yields the OLS estimator $\tilde{\tilde{\beta}}$. How is $\tilde{\tilde{\beta}}$ related to $\tilde{\beta}_1$? (Provide a mapping between $\tilde{\tilde{\beta}}$ and $\tilde{\beta}_1$ that does not involve X.)

   (d) What is the R^2 for the OLS regression of $\tilde{y}$ on X (from part (c))? Simplify your answer.

   (e) What is $\text{Var}(\tilde{\tilde{\beta}}|X)$? Simplify your answer.