Problem Set #1

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Question 1

The social planner's problem is:

$$\max_{\{I_t,C_t,K_{t+1}\}_{t=0}^{\infty}} \beta^t U(C_t) \text{ s.t. } K_{t+1} = (1-\delta)K_t + I_t - D_t,\ C_t + I_t \leq F(K_t)$$

At any optimum, the resource constraint will hold with equality, so we can combine the law of motion and the resource constraint to obtain a single constraint:

$$F(K_t) = K_{t+1} - (1-\delta)K_t + C_t + D_t$$

We can derive the Euler equation by taking the first-order conditions of the Lagrangian function.:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t U(C_t) - \lambda_t \left(F(K_t) - K_{t+1} + (1-\delta)K_t - C_t - D_t\right)$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t U'(C_t) + \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \lambda_t - \lambda_{t+1}\left(F'(K_{t+1}) + 1 - \delta\right) = 0$$

Taken together, these first-order conditions give us the Euler equation for the SPP:

$$U'(C_t) = \beta U'(C_{t+1})\left[F(K_{t+1}) + 1 - \delta\right]$$

Question 2

Question 3

Question 4