

Homework #1

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Question 1

Suppose $t_i \in S_i$ is strictly dominated by $s_i \in S_i$, but that $\sigma_i \in \Delta S_i$, which is supported by t_i is not strictly dominated. Let $\sigma'_i \in \Delta S_i$ be a mixed strategy that has the same support as σ_i , but with s_i played with the same frequency as t_i instead of t_i . Since s_i strictly dominates t_i , this strategy results in a strictly higher payoff than σ_i . Therefore, σ'_i strictly dominates σ_i .

\therefore by contradiction, any mixed strategy that contains a strictly-dominated pure strategy in its support is strictly dominated ■

Question 2

- (a) This scenario is a game with two players ($N = \{1, 2\}$) with identical strategy sets $S_i = \{2, 3, \dots, 499, 500\}$, $i = 1, 2$, and payoff functions:

$$u_i(s_i, s_j) = \begin{cases} s_i + 2, & s_i < s_j \\ s_i, & s_i = s_j \\ s_j - 2, & s_i > s_j \end{cases}, i \in \{1, 2\}, j \neq i$$

- (b) Player 1's payoff maximization problem is

$$\max_{s_1} u_1(s_1, s_2)$$

Where player 1's payoff matrix is:

	$s_2 < \bar{s}_2$	$s_2 = \bar{s}_2$
$s_1 < \bar{s}_2$	$[s_2 - 2, s_1 + 2]$	$s_1 + 2$
$s_1 = \bar{s}_2$	$s_2 - 2$	s_1
$s_1 > \bar{s}_2$	$s_2 - 2$	$s_2 - 2$

Thus, if $s_2 = \bar{s}_2$, then player 1's best response is clearly less than \bar{s} , and if $s_2 < \bar{s}_2$, then choosing $s_1 < \bar{s}_2$ will, at worst, make player 1 as poor-off as if they chose $s_1 \geq \bar{s}_2$ but will possibly make them better-off. Thus, player 1's best response is $s_1 < \bar{s}_2$.

- (c) Player 2 faces the same best response function that player 1 does. If you begin with the presumption that player 1 believes $\bar{s}_2 \in [2, 500]$ and iteratively remove strictly dominated strategies for each player, then you arrive at $s_1 = s_2 = 2$ regardless of your initial choice of \bar{s}_2 .

Question 2

- (a) No pure strategies in this game dominate any other pure strategies. However, player 2's pure strategy of C is dominated by a mixed strategy of L and R . Specifically, any strategy $\sigma_2 = \pi L + (1 - \pi)R$, where $\pi \in (\frac{1}{3}, \frac{1}{2})$ dominates C .
- (b) If there is common knowledge of rationality between the players, then no player will play a strictly dominated strategy, and each player will choose a strategy as if the other player will not play a strictly dominated strategy. Thus, player 2 will choose π such that player 1 is indifferent between T and B . Player 1's payoff, based on π is:

$$u_1(T) = 8(1 - \pi), u_1(B) = 2\pi + (1 - \pi)5$$

Player 1 is indifferent between T and B when $\pi = \frac{3}{5}$. Meanwhile, player 2 is indifferent between L and R when player 1 plays T with probability $\frac{8}{11}$. Thus, our prediction of play in this game is

$$\left(\frac{8}{11}T + \frac{3}{11}B, \frac{3}{5}L + \frac{2}{5}R \right)$$