Problem Set #5

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Questions 1

If we begin with a flexible-price version of this model, we being by solving the household problem with the consumption bundle as a choice variable alongside labor and investment. Households, then, maximize the Lagrangian:

$$\mathcal{L} = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\log\left(C_t\right) - L_t\right) - \lambda_t \left(P_t C_t + B_t - W_t L_t + \Pi_t - (1+i_{t-1})B_{t-1} + \mathrm{T}_t\right)\right]$$

This problem has three first-order conditions, for each choice variable:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= \frac{\beta^t}{C_t} - \lambda_t P_t = 0 \\
\frac{\partial \mathcal{L}}{\partial L_t} &= -\beta^t + \lambda_t w_t = 0 \\
\frac{\partial \mathcal{L}}{\partial B_t} &= -\lambda_t + \lambda_{t+1}(1+i_t) = 0
\end{aligned}$$

Combining the FOCs for consumption and labor yield a static labor supply curve, while combining the FOCs for consumption yield the model's Euler equation:

$$C_t = \frac{W_t}{P_t} \qquad\qquad C_t^{-1} = \beta \mathbb{E}_t\left[\left(\frac{P_t}{P_{t+1}}\right)\frac{1+i_t}{C_{t+1}}\right]$$

Since C_t is a standard CES aggregator and Y_{it} is a standard production function with only labor as an input, we know that the firm problem is solved by the

Dixit-Stiglitz price equations:[1]

$$P_{it} = \left(\frac{\theta}{\theta-1}\right)\frac{W_t}{A_t} \qquad\qquad P_t = \left(\int P_{it}^{1-\theta} di\right)^{\frac{1}{1-\theta}}$$

The steady-state of this model is determined by simply making each variable static across periods and combining the above equations, which yields:

$$\overline{C} = \frac{\overline{W}}{\overline{P}} \qquad\qquad \bar{i} = \frac{1}{\beta} - 1$$

$$\overline{P_i} = \left(\frac{\theta}{\theta-1}\right)\frac{\overline{W}}{\overline{A}} \qquad\qquad \overline{P} = \overline{P_i} N^{\frac{1}{1-\theta}}$$

Log-linearizing our equations for this model provides the following, which describe the dynamics of the model:

$$p_{it} = w_t - a_t \qquad\qquad c_t = w_t - p_t$$

$$p_t = p_{it} \qquad\qquad c_t = \mathbb{E}_t\left[c_{t+1} + p_{t+1} - p_t - i_t\right]$$

Finally, since labor is inelastically supplied at $L_t = 1$, $\ell_t = 0$. Note that we can combine the equations for p_t, p_{it}, and c_t to determine that $c_t = a_t$, which shows that all fluctuations in consumption come exclusively from productivity shocks.

[1] An unstated assumption is that $L_t = 1$. This comes from the fact that $\varphi = 0$ and thus firms are able to adjust wages and prices instantaneously to ensure a perfectly inelastic labor supply.

Question 2

(a) The problem for a firm in this market is:

$$\max_{\{P_{it}\}} \mathbb{E}_t \left[\sum_{j=0}^{\infty} \Theta_{t,t+j} \left(P_{it+j}C_{it+j} - W_{t+j}L_{it+j} - \frac{\varphi W_{t+j}}{2} \left(\frac{P_{it+j} - P_{it+j-1}}{P_{it+j-1}} \right)^2 \right) \right]$$

$$\text{s.t. } C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\theta} C_t,\ C_{it} = A_t L_{it}$$

Where $\Theta_{t,t+j}$ is taken from the household's Euler equation as the stochastic discount factor,

$$\Theta_{t,t+j} = \beta^j \left(\frac{P_t}{P_{t+j}}\right) \frac{C_t}{C_{t+j}}$$

and P_{it} is the price charged by the firm i in period t. Then, the constraints can be consolidated into the objective function to yield the problem:

$$\max_{\{P_{it}\}} \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{P_t}{P_{t+j}}\right) \frac{C_t}{C_{t+j}} \left(P_{it+j}^{1-\theta} P_{t+j}^{\theta} C_{t+j} - \frac{W_{t+j}}{A_{t+j}} \left(\frac{P_{it+j}}{P_{t+j}}\right)^{-\theta} C_{t+j} - \frac{\varphi W_{t+j}}{2} \left(\frac{P_{it+j} - P_{it+j-1}}{P_{it+j-1}} \right)^2 \right) \right]$$

The firm has a single first-order condition, for P_{it}:

$$(1-\theta)\left(\frac{P_{it}}{P_t}\right)^{-\theta} C_t + \theta \frac{W_t}{A_t} P_{it}^{-\theta-1} P_t^{\theta} C_t - \frac{\varphi W_t}{P_{it-1}} \left(\frac{P_{it}}{P_{it-1}} - 1\right) = -\mathbb{E}_t \left[\Theta_{t,t+1} \frac{\varphi W_{t+1} P_{it+1}}{P_{it}^2} \left(\frac{P_{it+1}}{P_{it}} - 1\right)\right]$$

(b) Symmetry across producers implies $P_{it} = P_t$ and $C_{it} = C_t$. Imposing this condition and letting $\pi_t = \frac{P_t}{P_{t-1}} - 1$, the firm FOC becomes:

$$(1-\theta)C_t + \theta \frac{W_t}{P_t A_t} C_t - \frac{\varphi W_t}{P_{t-1}} \pi_t = -\mathbb{E}_t \left[\Theta_{t,t+1} \frac{\varphi W_{t+1} P_{t+1}}{P_t^2} \pi_{t+1}\right]$$

$$(1-\theta)P_t C_t + \theta \frac{W_t}{A_t} C_t - \varphi W_t \pi_t(\pi_t + 1) = -\mathbb{E}_t \left[\Theta_{t,t+1} \varphi W_{t+1} \pi_{t+1}(\pi_{t+1} + 1)\right]$$

$$(1-\theta)P_t + \theta \frac{W_t}{A_t} = \varphi \mathbb{E}_t \left[\frac{W_t}{C_t} \pi_t(\pi_t + 1) - \frac{\Theta_{t,t+1}}{C_t} W_{t+1} \pi_{t+1}(\pi_{t+1} + 1)\right]$$

Where:

$$\frac{\Theta_{t,t+1}}{C_t} = \beta \left(\frac{P_t C_t}{C_t P_{t+1} C_{t+1}}\right) = \beta C_{t+1}^{-1} (\pi_{t+1} + 1)^{-1}$$

Thus, the simplified FOC is:

$$(1-\theta)P_t + \theta \frac{W_t}{A_t} = \varphi \mathbb{E}_t \left[\frac{W_t}{C_t} \pi_t(\pi_t + 1) - \beta \frac{W_{t+1}}{C_{t+1}} \pi_{t+1}\right]$$

(c)

(d)

Question 3

Question 4