Homework 1

Due Monday night, November 2, at midnight.
Feel free to work together on these problems, but each student needs to write up his/her own answers at the end, rather than directly copying from one master solution.

1. Show that if pure strategy $t_i \in S_i$ is strictly dominated, then so is any mixed strategy $\sigma_i \in \Delta S_i$ with t_i in its support.

2. Two travelers returning home from a remote island where they bought identical antiques discover that the airline has managed to smash them. The airline manager decides to use the following scheme to determine the compensation that the airline will give each traveler: Each traveler will separately report the cost of the antique, stating an integer number of dollars between 2 and 500. If both report the same number of dollars, then the airline will give each of them that number of dollars. If they report different numbers of dollars, then the traveler who stated the smaller number will be given the amount he stated plus a bonus of 2 dollars; the traveler who stated the higher number will be given the amount stated by the other traveler minus a penalty of 2 dollars. Suppose that each traveler cares only about maximizing the expected number of dollars he receives from the airline.

   Hint: You have not been told the actual values of the antiques. These are sunk costs, and the strategies, payoffs and outcomes in the game do not depend on these values.

   (a) Define the normal form game corresponding to the story above by specifying the set of players N, their strategy sets S_i, and their payoff functions u_i.

   (b) Suppose player 1 believes that the highest action in S_2 that receives positive probability is $\bar{s}_2 > 2$. Show that given this belief, player 1's best response must be less than $\bar{s}_2$.

   (c) If there is common knowledge of rationality between the players, what is the appropriate prediction of play in this game?

3. Consider the following normal form game:

	L	C	R
T	0, 4	5, 6	8, 7
B	2, 9	6, 5	5, 1

(a) Are any pure strategies in this game strictly dominated? If so, then for each such strategy s_i, identify all dominating strategies that do not put positive probability on s_i.

(b) If there is common knowledge of rationality between the players, what should our prediction of play in this game be?

4. Which pure strategies are rationalizable in the following normal form game? Explain.

<div align="center">2</div>

		a	b	c	d	e
	A	2, 1	1, 2	−1, −6	−3, −4	−1, −6
	B	0, 0	2, 1	1, 2	0, 0	−1, −6
1	C	−5, −1	−4, −1	2, 1	1, 2	−1, −6
	D	1, 2	−4, −1	0, 0	2, 1	−1, −6
	E	−5, −1	−4, −1	0, 0	0, 0	2, 1

5. Two firms have developed drugs to cure a rare disease. For each firm, if it chooses to seek FDA approval for the drug, it pays a cost $c > 0$. If only one firm seeks approval, it captures the entire market for this cure and gets revenue $R > 0$. If both firms seeks approval, each only gets a smaller amount of revenue $0 < r < R$. If a firm decides not to seek approval, it gets a payoff of zero.

(a) Set up this simultaneous-move, two player game. Carefully define the players, their strategy sets, and their payoffs.

(b) Under what conditions on (R, r, c) is it strictly dominant for each firm to seek approval for its drug? Under what conditions is seeking approval strictly dominated?