Problem Set #1

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Question 1

(a)

Question 2

(a) Letting $\widetilde{\theta}$ be some value between θ_0 and $\hat{\theta}$, the mean-value expansion of the first-order condition of the problem, at $\hat{\theta}$, is:

$$\frac{\partial \hat{Q}(\hat{\theta}_n)}{\partial \theta} = \frac{\partial \hat{Q}(\theta_0)}{\partial \theta} + \frac{1}{n}\sum_{i=1}^{n} \frac{\partial^2 g(W_i, \widetilde{\theta}_n, \hat{\gamma})}{\partial \theta \partial \theta'}(\hat{\theta}_n - \theta_0)$$

Denote $B_n = \frac{1}{n}\sum_{i=1}^{n} \frac{\partial^2 g(W_i, \widetilde{\theta}_n, \hat{\gamma})}{\partial \theta \partial \theta'}$, where:

$$B_n \to_p B_0 = \frac{\partial^2 g(W_i, \theta_0, \gamma_0)}{\partial \theta \partial \theta'}$$

Then:

$$\sqrt{n}\frac{\partial \hat{Q}(\theta_0)}{\partial \theta} = \frac{1}{\sqrt{n}}\frac{\partial g(W_i, \theta_0, \hat{\gamma})}{\partial \theta} \to_d \mathcal{N}(0, \Omega_0)$$

$$\text{Where } \Omega_0 = \mathbb{E}\left[\frac{\partial g(W_i, \theta_0, \gamma_0)}{\partial \theta}\frac{\partial g(W_i, \theta_0, \gamma_0)}{\partial \theta'}\right]$$

Thus, since the conditions for ULLN are satisfied,

$$\sqrt{n}\frac{\partial \hat{Q}(\hat{\theta}_n)}{\partial \theta} = \sqrt{n}\frac{\partial \hat{Q}(\theta_0)}{\partial \theta} + \frac{1}{n}\sum_{i=1}^{n} \frac{\partial^2 g(W_i, \widetilde{\theta}_n, \hat{\gamma})}{\partial \theta \partial \theta'}\sqrt{n}(\hat{\theta}_n - \theta_0) \to_d \mathcal{N}(0, \Omega_0)$$

Simplifying, this yields:

$$\sqrt{n}(\hat{\theta}_n - \theta_0) \to_d \mathcal{N}\left(0, B_0^{-1}\Omega_0 B_0^{-1}\right)$$