Econ 712: Macroeconomics I
Fall 2020, University of Wisconsin-Madison
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Problem Set #5 - Due 10/1/20 - Solution

In this problem set we study the macroeconomic consequences of eliminating the Social Security system in the U.S. To do so, we set up and solve a simple general equilibrium overlapping generations model. This model is a simplified version of the model by Conesa and Krueger (1999). You are not required to write the code from scratch to solve this model, we provide you with already written code with some missing parts. You are asked to understand the logic of the code, complete missing parts and use it to run policy experiment.

1 Model

Consider the model presented during Friday's discussion section.

Each period a continuum of agents is born. Agents live for J periods after which they die. The population growth rate is n per year (which is the model period length). Thus, the relative size of each cohort of age j, ψ_j, is given by:

$$\psi_{i+1} = \frac{\psi_i}{1+n}, \text{ for } i = 1, ..., J-1$$

with $\psi_1 = \tilde{\psi} > 0$. It is convenient to normalize ψ, so that it sums up to 1 across all age groups.

Newly born agents (i.e. $j = 1$) are endowed with no initial capital (i.e. $k_j = 0$) but can subsequently save in capital which they can rent to firms at rate r. A worker of age j supplies labor $\ell_j \in [0, 1]$ and pays proportional social security taxes on her labor income $\tau w e_j \ell_j$ until she retires at age $J^R < J$, where e_j is the age-efficiency profile. Upon retirement, agent receives pension benefits b.

The instantaneous utility function of a worker at age $j = 1, 2, ..., J^R - 1$ is given by:

$$u^W(c_j, \ell_j) = \frac{\left(c_j^\gamma (1-\ell_j)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma}$$

with c_j denoting consumption and ℓ_j denoting labor supply at age j. The weight on consumption is γ and the coefficient of relative risk aversion is σ. The instantaneous utility function of a retired agent at age $j = J^R, ..., J$ is given by:

$$u^R(c_j) = \frac{c_j^{1-\sigma}}{1-\sigma}. \quad (1)$$

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Preferences are then given by

$$\sum_{j=1}^{J^R-1} \beta^{j-1} u^W(c_j, \ell_j) + \sum_{j=J^R}^{J} \beta^{j-1} u^R(c_j)$$

There is a constant returns to scale production technology $Y = F(K, L) = K^\alpha L^{1-\alpha}$ with α denoting capital share, Y denoting aggregate output, K denoting aggregate capital stock and L denoting aggregate effective labor supply. The capital depreciates at rate δ. Capital and labor markets are perfectly competitive.

1.1 Parametrization

$$J = 66$$
$$J^R = 46$$
$$n = 0.011$$
$$k_1 = 0$$
$$\tau = 0.11$$
$$\gamma = 0.42$$
$$\sigma = 2$$
$$\beta = 0.97$$
$$\alpha = 0.36$$
$$\delta = 0.06$$

2 Questions

2.1

Derive the below equation for labor supply, used in the solution of workers' recursive problem (refer to lecture notes for details):

$$l = \frac{\gamma(1-\tau)e_j w - (1-\gamma)\left[(1+r)k - k'\right]}{(1-\tau)e_j w}$$

Value function of a working household:

$$V_j(k) = \max_{k', l} \left\{ u^W(c, l) + \beta V_{j+1}(k') \right\}$$
$$\text{s.t. } c = (1-\tau)wel + (1+r)k - k'$$

Take first order condition and solve for l.

$$
\begin{gathered}
(c^{\gamma}(1-l)^{1-\gamma})^{-\sigma}\gamma c^{\gamma-1}(1-l)^{1-\gamma}(1-\tau)we-(c^{\gamma}(1-l)^{1-\gamma})^{-\sigma}c^{\gamma}(1-\gamma)(1-l)^{-\gamma}=0\\
\gamma c^{\gamma-1}(1-l)^{1-\gamma}(1-\tau)we=c^{\gamma}(1-\gamma)(1-l)^{-\gamma}\\
\gamma(1-l)(1-\tau)we=c(1-\gamma)\\
\gamma(1-\tau)we-l\gamma(1-\tau)we=((1-\tau)wel+(1+r)k-k')(1-\gamma)\\
\gamma(1-\tau)we-l\gamma(1-\tau)we=l(1-\tau)we(1-\gamma)+((1+r)k-k')(1-\gamma)\\
\gamma(1-\tau)we-((1+r)k-k')(1-\gamma)=l[(1-\tau)we(1-\gamma)+\gamma(1-\tau)we]\\
l=\frac{\gamma(1-\tau)we-(1-\gamma)\left[(1+r)k-k'\right]}{(1-\tau)we}
\end{gathered}
$$

2.2

Matlab code for this problem set is provided in file `ps5.m`. Look for areas in the code that asks you to fill in the blanks and fill them in. Remember, the algorithm consists of the following steps:

1. Make initial guesses of the steady state values of the aggregate capital stock K and aggregate labor L.
2. Compute social security benefits, b, and the prices, w and r, implied by these guesses.
3. Compute the household's decision functions using dynamic programming.
4. Compute the optimal path for savings and labor supply for every cohort j by forward induction given that the initial capital stock is $k_1 = 0$.
5. Compute the aggregate capital stock and aggregate labor supply.
6. Update K and L and return to step 2 until convergence.

   If the guess was "far off" the obtained values, we update our initial guess with $K^1 = 0.9K^0 + 0.1K^{new}$ and $N^1 = 0.9N^0 + 0.1N^{new}$ and repeat the procedure. We proceed so until the guess and the updated values for K and N are "sufficiently close".

Completed Matlab code for this problem set is in `ps5_sol.m`. Note that you have to change the codes on lines 309-310 and where you set the social security tax rate.

2.3

Explain in words what each `while` and `for` loop does (lines 102-258), about 1-3 sentences per loop. Thus, write for example: The first while loop will iterate until both the absolute difference between the updated gross capital (labor) equals the initial level of capital (labor). The loop stops once the number if iterations equals the maximum number allowed. *Hint:* Do this before you start programming.

1. The first while loop will iterate until both the absolute difference between the updated gross capital (labor) equals the initial level of capital (labor). The loop stops once the number if iterations equals the maximum number allowed.

2. The first j-loop loops backward from the the year you die to you retire

3. the ik0-loop loops over today's level of capital. That is to say, we fix an age and a capital level.

4. the ik1-loop loops over tomorrows level of capital. That is to say, for fixed age and capital level we calculate the value of picking a specific capital level tomorrow.

5. The second j-loop loops backward from retirement to the start of working age, while the ik0 and ik1 loops does the same as before.

6. the third j-loop loops over all ages to find the optimal decision for labor and capital for each age.

Using your code, solve household optimization problem with $K = 3.1392$, $L = 0.3496$ and social security in place (Hint: run just one iteration of the main loop). Plot the value function over k for a retired agent at the model-age $j = 50$, $V_{50}(k)$. Is it increasing and concave? Plot the savings function for a worker at the model-age $j = 20$, $k'_{20}(k)$. Is saving increasing in k? What about net saving, $k' - k$?

Value function is shown in Figure 1, it is increasing and concave.

Saving function $k'_{20}(k)$ is shown in Figure 2. It is increasing in k, but net saving is decreasing.

2.4

Evaluate the macroeconomic consequences of eliminating social security. You can use Table 1 to support your answers. Note: when initially solving the model, lines 300 on wards will give you an error. This is because the last part is used to compare the two models after you have solved for it already. Either skip or ignore that part and use it afterwards.

1. Solve for the stationary competitive market equilibrium of the benchmark model with social security, i.e., solve the whole model with initial guess of $(K, L) = (3.1392, 0.3496)$. The model should converge after ≈ 25 iterations. Is this economy dynamically efficient (compare the interest rate with the implicit return from social security, which is equal to the population growth rate)?

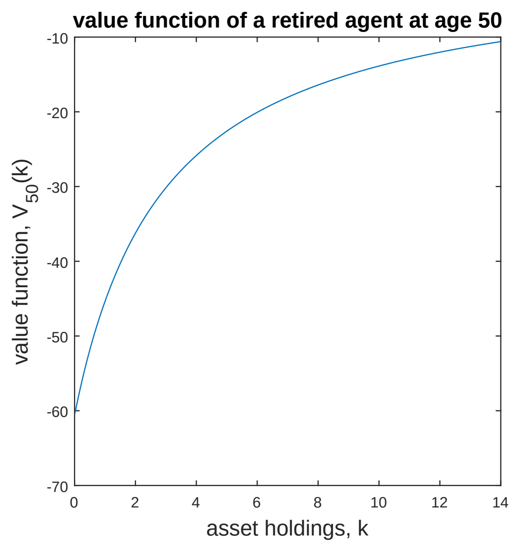

Figure 1: Value function of a retired agent of cohort $j = 50$.

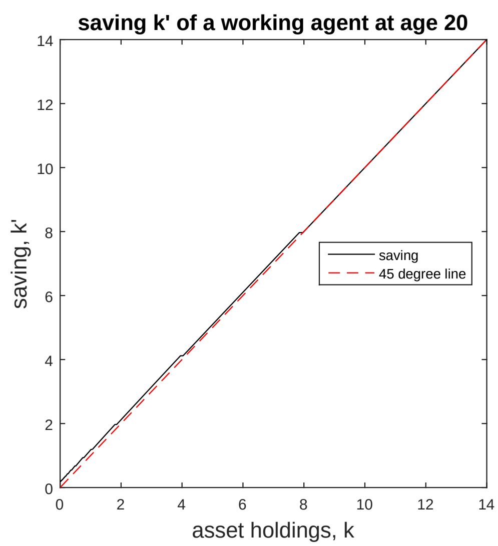

Figure 2: Saving function of cohort $j = 20$.

2. Eliminate social security by setting $\tau = 0$ and solve for the new stationary equilibrium, i.e., solve the model again, but this time set $\tau = 0$ and use initial guess of $(K, L) = (3.9288, 0.3663)$. The model should converge after

$\approx$ 15 iterations.

- The code should have saved the results using lines 291. Check how aggregate capital and labor supply change as a result of the tax reform. Fill in the appropriate column of Table 1 for your answers.
- Plot and compare the profiles of wealth by age groups for the case with and without social security. Provide intuition for observed differences.
- Will a newborn generation prefer to start in a steady state with or without social security? (Hint: lines 332 & 347)
- Assuming that the reform instantly takes every cohort into the new steady state, will it be supported by a majority voting? (Hint: Compare aggregate welfare $W = \sum_{j=1}^{J} \psi_j V_j(k_j)$) (More hint: lines 341 & 356)

Economy with social security is dynamically efficient because $n < r$.

Capital increases substantially, because agents now make private savings for their retirement. Labor increases a little because removel of social security tax increases after-tax wage, and higher capital increases marginal productivity of labor.

Age-wealth profile shown in Figure 3 demonstrates that households need to save more for their consumption smoothing needs if there is no social security.

Welfare comparison shows that newborn generations would prefer the economy without social security, but on aggregate population will be worse off.

	Benchmark model	
	with SS	without SS
capital K	2.98	3.91
labor L	0.35	0.37
wage w	1.38	1.50
interest r	0.031	0.019
pension benefit b	0.22	0
newborn welfare $V_1(k_1)$	-54.61	-52.03
aggregate welfare W	-35.07	-36.03

Table 1: Results of the policy experiment

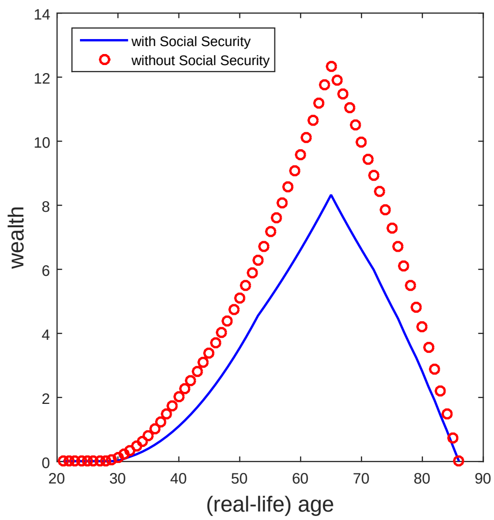

Figure 3: Age-wealth profiles.