Problem Set #6

Danny Edgel
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Collaborated with Sarah Bass, Emily Case, Michael Nattinger, and Alex Von Hafften

1 (Non-) Commitment in a black-box example with discrete choice sets

For clarity, the choice of the individual household is diplayed below. The household chooses between $\xi_k = x_L$ (left) and $\xi_k = x_H$ (right).

$\xi_k = x_L$

	x_L	x_H
y_L	12	30
y_H	0	-1

$\xi_k = x_H$

	x_L	x_H
y_L	-1	25
y_H	30	24

1. The Ramsey outcome is reached when the government moves first and is thus able to optimize based on predicted household behavior. To find this outcome, we induct backward from the household problem, where households maximize utility at each x_i and y_j:

$$\xi_k = \underset{\xi \in X}{\text{argmax}} u(\xi, x, y) = \begin{cases} x_L, & x_i = X_L \wedge y_j = y_L \\ x_L, & x_i = X_L \wedge y_j = y_H \\ x_H, & x_i = X_H \wedge y_j = y_H \\ x_L, & x_i = X_H \wedge y_j = y_L \end{cases}$$

   The government then chooses y_j to maximize utility, given the HH solution and $\xi_k = x_i$. If the government chooses y_L, then households will chose x_L, resulting in a total utility of 12. If the government chooses y_H, then households will choose x_H, resulting in a total utilty of 24. Thus, (x_H, x_H, y_H) is the Ramsey outcome.

   In the no-commitment outcome, households move first, choosing ξ_k after inducting backward from the government's response to their choice. After

households choose, government will be faced with maximizing total utility given $\xi_k = x_i$:

$\xi_k = x_i$

	x_L	x_H
y_L	12	25
y_H	0	24

y_L clearly dominates y_H, so households maximize utility assuming that $y_j = y_L$:

x_i

	x_L	x_H
$\xi_k = x_L$	12	30
$\xi_k = x_H$	-1	25

Regardless of x_i, households will be better-off choosing x_L, so the equilibrium in this case is (x_L, x_L, y_L).

2. In a repeated economy, the Ramsey outcome cannot be supported because the government can achieve a higher level of utility by deviating from (x_H, x_H, y_H) to choose y_L such that $u(x_H, x_H, y_L) = 25$. It would choose to do so in the final period, knowing that there will not be an opportunity for households to respond by moving to (x_L, x_L, y_L), where utility is lower, at 12. Households know that this is the government's optimal choice, so they factor this into their maximization problem by choosing x_L in the fourth period. Governments then anticipate this in the third period, and so on. As a result, there is no period where the Ramsey equilibrium can be supported.

3. The tables below display the full incentive structure of the agents in this economy. The pure strategy Nash equilibria are (x_{LL}, x_{LL}, y_{LL}) and (x_L, x_L, y_L), since at these bundles, neither households nor the government can inprove household utility unilaterally.

$\xi_k = x_{LL}$

	x_{LL}	x_L	x_H
y_{LL}	2	30	30
y_L	1	-1	30
y_H	-1	30	-1

$\xi_k = x_L$

	x_{LL}	x_L	x_H
y_{LL}	-1	6	30
y_L	30	12	30
y_H	30	0	-1

$\xi_k = x_H$

	x_{LL}	x_L	x_H
y_{LL}	-1	30	10
y_L	30	-1	25
y_H	30	30	24

2 Static taxation