Problem Set #4

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Question 1

(i) Each of the series is covariance stationary if their autocovariance can be represented by some constant function $\gamma(k)$:

$$\begin{aligned}
Cov(U_t, U_{t+k}) &= \mathbb{E}\left[(U_t - \mathbb{E}\left[U_t\right])(U_{t+k} - \mathbb{E}\left[U_{t+k}\right])\right] \\
&= \mathbb{E}\left[(\varepsilon_t\varepsilon_{t-1} - \mathbb{E}\left[\varepsilon_t\varepsilon_{t-1}\right])(\varepsilon_{t+k}\varepsilon_{t+k-1} - \mathbb{E}\left[\varepsilon_{t+k}\varepsilon_{t+k-1}\right])\right] \\
&= \mathbb{E}\left[\varepsilon_t\varepsilon_{t-1}\varepsilon_{t+k}\varepsilon_{t+k-1}\right] = 0 \ \forall k \in \{1, T-t\} \\
\Rightarrow \gamma_U(k) &= \begin{cases} \sigma^4, & k = 0 \\ 0, & k \geq 1 \end{cases} \\
Cov(W_t, W_{t+k}) &= \mathbb{E}\left[(W_t - \mathbb{E}\left[W_t\right])(U_{W+k} - \mathbb{E}\left[W_{t+k}\right])\right] \\
&= \mathbb{E}\left[(\varepsilon_t\varepsilon_0 - \mathbb{E}\left[\varepsilon_t\varepsilon_0\right])(\varepsilon_{t+k}\varepsilon_0 - \mathbb{E}\left[\varepsilon_{t+k}\varepsilon_0\right])\right] \\
&= \mathbb{E}\left[\varepsilon_t\varepsilon_0\varepsilon_{t+k}\varepsilon_0\right] = 0\sigma^2 = 0 \ \forall k \in \{1, T-t\} \\
\Rightarrow \gamma_W(k) &= \begin{cases} \sigma^4, & k = 0 \\ 0, & k \geq 1 \end{cases} \\
Cov(V_t, V_{t+k}) &= \mathbb{E}\left[(V_t - \mathbb{E}\left[V_t\right])(U_{V+k} - \mathbb{E}\left[V_{t+k}\right])\right] \\
&= \mathbb{E}\left[(\varepsilon_t^2\varepsilon_{t-1} - \mathbb{E}\left[\varepsilon_t^2\varepsilon_{t-1}\right])(\varepsilon_{t+k}^2\varepsilon_{t+k-1} - \mathbb{E}\left[\varepsilon_{t+k}^2\varepsilon_{t+k-1}\right])\right] \\
&= \mathbb{E}\left[\varepsilon_t^2\varepsilon_{t-1}\varepsilon_{t+k}^2\varepsilon_{t+k-1}\right] = 0 \ \forall k \in \{1, T-t\} \\
\Rightarrow \gamma_V(k) &= \begin{cases} \sigma^6, & k = 0 \\ 0, & k \geq 1 \end{cases}
\end{aligned}$$

(ii) $\overline{U}$, $\overline{W}$, and $\overline{V}$ converge in probability if their variance, divided by T, converges to zero as $T \to \infty$. Since, as we showed in (i), each series' variance is constant, this is true. Therefore, each sample mean converges to its expectation.

(iii) As with (ii), the weak law of large numbers holds that any estimator will converge to its expectation if its variance, divided by sample size, converges to zero as sample size approaches infinity. By assumption, $\mathbb{E}\left[\varepsilon^8\right] < \infty$, where $\mathbb{E}\left[\varepsilon^8\right]$ is the variance of $\hat{\gamma}_U(0)$ and $\hat{\gamma}_W(0)$. Thus, the sample second moments of U and V converge in probability to their expectations. However, we do not have enough information to determine whether $\hat{\gamma}_W(0)$ has a finite second moment and thus converges to its expectation in probability.

(iv) Since each of the three series are mean zero with finite variance, drawn from a random sample and serially independent, the Central Limit Theorem holds that all three have asymptotically normal scaled sample means.

Question 2

(i)

T	ρ_0	α_0	β_0	ρ_1
50	0.7	0.818	0.015	0.751
		[-0.010,1.646]	[-0.009,0.039]	[0.576,0.926]
50	0.9	1.415	0.006	0.857
		[0.691,2.140]	[-0.015,0.028]	[0.761,0.953]
50	0.95	1.026	0.013	0.950
		[0.220,1.832]	[-0.031,0.056]	[0.861,1.040]
150	0.7	1.169	0.009	0.685
		[0.754,1.584]	[0.005,0.013]	[0.615,0.756]
150	0.9	1.009	0.017	0.860
		[0.639,1.379]	[0.010,0.025]	[0.812,0.907]
150	0.95	1.212	0.010	0.942
		[0.741,1.682]	[0.003,0.017]	[0.916,0.969]
250	0.7	0.784	0.010	0.723
		[0.491,1.077]	[0.007,0.012]	[0.666,0.781]
250	0.9	0.903	0.015	0.874
		[0.552,1.254]	[0.010,0.021]	[0.836,0.912]
250	0.95	1.277	0.009	0.949
		[0.866,1.687]	[0.004,0.014]	[0.927,0.971]

(ii)

(iii)