Problem Set #8

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Discussed and/or compared answers with Sarah Bass, Emily Case, Katherine Kwok, Michael Nattinger, and Alex Von Hafften

Exercise 18.2

(a) Since there are four observations with four possible outcomes, each outcome is determined in this model. Let i and t each take on values of 0 and 1, with $State_1 = 1$ and $Time_1 = 1$. Then we can remove time and state fixed effects from the model by taking averages across the state variable:

$$Y_{it} - \overline{Y}_i = \theta(D_{it} - \overline{D}_i) + \varepsilon_{it} - \overline{\varepsilon}_i$$

Then this model can be estimated with OLS, where

$$\hat{\theta} = \frac{\sum_{i=0}^{1}\sum_{t=0}^{1}(Y_{it} - \overline{Y}_i)(D_{it} - \overline{D}_i)}{\sum_{i=0}^{1}\sum_{t=0}^{1}(D_{it} - \overline{D}_i)^2}$$

(b) Recall how our $Time$ and $State$ are defined. The sample has only four observations, with $\{it\} \in \{\{00\}, \{10\}, \{01\}, \{11\}\}$. Then,

$$D_{0t} = State_0 Time_t = 0 Time_t = 0 \ \forall t$$

Furthermore,

$$\begin{aligned}\sum_{t=0}^{1}(D_{1t} - \overline{D}_i)^2 &= \sum_{t=0}^{1}\left(State_1 Time_t - \frac{1}{2}\sum_{t=0}^{1} State_1 Time_t\right)^2 \\ &= \left(1 * 0 - \frac{1}{2}\right)^2 + \left(1 * 1 - \frac{1}{2}\right)^2 = \frac{1}{2}\end{aligned}$$

Thus, our estimate for $\hat{\theta}$ simplifies to:

$$\hat{\theta} = \frac{(Y_{10} - \overline{Y}_1)\left(-\frac{1}{2}\right) + (Y_{11} - \overline{Y}_1)\left(\frac{1}{2}\right)}{\frac{1}{2}} = Y_{11} - Y_{10}$$

(c) No. This is a single difference estimator.

(d) $\hat{\theta}$ would be an appropriate estimator of the treatment effect if there is no omitted variable, such as a time trend, causing a change in Y_{1t} from $t = 0$ to $t = 1$.

Exercise 18.4

If N_2 interaction dummies are included, then no observation will be used as the omitted group in the fixed effects estimation, leading to multicollinearity. The regression will fail because the independent variable matrix will not be invertible. This is also the reason that only $N_1 - 1$ interaction dummy variables are included in the regression test for equal control effects.

Exercise 18.5

(a) The table below displays how the economist would calculate a difference in difference estimate for this data, with her point estimate in bold.

	Wisconsin	Minnesota	Difference
Before	15.23	16.42	-1.19
After	16.72	18.10	-1.38
Difference	1.49	1.68	**-0.19**

Thus, her point estimate is -0.19.

(b) Since the economist does not add any fixed effects, her point estimate of the difference-in-difference is the same, and β is the value for Minnesota in the "After" period. Thus, $\hat{\beta} = 18.10$.

(c) γ represents the value for Wisconsin in the "Before" period. Thus, $\hat{\gamma} = 15.23$.

Exercise 17.1

$$\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right)$$

(a)

$$
\begin{aligned}
\mathbb{E}\left[X^*\right] &= \int x\hat{f}(x)dx = \int \frac{x}{nh}\sum_i^n K\left(\frac{x-X_i}{h}\right)dx \\
&= \int \left(\frac{X_i+uh}{n}\right)\sum_i^n K(u)du \\
&= \frac{1}{n}\sum_i^n X_i \int K(u)du + \int \left(\frac{1}{n}\right)\sum_i^n huK(u)du \\
&= \overline{X}_n
\end{aligned}
$$

(b)

$$
\begin{aligned}
Var(X^*) &= \mathbb{E}\left[(X^*)^2\right] - \mathbb{E}\left[X^*\right]^2 = \mathbb{E}\left[(X^*)^2\right] - \overline{X}_n^2 \\
\mathbb{E}\left[(X^*)^2\right] &= \int \frac{x^2}{nh}\sum_i^n K\left(\frac{x-X_i}{h}\right)dx \\
&= \frac{1}{n}\sum_i^n \int (X_i+uh)^2 K(u)du
\end{aligned}
$$

Exercise 17.3

Exercise 17.4

Exercise 19.3

Exercise 19.4

Exercise 19.9

Exercise 19.11