Calibration Matrix Notes

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1 Measurement

A quantum computer contains a transmon qubit coupled to a coplaner waveguide mircowave resonator. The full description of the interaction is given by the Jaynes-Cummings model.

$$H_{JC} = \hbar w_r (a^\dagger a + \frac{1}{2}) + \frac{1}{2}\hbar w_a \sigma_z + \hbar g(\sigma_+ a + a^\dagger \sigma_-) \tag{1}$$

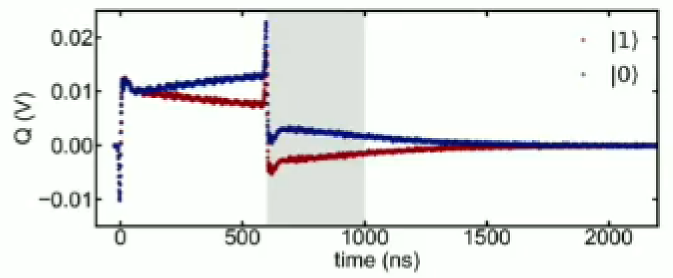
The first term describes the cavity, where w_r is the resonance frequency of the cavity. The second term is the atomic term that describes the qubit, where w_a is the transition frequency. The third term is a harmonic oscillator like term.

For the case of zero detuning ($w_r = w_a$) the interaction lifts the degeneracy of the photon number state $|n\rangle$ and the atomic states $|g\rangle >$ and $|e\rangle$ and pairs of dressed states are formed. The new states are superposition of cavity and atom states.

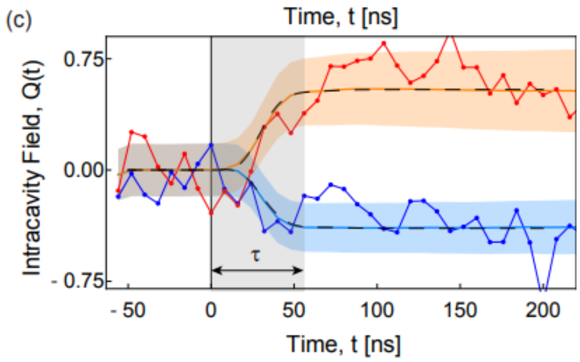
$$|n, \pm\rangle = \frac{1}{\sqrt{2}}(|g\rangle|n\rangle \pm |e\rangle|n-1\rangle) \tag{2}$$

which are energetically split by $2g\sqrt{n}$. If the detuning is significantly larger than the combined cavity and atomic linewidth, the cavity states are merely shifted by $^2/\Delta$, where $\Delta = w_a - w_r$. This provides the possibility to read out the qubit state by measuring the transition frequency.

For a general Quantum computer which contains a transmon qubit coupled to a resonator. The resonance frequency of the resonator is quite far away from the qubit transition frequency in order of GHz. However, due to the coupling, there is a shift in the resonator's frequency depending on the qubit state. The shifts is shown by the measured resonator transmission dips of the qubit in the ground and excited state. This shift is typically on the order of a few MHz, three orders of magnitude smaller than the detuning. Thus one can observe this shift and as such the qubit state, by injecting the resonator with a pulse near the resonator frequency.



The figure above looks clean for it is averaged over thousand of measurements. However, for most real quantum protocols, it is required to discern the qubit state in just a single run.



Where the blue is the ground and the red is the excited single shot state. The black dashed line shows the theoretical expected dynamics. Which shows that the single is hampered by quantum noise. To quantify how well qubit measurement is performing in the presence of this noise, they record the integrated readout quadrature value.

$$q_\tau = \sqrt{k_p} \int_0^\tau dt Q(t) W(T) \tag{3}$$

where the weighting function $W(t) \propto |\langle Q_e(t) - Q_g(t) \rangle|$. Each experiment alternates the qubit preparation between the ground and excited state, and populate histogram of q_τ, extracting the fidelity of the measurement.

$$F = 1 - \frac{1}{2}(P(0|1) + P(1|0)) \tag{4}$$

The fidelity expresses the probability that the measurement returns the right outcome averaged over the two possible qubit input states.

2 Unfolding

Measuring a qubit takes significantly longer than unitary operations on qubits. Thus, during measurement, the qubit being measured may change their states because of decoherence introducing readout.

When data is measured, the measuring device may introduce errors, i.e. the measured value does not exactly reflect the true state of the system measured. Repeating such a measurement results in a distribution of measured values which deviates from the true distribution of values that would have resulted if the measurement device would be error free. Unfolding is a method to determine the true, undisturbed distribution from the measured, disturbed distribution.

Let t be the vector corresponding to the true, undisturbed distribution and m be the vector of the measured, disturbed distribution.The method of matrix inversion assumes that t and m are related by a calibration matrix C such that m=Ct. The coefficient C_{ij} of the matrix C is the probability of measuring the value i under the condition that the true value is j

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\begin{aligned}
C_{ij} &= \mathbb{P}(\text{measured value is i}|\text{true value is j}) \\
C &= \begin{bmatrix} P^k_{0,0} & P^k_{0,1} \\ P^k_{1,0} & P^k_{1,1} \end{bmatrix}
\end{aligned} \tag{5}
$$

Thus the assumption m=Ct gives

$$m_i = \sum_k C_{ik} t_k = \sum_k \mathbb{P}(i|k) t_k \tag{6}$$

and with $m_i \approx \mathbb{P}(k)$ the assumption becomes the law of total probability, i.e.:

$$\mathbb{P}(i) = \sum_k \mathbb{P})(i|k)\mathbb{P}(k) \tag{7}$$

Thus the assumption of the matrix inversion method is that the calibration matrix C is regular, and so C^{-1} exists. Then,

$$t = C^{-1} m \tag{8}$$

3 ref

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- The bitter truth about gate-based quantum algorithms in the NISQ era
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