Importance Sampling

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Warm-up discussion

How did the lab go?

What did you learn about issues with MC?

Learning objectives

- Review the difference between a PDF and an RV. (Lecture)
- Understand the two cases in which Importance Sampling is useful. (Lecture+Lab)
- Know what the proposal distribution is for Importance Sampling. (Lecture+Lab)
- Be able to describe the Importance Sampling procedure. (Lecture+Lab)

Recall Random Variables

Recall that an RV is made of two components:

$p(x)$: the probability density function

$f(x)$: the outcome function

So, the expectation is defined as:

$$E[X] = \int_a^b f(x)p(x)dx$$

We will use this separation of concerns in Importance Sampling.

Querying revisited

With Monte Carlo integration, we can approximate $E[X]$.

But MCI relies on drawing from X.

What if we can't even do that?

- $E[X]$: difficult
- $a \sim X$: difficult
- $P(X = a)$: easy

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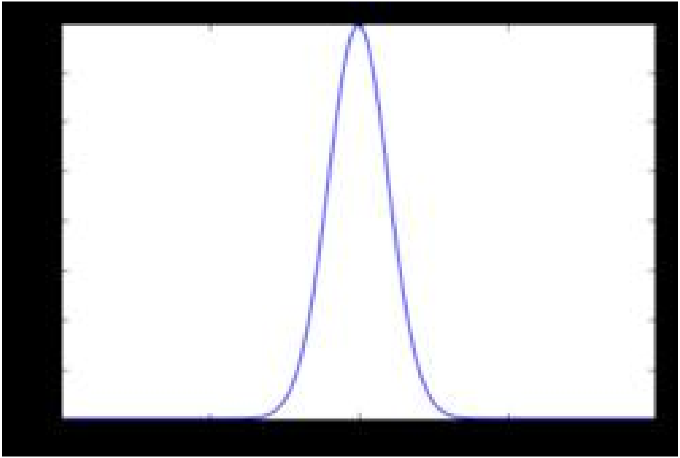
Importance Sampling to the rescue.

Big idea:

We use an easy-to-sample distribution to approximate a difficult-to-sample one.

Intuitive example

Let's say we have $X \sim \mathcal{N}(0,1)\ldots$



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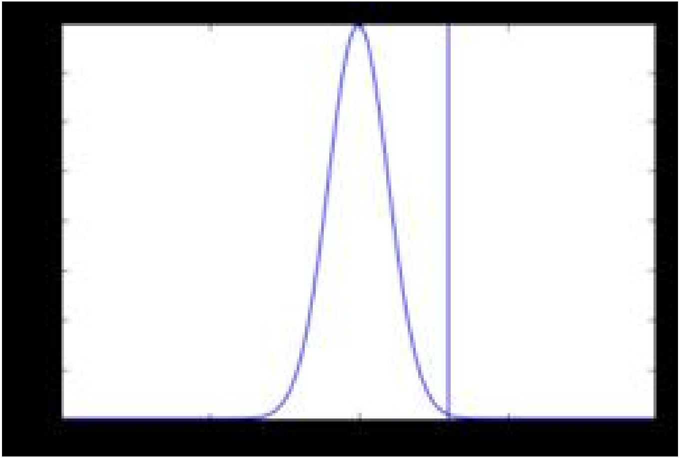
And we want to know $P(X >= 3)$:



Discuss: what will MCI do?

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General procedure

We can posit an *easy approximating distribution*, call it q.

Then...

We sample from q, then...

We *correct* the samples from q towards our original distribution.

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How do we correct the samples?

Let's draw a sample from q: $a \sim q$

That sample naturally has a probability in both p and q:

$p(a)$

$q(a)$

(Remember, we can use the CDF of both distributions.)

Then, we weight a by the ratio

$$\frac{p(a)}{q(a)}$$

.

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We call that ratio the *importance weight* of a:

$$w(a) = \frac{p(a)}{q(a)}$$

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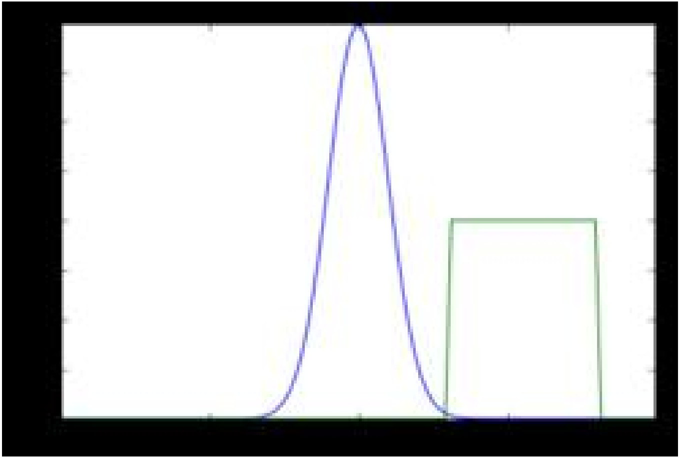
Example: $p = \mathcal{N}(0,1)$, $q = \mathcal{U}(3,8)$



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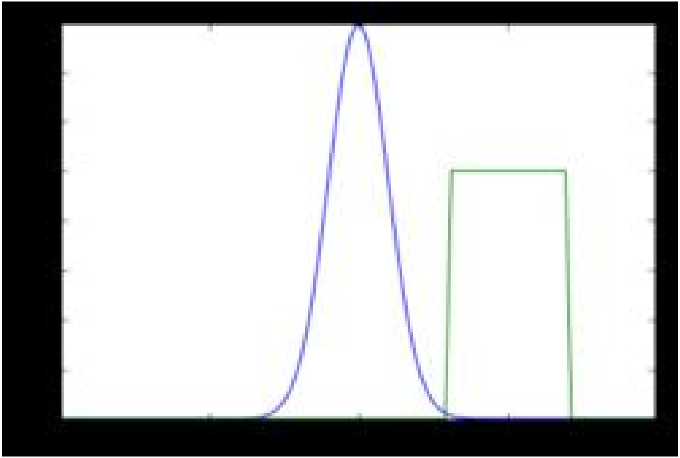
Example: $p = \mathcal{N}(0,1), q = \mathcal{U}(3,6)$



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Example: $p = \mathcal{N}(0,1), q = \mathcal{U}(3,10)$