Riemann Sum Practice Problems

Questions:

1. Approximate the area under the curve

$$f(x) = x^2 + 2, \quad -2 \leq x \leq 1$$

with a Riemann sum, using six sub-intervals and right endpoints.

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2 Approximate the area under the curve

$$f(x) = \sqrt{x+1}, \quad -1 \leq x \leq 0$$

with a Riemann sum, using 4 sub-intervals and left endpoints.

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3. **a)** Approximate the value of the integral:

$$\int_1^3 x^3 - 3\,dx$$

with a Riemann sum, using three sub-intervals and right endpoints.

b) using five sub-intervals and left endpoints.

c) Compute the exact value of the integral using the Fundamental Theorem of Calculus.

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4. A rectangular canal, 5m wide and 100m long has an uneven bottom. Depth measurements are taken at every 20m along the length of the canal. Use these depth measurements to construct a Riemann sum using right endpoints to estimate the volume of water in the canal.

Distance	0m	20m	40m	60m	80m	100m
Depth	2.0m	1.6m	1.8m	2.1m	2.1m	1.9m

Answers:

1. $a = -2,\ \ b = 1,\quad \Delta x = \dfrac{b-a}{n} = \dfrac{1-(-2)}{6} = \dfrac{1}{2},\quad f(x) = x^2 + 2$

$$\text{Area} \simeq \left(\sum_i f(x_i)\Delta x\right) = \Delta x\big(f(-1.5) + f(-1) + f(-0.5) + f(0) + f(0.5) + f(1)\big)$$

$$= \frac{1}{2}\big((-1.5)^2 + 2 + (-1)^2 + 2 + (-0.5)^2 + (0)^2 + 2 + (0.5)^2 + 2 + (1)^2 + 2\big)$$

$$= 8.375$$

2. $a = -1,\ \ b = 0,\quad \Delta x = \dfrac{b-a}{n} = \dfrac{0-(-1)}{4} = \dfrac{1}{4},\quad f(x) = \sqrt{x+1}$

$$\text{Area} \simeq \left(\sum_i f(x_i)\Delta x\right) = \Delta x\big(f(-1) + f(-0.75) + f(-0.5) + f(-0.25)\big)$$

$$= 0.25\left(\sqrt{0} + \sqrt{0.25} + \sqrt{0.5} + \sqrt{0.75}\right) \simeq 0.5183$$

3. a) $a = 1,\ \ b = 3,\quad \Delta x = \dfrac{3-1}{3} = \dfrac{2}{3},\quad f(x) = x^3 - 3$

$$\text{Area} \simeq \left(\sum_i f(x_i)\Delta x\right) = \frac{2}{3}\left(f\left(\frac{5}{3}\right) + f\left(\frac{7}{3}\right) + f(3)\right)$$

$$= \frac{2}{3}\left(\left(\left(\frac{5}{3}\right)^3 - 3\right) + \left(\left(\frac{7}{3}\right)^3 - 3\right) + \left((3)^3 - 3\right)\right) \simeq 23.556$$

b) As before, $a = 1,\ \ b = 3,\ \ f(x) = x^3 - 3$, but $\Delta x = \dfrac{3-1}{5} = \dfrac{2}{5}$

$$\text{Area} \simeq \left(\sum_i f(x_i)\Delta x\right) = \frac{2}{5}\left(f(1) + f\left(\frac{7}{5}\right) + f\left(\frac{9}{5}\right) + + f\left(\frac{11}{5}\right) + f\left(\frac{13}{5}\right)\right)$$

$$= \frac{2}{5}\left(\left((1)^3 - 3\right) + \left(\left(\frac{7}{5}\right)^3 - 3\right) + \left(\left(\frac{9}{5}\right)^3 - 3\right) + \left(\left(\frac{11}{5}\right)^3 - 3\right) + \left(\left(\frac{13}{5}\right)^3 - 3\right)\right)$$

$$\simeq 9.120$$

c) $$\int_1^3 x^3 - 3\,dx = \frac{1}{4}x^4 - 3x\Big|_1^3 = \left(\frac{81}{4} - 9\right) - \left(\frac{1}{4} - 3\right) = 20 - 6 = 14$$

4. Volume $=$ (width)(cross section area)

$$\simeq (5\text{m})\left(\sum_i f(x_i)\Delta x\right) = (5\text{m})\Delta x\big(f(20) + f(40) + f(60) + f(80) + f(100)\big)$$

Δx here is the distance between our measurements, 20m, so we get:

$$\begin{aligned}\text{Volume} &\simeq 100\big(f(20) + f(40) + f(60) + f(80) + f(100)\big)\\ &= 100(1.6 + 1.8 + 2.1 + 2.1 + 1.9)\\ &= 950\text{m}^3\end{aligned}$$