Matrix Factorization for Recommendation Engines

Dan Becker

April 1, 2015

Goals

I. Conceptual Understanding of Latent Factor Models for Recommender Systems

II. High level understanding of Latent Factor Model implementation using FunkSVD algorithm

III. Ability to tune model performance

The Setup

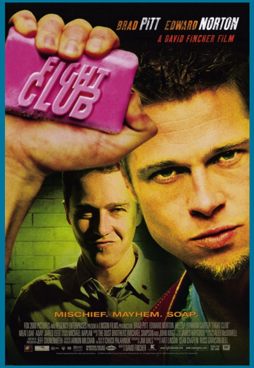
Movie

User

	A	B	C	D	…
Alice	1	?	2	?	
Bob	?	2	3	4	
Charlie	3	?	1	5	
Dan	?	2	?	?	
…					

The Problem With Item-Item Recommendations

I Like Surprising Endings



Violent movie recommendations

Movies (And Everything Else) Have Many Attributes

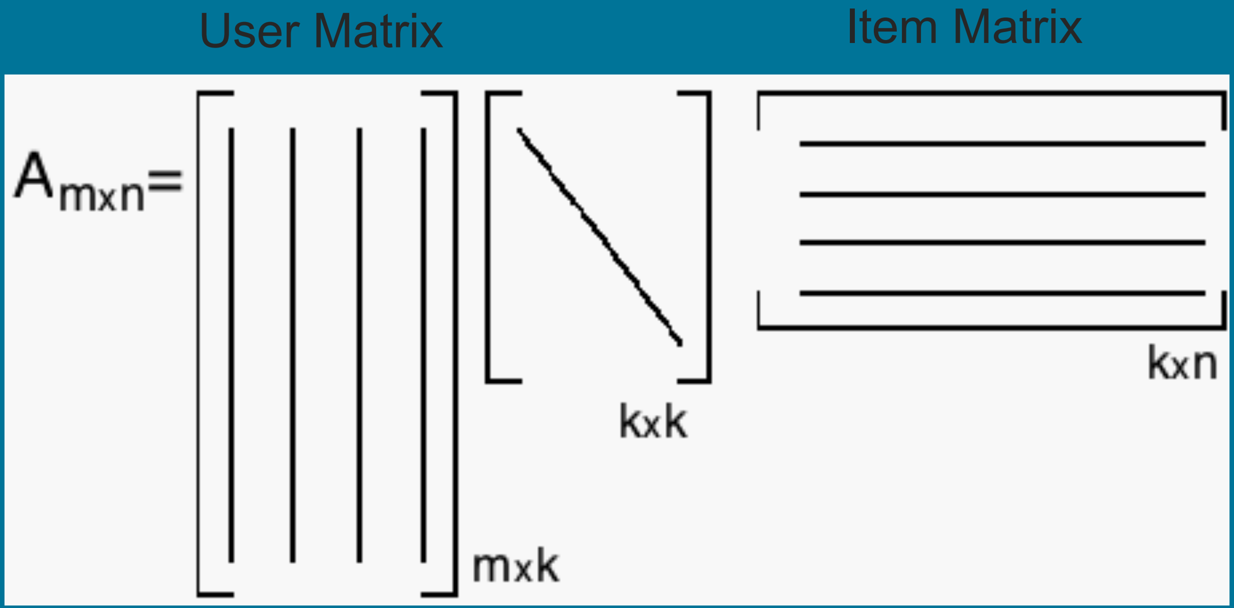
Movie Attributes

- Action
- Comedy
- Drama
- …
- Tom Hanks
- Brad Pitt
- Jeff Bridges
- …
- Happy ending
- Sad Ending
- …
- Movie length
- Subtitles
- …

A Familiar Looking Model

$$
\begin{aligned}
\text{Predicted rating} = \quad & B_0 + B_1 * \text{level of action} \\
& + B_2 * \text{level of comedy} \\
& + B_3 * \text{level of drama} \\
& + \ldots \\
& + B_n * \text{Tom Hanks} \\
& + B_{n+1} * \text{Brad Pitt} \\
& + \ldots \\
& + B_k * \text{length} \\
& + \ldots \\
& + e
\end{aligned}
$$

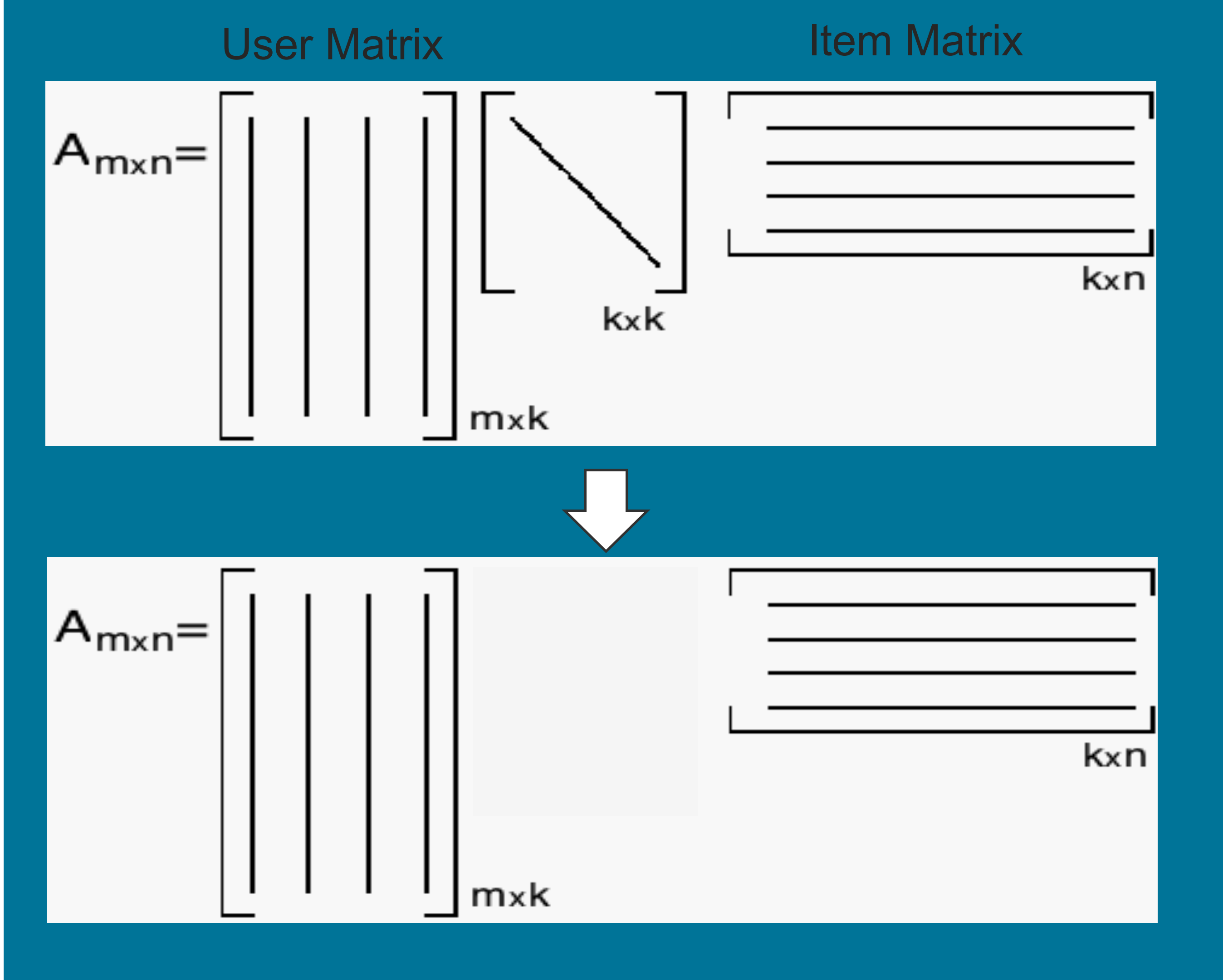
Singular Value Decomposition



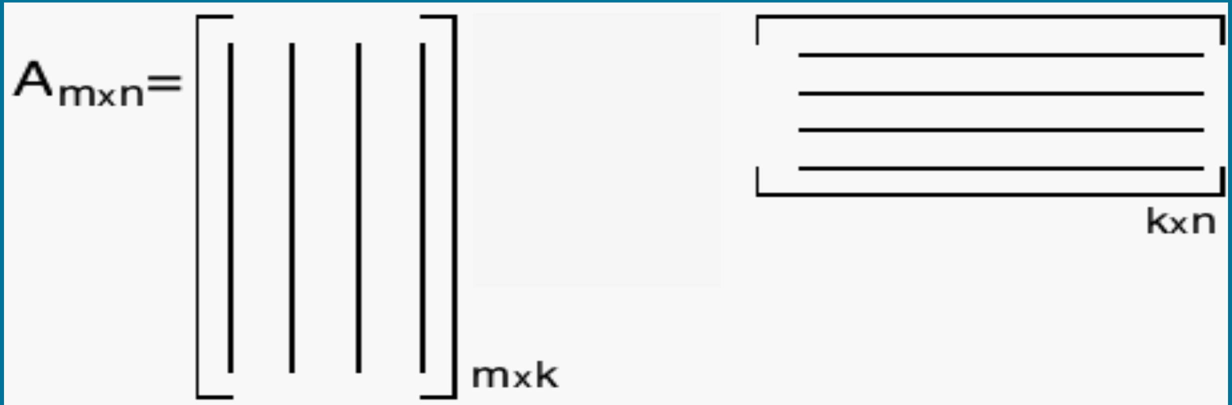
SVD Doesn't Work With Missing Values

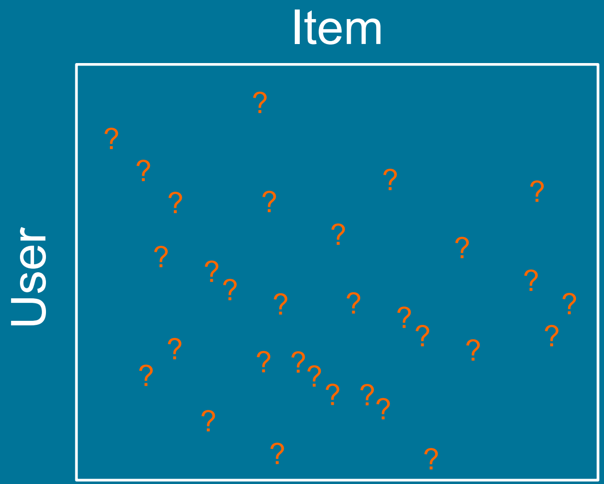
Start By Simplifying the Problem

UV Decomposition

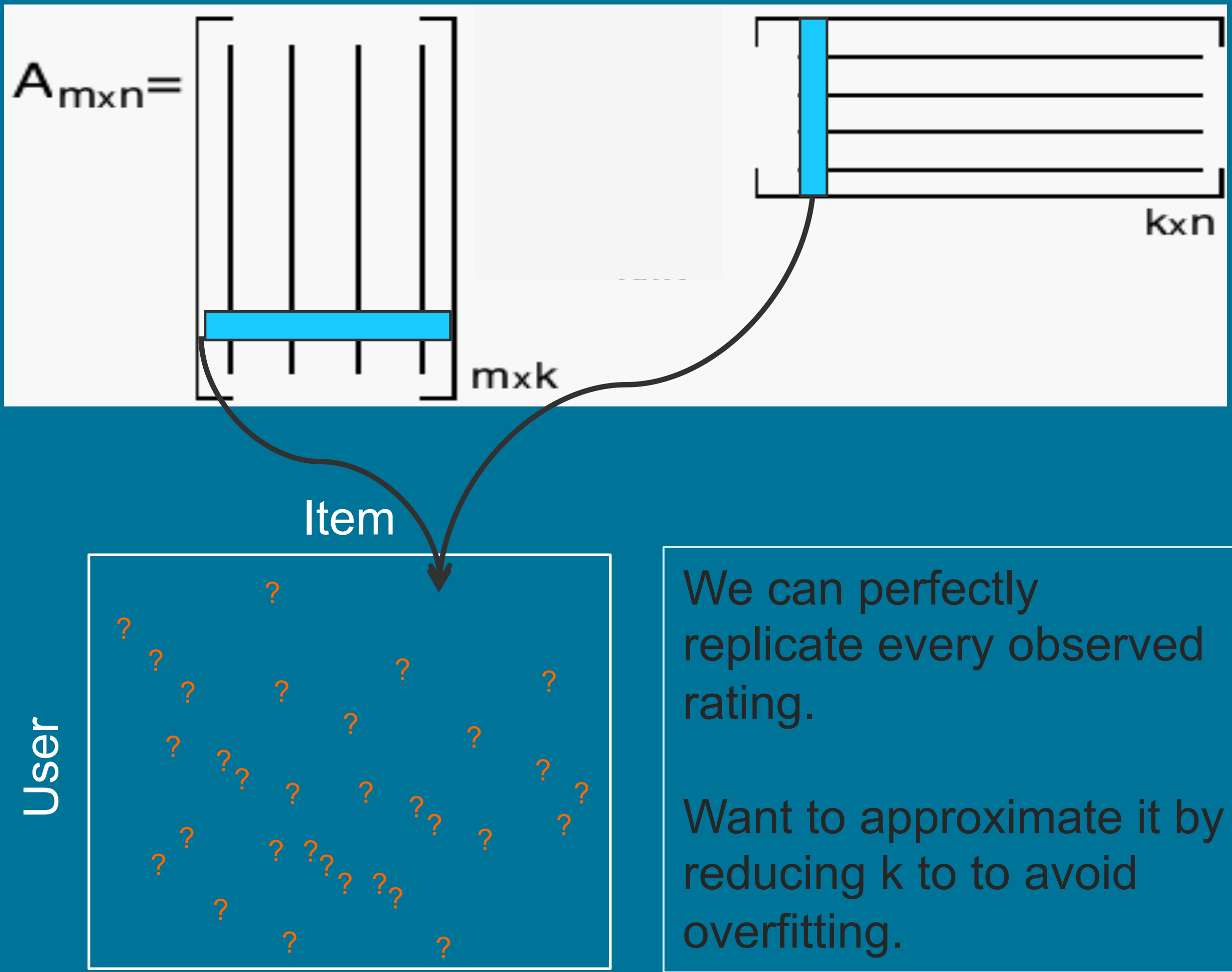


Evaluate Where we Can





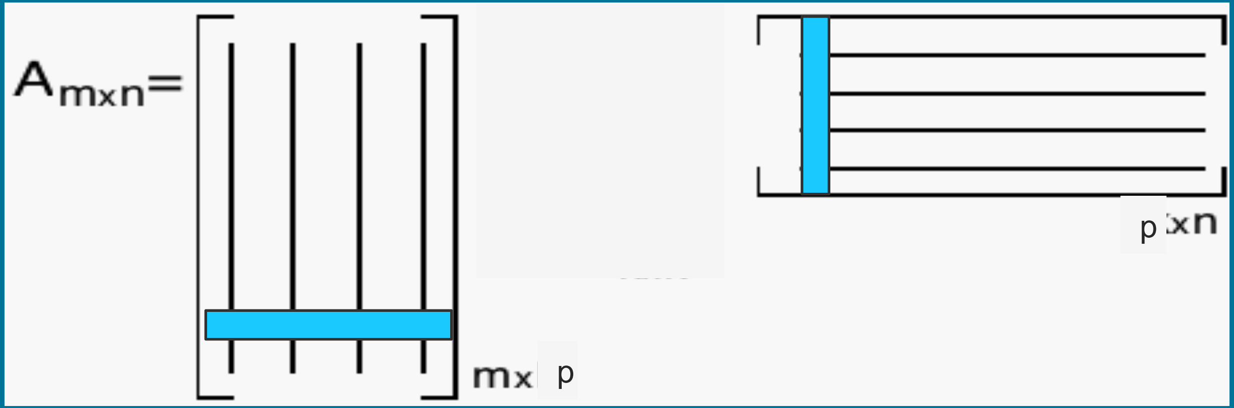
Every Rating is Product of Two Vectors



We can perfectly replicate every observed rating.

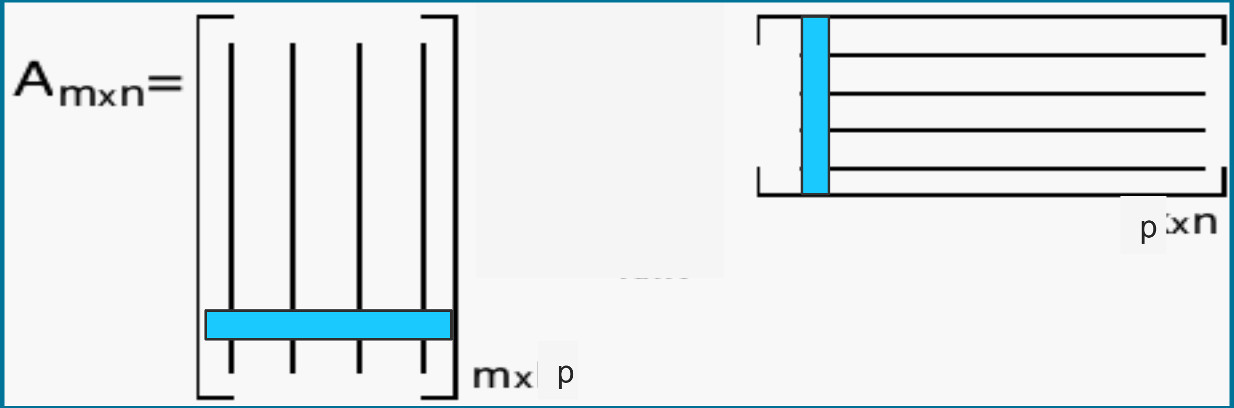
Want to approximate it by reducing k to to avoid overfitting.

We Can Optimize Matrices



Change element in either matrix → Change predicted rating → Change prediction error

We Can Optimize Matrices



$$\overline{R}_{ai} = \mu + b_a + b_i + u_a + v_i$$

$$e_{ai} = \overline{R}_{ai} - R_{ai}$$

SGD Update Rules for Mean Squared Error

Without Regularization

$$\Delta u_{af} = \lambda e_{ai} v_{if}$$

$$\Delta v_{if} = \lambda e_{ai} u_{af}$$

With Regularization

$$\Delta u_{af} = \lambda (e_{ai} v_{if} - \gamma u_{af})$$

$$\Delta v_{if} = \lambda (e_{ai} u_{af} - \gamma v_{if})$$

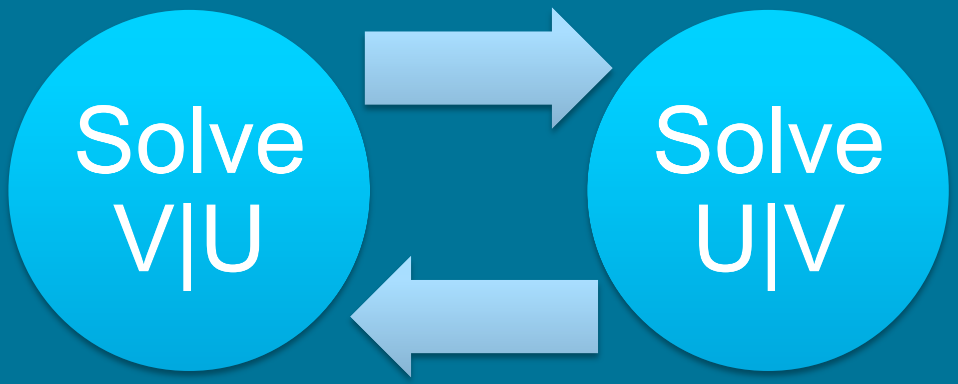
FunkSVD Algorithm

```
Initialize U and V matrices
for each feature:
    while objective function improves:
        for each user:
            for each item that user has rated:
                predict rating
                calculate error
                update U element for that user-feature pair
                update V element for that item-feature pair
```

Hints on Metaparameters:

- Learning rate around 0.001
- Regularization factor around 0.01

Alternating Least Squares SVD

- Initialize U and V
- 

until convergence

Alternating Least Squares
vs
Stochastic Gradient Descent