Gibbs Sampling

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Review

Lab questions?

Learning objectives

- Know the Gibbs sampling algorithm. (Lecture + Lab)
- Know when you can and cannot use Gibbs sampling. (Lecture + Lab)
- Implement a Gibbs sampler. (Lab)

Context

This morning, we looked at our first MCMC algorithm: the Metropolis algorithm.

What's a definition of the Metropolis algorithm from the lab?

Context

The Metropolis algorithm is useful when we cannot sample at all from a distribution.

It is naturally extended to the *Metropolis-Hastings* algorithm, which allows for asymmetric proposal distributions g.

In plain English, M-H lets us use more-complicated distributions for the proposal step, such as the Gamma or Bernoulli distributions.

Context

However, there are optimizations available when we can do *some* exact sampling.

One we'll look at today is *Gibbs sampling*.

Context

Gibbs sampling is not widely applicable.

But, when it is, we can use it to get big speedups.

An excellent example is Gibbs sampling for Latent Dirichlet Allocation.

- Fast Collapsed Gibbs Sampling For Latent Dirichlet Allocation, Porteous et al., 2008
- Blocking Collapsed Gibbs Sampler for Latent Dirichlet Allocation Models, Zhang, 2016
- GLDA: Parallel Gibbs Sampling for Latent Dirichlet Allocation on GPU, Xue et al., 2016

Gibbs sampling

Gibbs sampling is an MCMC algorithm.

As we discussed this morning, MCMC algorithms are implementations that satisfy the ergodic properties required by MCMC.

Gibbs sampling

The key requirement of Gibbs sampling is that we must be able to sample from the conditional probability distributions of our model.

This is useful when we can sample from the conditional distributions but not the full joint distribution.

Gibbs sampling

$P(x, y, z)$ NO

$P(x|y, z)$ YES

$P(y|x, z)$ YES

$P(z|x, y)$ YES

If we can sample the conditional distributions, then we can use Gibbs sampling

Gibbs sampling

Intuition:

Initialize values for our variables.

Sample from conditional distributions in a loop, using the most up-to-date values we have seen.

Gibbs sampling

Algorithm

1. Initialize variables x, y, z with starting values x_0, y_0, z_0.
2. For each iteration, update each variable with a sample of it pulled from the most up-to-date conditional distribution.

Gibbs sampling

Example

$x_1 \leftarrow x_1 | y_0, z_0$

$y_1 \leftarrow y_1 | x_1, z_0$

$z_1 \leftarrow z_1 | x_1, y_1$

$\ldots$

$x_t \leftarrow x_t | y_{t-1}, z_{t-1}$

$y_t \leftarrow y_t | x_t, z_{t-1}$

$z_t \leftarrow z_t | x_t, y_t$